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Agricultural and Forest Meteorology 147 (2007) 80-87

www.elsevier.com/locate/agrformet

### Moving towards a more mechanistic approach in the determination of soil heat flux from remote measurements I. A universal approach to calculate thermal inertia

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#### Abstract

In this paper we pledge that physically based equations should be combined with remote sensing techniques to enable a more theoretically rigorous estimation of area-average soil heat flux, G. A standard physical equation (i.e. the analytical or exact method) for the estimation of G, in combination with a simple, but theoretically derived, equation for soil thermal inertia ( $\Gamma$ ), provides the basis for a more transparent and readily interpretable method for the estimation of G; without the requirement for in situ instrumentation. Moreover, such an approach ensures a more universally applicable method than those derived from purely empirical studies (employing vegetation indices and albedo, for example).

Hence, a new equation for the estimation of  $\Gamma$  (for homogeneous soils) is discussed in this paper which only requires knowledge of soil type, which is readily obtainable from extant soil databases and surveys, in combination with a coarse estimate of moisture status. This approach can be used to obtain area-averaged estimates of  $\Gamma$  (and thus G, as explained in paper II) which is important for large-scale energy balance studies that employ aircraft or satellite data. Furthermore, this method also relaxes the instrumental demand for studies at the plot and field scale (no requirement for in situ soil temperature sensors, soil heat flux plates and/or thermal conductivity sensors).

In addition, this equation can be incorporated in soil-vegetation-atmosphere-transfer models that use the force restore method to update surface temperatures (such as the well-known ISBA model), to replace the thermal inertia coefficient. © 2007 Elsevier B.V. All rights reserved.

Keywords: Energy balance; Thermal conductivity; Thermal inertia; Soil heat flux; Remote sensing

### 1. Introduction

The surface energy balance describes the energy exchanges between the land surface and the atmosphere. Land surface characteristics determine which energy transfer process will predominate and these processes in turn will affect the atmospheric state variables (and vice

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versa). The energy budget at the land surface is given by

$$R_{\rm n} = Q + E + G \tag{1}$$

where  $R_n$  is the net radiation, Q the sensible heat flux, E the latent heat flux and G is the soil heat flux.

Mounting evidence of the failure of current measurement systems to capture the closure of the energy balance, has urged workers to review their measurement and correction methods for determination of E and Q(e.g. Foken et al., 2006). Often in such studies, the importance of G within Eq. (1) is not addressed or assumed not significant enough to warrant a more

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<sup>0168-1923/\$ –</sup> see front matter  $\odot$  2007 Elsevier B.V. All rights reserved. doi:10.1016/j.agrformet.2007.07.004

accurate determination. However, when vegetation is relatively sparse G can consume a significant proportion of the recorded  $R_n$ . Moreover, during the night G is an important term in the energy balance, when low values of  $R_n$  and stable atmospheric conditions cause Q and E to be small. Thus, a more accurate determination of G is vital in improving the closure of the surface energy balance.

Hydrological and meteorological studies are frequently undertaken over large areas, where in situ measurements of G are impractical or unrepresentative and thus researchers rely on remotely sensed surface variables to infer suitable estimates of G.

The remote estimation of *G* is commonly approached through the derivation of empirical equations employing remotely measurable surface variables (such as surface temperature, *T*) and/or proxies of key variables, e.g. vegetation indices, VI, to represent leaf area index, *L*, or albedo,  $\alpha$ , to represent soil moisture content (Clothier et al., 1986; Choudhury et al., 1987; Kustas and Daughtry, 1990; Kustas et al., 1993; Friedl, 1995, 1996, 2002; Jacobsen and Hansen, 1999).

The rationale behind using VI is that the canopy exerts a significant influence on G, through the reduction of incident net radiation reaching the soil surface,  $R_{n,s}$ , during daytime, and by insulation at night.

The majority of papers focus on the determination of  $G/R_n$ , from which G can be derived, provided  $R_n$  is known. Choudhury et al. (1987) use a Beer's law extinction expression (dependent on L) to predict  $R_{n,s}$ , combined with an assumed value of  $G/R_n$  for bare soil, to predict estimates of  $G/R_n$  for soil under vegetation. This approach is summarised by

$$\frac{G}{R_{\rm n}} = \eta_{\rm g} \,\mathrm{e}^{-\beta L} \tag{2}$$

where  $\eta_g$  is the assumed  $G/R_n$  value for bare soil and  $\beta$  is the canopy extinction coefficient.

