LG5 Function Errata

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1 LG5 model formula typo

The formula is written as follows in McMahon and Parker 2015 Eq 1 (page 245). Note how in the numerator the L's cancel out, leaving only K being included in the formula:

$$dbh = \frac{L + (K - L)}{1 + 1/\theta * \exp(-r(doy - doy_{ip})/\theta)^{\theta}}$$

The formula is written as follows in the D'Orangeville et al. draft manuscript (page 9). Note the numerator is different than McMahon and Parker's formulation:

$$dbh = \frac{K - L}{1 + 1/\theta * \exp(-r(doy - doy_{ip})/\theta)^{\theta}}$$

However the `lg5.pred()` function from the RDendrom package computes DBH as follows:

```
lg5.pred <- function(params, doy) {
  L <- params[1] # min(dbh, na.rm = T)
  K <- params[2]
  doy.ip <- params[3]
  r <- params[4]
  theta <- params[5]
  dbh <- vector(length = length(doy))
  dbh <- L + ((K - L) / (1 + 1 / theta * exp(-(r * (doy - doy.ip) / theta))^theta))
  return(dbh)
}
```

which corresponds to the following formula:

$$dbh = L + \frac{K - L}{1 + 1/\theta * \exp(-r(doy - doy_{ip})/\theta)^{\theta}}$$

Also note that the denominator of the LG5 function can be simplified since

$$\exp(-r(doy - doy_{ip})/\theta)^{\theta} = \exp(-r(doy - doy_{ip}))$$

2 r parameter from LG5 model

I have a suspicion both these interpretations of the r parameter of LG5 model are incorrect:

1. McMahon and Parker 2015 state just above Eq 1 (page 245): "the rate parameter r describes the slope of the curve at the inflection point"

2. D'Orangeville et al. state (page 9): "r represents the maximum growth rate" and "doy_{ip} is the day of year when maximum growth rate occurs"

Rather, I believe that the correct formulation of the slope/the maximum growth rate is instead:

$$r_{new} = \frac{K - L}{\left(1 + \frac{1}{\theta}\right)^2} \frac{r}{\theta}$$

2.1 Plot LG5 diameter and growth curves

Let's set arbitrarily chosen parameter values then

1. Use `lg5.pred()` to compute all diameters for all DOY $= 1, \ldots, 365$
2. Compute growth by taking differences of diameters

```
L <- 13
K <- 14
doy.ip <- 175
r <- 0.05
theta <- 1
```

We then plot both the diameter curve as well as the growth curve, with `doy.ip` marked with a vertical dotted line. Observe how growth peaks at `doy.ip`.
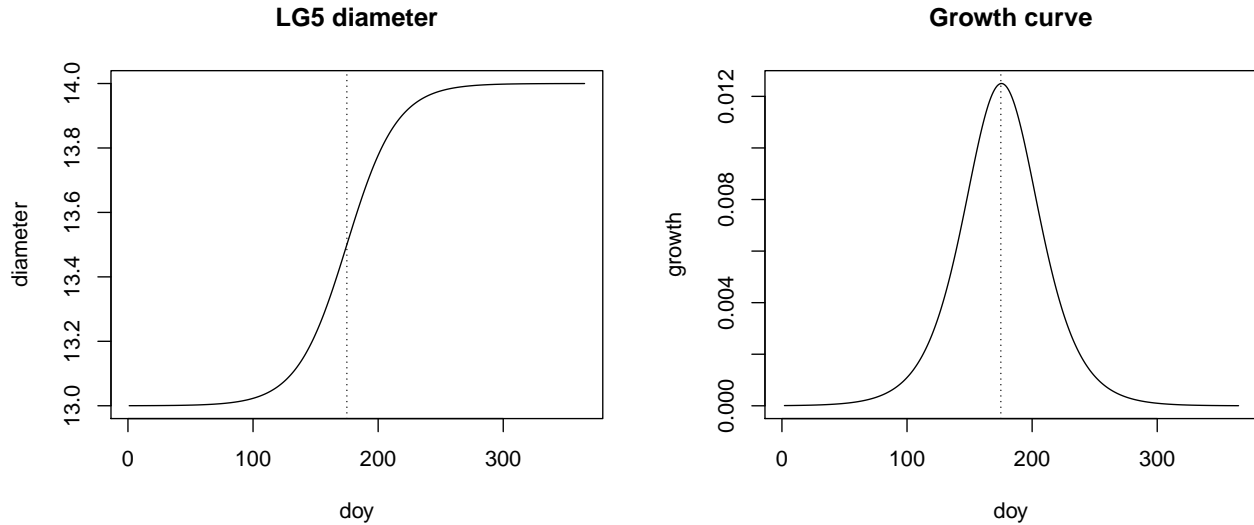
2.2 Original interpretations of r

