

# LG5 Function Errata

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## 1 LG5 model formula typo

The formula is written as follows in [McMahon and Parker 2015 Eq 1 \(page 245\)](#). Note how in the numerator the  $L$ 's cancel out, leaving only  $K$  being included in the formula:

$$dbh = \frac{L + (K - L)}{1 + 1/\theta * \exp(-r(doy - doy_{ip})/\theta)^\theta}$$

The formula is written as follows in the D'Orangeville et al. draft manuscript (page 9). Note the numerator is different than McMahon and Parker's formulation:

$$dbh = \frac{K - L}{1 + 1/\theta * \exp(-r(doy - doy_{ip})/\theta)^\theta}$$

However the `lg5.pred()` function from the [RDendrom package](#) computes DBH as follows:

```
lg5.pred <- function(params, doy) {  
  L <- params[1] # min(dbh, na.rm = T)  
  K <- params[2]  
  doy.ip <- params[3]  
  r <- params[4]  
  theta <- params[5]  
  dbh <- vector(length = length(doy))  
  dbh <- L + ((K - L) / (1 + 1 / theta * exp(-(r * (doy - doy.ip) / theta))^theta))  
  return(dbh)  
}
```

which corresponds to the following formula:

$$dbh = L + \frac{K - L}{1 + 1/\theta * \exp(-r(doy - doy_{ip})/\theta)^\theta}$$

Also note that the denominator of the LG5 function can be simplified since

$$\exp(-r(doy - doy_{ip})/\theta)^\theta = \exp(-r(doy - doy_{ip}))$$

## 2 $r$ parameter from LG5 model

I have a suspicion both these interpretations of the  $r$  parameter of LG5 model are incorrect:

1. McMahon and Parker 2015 state just above Eq 1 (page 245): “the rate parameter  $r$  describes the slope of the curve at the inflection point”

2. D’Orangeville et al. state (page 9): “ $r$  represents the maximum growth rate” and “ $doy_{ip}$  is the day of year when maximum growth rate occurs”

Rather, I believe that the correct formulation of the slope/the maximum growth rate is instead:

$$r_{new} = \frac{K - L}{\left(1 + \frac{1}{\theta}\right)^2} \frac{r}{\theta}$$

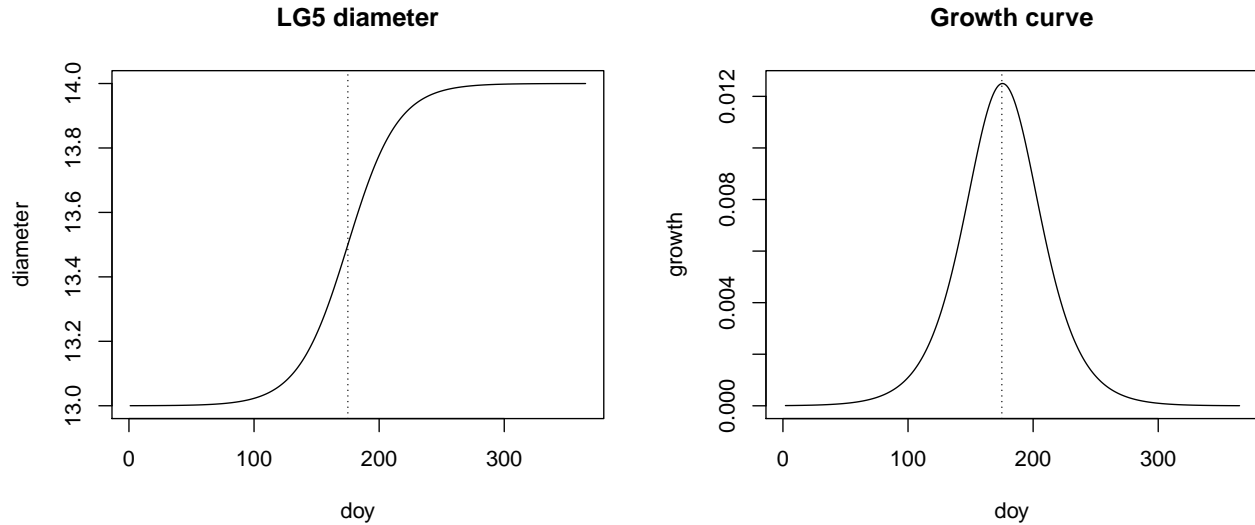
## 2.1 Plot LG5 diameter and growth curves

Let’s set arbitrarily chosen parameter values then

1. Use `lg5.pred()` to compute all diameters for all  $DOY = 1, \dots, 365$
2. Compute growth by taking differences of diameters