This approach has been adopted and improved by the work of Friedl (2002) and Kustas et al. (1993) which led to the development of a generic non-linear equation, to express the relationship between  $G/R_n$  and VI, which has been subsequently supported by the findings of Jacobsen and Hansen (1999), for example:

$$\frac{G}{R_{\rm n}} = a({\rm VI})^{\rm b} \tag{3}$$

where *a* and *b* are empirically determined coefficients. However, the approach adopted by these workers negates the importance of soil thermal properties as a determinate influence on G – soil surface variables (such as soil surface temperature) are also not explicitly incorporated into the model. Therefore, to satisfy theoretical understanding, we must assume that the unitless coefficients (*a* and *b*) must implicitly account for factors in the near-surface soil layer (such as soil thermal properties) to enable the satisfactory estimates of  $G/R_n$  found in both the studies by Kustas et al. and Jacobsen and Hansen.

An alternative method for the remote estimation of *G* was proposed in the work of Santanello and Friedl (2003). Here, the maximum daytime  $G/R_n$  value is predicted from observations of the diurnal change in surface temperature (i.e. maximum minus minimum recorded temperatures,  $\Delta T$  (K)) by an empirically derived equation, which is valid for bare soil and sparse vegetation:

$$\left(\frac{G}{R_{\rm n}}\right)_{\rm max} = 0.0074\Delta T + 0.088\tag{4a}$$

To determine the diurnal course of  $G/R_n$ , Santanello and Friedl use a weighted cosine model to place the occurrence of the maximum value at mid-morning, in accordance with the findings from their datasets:

$$\frac{G}{R_{\rm n}} = \left(\frac{G}{R_{\rm n}}\right)_{\rm max} \cos\left[\frac{2\pi(t+10800)}{B}\right] \tag{4b}$$

where *B* is a variable that depends on  $\Delta T$  and *t* is time (s).

In contrast to vegetation indices, soil surface temperature represents a variable that is reactive to both the overriding atmospheric and surface conditions, and may therefore provide a more suitable indicator of  $G/R_n$ values for a given surface. As Santanello and Friedl (2003) argue, the diurnal change in surface temperature allows for the integrated effect of soil type and soil surface moisture content (thus soil thermal properties) to be implicitly accounted for in estimates of maximum  $G/R_n$ . Although skin surface temperatures (a combination of soil and canopy temperature) were used by Santanello and Friedl, the sites examined were sparsely vegetated and thus soil surface temperature would predominate over canopy temperatures in the used temperature signal.

However, the Santanello and Friedl approach (Eqs. (4a) and (4b)) is theoretically limited as the relationship between  $\Delta T$  and G should depend on the soil thermal properties. As discussed earlier (when examining Eq. (3)) this information must be contained in the coefficients in Eq. (4a). To illustrate this we quote the physically based equation of Hares et al. (1985) which computes the total positive soil heat flux ( $G^+$ ) for bare soil (i.e. total G moving into the soil on a given day), from  $\Delta T$ , angular frequency,  $\omega$  (s<sup>-1</sup>), and the soil thermal inertia,  $\Gamma$ , which is given by  $\Gamma = H_c \sqrt{D_h}$  or

 $\sqrt{\lambda H_c}$  (both in J m<sup>-2</sup> K<sup>-1</sup> s<sup>-0.5</sup>), where  $H_c$  is heat capacity (J m<sup>-3</sup> K<sup>-1</sup>),  $D_h$  is the soil diffusivity (m<sup>2</sup> s<sup>-1</sup>) and  $\lambda$  is the thermal conductivity (W m<sup>-1</sup> K<sup>-1</sup>):

$$G^{+} = \frac{\Gamma \,\Delta T}{\sqrt{\omega}} \tag{5a}$$

If we divide Eq. (5a) by  $R_n^+$  (the total positive net radiation) we get

$$\frac{G^+}{R_n^+} = \Delta T \frac{\Gamma}{\sqrt{\omega}R_n^+}$$
(5b)

We would expect there to be a strong relationship between  $G^+/R_n^+$  and the maximum recorded daytime value of  $G/R_n$ , which implies that the form of Eq. (4a), i.e.  $y = m\Delta T + c$ , given by Santanello and Friedl method (Eq. (4a)) could be misleading; in that there should be no *c* value and the coefficient (*m*) is a combination of terms shown in Eq. (5b).

