

# **muskellungeSmallLakeSpwnVegCurveB model summary**

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## Overview

This document summarizes the results of a global sensitivity and uncertainty analysis for the **muskellungeSmallLakeSpwnVegCurveB** habitat suitability index (HSI) model for *Esox masquinongy*. Metadata for the model is stored in the `ecorest` package in R.

The original documentation for this model can be found here<sup>1</sup>.

Sub-model: **Small lake spawning vegetation curve B**

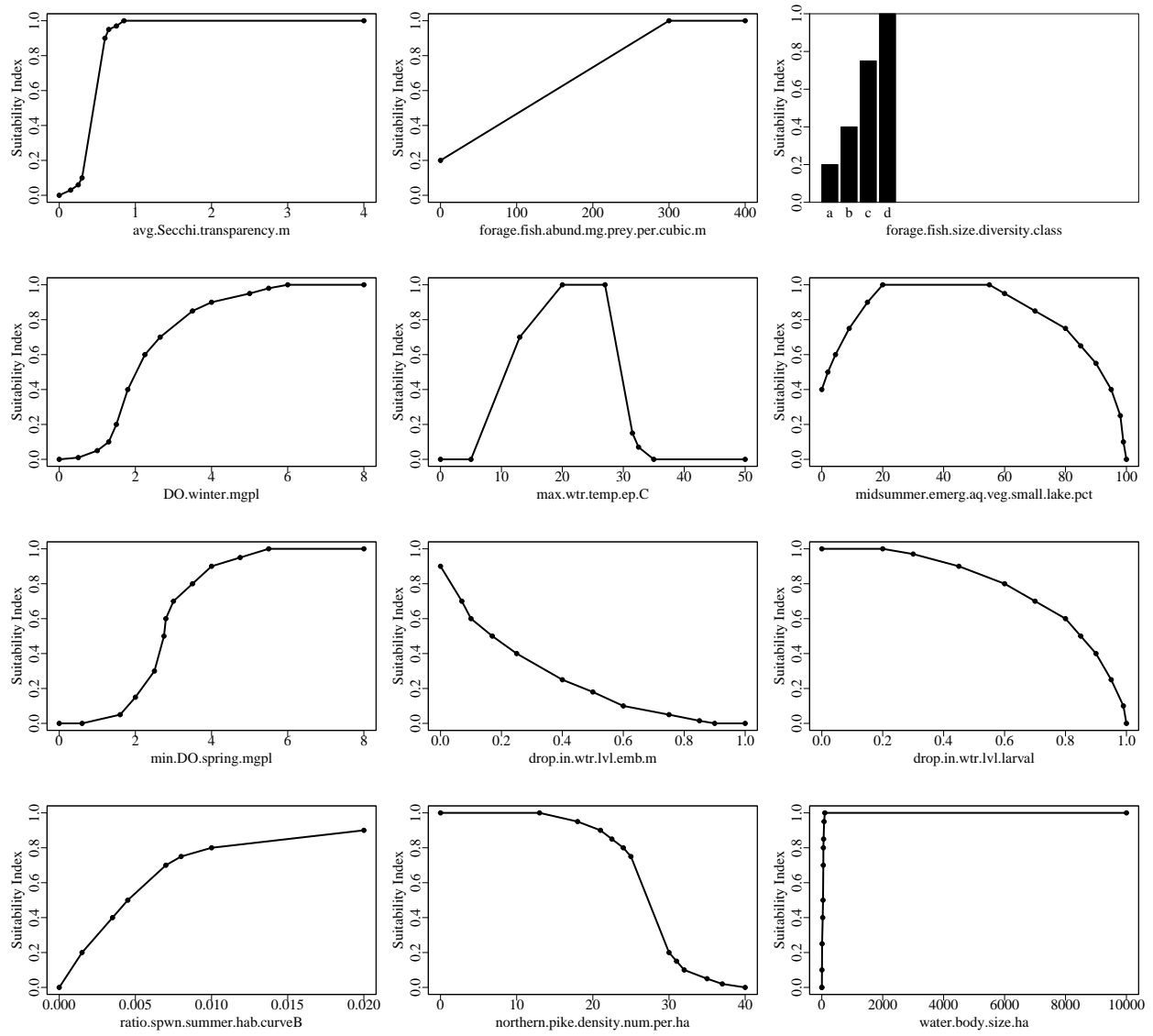
The muskellungeSmallLakeSpwnVegCurveB model is comprised of **12** variables and **4** components.