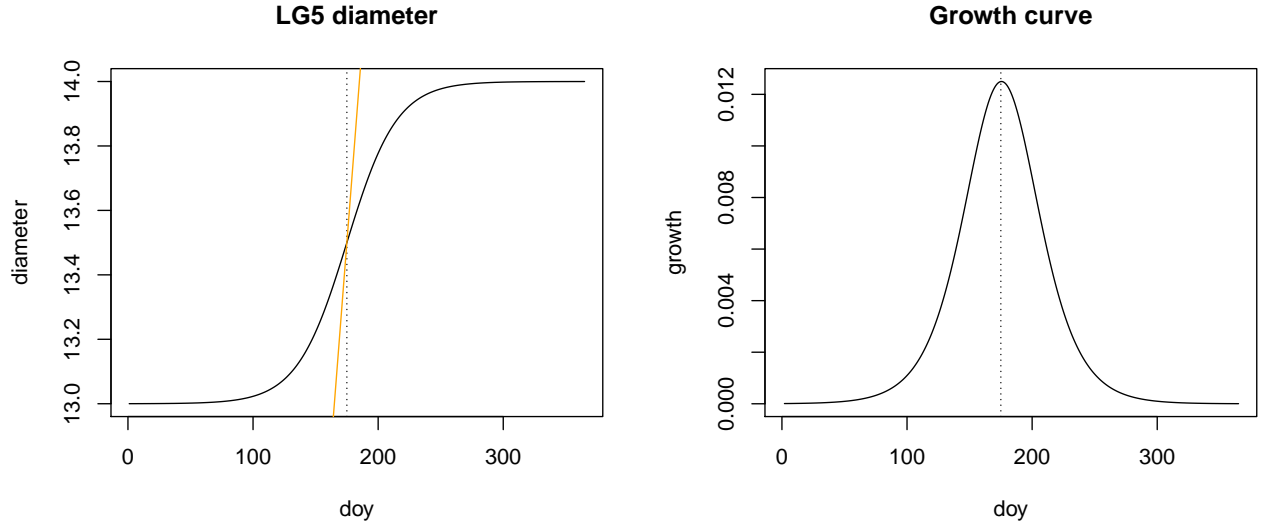
```
L <- 13
K <- 14
doy.ip <- 175
r <- 0.05
theta <- 1
```

We then plot both the diameter curve as well as the growth curve, with `doy.ip` marked with a vertical dotted line. Observe how growth peaks at `doy.ip`.



## 2.2 Original interpretations of $r$

1. McMahon and Parker 2015 state: “the rate parameter  $r$  describes the slope of the curve at the inflection point”. We mark the appropriate line with slope  $r$  in orange in the diameter plot.
2. D’Orangeville et al. state: “ $r$  represents the maximum growth rate” and “ $doy_{ip}$  is the day of year when maximum growth rate occurs”. Let’s mark this maximum growth rate with a horizontal orange line in the growth plot.



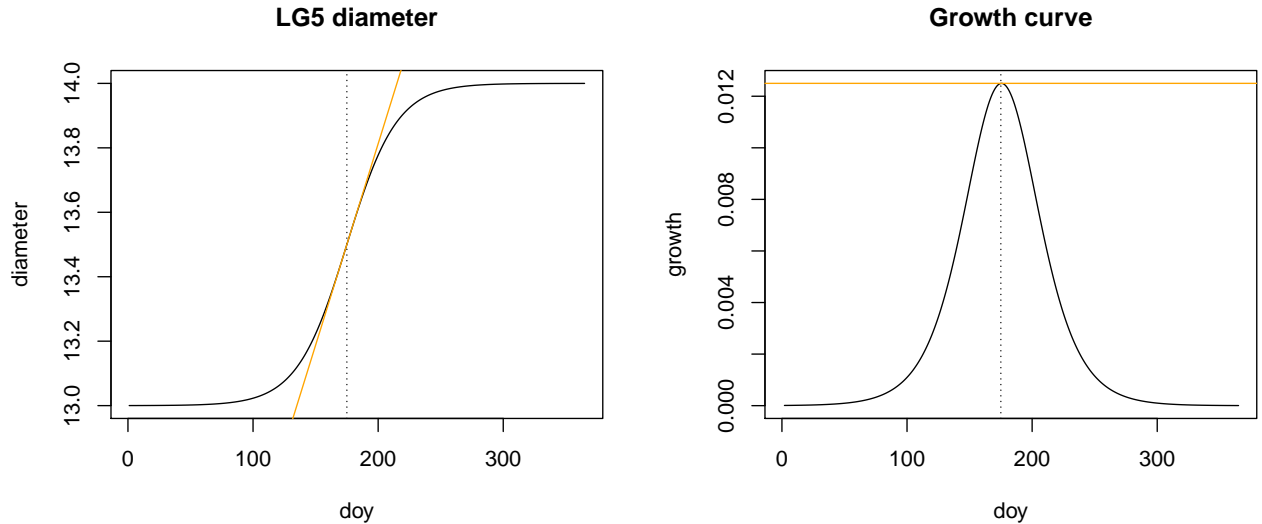
Observe

1. In the left plot how the orange line does not appear to be tangent to the LG5 curve at `day.ip` and hence is not the slope at that point.
2. In the right plot  $r = 0.05$  does not correspond to the growth rate at `day.ip` which should be the max growth rate. Rather it appears to be around 0.012

### 2.3 Proposed new $r$

Let's repeat the above with our proposed growth rate  $r_{new}$ , which is  $\frac{d}{dx}LG5(x)$  evaluated at  $x = \text{day.ip}$ . See this [PDF](#) for the analytic derivation of the derivative.

$$r_{new} = \frac{K - L}{\left(1 + \frac{1}{\theta}\right)^2} \frac{r}{\theta}$$



Observe:

1. In the left plot how the orange line is now tangent to the LG5 curve at `day.ip` and thus is the correct slope.
2. In the right plot  $r_{new} = 0.0125$  now corresponds to the max growth rate occurring at `day.ip`.