

Sistemas Complejos en Ecología, Análisis y Modelos

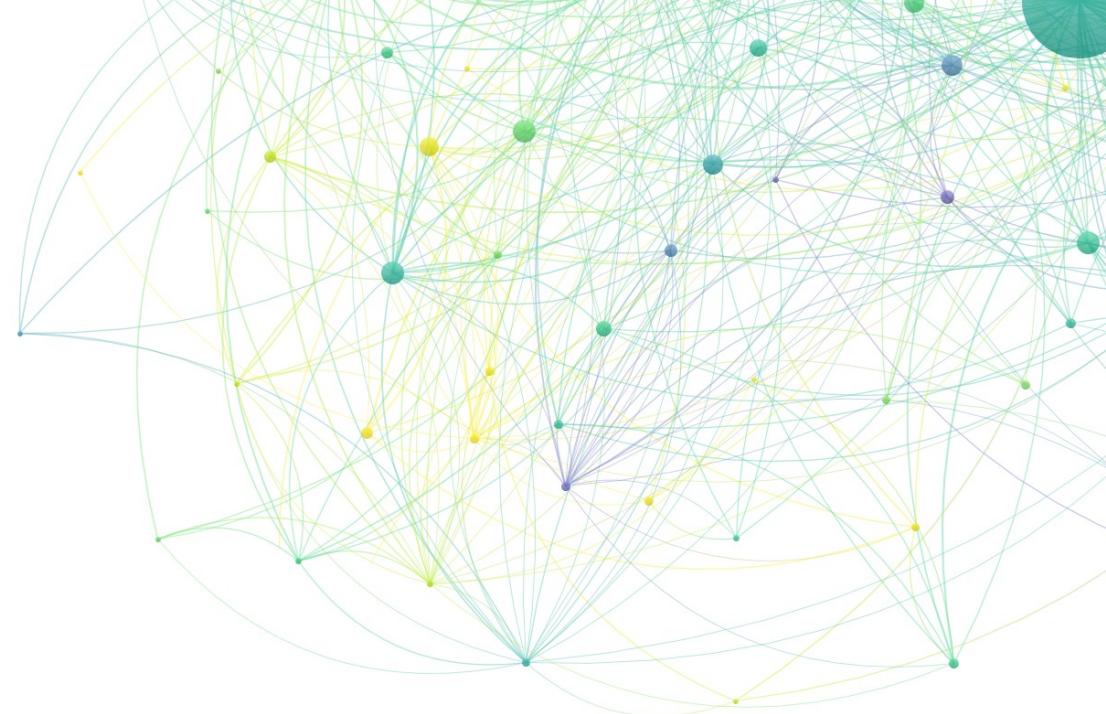


Rebeca de la Fuente

Investigadora Postdoctoral (CADIC-CONICET)

ÍNDICE

1. Modelización de procesos en ecología
2. Modelos deterministas
3. Evolución, estabilidad y bifurcaciones
4. Modelos estocásticos
5. Teoría de campo medio
6. Dinámica de fuegos



Resolución Ejercicios

1.

$$n_{t+1} = 4n_t$$

$$n_0 = 1$$

Resolución Ejercicios

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$$n_{t+1} = 4n_t$$

$$n_0 = 1$$

$$n_1 = 4n_0 = 4$$

Resolución Ejercicios

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$$n_2 = 4n_1 = 4^2$$

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$$n_3 = 4n_2 = 4^3$$

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•
⋮

$$n_t = 4^t$$

Resolución Ejercicios

2.

$$n_{t+1} = R n_t \left(\frac{K - n_t}{K} \right) \rightarrow n_{t+1} = 4 n_t \left(\frac{20 - n_t}{20} \right)$$

Resolución Ejercicios

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$$n_{t+1} = R n_t \left(\frac{K - n_t}{K} \right) \rightarrow n_{t+1} = 4 n_t \left(\frac{20 - n_t}{20} \right)$$
$$n_0 = 20$$

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$$n_0 = 20$$

$$n_1 = 0$$

Resolución Ejercicios

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$$n_2 = 0$$

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$$n_1 = 0$$

$$n_2 = 0$$

⋮
⋮

$$n_{12} = 0$$

Resolución Ejercicios

3.

$$n_{t+1} = 5n_t \left(\frac{100 - n_t}{100} \right)$$

$$n_0 = 20$$

Resolución Ejercicios

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$$n_{t+1} = 5n_t \left(\frac{100 - n_t}{100} \right)$$

$$n_0 = 20$$

$$n_1 = 5 * 20 * \left(\frac{100 - 20}{100} \right) = 80$$

$$n_2 = 5 * 80 * \left(\frac{100 - 80}{100} \right) = 80$$

•
⋮

$$n_t = 80$$

Resolución Ejercicios

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•
⋮

$$n_t = 80$$

$$n_{t+1} = n_t$$

$$5n_t \left(\frac{100 - n_t}{100} \right) = n_t$$


$$\begin{array}{l} n_t^* = 0 \\ n_t^* = 80 \end{array}$$

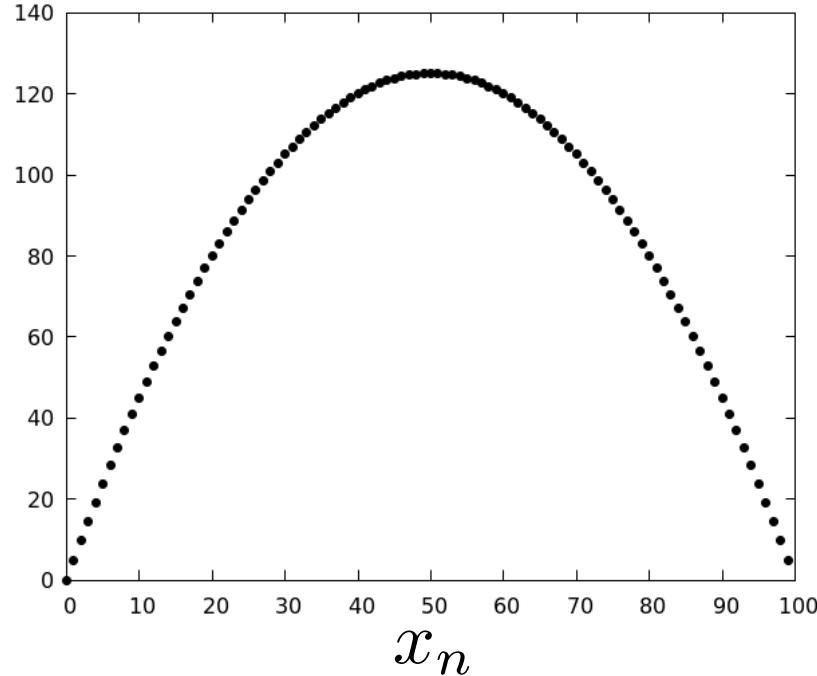
Resolución Ejercicios

3.

$$n_{t+1} = 5n_t \left(\frac{100 - n_t}{100} \right)$$

$$n_0 = 20$$

$$x_{n+1}$$



$$n_{t+1} = n_t$$

$$5n_t \left(\frac{100 - n_t}{100} \right) = n_t$$

$$\begin{array}{l} \xrightarrow{\hspace{1cm}} n_t^* = 0 \\ \xleftarrow{\hspace{1cm}} n_t^* = 80 \end{array}$$