## Variables:

**Table 1.** SIV variables included in the muskellungeSmallLakeSpwnVegCurveB model. Type indicates whether a variable is numeric or categorical and breakpoints indicates the number of distinct breakpoints in suitability graphs.

	Variable name	Type	Breakpoints
SIV1	avg.Secchi.transparency.SIV	numeric	9
SIV2	forage.fish.abund.SIV	numeric	3
SIV3	forage.fish.size.diversity.SIV	categorical	4
SIV4	DO.winter.SIV	numeric	14
SIV5	max.wtr.temp.ep.SIV	numeric	9
SIV6	midsummer.emerg.aq.veg.small.lake.SIV	numeric	16
SIV7	min.DO.spring.SIV	numeric	13
SIV8	drop.in.wtr.lvl.emb.SIV	numeric	12
SIV8B	drop.in.wtr.lvl.larval.SIV	numeric	12
SIV9	ratio.spwn.summer.hab.curveB.SIV	numeric	8
SIV10	northern.pike.density.SIV	numeric	13
SIV11	water.body.size.SIV	numeric	12

<sup>1</sup><https://ecolibrary.sec.usace.army.mil/resource/72d73173-f056-48e3-fdfd-2d6e6b471366>



**Figure 1.** Suitability index graphs for variables included in the muskellungeSmallLakeSpwnVegCurveB model in ecorest.

## Components:

**Table 2.** Components included in the muskellungeSmallLakeSpwnVegCurveB model in ecoREST.

Component		Equation
CF	Food component	$\min(\text{SIV1}, (\text{SIV2} * \text{SIV3})^{1/2})$
CWQ	Water quality component	$\min(\text{SIV4}, \text{SIV5})$
CCR	Reproduction cover component	$\min(\text{SIV6}, \text{SIV7}, \text{SIV8}, \text{SIV8B}, \text{SIV9})$
COT	Other component	$(\text{SIV10} * \text{SIV11})^{1/2}$

## Model equation:

The equation to calculate an overall HSI index for the muskellungeSmallLakeSpwnVegCurveB model is:

$\min(\text{CF}, \text{CWQ}, \text{CCR}, \text{COT})$

According to our classification, this model's format is: **author-specified**

## Global sensitivity and uncertainty analysis:

We ran global sensitivity and uncertainty analyses on the muskellungeSmallLakeSpwnVegCurveB model using the sensobol package in R (Puy et al. 2022). The following parameters were used for the sensobol analysis:

