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$$LG5(x) = L + \frac{K-L}{1 + \frac{1}{\Theta} \exp\left(-r \frac{(x-d)}{\Theta}\right)} \quad (1)$$

$$= L + \frac{K-L}{1 + \frac{1}{\Theta} \exp(-r(x-d))} \quad (2)$$

$$\frac{d}{dx} LG5(x) = 0 + \frac{(K-L) \times (-1)}{\left(1 + \frac{1}{\Theta} \exp(-r(x-d))\right)^2} \frac{d}{dx} \left(1 + \frac{1}{\Theta} \exp(-r(x-d))\right)$$

$$= - \frac{(K-L)}{\left(1 + \frac{1}{\Theta} \exp(-r(x-d))\right)^2} \left(0 + \frac{1}{\Theta} \frac{d}{dx} \exp(-r(x-d))\right)$$

$$= \quad \quad \quad \left(\frac{1}{\Theta} \exp(-r(x-d)) \frac{d}{dx} (-r(x-d))\right)$$

$$= \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (-r)$$

$$= - \frac{K-L}{\left(1 + \frac{1}{\Theta} \exp(-r(x-d))\right)^2} \left(\frac{1}{\Theta} \exp(-r(x-d))\right) (-r)$$

Evaluated at $x=d$

$$\left. \frac{d}{dx} LG5(x) \right|_{x=d} = - \frac{K-L}{\left(1 + \frac{1}{\Theta} \exp(0)\right)^2} \frac{1}{\Theta} \exp(0) (-r)$$

$$= \frac{K-L}{\left(1 + \frac{1}{\Theta}\right)^2} \frac{r}{\Theta} \quad (3)$$