Resolución Ejercicios

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$$x_{n+1}$$

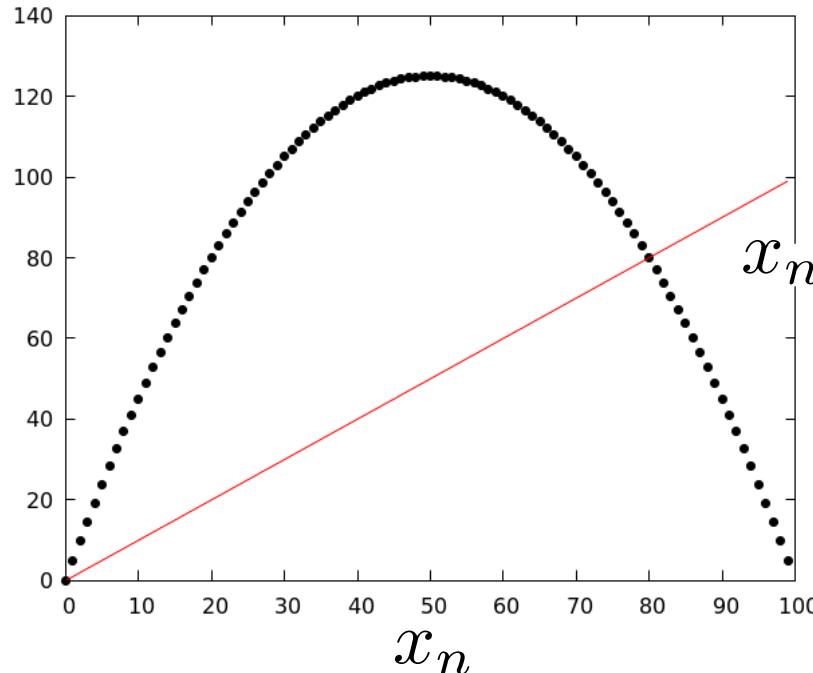
$$n_{t+1} = n_t$$

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$$\begin{array}{l} n_t^* = 0 \\ n_t^* = 80 \end{array}$$



$$x_{n+1} = x_n$$



Resolución Ejercicios

4.

$$\frac{dx(t)}{dt} = ax(t) - bx(t)y(t)$$

$$\frac{dy(t)}{dt} = cx(t)y(t) - dy(t)$$

Resolución Ejercicios

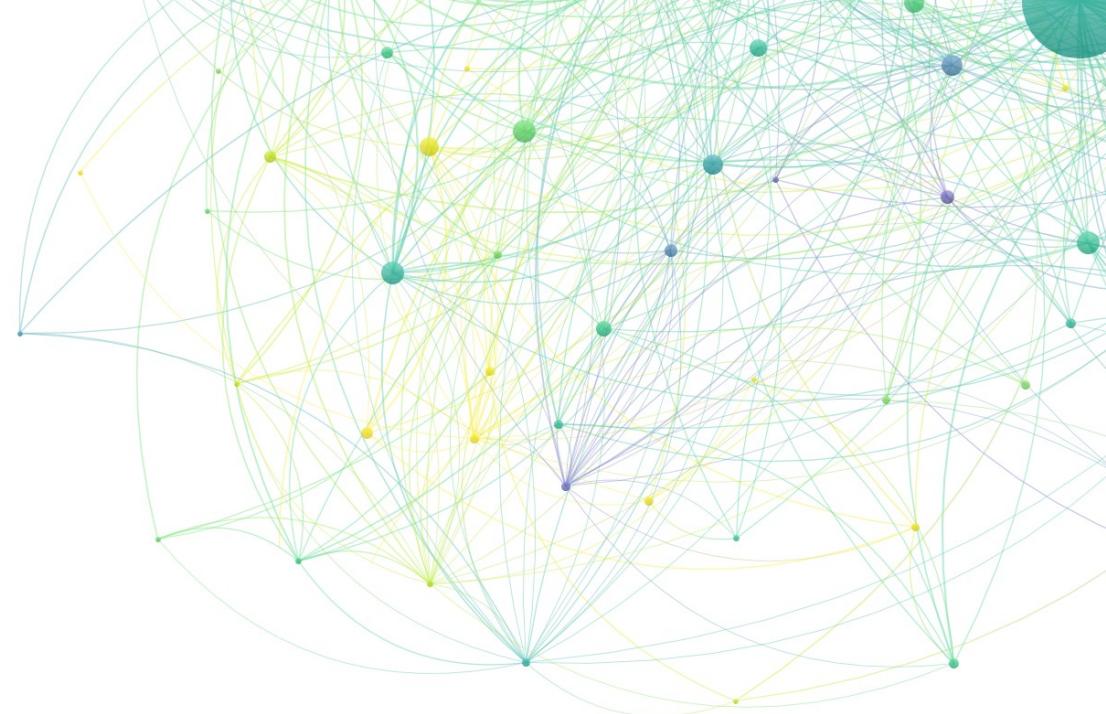
4.

$$\frac{dx(t)}{dt} = ax(t) - bx(t)y(t) - ex^2$$

$$\frac{dy(t)}{dt} = cx(t)y(t) - dy(t) - h$$

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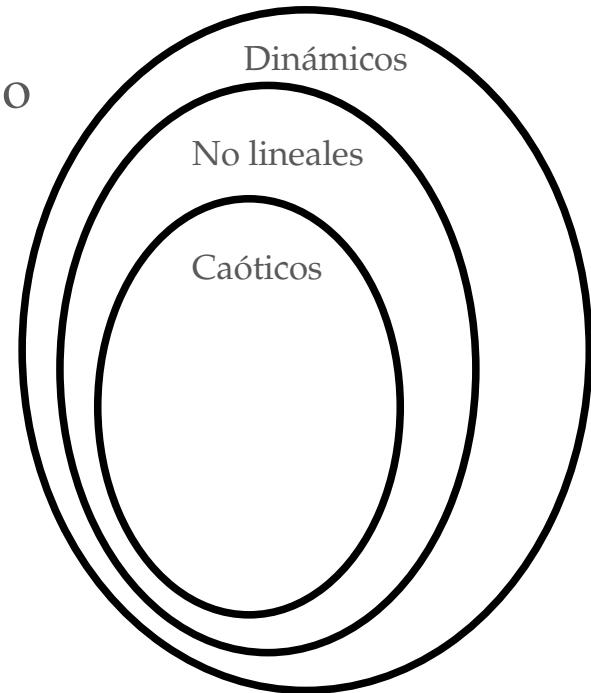
Sistemas dinámicos: sistemas que evolucionan con el tiempo

CONCEPTOS

- Variables (estado sistema)
- Parámetros
- Condiciones iniciales
- Trayectoria
- Atractor
- Base de atracción
- Régimen transitorio
- Punto fijo (estable / inestable)
- Órbita periódica
- Bifurcación

REPRESENTACIÓN

- Espacio físico
- Gráfica evolución temporal
- Gráfica de correlación (return map)
- Espacio de estados / fases (phase/state space)
- Diagrama de bifurcación