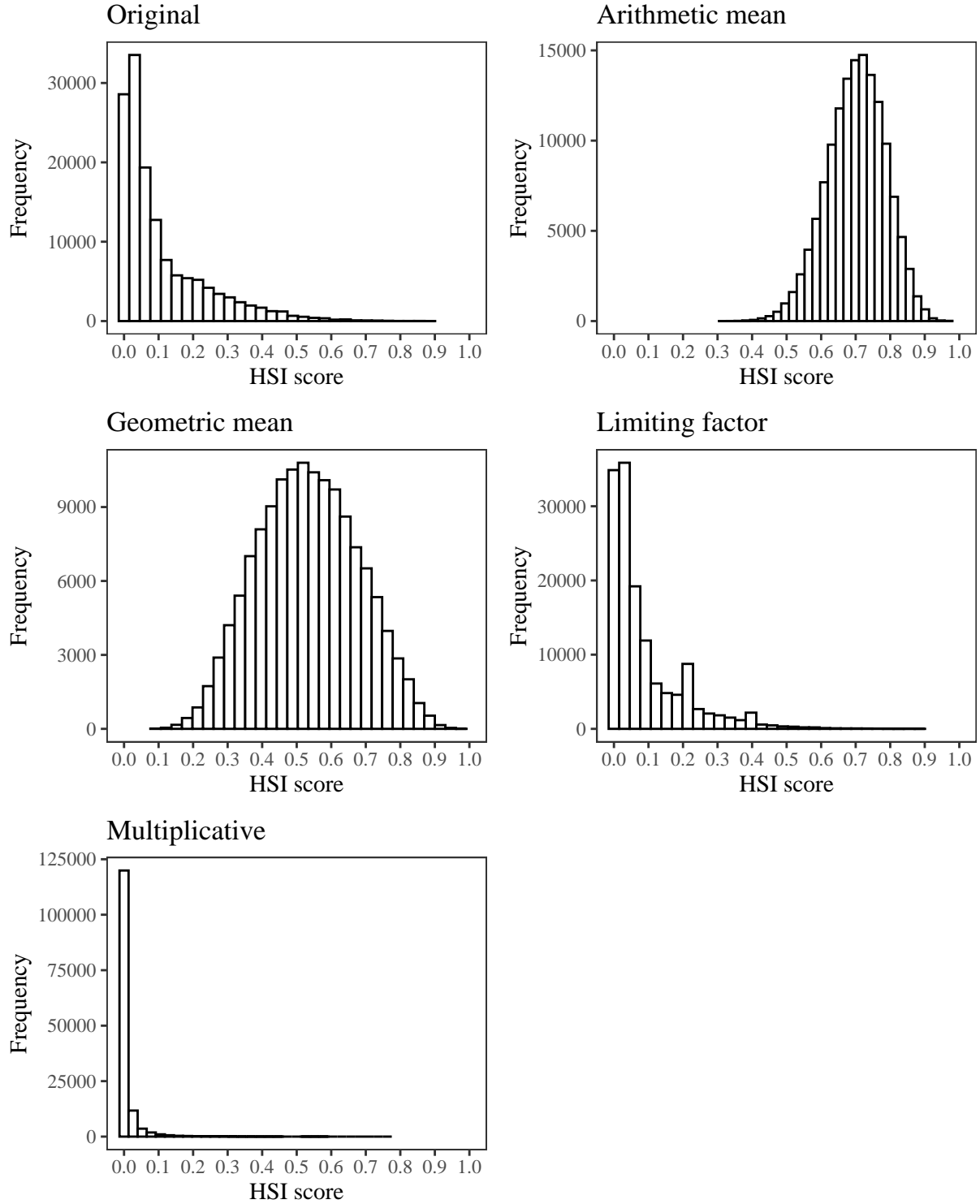
**Table 3.** Parameters and settings used for sensobol sensitivity and uncertainty analyses.

Parameter	Equation	Value
Number of input variables (M)	-	12
Base sample size (n)	-	10000
Number of model evaluations (N)	$n * (M + 2)$	140000
First order estimator	See Puy et al. (2022)	Saltelli
Total order estimator	See Puy et al. (2022)	Jansen
Number of bootstrap replications	-	1000
Sampling scheme	-	Quasi-random
Matrices	-	A, B, AB

We ran a sensitivity and uncertainty analysis for the muskellungeSmallLakeSpwnVegCurveB model using the original equation outlined in the documentation from Cook and Solomon (1987) and using arithmetic mean, geometric mean, limiting factor, and multiplicative equations to contrast the results across different equation structures.