In Eq. (4a) the coefficients adopted by Santanello and Friedl are constant (i.e. m = 0.0074 and c = 0.088) which indicates that subsequently the implicit value of soil thermal inertia is interpreted as having a relatively constant value. Moreover, it stipulates that  $\Gamma$  must change in proportion to  $R_n^+$  to retain a constant value of 0.0074 (see Eq. (5b)). This presumption has little physical basis and the relative success of Eq. (4a) is most likely due to the fact that all sites examined were situated in semi-arid climates, exhibiting similar vegetation densities (sparse savannah-type canopies), moisture contents (relatively dry) and soil types (predominantly sandy), so that both  $\Gamma$  and  $R_n^+$  remained relatively constant during the period of analysis.

Another empirical approach has been proposed by Bastiaansen (1995) where estimates of diurnal  $G/R_n$  were determined using surface (skin) temperature, *T*, normalised vegetation indices, NDVI, and instantaneous and averaged (daytime) albedo,  $\alpha$  and  $\bar{\alpha}$ , respectively. For soil under vegetation estimates of  $G/R_n$  were computed using

$$\frac{G}{R_{\rm n}} = \frac{T}{100\alpha} (0.32\bar{\alpha} + 0.62\bar{\alpha}^2) [1 - 0.978(\rm NDVI)^4] \quad (6)$$

This methodology uses albedo (and to some extent surface temperature) to infer some description of the surface wetness, thus gaining some indication of soil thermal properties. However, this link is somewhat tenuous for soils under extensive vegetation, where the contribution of the soil component to the overall albedo and temperature values would be significantly lower than under sparse vegetation. Moreover, from a mathematical perspective, the imbalance of units, where the RHS of Eq. (6) has units of  $K^{-1}$  and the LHS is unit-less, signifies the highly empirical nature of the equation. This makes the universal applicability of Eq. (6) more difficult to justify.

Nonetheless, the empirical approaches provide practical methodologies for workers examining G remotely. In this current work, however, we endeavour approaching the estimation of G from a more theoretical starting point. We aim to improve estimates of G by using a physical equation and assessing the possibility of estimating the required input variables. In this regard we can take explicit account of soil thermal properties and propose methods to estimate suitable values, without the requirement of in situ instrumentation. This would enable a more universally acceptable approach to determine G over a variety of soil types and conditions (different cover types, i.e. bare or vegetated; varying canopy densities; variable soil moisture conditions). Furthermore, it means that errors and uncertainty in subsequent estimates of G are more transparent and more readily interpreted. This is important for sensitivity studies, for example.

An analytical equation for G (often called the analytical or exact method) is based on a harmonic analysis of soil surface temperatures (Van Wijk and De Vries, 1963; Horton and Wierenga, 1983; Verhoef, 2004):

$$G = \Gamma \sum_{n=1}^{M} A_n \sqrt{n\omega} \sin\left[n\omega t + \phi_n + \left(\frac{\pi}{4}\right)\right]$$
(7)

where *M* is the total number of harmonics (*n*) used,  $A_n$  the amplitude of the *n*th harmonic,  $\varphi_n$  the phase shift of the *n*th harmonic and *t* is time.

For Eq. (7) to be used without in situ equipment, requires the estimation of both  $\Gamma$  and soil surface temperatures without direct instrumentation. A method has been proposed by Verhoef (2004) to determine  $\Gamma$ from the nighttime drop in surface temperature and average nighttime  $R_n$  values. However, this approach only works for bare soil and windless nights and when we tested this equation for vegetated soil, it was shown to be unsuitable. To allow for the use of Eq. (7) for surfaces ranging between bare soil and dense canopy requires a robust method to estimate  $\Gamma$  and the soil surface temperature required for the calculation of the harmonic terms in Eq. (7). In both cases, we are limited by the fact that (dense) canopy will obscure the soil surface and hence hamper direct estimates of soil surface temperature.

This paper (paper I) will focus on the first aspect (determination of  $\Gamma$ ), whereas a companion paper (paper II: Murray and Verhoef, 2007) will address the

determination of the harmonic terms and test the validity of Eq. (7) for various experimental datasets.