Sistemas dinámicos: sistemas que evolucionan con el tiempo

CONCEPTOS

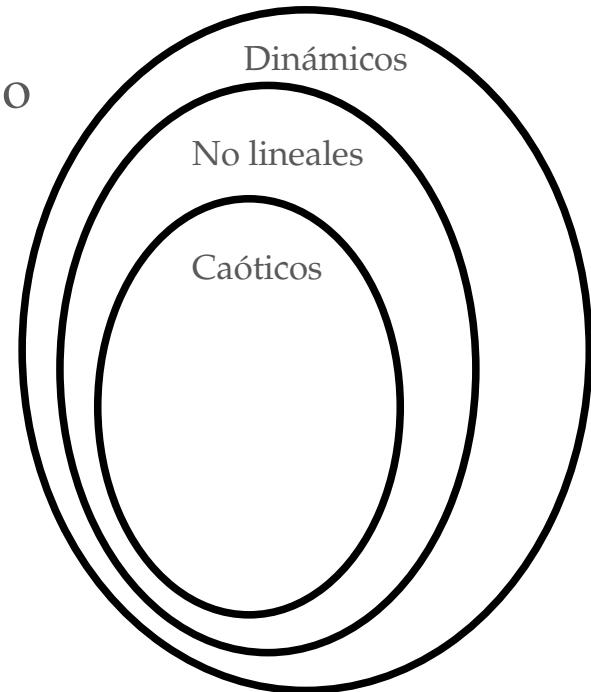
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$$x_{n+1} = f(x_n)$$

$$\frac{dx(t)}{dt} = f(x(t))$$



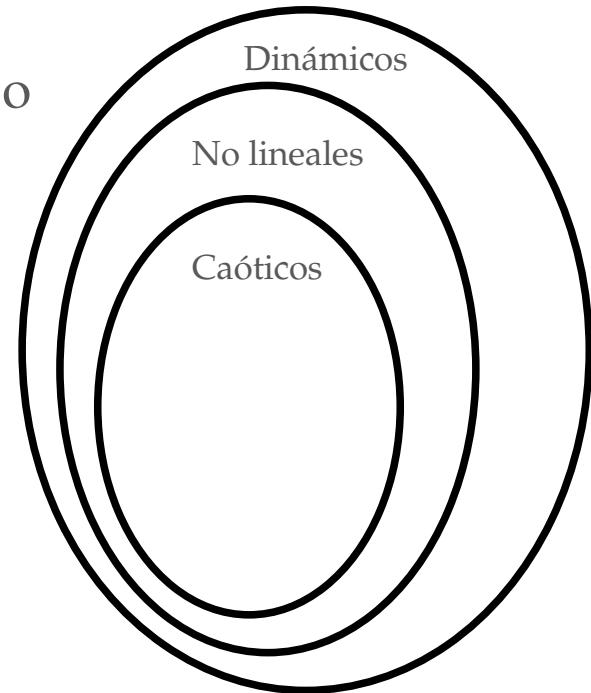
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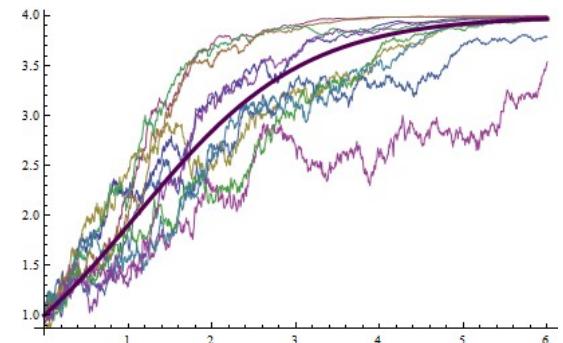
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$$x_{n+1} = f(x_n)$$