1. McMahon and Parker 2015 state: "the rate parameter r describes the slope of the curve at the inflection point". We mark the appropriate line with slope `r` in orange in the diameter plot.
2. D'Orangeville et al. state: "r represents the maximum growth rate" and "doy_{ip} is the day of year when maximum growth rate occurs". Let's mark this maximum growth rate with a horizontal orange line in the growth plot.
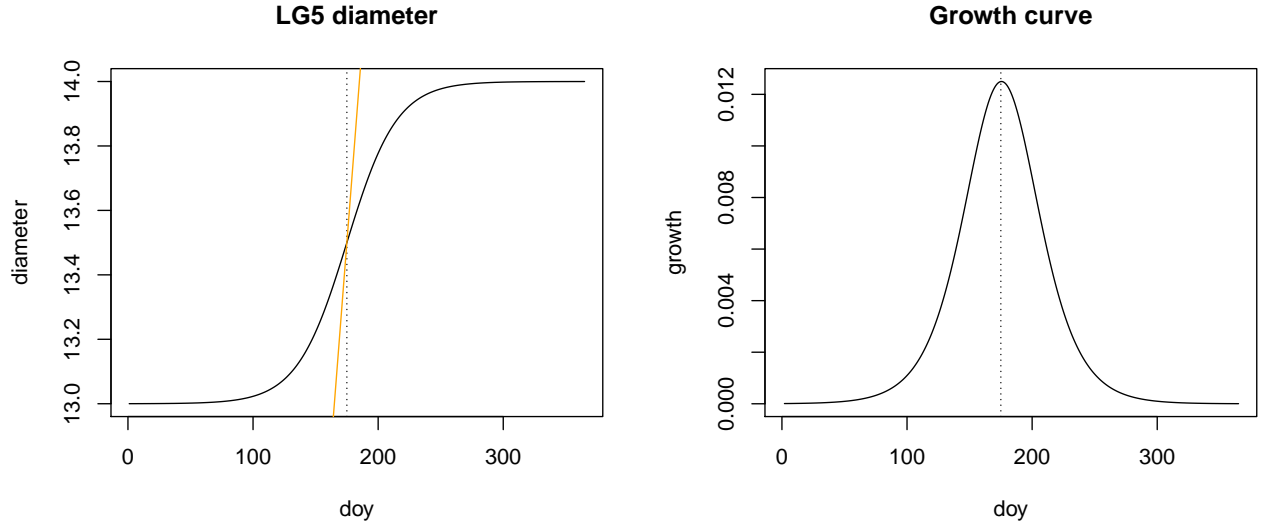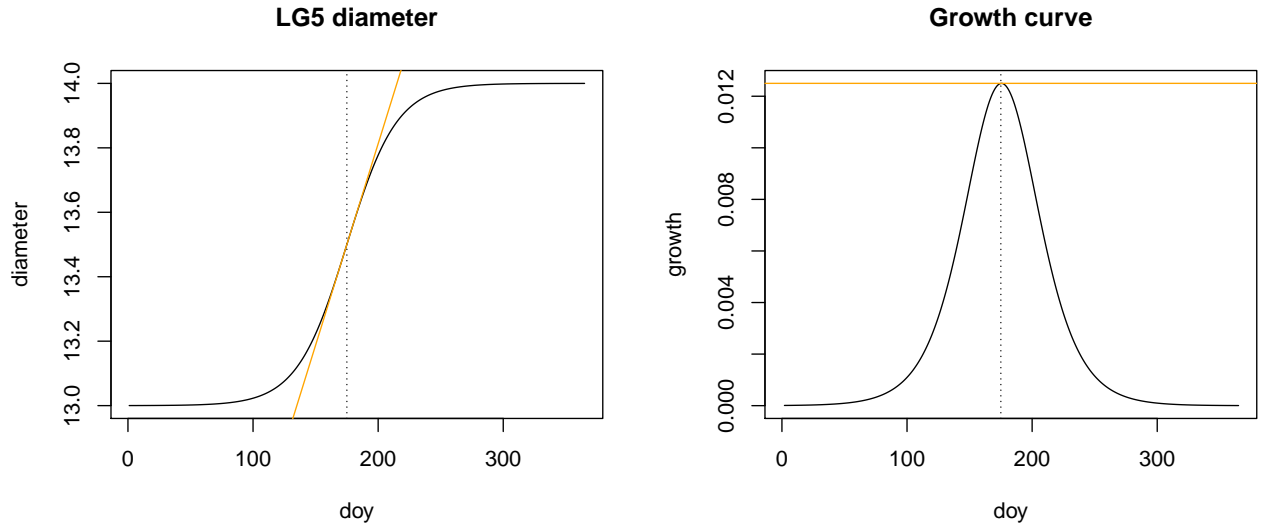
Observe

1. In the left plot how the orange line does not appear to be tangent to the LG5 curve at `doy.ip` and hence is not the slope at that point.
2. In the right plot $r = 0.05$ does not correspond to the growth rate at `doy.ip` which should be the max growth rate. Rather it appears to be around 0.012

2.3 Proposed new r

Let's repeat the above with our proposed growth rate r_{new}, which is $\frac{d}{dx}LG5(x)$ evaluated at $x = $ `doy.ip`. See this PDF for the analytic derivation of the derivative.

$$r_{new} = \frac{K - L}{\left(1 + \frac{1}{\theta}\right)^2} \frac{r}{\theta}$$

Observe:

1. In the left plot how the orange line is now tangent to the LG5 curve at `doy.ip` and thus is the correct slope.
2. In the right plot $r_{new} = 0.0125$ now corresponds to the max growth rate occurring at `doy.ip`.

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