## Model uncertainty

We ran the muskellungeSmallLakeSpwnVegCurveB model using 140000 combinations of its SIV variables, which were sampled from a uniform distribution spanning the range of possible values listed in the muskellungeSmallLakeSpwnVegCurveB documentation. We limited the range of possible values for each parameter to the range in which the SIV values were greater than zero to prevent HSI score distributions with primarily zero values.



**Figure 2.** Empirical distributions of HSI scores for the muskellungeSmallLakeSpwnVegCurveB model using the original author-specified model equation from Cook and Solomon (1987), and an arithmetic mean, geometric mean, limiting factor, and multiplicative structure incorporating all SIV variables. Note differences in the y axis.

We assumed a uniform distribution for all parameters because we evaluated all ecorest models in batch. Should you decide to run your own sensitivity analysis, this assumption should be evaluated independently for each parameter in the model.

**Table 4.** Quantiles from the empirical distribution of HSI scores for the original muskellungeSmallLakeSpwnVegCurveB model structure, an arithmetic mean equation, a geometric mean equation, a limiting factor equation, and a multiplicative equation structure.

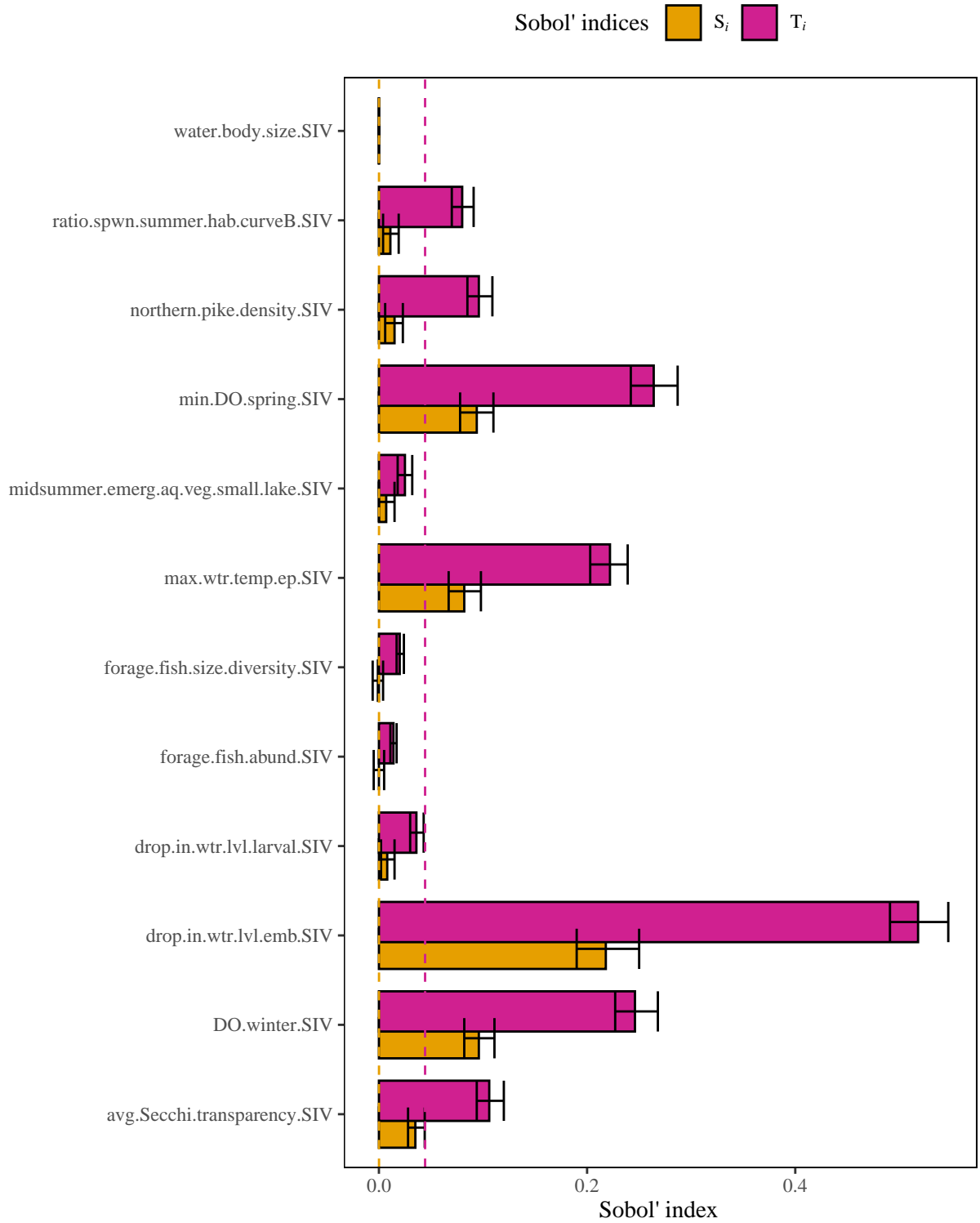
	1%	2.5%	5%	25%	50%	75%	95%	97.5%	99%	100%
Original	0.00	0.00	0.00	0.02	0.06	0.15	0.38	0.45	0.54	0.89
Arithmetic	0.49	0.53	0.56	0.65	0.71	0.76	0.83	0.86	0.88	0.98
Geometric	0.23	0.26	0.30	0.43	0.53	0.64	0.78	0.81	0.85	0.98
Limiting	0.00	0.00	0.00	0.02	0.04	0.12	0.33	0.40	0.47	0.89
Multiplicative	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.08	0.14	0.76