$$\frac{dx(t)}{dt} = f(x(t))$$

$$\frac{dx(t)}{dt} = f(x(t)) + q(x(t))\eta(t)$$



Mapa logístico

$$L : (0, 1) \longrightarrow (0, 1)$$

$$x_{n+1} = Rx_n(1 - x_n)$$

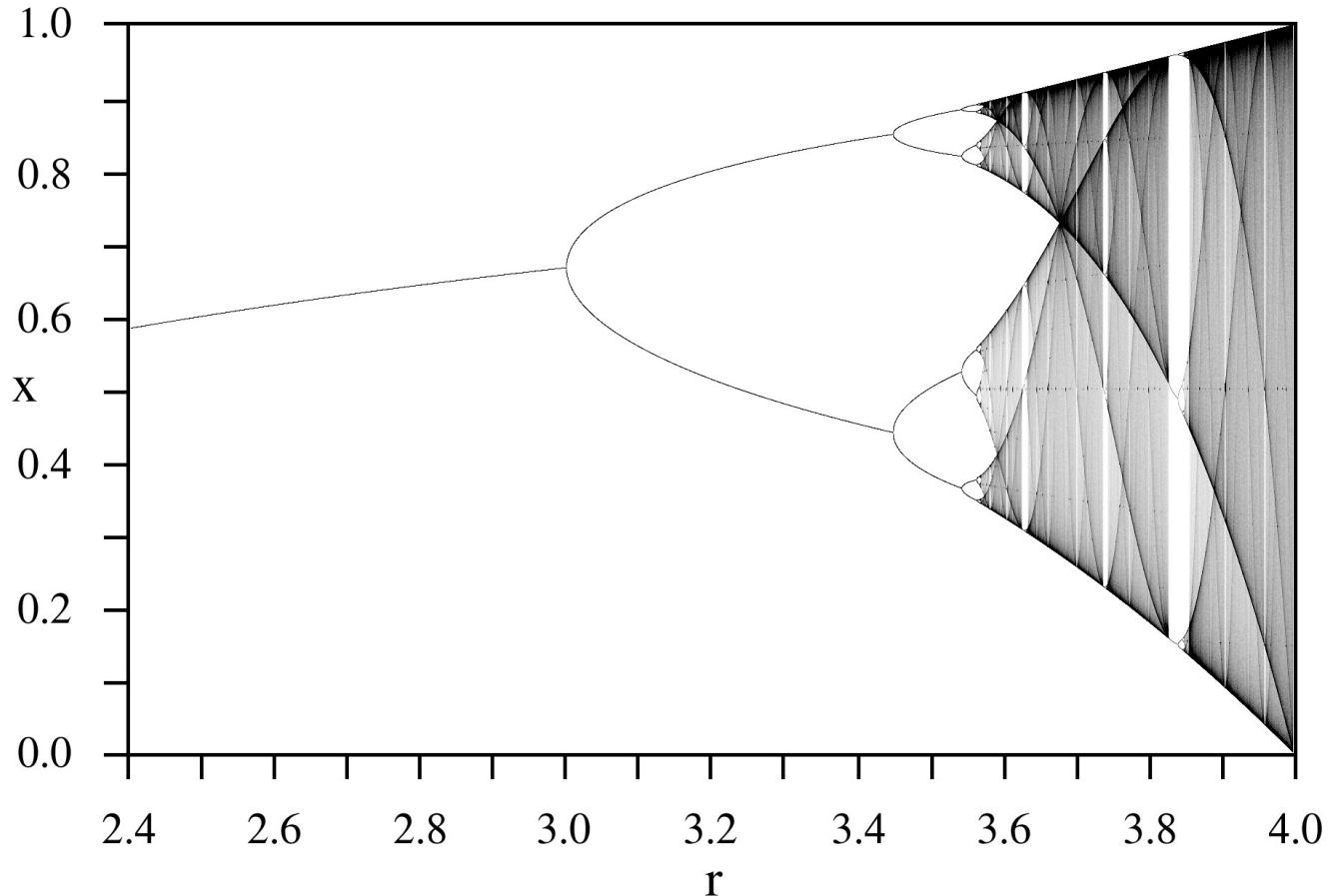
$$R \in (0, 4)$$

Mapa logístico

$$L : (0, 1) \longrightarrow (0, 1)$$

$$x_{n+1} = Rx_n(1 - x_n)$$

$$R \in (0, 4)$$



Mapa logístico

$$L : (0, 1) \longrightarrow (0, 1)$$

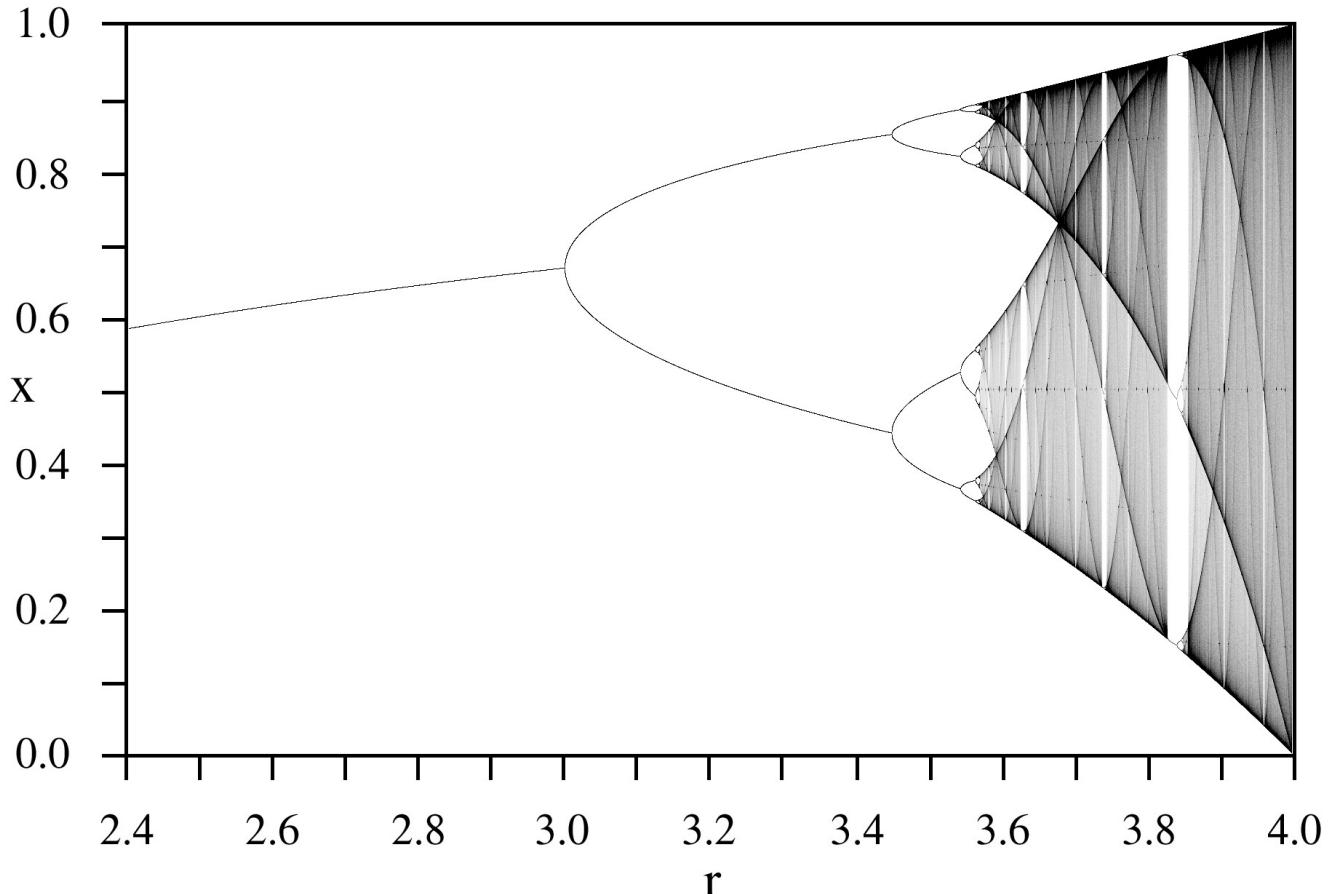
$$x_{n+1} = Rx_n(1 - x_n)$$

$$R \in (0, 4)$$

$$\frac{\Delta_2}{\Delta_3} \approx \frac{\Delta_1}{\Delta_2}$$

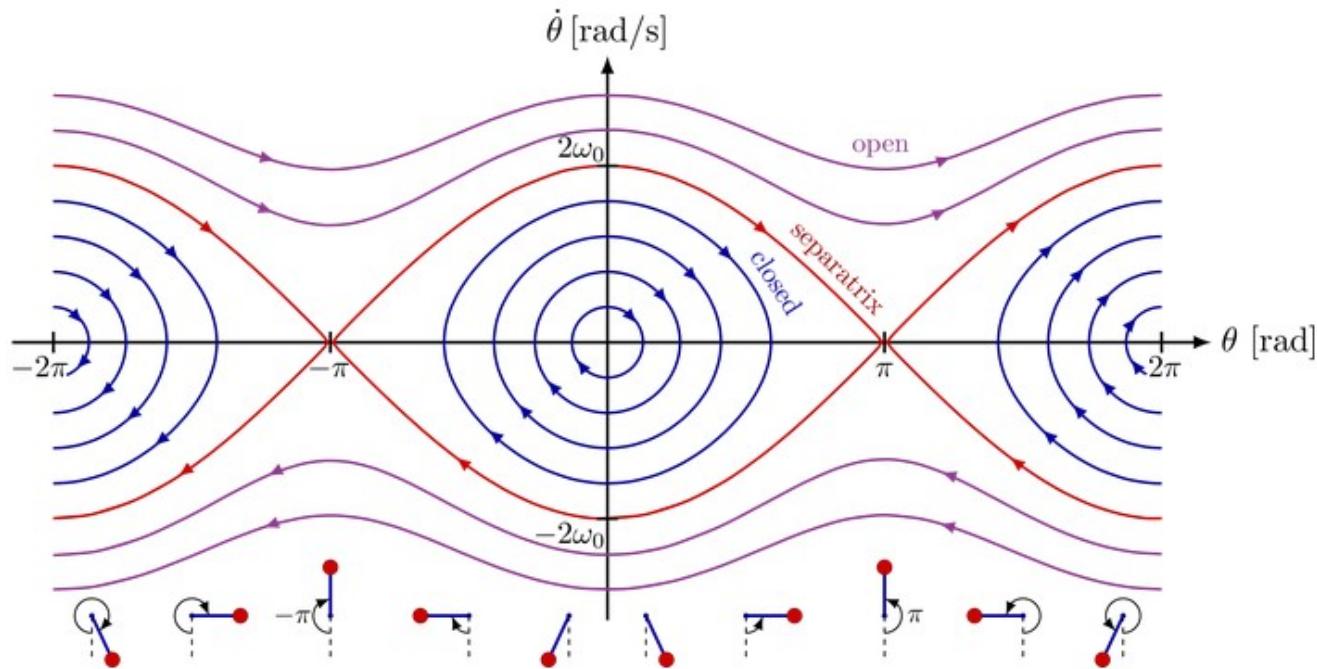
Feigenbaum number

$$\lim_{n \rightarrow \infty} \frac{\Delta_n}{\Delta_{n+1}} = 4.66$$



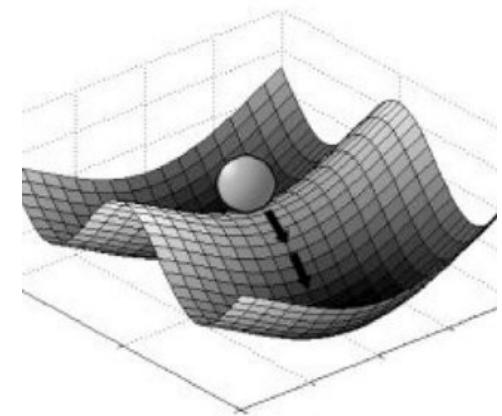
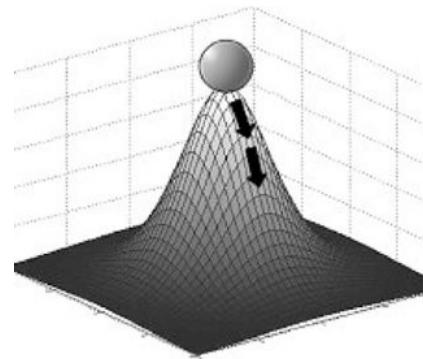
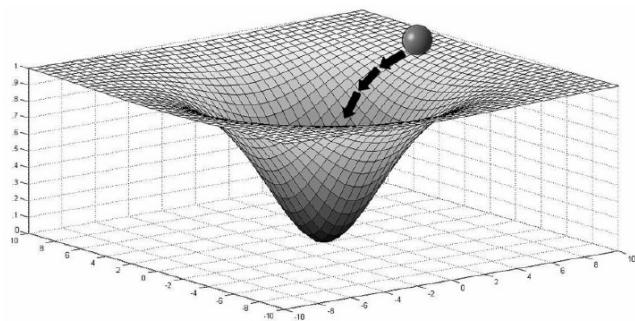
Espacio de fases