Rather than trying to obtain  $\Gamma$  purely from remotely sensed data (e.g. *T* and *R*<sub>n</sub>) we decided to use existing theoretical equations to calculate  $\Gamma$  (i.e. from *H*<sub>c</sub> and  $\lambda$ ). These equations need soil physical information (texture, dry bulk density, quartz content, and soil moisture content). However, we simplified these equations into one equation for the calculation of  $\Gamma$  that depends on soil porosity and an estimate of soil water status only.

# 2. Estimation of thermal inertia from soil properties

Very few equations to estimate thermal inertia,  $\Gamma$ , directly from texture or other soil properties can be found in the literature. Noilhan and Planton (1989) calculate a thermal inertia coefficient,  $C_g$  (K m<sup>2</sup> J<sup>-1</sup>), see also Giard and Bazile (2000), which depends on texture. This coefficient is used by these authors in the force restore method (Bhumralkar, 1976), a prognostic equation for surface temperature, as used in their interaction soil biosphere atmosphere (ISBA) scheme.

The relationship between  $C_g$  and  $\Gamma$  (J m<sup>-2</sup> K<sup>-1</sup> s<sup>-0.5</sup>) is as follows:

$$C_{\rm g} = \frac{2}{\Gamma \sqrt{P/\pi}} \tag{8}$$

where  $P = 24 \times 60 \times 60$  s.

Noilhan and Planton (1989) and Giard and Bazile (2000) employ the Clapp and Hornberger (1978) hydraulic parameters b (–) and  $\theta_*$  (m<sup>3</sup> m<sup>-3</sup>), the saturated soil moisture content, to estimate  $C_g$  for the 11 USDA soil types from soil physical information

(see Table 1):

$$C_{\rm g} = C_{\rm g,*} \left( \frac{\theta_*}{\max(\theta, \theta_{\rm w})} \right)^{(b/2\log 10)} \tag{9}$$

where  $C_{g,*}$  is the value of  $C_g$  at saturation (see Table 1),  $\theta$  the actual soil moisture content (m<sup>3</sup> m<sup>-3</sup>) and  $\theta_w$  is the volumetric water content at wilting point (m<sup>3</sup> m<sup>-3</sup>). This equation is based on regression fits of  $C_g$  (calculated via Eq. (8)) against  $\theta$ , where  $\Gamma$  required in Eq. (8) is obtained using the theoretical definition of  $\Gamma$ :

$$\Gamma = \sqrt{\lambda H_{\rm c}} \tag{10}$$

Here,  $\lambda$  is the soil thermal conductivity (W m<sup>-1</sup> K<sup>-1</sup>), and  $H_c$  is the heat capacity (J m<sup>-3</sup> K<sup>-1</sup>), see also the section just before Eq. (5a), where we already briefly discussed the definition(s) of  $\Gamma$ .

Noilhan and Planton (1989) used the equations given by McCumber and Pielke (1981) to calculate  $\lambda$ :

$$\lambda = 418 e^{[-(\log|100\psi|+2.7)]}, \quad \text{if } \log|100\psi| < 5.1,$$
  
 
$$\lambda = 0.171, \quad \text{if } \log|100\psi| > 5.1. \tag{11}$$

where  $\psi$  is the matric potential (m) which can be calculated using

$$\psi = \psi_* \left(\frac{\theta}{\theta_*}\right)^{-b} \tag{12}$$

Here,  $\psi_*$  is the matric potential at saturation (m), which can also be found for the 11 soil types in Table 1.

The standard equation given by e.g. Van Wijk and De Vries (1963) is used to calculate  $H_c$ :

$$H_{\rm c} = H_{\rm c,w}\theta + (1 - \theta_*)H_{\rm c,s} \tag{13}$$

where  $H_{c,w}$  and  $H_{c,s}$  are the heat capacity of water and solid soil minerals, respectively (i.e.  $4.2 \times 10^6$  and  $2.0 \times 10^6$  J m<sup>-3</sup> K<sup>-1</sup>).