The empirical distribution of the original muskellungeSmallLakeSpwnVegCurveB model has a coefficient of variation (CV) of **1.159**, while the arithmetic mean model has a CV of **0.12**, the geometric mean model has a CV of **0.273**, the limiting factor model has a CV of **1.218**, and the multiplicative model has a CV of **3.119**. Hence, the **Multiplicative** model is the most uncertain, while the **Arithmetic mean** model is the least uncertain.

### Model sensitivity

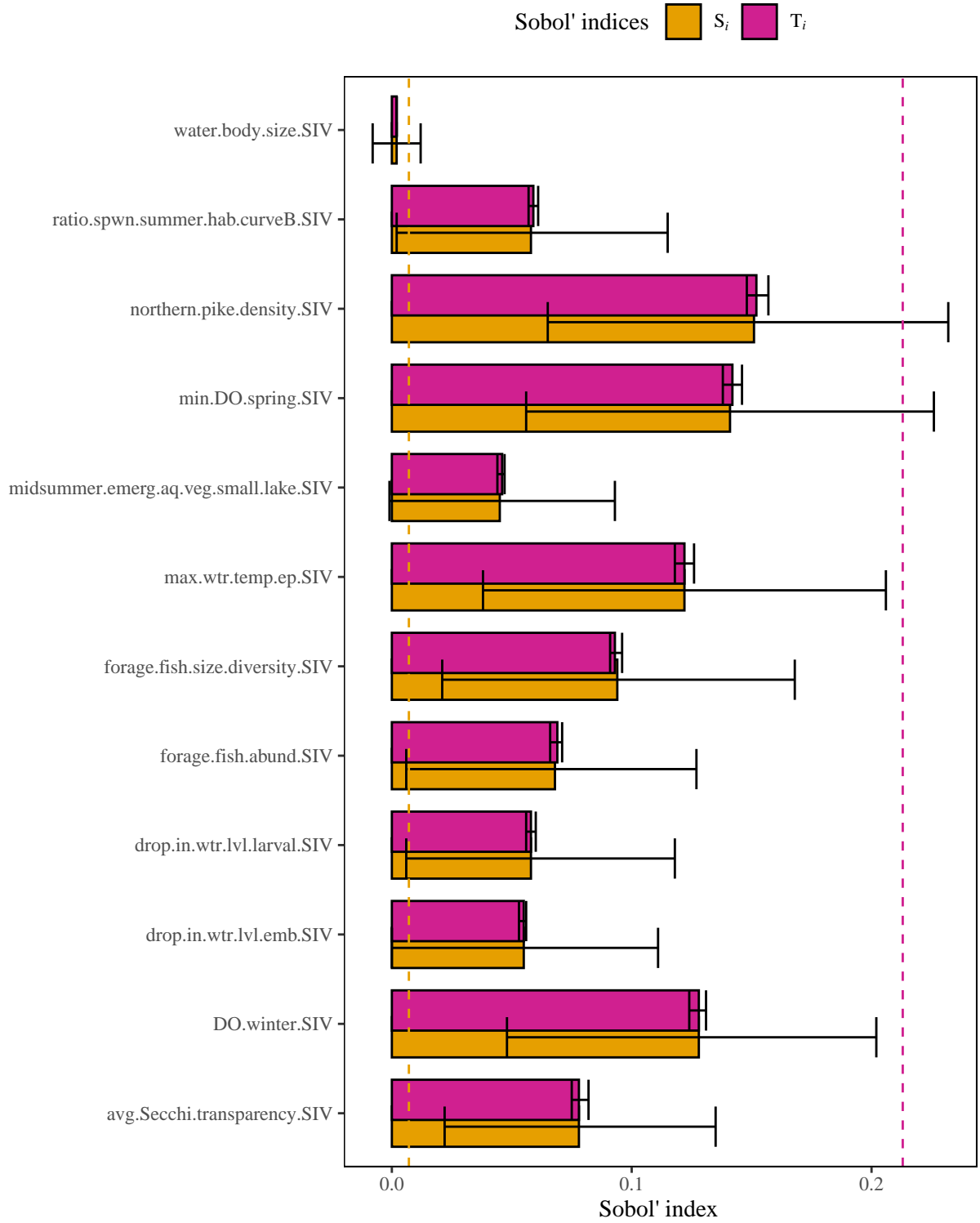
Below are the results of the global sensitivity analysis for the muskellungeSmallLakeSpwnVegCurveB model using the original equation, an arithmetic mean, a geometric mean, a limiting factor, and a multiplicative model structure. The sensobol package uses variance-based sensitivity metrics, so the model’s sensitivity to a given parameter is a measure of how much variance in the HSI score decreases in response to that parameter being fixed (Puy et al. 2022). For each parameter, the observed changes in the variance of the HSI score can be described with a first order sensitivity index ( $S_i$ ) that accounts for the influence of a single parameter of interest on variance in HSI, or with a total order index ( $T_i$ ) that accounts for the influence of a single parameter on its own and in combination with all other parameters (*i.e.*, interactions) (Puy et al. 2022). We can compare the 95% confidence intervals for the first and total order indices to a dummy parameter, which represents a parameter that has no influence on the variance in a model’s output. While an uninfluential variable should theoretically have an  $S_i$  and  $T_i$  of zero, small approximation errors can lead variables to have a non-zero influence on a model’s output (Puy et al. 2022). If the confidence interval of the  $S_i$  and  $T_i$  index for a given parameter overlaps the confidence interval of the dummy parameter, we can deduce that the parameter has a negligible effect on variance in HSI scores, both on its own and in combination with all other variables.

## Original model



**Figure 3.** Results of a sensitivity analysis for the muskellungeSmallLakeSpwnVegCurveB model based on the original author-specified model outlined in Cook and Solomon (1987). Dashed lines represent baseline numerical approximation error for  $S_i$  and  $T_i$  (*i.e.*, dummy variables).