Espacio de fases

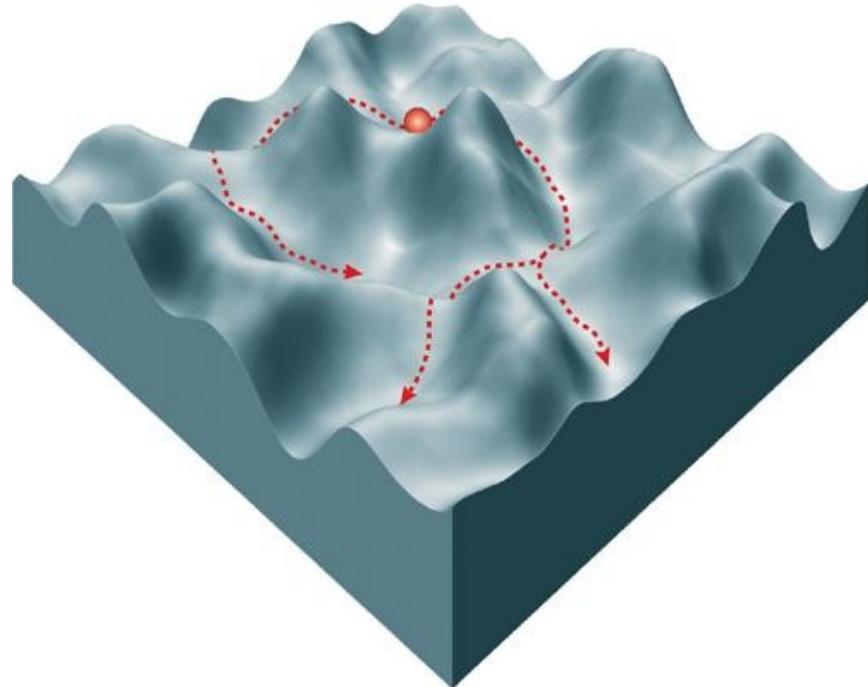


Paisaje dinámico

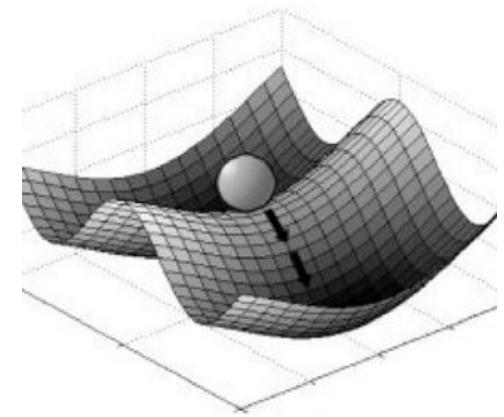
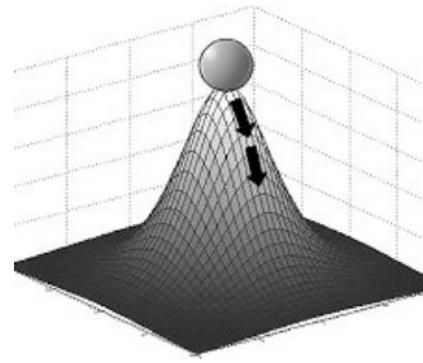
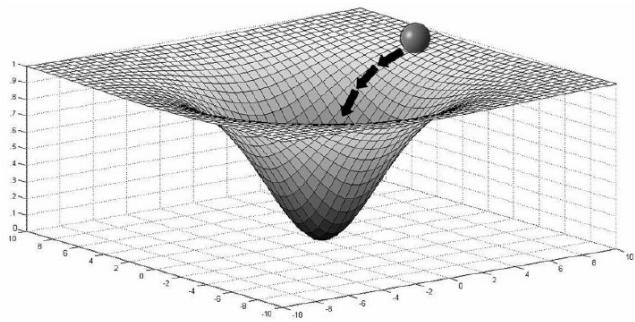
Paisaje dinámico (Dynamical landscape)



Paisaje dinámico (Dynamical landscape)



Paisaje dinámico (Dynamical landscape)



Variedad estable → Conjunto de condiciones iniciales x que convergen a x^* cuando $t \rightarrow +\infty$

Variedad inestable → Conjunto de condiciones iniciales x que convergen a x^* cuando $t \rightarrow -\infty$

Análisis estabilidad local

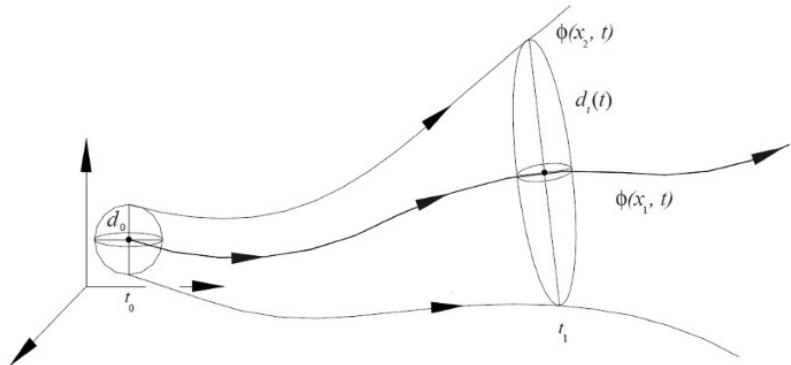
$$\frac{dx_i(t)}{dt} = f_i(x(t))$$

Puntos equilibrio

$$M = J|_{x^*}$$

$$\frac{dx_i(t)}{dt} = 0 \longrightarrow x^*$$

$$\frac{d\Delta x(t)}{dt} = M\Delta x(t) \longrightarrow \Delta x(t) = e^{Mt}\Delta x(0)$$



Linearización alrededor del punto x → Expansión Taylor

$$f(x^* + \Delta x) \approx f(x^*) + \frac{df}{dx}(x^*)\Delta x + \frac{1}{2} \frac{d^2 f}{dx^2}(x^*)\Delta^2 x + \dots \longrightarrow f(x^* + \Delta x) \approx \frac{df}{dx}(x^*)\Delta x$$

$$f(x^* + \Delta x) \approx f(x^*) + J|_{x^*}\Delta x(t) \longrightarrow f(x^* + \Delta x) \approx J|_{x^*}\Delta x(t) \quad J_{ij} = \frac{\partial f_i(x)}{\partial x_j}$$

