

multigraf

$$G = (V, E, g)$$

$$A \otimes B = \{(a, b) \mid a \in A, b \in B \text{ nu au designat}\}$$

$$V \neq \emptyset$$

$$E \cap V = \emptyset$$

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

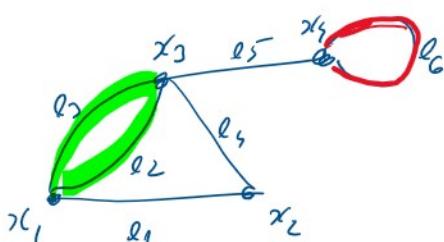
$\downarrow a, b \quad a = s$

$$g : E \rightarrow V \otimes V$$

$$G = (V(G), E(G), g(G))$$

$n = |V|$ ordinal grafului

$m = |E|$ dimensiunea grafului



$$g(e) = \{a, a\} \quad \text{bucle} \quad e_6$$

$g(e_1) = g(e_2) \Rightarrow e_1, e_2 \text{ sunt paralele}$

$$V = \{x_1, \dots, x_5\}$$

$$g(e_1) = \{x_1, x_2\}$$

$$E = \{e_1, \dots, e_6\}$$

$$g^{-1}(a, b) = \{e \in E \mid g(e) = \{a, b\}\}$$

x un nod din G

$$N_G(x) = \{y \in V \mid \exists e \in E, g(e) = \{x, y\}\}$$

num

$$N_G(x) = \{y \in V \mid g^{-1}(x, y) \neq \emptyset\}$$

pt un multigraf număr multilor incident la x

$$I_G(x) = \{e \in E \mid \exists y \in V, y \neq x, g(e) = \{x, y\}\}$$

număr buclelor incidente de x

vertul buclelor incidente vrf. x

$$\mathcal{L}_G(x) = \{ e \in E \mid g(e) = \{x, x\} \}$$

$d(x)$ - gradul vrf. x

$$d(x) = |I_G(x)| + 2|\mathcal{L}_G(x)|$$

$$d(x) = \begin{cases} 0, & \text{vrf izolat} \\ 1, & \text{vrf unei muchii} \\ >1, & \end{cases}$$

Graf simplu

multgraf fară muchii paralele și bucle

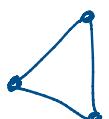
$$|g^{-1}(a, b)| \leq 1 \quad \forall a, b \in V$$

$$\{a, b\} \quad g(e) = \{a, b\}$$

$$G = (V, E)$$

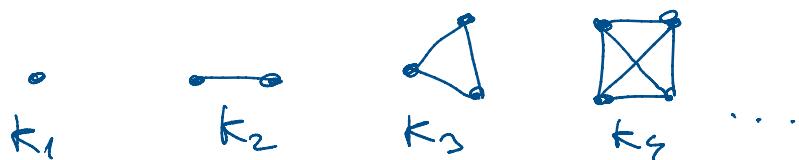
$$d(x) = |N_G(x)|$$

- graf regular \Rightarrow toate vrf. au același grad
graf k -regular $d(x) = k \quad \forall x \in V$



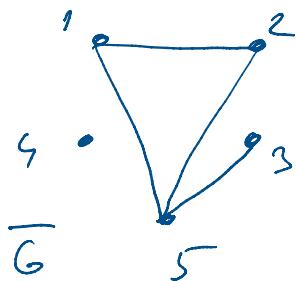
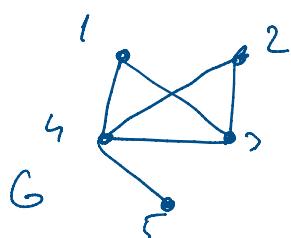
$$d(x) = 2 \quad \forall x \in V$$

- graf complet K_n



- $\bar{G} = (\bar{V}, \bar{E})$ este complementul grafului $G = (V, E)$ dacă

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 $\bar{V} = V$ și $\bar{E} = \{(u, v) \mid \{u, v\} \notin E\}$



• graf de ordinul $n \Rightarrow E_G \cup E_{\bar{G}} = E_{K_n}$

Ex. $m, n \in \mathbb{N}$

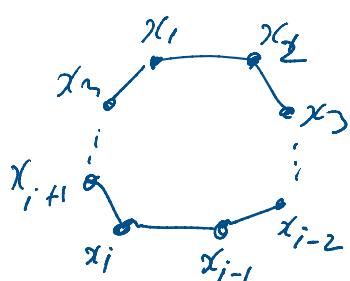
- graf nul $G = \emptyset$ $V_{\emptyset} = \{x_1, \dots, x_m\}$
 $E_{\emptyset} = \emptyset$