Table 1

Soil hydraulic parameters (b,  $\theta_*$  and  $\psi_*$ ), quartz content (QC, see Peters-Lidard et al., 1998), sand fraction ( $f_s$ , taken from Cosby et al., 1984) and saturated thermal inertia coefficient for the 11 soil types of the USDA textural classification

Soil texture	b	$\theta_* (m^3 m^{-3})$	ψ* (m)	QC	$f_{ m s}$	$C_{\rm g,*} \ (10^{-6} \ {\rm K \ m^{-2} \ J^{-1}})$	
1. Sand	4.05	0.395	-0.121	0.92	0.92	3.22	
2. Loamy sand	4.38	0.410	-0.090	0.82	0.82	3.06	
3. Sandy loam	4.90	0.435	-0.218	0.60	0.58	3.56	
4. Silt loam	5.30	0.485	-0.786	0.25	0.17	4.42	
5. Loam	5.39	0.451	-0.478	0.40	0.43	4.11	
6. Sandy clay loam	7.12	0.420	-0.299	0.60	0.58	3.67	
7. Silty clay loam	7.75	0.477	-0.356	0.10	0.10	3.59	
8. Clay loam	8.52	0.476	-0.630	0.35	0.32	4.00	
9. Sandy clay	10.40	0.426	-0.153	0.52	0.52	3.06	
10. Silty clay	10.40	0.492	-0.490	0.10	0.06	3.73	
11. Clay	11.40	0.482	-0.405	0.25	0.22	3.60	
•							

However, Eq. (11) gives  $\lambda$  values which are often larger than 2.5 (the maximum value of  $\lambda$  recorded for in situ soils), see Peters-Lidard et al. (1998) and Yang and Koike (2005); using this equation, values as large as 6 are reached for sandy soils. This means that the related values of  $\Gamma$  as calculated using Eq. (10), and hence values of  $C_g$  (from Eq. (8)) to which Eq. (9) was fitted, are seriously overestimated when using  $\lambda$ values from the McCumber and Pielke equation (Eq. (11)). We therefore chose to also calculate  $\Gamma$ values (with Eq. (10)) using  $\lambda$  values as calculated using the Johansen (1975) model for which Peters-Lidard et al. (1998) and Lu et al. (2007) found very good agreement between measured and modelled  $\lambda$ values.

In Section 3 we will compare  $\Gamma$  as estimated from Eqs. (8) and (9), and as estimated from Eq. (10) (using the Johansen  $\lambda$  model, as described in Peters-Lidard et al., 1998, with some adaptations as suggested in Lu et al., 2007, and Eq. (13) to find  $H_c$ ).

In the Johansen (1975) model the thermal conductivity is calculated using

$$\lambda = K_{\rm e}(\lambda_* - \lambda_0) + \lambda_0 \tag{14}$$

where  $\lambda_*$  and  $\lambda_0$  are the thermal conductivity for saturated and air-dry soil conditions, respectively.  $K_e$ is the Kersten number for which we use the amended equation as given by Lu et al. (2007):

$$K_{\rm e} = \exp\{\gamma [1 - S_{\rm r}^{\gamma - \delta}]\} \tag{15}$$

where  $S_r$  is  $\theta/\theta_*$ ,  $\gamma$  a soil-texture dependent parameter ( $\gamma = 0.96$  for soils with sand fraction,  $f_s$ , >0.40, and  $\gamma = 0.27$  for soils with  $f_s \le 0.40$ ;  $f_s$  is given in Table 1) and  $\delta$  (=1.33) is a shape parameter. The advantage of

this equation compared to the original ( $K_e = \log(S_r) + 1$ ) is that Eq. (15) can be used for the whole range of  $S_r$ , i.e. also for  $S_r < 0.10$ . Furthermore, it gives a more accurate description of  $\lambda$  near  $S_r = 0$ , for fine-textured soils.

The dry thermal conductivity was estimated by a linear equation also given by Lu et al. (2007):

$$\lambda_0 = a\phi + b \tag{16}$$

where  $\phi$  is the porosity, here assumed to equal  $\theta_*$  (see Table 1) and *a* are *b* are constants with values of -0.56 and 0.51, respectively. For most soils  $\phi$  ranges between 0.25 and 0.55 and hence  $\lambda_0$  takes on values between 0.38 and 0.2. Johansen (1975) used a non-linear equation to calculate  $\lambda_0$ , but a superior fit, when comparing to measured values of  $\lambda_0$ , was found by Lu et al. (2007) when using Eq. (16).