Análisis estabilidad local

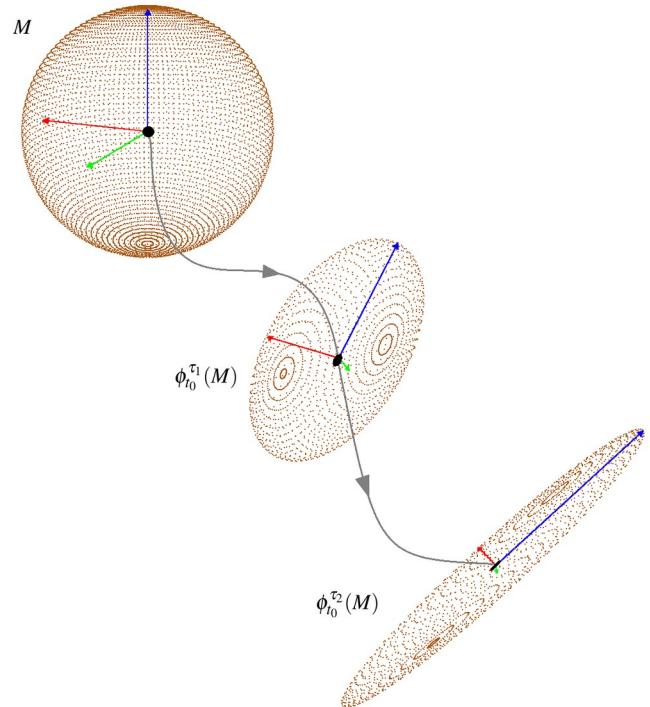
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$$\frac{d\Delta x(t)}{dt} = M\Delta x(t) \longrightarrow \Delta x(t) = e^{Mt}\Delta x(0)$$

$$\lambda_1 > \lambda_2 > \dots > \lambda_n$$

$$\Delta x(t) = e^{\lambda t}\Delta x(0)$$



Análisis estabilidad local

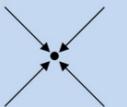
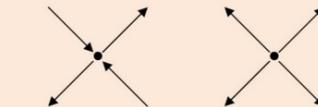
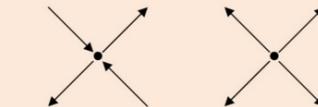
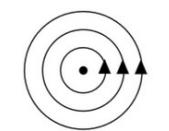
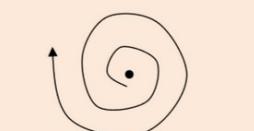
$$\frac{dx_i(t)}{dt} = f_i(x(t))$$

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$$\frac{d\Delta x(t)}{dt} = M\Delta x(t) \longrightarrow \Delta x(t) = e^{Mt}\Delta x(0)$$

$$\lambda_1 > \lambda_2 > \dots > \lambda_n$$

$$\Delta x(t) = e^{\lambda} \Delta x(0)$$

	Stable $\text{Re}(\lambda_d) < 0$	Lyapunov stable* $\text{Re}(\lambda_d) = 0$	Unstable $\text{Re}(\lambda_d) > 0$	
Real eigenvalues	Stable point $\text{Re}(\lambda_1) < 0, \text{Re}(\lambda_2) < 0$ 	Neutral point $\text{Re}(\lambda_1) = 0, \text{Re}(\lambda_2) < 0$ $\text{Re}(\lambda_1) = 0, \text{Re}(\lambda_2) = 0$ 	Saddle point $\text{Re}(\lambda_1) > 0, \text{Re}(\lambda_2) < 0$ 	Unstable point $\text{Re}(\lambda_1) > 0, \text{Re}(\lambda_2) > 0$ 
Complex conjugate eigenvalues	Stable spiral focus $\text{Re}(\lambda_1) = \text{Re}(\lambda_2) < 0$ 	Neutral center $\text{Re}(\lambda_1) = \text{Re}(\lambda_2) = 0$ 	Unstable spiral focus $\text{Re}(\lambda_1) = \text{Re}(\lambda_2) > 0$ 	

Análisis estabilidad local

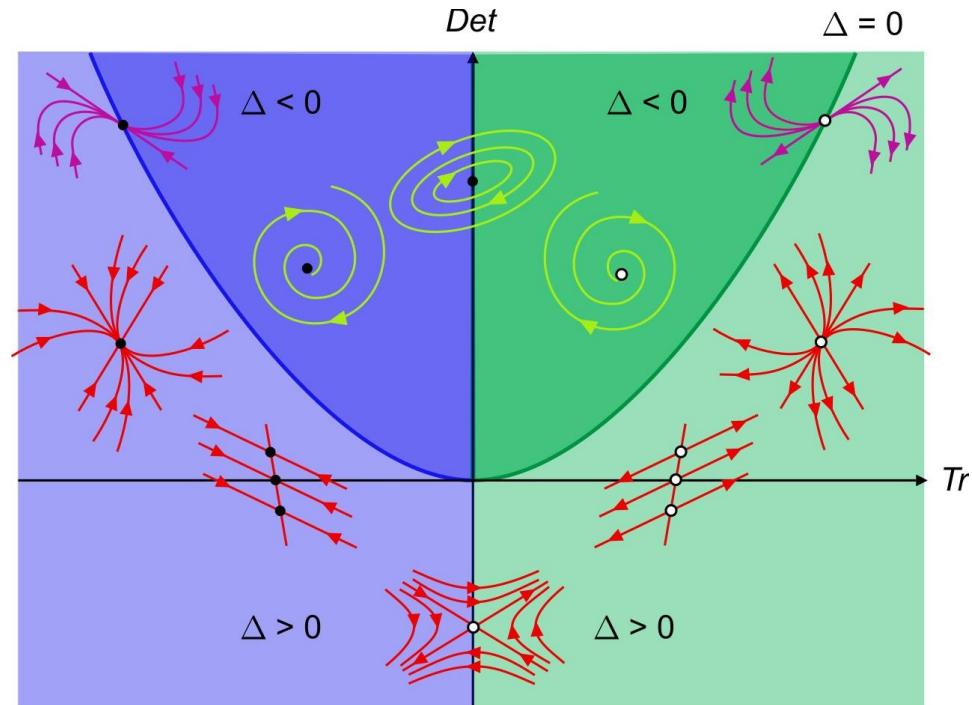
$$\frac{dx_i(t)}{dt} = f_i(x(t))$$

$$M = J|_{x^*}$$

$$\frac{d\Delta x(t)}{dt} = M\Delta x(t) \longrightarrow \Delta x(t) = e^{Mt}\Delta x(0)$$

$$\lambda_1 > \lambda_2 > \dots > \lambda_n$$

$$\Delta x(t) = e^{\lambda} \Delta x(0)$$



$$\begin{aligned} \frac{dx}{dt} &= A_{11}x + A_{12}y \\ \frac{dy}{dt} &= A_{21}x + A_{22}y \end{aligned}$$

$$\begin{aligned} \text{Tr } A &= A_{11} + A_{22} \\ \text{Det } A &= A_{11}A_{22} - A_{21}A_{12} \\ \Delta &= \text{Tr}^2 - 4\text{Det} \end{aligned}$$

Análisis estabilidad local

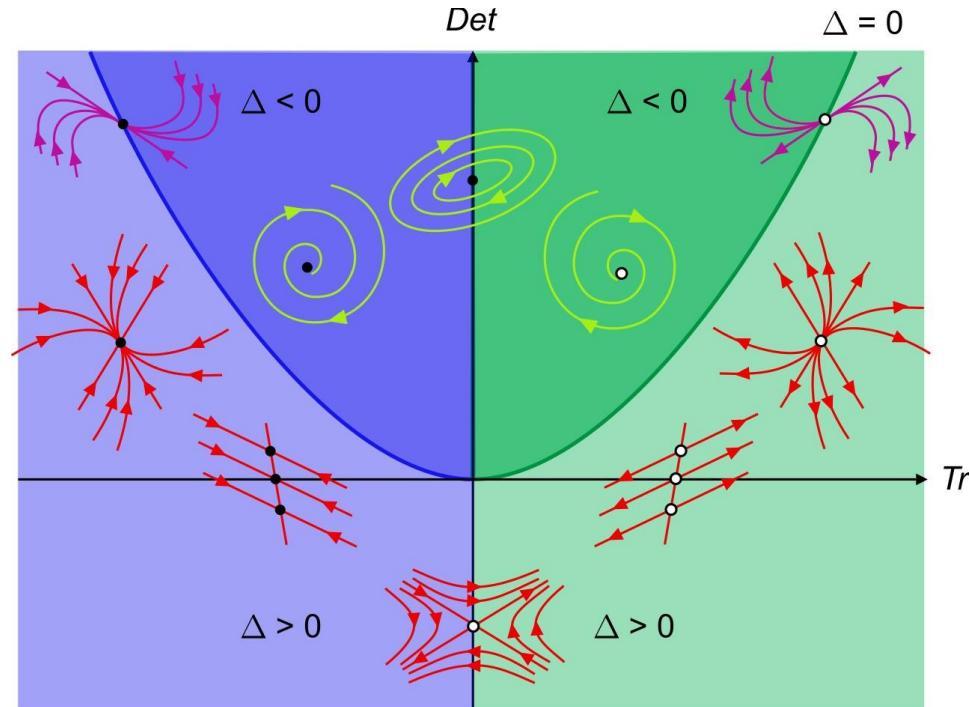
$$\frac{dx}{dt} = 2x - y$$

$$\frac{dy}{dt} = x + 2y$$

$$J = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$$

$$\Delta = \text{tr}^2 - 4\text{Det} = 4^2 - 4 * 5 = -4 < 0$$

$$\det \begin{bmatrix} 2 - \lambda & -1 \\ 1 & 2 - \lambda \end{bmatrix} = 0 \longrightarrow \lambda = 2 \pm i$$



