LG5 Function Errata Updated

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Last updated on 2020-11-16

1 LG5 model formula typo

The formula is written as follows in McMahon and Parker 2015 Eq 1 (page 245). Note how in the numerator the L's cancel out, leaving only K being included in the formula:

$$dbh = \frac{L + (K - L)}{1 + 1/\theta * \exp(-r(doy - doy_{ip})/\theta)^{\theta}}$$

The formula is written as follows in the D'Orangeville et al. draft manuscript (page 9). Note the numerator is different than McMahon and Parker's formulation:

$$dbh = \frac{K - L}{1 + 1/\theta * \exp(-r(doy - doy_{ip})/\theta)^{\theta}}$$

However the `lg5.pred()` function from the `RDendrom` package computes DBH as follows:

```
lg5.pred <- function(params, doy) {
  L <- params[1] # min(dbh, na.rm = T)
  K <- params[2]
  doy.ip <- params[3]
  r <- params[4]
  theta <- params[5]
  dbh <- vector(length = length(doy))
  dbh <- L + ((K - L) / (1 + 1 / theta * exp(-(r * (doy - doy.ip) / theta))^theta))
  return(dbh)
}
```

which corresponds to the following formula:

$$dbh = L + \frac{K - L}{1 + 1/\theta * \exp(-r(doy - doy_{ip})/\theta)^{\theta}}$$

Also note that the denominator of the LG5 function can be simplified since

$$\exp(-r(doy - doy_{ip})/\theta)^{\theta} = \exp(-r(doy - doy_{ip}))$$

2 doy_{ip} and r parameter from LG5 model

I have a suspicion both these interpretations of the r and doy_{ip} parameters of LG5 model are incorrect:

1. McMahon and Parker 2015 state just above Eq 1 (page 245): "doy_{ip} marks the day of the year when the inflection of the curve is predicted to occur, and the rate parameter r describes the slope of the curve at the inflection point"

2. D'Orangeville et al. state (page 9): "r represents the maximum growth rate" and "doy_{ip} is the day of year when maximum growth rate occurs" and "doy_{ip} is the day of year (doy) when maximum growth rate occurs..."

Rather, I believe that the correct formulations of (1) the slope/the maximum growth rate and (2) inflection points are instead:

$$doy_{ip}^{*} = -\frac{\log(\theta)}{r} + doy_{ip}$$
$$r^{*} = \frac{r(K-L)}{4}$$

2.1 Plot LG5 diameter and growth curves

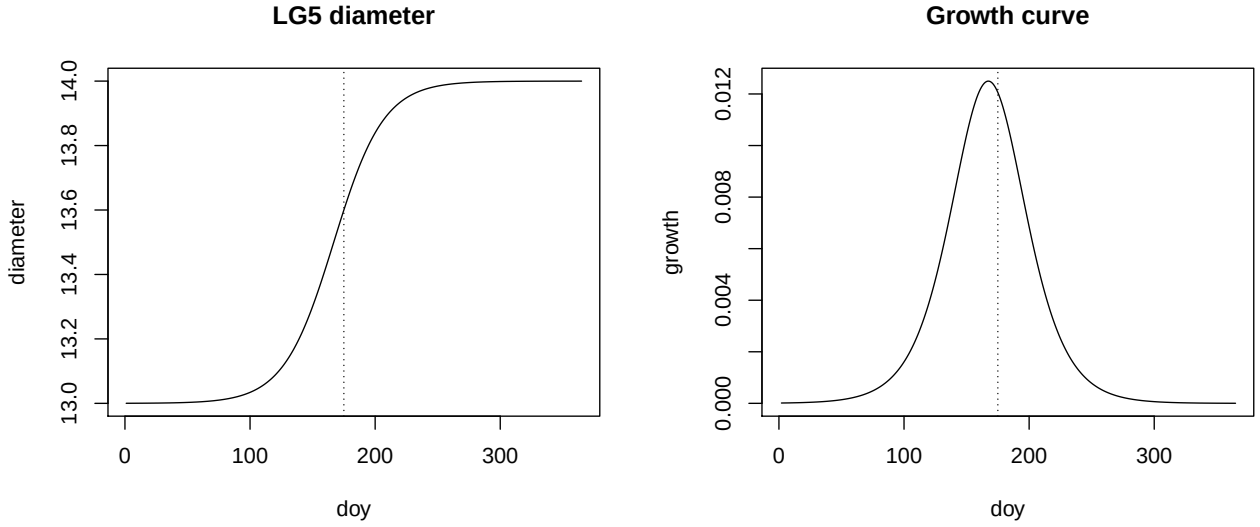
Let's set arbitrarily chosen parameter values **but making sure to use a value other than $\theta = 1$**:

1. Use `lg5.pred()` to compute all diameters for all DOY $= 1, \ldots, 365$
2. Compute growth by taking differences of diameters

```
L <- 13
K <- 14
doy.ip <- 175
r <- 0.05
# Use a value other than theta = 1:
theta <- 1.5
```

2.2 Original interpretations of doy_{ip}

We then plot both the diameter curve as well as the growth curve, with `doy.ip` marked with a vertical dotted line.

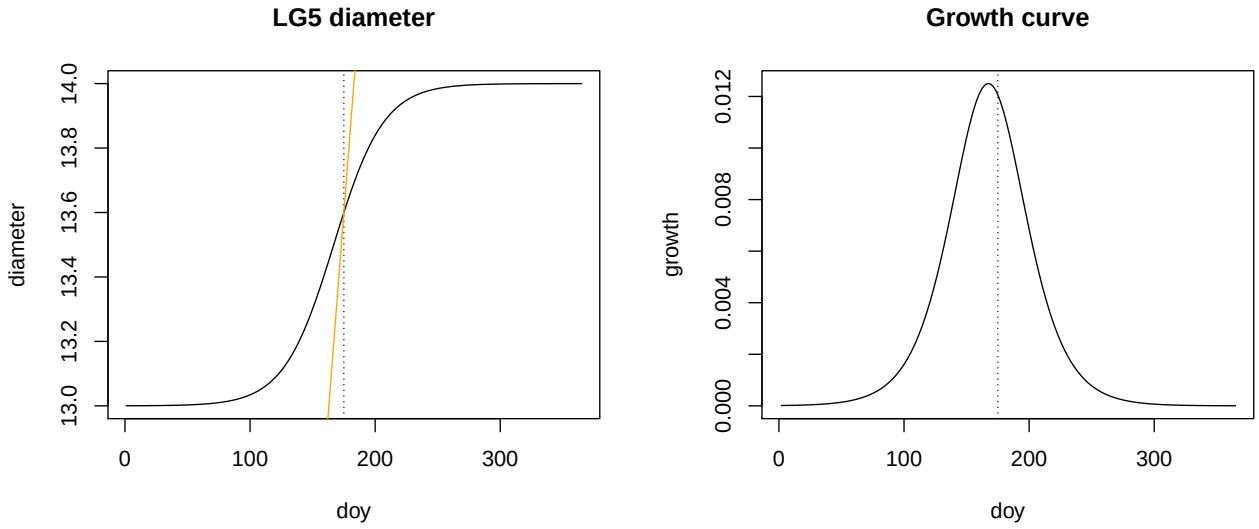


Observe how `doy.ip` is not the day the peak growth occurs.

2.3 Original interpretations of r

1. McMahon and Parker 2015 state: "the rate parameter r describes the slope of the curve at the inflection point". We mark the appropriate line with slope `r` in orange in the diameter plot.

2. D'Orangeville et al. state: "r represents the maximum growth rate" and "doy_{ip} is the day of year when maximum growth rate occurs". Let's mark this maximum growth rate with a horizontal orange line in the growth plot.