Saturated thermal conductivity is estimated using

$$\lambda_* = \lambda_s^{(1-\phi)} \lambda_w^\phi \tag{17}$$

with  $\lambda_s$  the thermal conductivity of the soil solids and  $\lambda_w$  that of water (0.57 W m<sup>-1</sup> K<sup>-1</sup>).  $\lambda_s$  is found from

$$\lambda_{\rm s} = \lambda_{\rm q}^{\rm QC} \lambda_{\rm o}^{\rm 1-QC} \tag{18}$$

where QC is the quartz content, which is very similar to  $f_s$  (see Table 1) and hence can be assumed equal to  $f_s$  if data for QC are not available,  $\lambda_q$  the thermal conductivity of quartz (7.7 W m<sup>-1</sup> K<sup>-1</sup>) and  $\lambda_o$  is the thermal conductivity of other minerals (2.0 W m<sup>-1</sup> K<sup>-1</sup> for QC > 0.2 and 3.0 W m<sup>-1</sup> K<sup>-1</sup> otherwise).

### 3. Results and discussion

Fig. 1a shows  $\Gamma$  as a function of  $S_r$  as calculated by inverting Eq. (8) (with  $C_g$  obtained from Eq. (9), i.e. the



Fig. 1. Thermal inertia as a function of relative saturation,  $S_r$  (a) As calculated using the combination of Eqs. (8) and (9) (requiring parameters  $C_{g,*}$ , b, and  $\theta_*$ ) and (b) using Eq. (10) with  $\lambda$  calculated with the adapted Johansen (1975) model and  $H_c$  from Eq. (13).

equation proposed by Noilhan and Planton) for the various soil types given in Table 1. For comparison, Fig. 1b shows  $\Gamma$  calculated using Eq. (10), the theoretical definition of  $\Gamma$  (using Eq. (13) to calculate  $H_c$  and the adapted Johansen model, i.e. Eqs. (14)–(18), to calculate  $\lambda$ ), plotted against  $S_r$  for the same soil types.

It is clear that estimating  $\Gamma$  following the approach in Noilhan and Planton (1989), i.e. Eqs. (8) and (9), leads to large overestimates of  $\Gamma$  at medium to high values of relative saturation. This is illustrated by comparison with values of in situ  $\Gamma$ . For example, Verhoef (2004) for a UK sandy loam bare soil found  $\Gamma$ = 1250 and 1650 J m<sup>-2</sup> K<sup>-1</sup> s<sup>-0.5</sup>, for S<sub>r</sub> values of 0.48 and 0.58, respectively. Furthermore, Verhoef et al. (1996) found  $\Gamma$ = 450 J m<sup>-2</sup> K<sup>-1</sup> s<sup>-0.5</sup> at a S<sub>r</sub> value of 0.07 for a sandy soil in a vineyard in Central Spain and 1543 J m<sup>-2</sup> K<sup>-1</sup> s<sup>-0.5</sup> (S<sub>r</sub> = 0.33) for a sandy soil (vegetation: sparse savannah) in Niger. These values fit much better on the curves shown in Fig. 1b compared to those shown in Fig. 1a.

The overestimation of  $\Gamma$  shown in Fig. 1a is mainly the result of the overestimates of  $\lambda$  by the McCumber and Pielke equation (Eq. (11)) on which Eq. (9) is based, as already discussed. Furthermore, this model does not allow for calculations of  $C_{g,s}$  below  $\theta_w$  (the latter has been calculated using Eq. (12), with  $\psi = -150$  m and the hydraulic parameters ( $\psi_*$  and b) as given in Table 1. Also, Giard and Bazile (2000) put a constraint on  $C_{g,s}$  (8 × 10<sup>-6</sup> K m<sup>2</sup> J<sup>-1</sup>). This corresponds to a minimum  $\Gamma$  of 1507 J m<sup>-2</sup> K<sup>-1</sup> s<sup>-0.5</sup> which is rather high compared to the average minimum values of ~500 J m<sup>-2</sup> K<sup>-1</sup> s<sup>-0.5</sup> (Fig. 1b), which are ultimately based on reliable measured values of  $\lambda_0$  (see Lu et al., 2007).