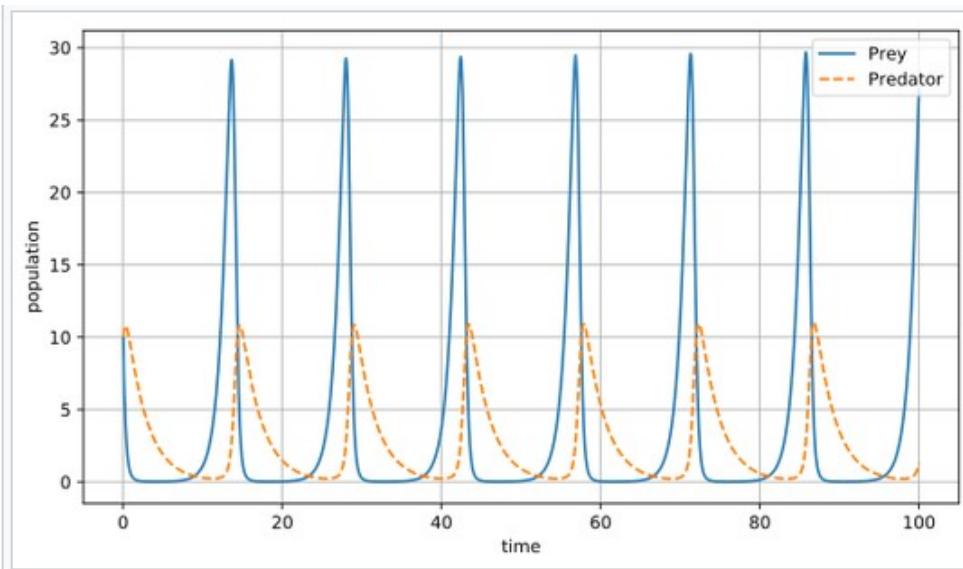
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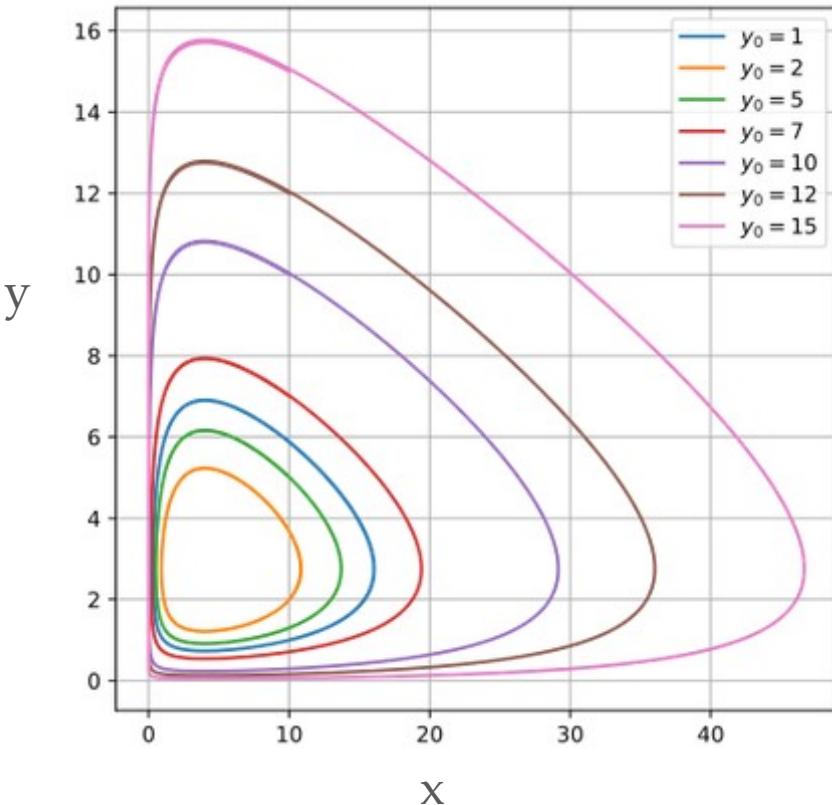
Depredador-presa Lokta-Volterra

$$\frac{dx(t)}{dt} = ax(t) - bx(t)y(t)$$

$$\frac{dy(t)}{dt} = cx(t)y(t) - dy(t)$$



Espacio de fases



Depredador-presa Lokta-Volterra

$$\frac{dx(t)}{dt} = ax(t) - bx(t)y(t)$$

$$\frac{dy(t)}{dt} = cx(t)y(t) - dy(t)$$

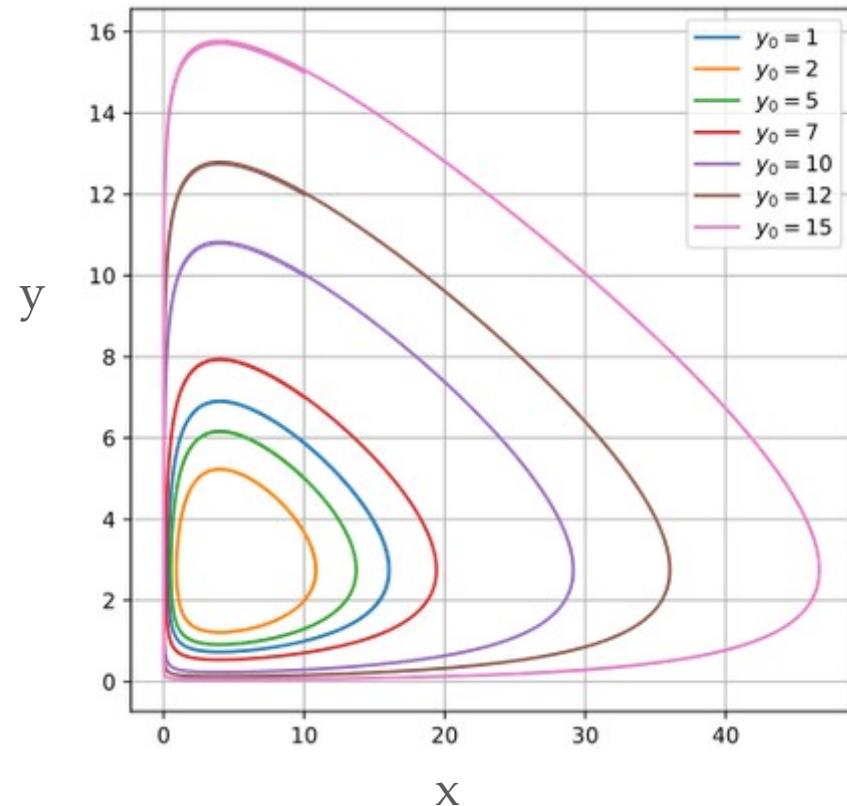
Puntos de equilibrio

$$\frac{dx}{dt} = 0 \quad \frac{dy}{dt} = 0$$

$$(x_0 = 0, y_0 = 0)$$

$$(x_0 = d/c, y_0 = a/b)$$

Espacio de fases



Depredador-presa Lokta-Volterra

$$\frac{dx(t)}{dt} = ax(t) - bx(t)y(t)$$

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Puntos de equilibrio

$$\frac{dx}{dt} = 0 \quad \frac{dy}{dt} = 0$$

$$(x_0 = 0, y_0 = 0)$$

$$(x_0 = d/c, y_0 = a/b)$$

$$J = \begin{bmatrix} \frac{\partial f_x}{\partial x} & \frac{\partial f_x}{\partial y} \\ \frac{\partial f_y}{\partial x} & \frac{\partial f_y}{\partial y} \end{bmatrix} = \begin{bmatrix} a - by & -bx \\ cy & cx - d \end{bmatrix}$$