- graf linii $n \geq 2$, $G = P_n$ $V_{P_n} = \{x_1, \dots, x_n\}$



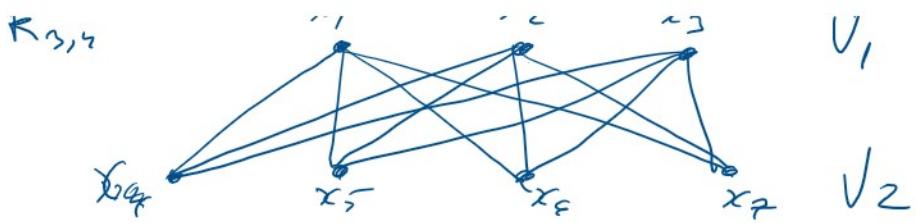
- graf ciclu $n \geq 3$ $G = C_n$



- graf complet $G = K_n$

- graf bipartit complet (m, n) $G = K_{m,n}$





$$\sum_{v \in V} d(v) = 2 |E|$$

G k -regular

$$\frac{km}{2}$$
 muchii

$$k_m = \frac{m(m-1)}{2}$$

Subgraf

$G_1 = (V_1, E_1, g_1)$ și $G = (V, E, g)$ sunt căi G_1 este un subgraf al lui G dacă:

$$V_1 \subseteq V$$

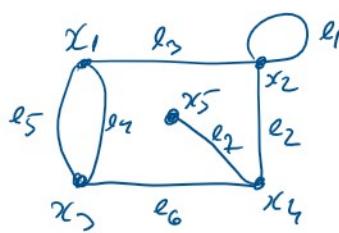
$$E_1 \subseteq E$$

$$g_1(e) = g(e) \quad \forall e \in E_1$$

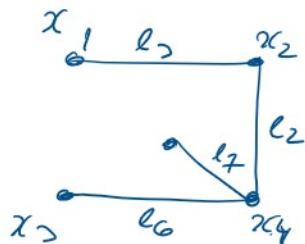
$$\text{re notam} \quad G_1 \subseteq G$$

→ pt un subgraf nenul $W \subseteq V$ de af., subgraful lui G indus de W , notat ca $G[W]$, este graful cu rețea af. W și cărui set de muchii constă din muchiile lui G ce au vf. în W

- $F \subseteq E$, subgraful lui G este indus de F , notat $G[F]$

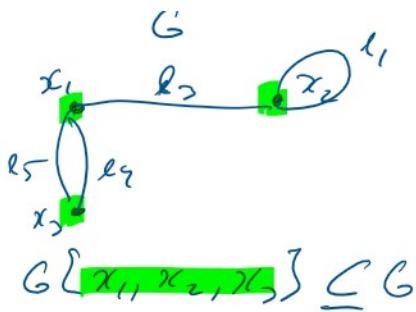


$$G$$



$$G_1 \subseteq G$$





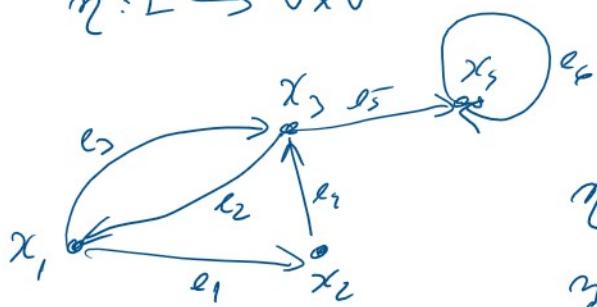
graf / multigraf orientat

$$\vec{G} = (V, E, \eta)$$

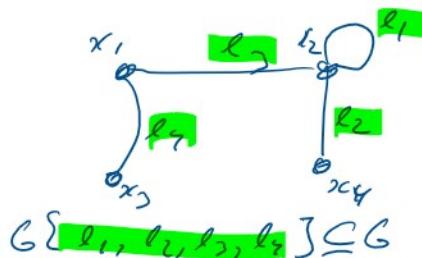
$$V \neq \emptyset$$

$$V \cap E = \emptyset$$

$$\eta : E \rightarrow V \times V$$



$$G_1 \subseteq G$$



$$\longrightarrow$$

$\eta(e) = \{u, v\}$ and e is incident upon exterior of u if v is incident upon interior of u



$$\eta(e_3) = \text{exterior}$$

$$\eta(e_2) = \eta(e_1) = \text{interior}$$

integrad interior/exterior pt \vec{G}

$$\vec{G} = (V, E, \eta) \quad \forall x \in V$$

$d^-(x) = \text{integrad interior wrt arcs incident upon interior of } x$

$$d^-(x) = |\{e \in E \mid \eta(e) = \{y, x\}, \text{ and } y \in V\}| = |N_G^{\text{in}}(x)|$$