The  $\Gamma$  values derived from the Johansen  $\lambda$  model (Fig. 1b) attain much more realistic values and the correct relationship with moisture content (a levelling off of  $\Gamma$  with increasing  $S_r$ ). In Fig. 1b two groups can be distinguished: the fine-textured soils with a very slow increase of  $\Gamma$  at low  $S_r$  values and generally lower values of  $\Gamma$  at  $S_r = 1.0$  ( $\Gamma_*$ ), and the coarse-textured soils, with  $f_s > 0.4$ , which increase faster when the soil is wetted up from air–dry thermal inertia ( $\Gamma_0$ ); they also reach larger values of  $\Gamma_*$ , and show more of a spread.

Following the approach taken in Eq. (14) we propose here a universal equation for  $\Gamma$  (based on the data shown in Fig. 1b) which allows the calculation of  $\Gamma$ , at each soil moisture content, with knowledge of the soil's textural type only (using Table 1):

$$\Gamma = K_{\rm e}(\Gamma_* - \Gamma_0) + \Gamma_0 \tag{19}$$

For each general soil type (see Table 1)  $\Gamma_0$  and  $\Gamma_*$  are given by

$$\Gamma_0 = -1062.4\theta_* + 1010.8, \quad [r^2 = 1.0, n = 11]$$
(20a)

$$\Gamma_* = 788.2\theta_*^{-1.29}, \quad [r^2 = 0.91, n = 11]$$
 (20b)

However, using the  $K_{\rm e}$  proposed in Eq. (14) (which represents the shape of normalised  $\lambda$ ) did not adequately describe the shape of normalised  $\Gamma$ . Hence, we optimised  $\gamma$  and  $\delta$  (yielding  $\gamma'$  and  $\delta'$ ) to fit the thermal inertia versus  $S_{\rm r}$  curve, using a least-square fitting procedure and found  $\delta' = 2.0/\gamma' = 1.78$  for coarse-textured soils ( $f_{\rm s} > 0.8$ , soils 1 and 2 in Table 1),  $\delta' = 1.5/\gamma' = 0.93$  for fine-textured soils ( $f_{\rm s} < 0.4$ , soils 4, 7, 8, 10 and 11 in Table 1) and  $\delta' = 4.0/\gamma' = 3.84$  for medium-textured soils (soils 3, 5, 6 and 9 in Table 1).

 $\Gamma$  predicted with these equations corresponds very well to  $\Gamma$  based on  $\lambda$  calculated from Eqs. (14)–(18) and  $H_c$  from Eq. (13) ( $r^2$  of  $\geq 0.98$ , linear fits forced through the origin, with slopes between 0.98 and 1.05). Hence, this calculation procedure of  $\Gamma$  only requires knowledge of  $\theta$ ,  $\theta_*$  and a rough estimate of sand content (not  $C_{g,s}$ , band  $\theta_w$  as well, as in the Noilhan and Planton case (Eqs. (8) and (9)), or quartz content and a larger number of equations when calculating  $\Gamma$  via Johansen's  $\lambda$ , and  $C_h$ ).

These universal estimates of  $\Gamma$  can be used in conjunction with a harmonic analysis of surface temperature, T (see Eq. (7)), to calculate soil heat flux, G, for bare soils. With  $\theta_*$  and  $f_s$  obtained from a suitable soil database,  $\theta$  from passive microwave remote sensing (Kerr et al., 2001), for example, and T from infrared thermometry, calculation of G in this way entails a true and accurate remote method. This avoids the use of the empirical relationships, such as those given by Bastiaansen (1995) and Kustas et al. (1993) to obtain  $G/R_n$ , and hence G with knowledge of  $R_n$ .

## 3.1. Pragmatic approach if exact soil moisture content or soil types are not known

If exact soil moisture content is unknown one can suffice with a categorisation of soil moisture status, expressed as the fraction of water-filled porosity ( $S_r = \theta/$  $\theta_*$ ), see Table 2, for example. If the user has a general idea of the soil moisture status, a moisture status label can be assigned (for example, with knowledge of precipitation events, or their absence, over a period of time). Eq. (19) can then be used at the midpoint of each category to determine probable values of  $\Gamma$  for the relevant soil type.