Estabilidad en torno al primer punto fijo

$$J(0, 0) = \begin{bmatrix} a & 0 \\ 0 & -d \end{bmatrix} \quad (x_0 = 0, y_0 = 0)$$

Hallar las raíces del polinomio característico

$$\det \begin{bmatrix} a - \lambda & 0 \\ 0 & -d - \lambda \end{bmatrix} = 0$$

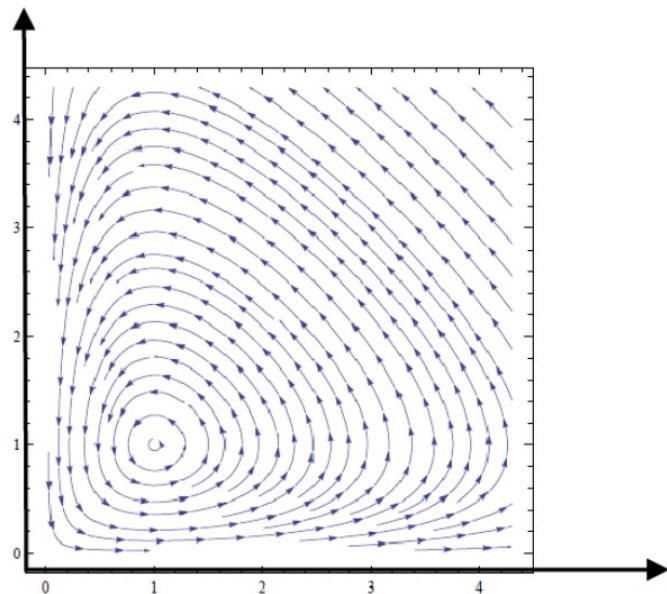
$$\lambda_1 = a > 0$$

$$\lambda_2 = -d < 0$$

Depredador-presa Lokta-Volterra

$$\frac{dx(t)}{dt} = ax(t) - bx(t)y(t)$$

$$\frac{dy(t)}{dt} = cx(t)y(t) - dy(t)$$



$$J = \begin{bmatrix} \frac{\partial f_x}{\partial x} & \frac{\partial f_x}{\partial y} \\ \frac{\partial f_y}{\partial x} & \frac{\partial f_y}{\partial y} \end{bmatrix} = \begin{bmatrix} a - by & -bx \\ cy & cx - d \end{bmatrix}$$

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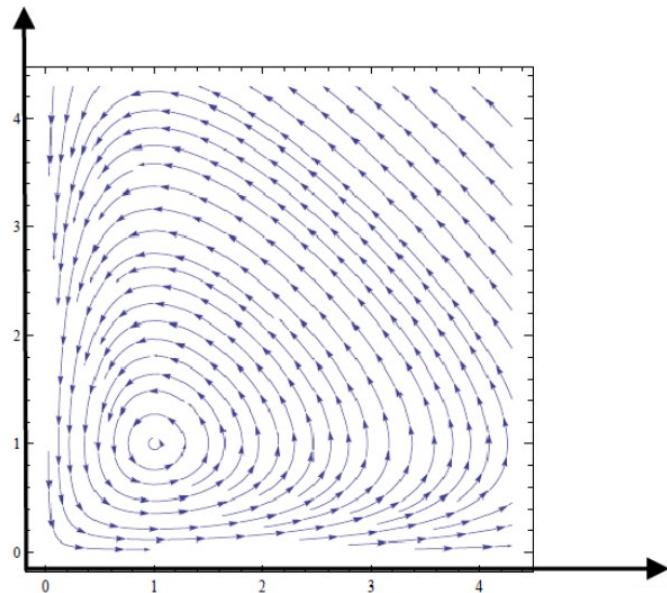
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Depredador-presa Lokta-Volterra

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$$\frac{dy(t)}{dt} = cx(t)y(t) - dy(t)$$



$$J = \begin{bmatrix} \frac{\partial f_x}{\partial x} & \frac{\partial f_x}{\partial y} \\ \frac{\partial f_y}{\partial x} & \frac{\partial f_y}{\partial y} \end{bmatrix} = \begin{bmatrix} a - by & -bx \\ cy & cx - d \end{bmatrix}$$

Estabilidad en torno al primer punto fijo

$$J(d/c, a/b) = \begin{bmatrix} 0 & -bd \\ \frac{ac}{b} & 0 \end{bmatrix} \quad (x_0 = d/c, y_0 = a/b)$$

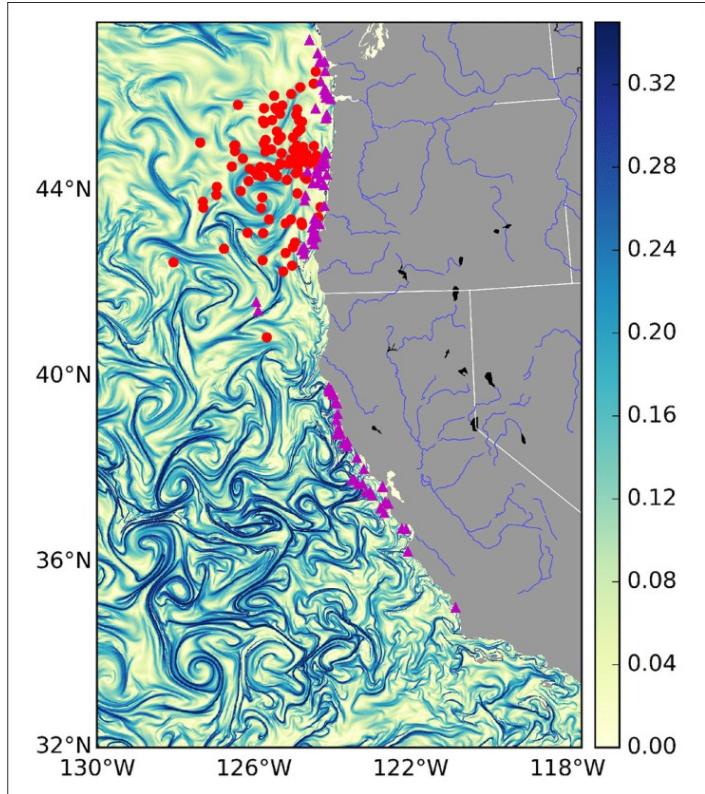
Hallar las raíces del polinomio característico

$$\det \begin{bmatrix} -\lambda & -\frac{bd}{c} \\ \frac{ac}{b} & -\lambda \end{bmatrix} = 0 \quad \longrightarrow \quad \lambda = \pm \sqrt{(-ad)}$$

$$\lambda_1 = +i\rho$$

$$\lambda_2 = -i\rho$$

Análisis estabilidad en un flujo natural



Preguntas... ?

Rebeca de la Fuente

Investigadora Postdoctoral (CADIC-CONICET)