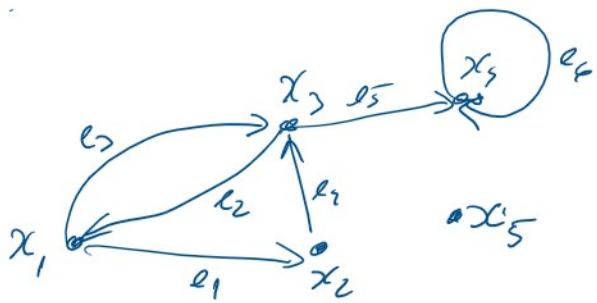
$d^+(x) - \text{integrad exterior} = \text{integrad arcs incident upon exterior of } x$

$$d^+(x) = |\{e \in E \mid \eta(e) = \{x, y\}, \text{ and } y \in V\}|$$

$$d(x) = d^-(x) + d^+(x)$$

T

$$\sum_{u \in V} d^-(u) = \sum_{u \in V} d^+(u) = |E|$$



| | |
|---------------------|----------------|
| $d^-(x_1) = 1$ | $d^+(x_1) = 2$ |
| $d^-(x_2) = 1$ | $d^+(x_2) = 1$ |
| $d^-(x_3) = 2$ | $d^+(x_3) = 2$ |
| $d^-(x_4) = 2$ | $d^+(x_4) = 1$ |
| $d^-(x_5) = 0$ | $d^+(x_5) = 0$ |
| $\sum \overline{G}$ | |

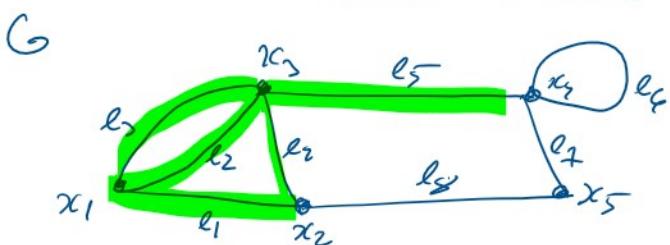
drum

$$\vec{G} = (V, E, \gamma)$$

draw sets or receive $(x_0, l_1, x_1, l_2, x_2, \dots, l_k, x_k)$

graf neorientat

draw → last
circuit → cieler



Want $\underline{d} = (x_1, d_2, x_3, d_3, x_4, d_4, \dots, x_r)$

Termo simple = $(x_1, l_2, x_3, l_4, x_1, l_1, x_2, l_8, x_5, l_7, x_6)$
 ↳ são estes elementos (x_i , apesar de dizer os j)

laut elementar $(x_1, \underline{l_2}, x_3, \underline{l_4}, x_2, \underline{l_8}, x_5, \underline{l_9}, x_3)$

lant elementar $(x_1, \underline{l_2}, x_2, \underline{l_3}, x_2, \underline{l_8}, x_5, \underline{l_4}, x_3)$

ciclu de lungime 6 $(x_3, l_2, x_1, l_2, x_3, l_3, x_2, l_2, x_5, l_7, x_4, l_5, x_3)$

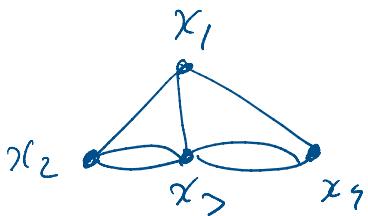
reprezentare matricială a unui graf

$$G = (V, E, g)$$

matr. adiacență

$$A = (a_{ij})_{i,j=1,n}$$

$$a_{ij} = \begin{cases} |g^{-1}(x_i, x_j)|, & \text{daca } i \neq j \\ 2|g^{-1}(x_i, x_j)|, & i = j \end{cases}$$



$$A = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 2 & 0 \\ 1 & 2 & 0 & 2 \\ 1 & 0 & 2 & 0 \end{pmatrix}$$

graf neorientat

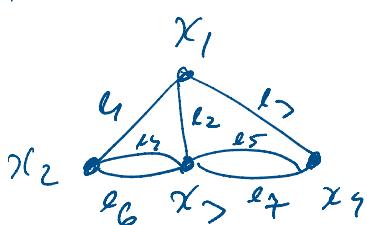
$$a_{ij} = \begin{cases} 1 & \text{daca } \{i, j\} \in E \\ 0 & \text{daca } \{i, j\} \notin E \end{cases} \rightarrow$$

• matr. incidentă

$$E = (1, 2, \dots, m) \quad m = |E|$$

$m \times m$

$$b_{ij} = \begin{cases} 1, & \exists j \in V : e = (i, j) \\ 0 & \end{cases}$$



$$\begin{matrix} & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 & l_7 \\ x_1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ x_2 & - & - & - & - & - & - & - \\ x_3 & - & - & - & - & - & - & - \\ x_4 & - & - & - & - & - & - & - \end{matrix}$$