Table 2 Assigned intervals of fractional water-filled porosity  $(S_r = \theta/\theta_*)$  for each soil moisture category

Soil moisture category	Fractional water-filled porosity, $S_r$			
Dry	<0.1			
Dry-moist	0.1-0.25			
Moist	0.26-0.50			
Moist-wet	0.51-0.75			
Wet	0.76–1.0			

Fig. 2 shows the values of  $\Gamma$  determined from the combination of Eqs. (19), (20a) and (20b). Knowledge of approximate texture (coarse, medium, and fine) and moisture status (dry, dry-moist, moist, moist-wet or wet) would already considerably narrow down the possible values of  $\Gamma$ .

The error in  $\Gamma$  (and hence in *G*, because of straightforward multiplication of the harmonic terms by  $\Gamma$  in Eq. (7)) incurred by using soil moisture status, rather than exact values of moisture content, clearly depends on soil type. Fig. 3 shows the relative error in  $\Gamma$  predicted from Eq. (19) as a function of relative saturation,  $S_{\Gamma}$  Representative soils in each texture class (coarse, medium, and fine) are shown. The largest relative error in  $\Gamma$  is found for the coarse-textured soils under dry soil moisture status, but this error rapidly declines with increasing values of relative saturation (errors dropping below 10%). The medium-textured soil shows a steady decline in error, ranging between just over 20% and ~5%. For the finetextured soil the largest error in  $\Gamma$  predicted will occur for the dry–moist category and values for the moist and



Fig. 2. Soil thermal inertia values calculated from Eq. (19) as a function of  $S_r$ . Typical soil moisture categories (see Table 2) are indicated by the vertical grid lines.



Fig. 3. The relative error in  $\Gamma$  predicted from Eq. (19) (when calculating  $\Gamma$  at the midpoint of each moisture category, rather than at exact  $S_r$  values) as a function of relative saturation. Representative soils in each texture class (coarse, medium, and fine) are shown.

moist-wet category are larger than those calculated for the other two texture groups. However, when the soil is dry, errors are considerably smaller than those for the coarse and medium-textured soils.

Although the errors in  $\Gamma$  can be considerable, the use of an assumed moisture status category may prove useful when considering a scale of land greater than that of the field. In these cases a representative value of  $\theta$  cannot be found without the use of intensive instrumentation and broad assumptions about the general moisture status may therefore prove more practical. At smaller scales (i.e. field) the use of a moisture category label would be less applicable (given the accompanying error), and in situderived  $\theta$  estimates would be more advantageous.

If the soil type is unknown,  $\Gamma$  can be averaged over all 11 soils presented in Table 1, resulting in an average error <10% for medium-textured soils and of ~20% on average for coarse and fine-textured soils (with corresponding errors in *G*).

To determine soil type one can use extant soil maps, soil databases or a soil textural analysis could be employed at a site. If only a limited amount of information on soil textural properties is available (e.g. sand fraction), information such as that given in Table 1 can be used to narrow down the soil textural type and hence a value of  $\Gamma$ . Although an exact soil thermal inertia value will not always be attainable, the worker can focus on a reasonable estimate to incorporate within a physical model such as that given in Eq. (7).

#### 4. Conclusions

The approach proposed for the estimation of soil thermal inertia,  $\Gamma$ , enables the use of a physical equation

(such as Eq. (7)), rather than empirical equations, to determine estimates of soil heat flux, *G*, provided some information on soil type and soil moisture status is available (and provided the assumption of homogeneity is valid).

Using Eq. (19) to obtain reliable estimates of  $\Gamma$ , in combination with Eq. (7), is helpful in the context of remotely sensed *G*, but it also avoids the use of some in situ sensors required to estimate  $\Gamma$ , i.e. those used to measure thermal diffusivity (soil temperature sensors at two depths, see e.g. Verhoef et al., 1996 for a summary of methods) or thermal conductivity (although the latter could be determined from Eq. (14) and related equations, if  $\theta$ -values, QC and  $\theta_*$  data were available). However, ideally in situ moisture sensors, to yield  $\theta$ , are used to determine  $\Gamma$  using Eq. (19).

Furthermore, this equation could be used to replace the less reliable  $C_g$  approach in the ISBA model, for example.

Moreover, we assert that the approach adopted in this paper to calculate G allows for a more universally applicable methodology. Explicitly accounting for soil thermal inertia and using, albeit a targeted, average value will lead to more accurate estimates of G. Therefore, unlike more empirically driven methodologies, the worker is able to interpret, evaluate and discuss estimates of G from a more theoretical stand point, whilst being explicitly aware of the cause of uncertainty in resultant estimates of G.

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