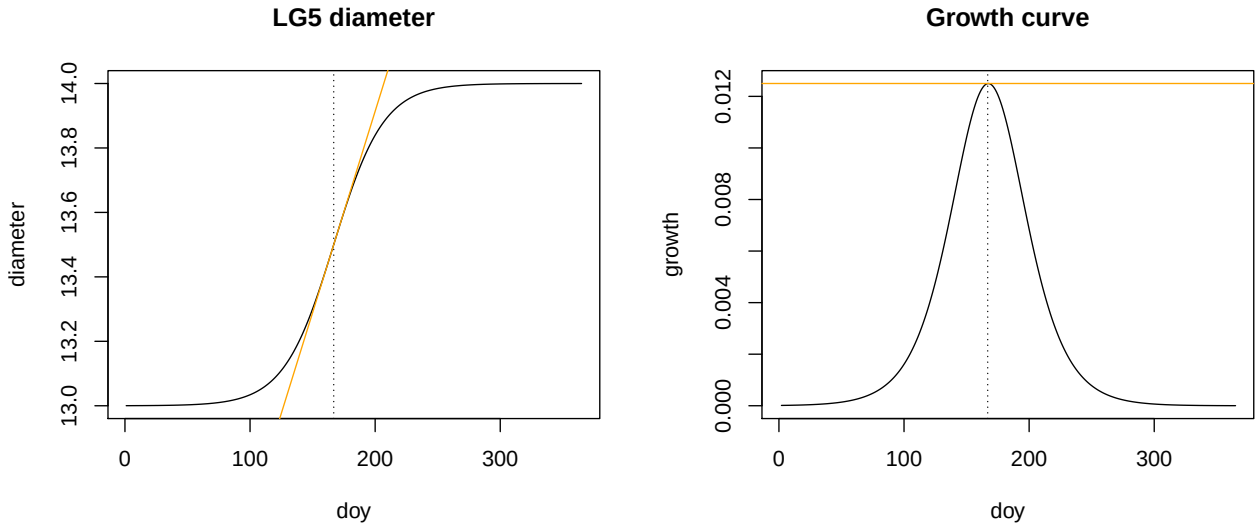
Observe

1. In the left plot how the orange line does not appear to be tangent to the LG5 curve at `doy.ip` and hence is not the slope at that point.
2. In the right plot $r = 0.05$ does not correspond to the growth rate at the peak of the curve, rather it appears to be around 0.012.

2.4 Proposed doy_{ip}^* and r^*

Let's repeat the above with our proposed doy_{ip}^*, which is the point x at which $\frac{d^2}{dx^2}LG5(x) = 0$ (per calculus on inflection points), and our proposed new growth rate r^*, which is $\frac{d}{dx}LG5(x)$ evaluated at $x = doy_{ip}^*$. See this and this PDF for all analytic derivative derivations.

$$r^* = \frac{r(K - L)}{4}$$
$$doy_{ip}^* = -\frac{\log(\theta)}{r} + doy_{ip}$$



Observe:

1. In the left plot how the orange line is now tangent to the LG5 curve at `doy.ip` and thus is the correct slope.
2. In the right plot $r^* = 0.0125$ now corresponds to the max growth rate correctly occurring at `doy.ip`.

3 Previous errata

In the previous errata PDF `LG5_errata.pdf` I suggested that the correct max growth rate was

$$r_{new} = \frac{K-L}{\left(1+\frac{1}{\theta}\right)^2}\frac{r}{\theta}$$

However, this result was based on the incorrect assumption that doy_{ip} was the correct inflection point, which we evaluated $\frac{d}{dx}LG5(x)$ to obtain the max growth rate. Furthermore, the empirical simulations/plots I showed were for $\theta = 1$. If we plug this value into the previous r_{new} we get

$$r_{new} = \frac{K-L}{\left(1+\frac{1}{\theta}\right)^2}\frac{r}{\theta} = \frac{K-L}{\left(1+\frac{1}{1}\right)^2}\frac{r}{1} = \frac{r(K-L)}{4}$$

which is our updated formulation above.