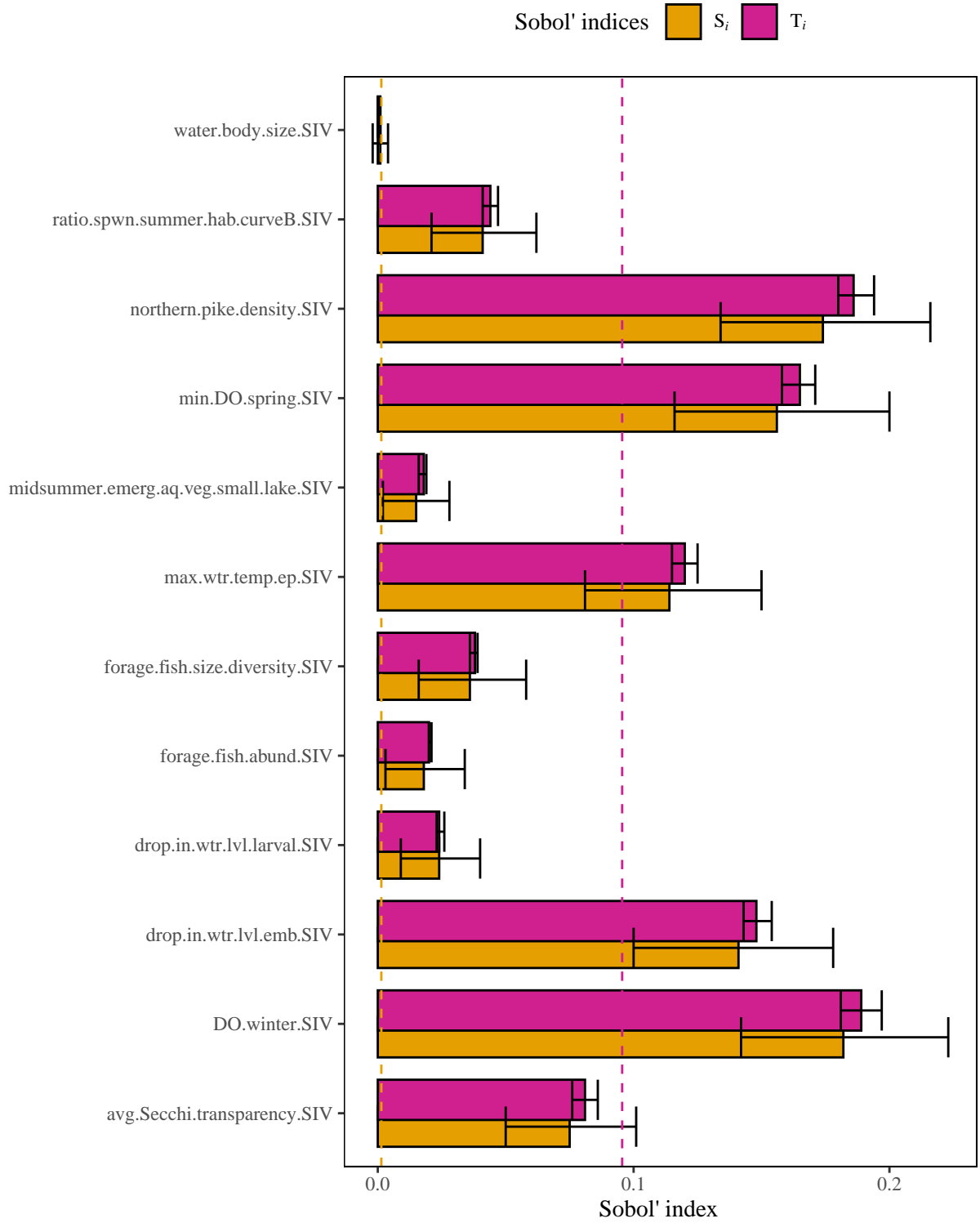
# Arithmetic mean model



**Figure 4.** Results of a sensitivity analysis for the muskellungeSmallLakeSpwnVegCurveB model based on an arithmetic mean structure. Dashed lines represent baseline numerical approximation error for  $S_i$  and  $T_i$  (*i.e.*, dummy variables).

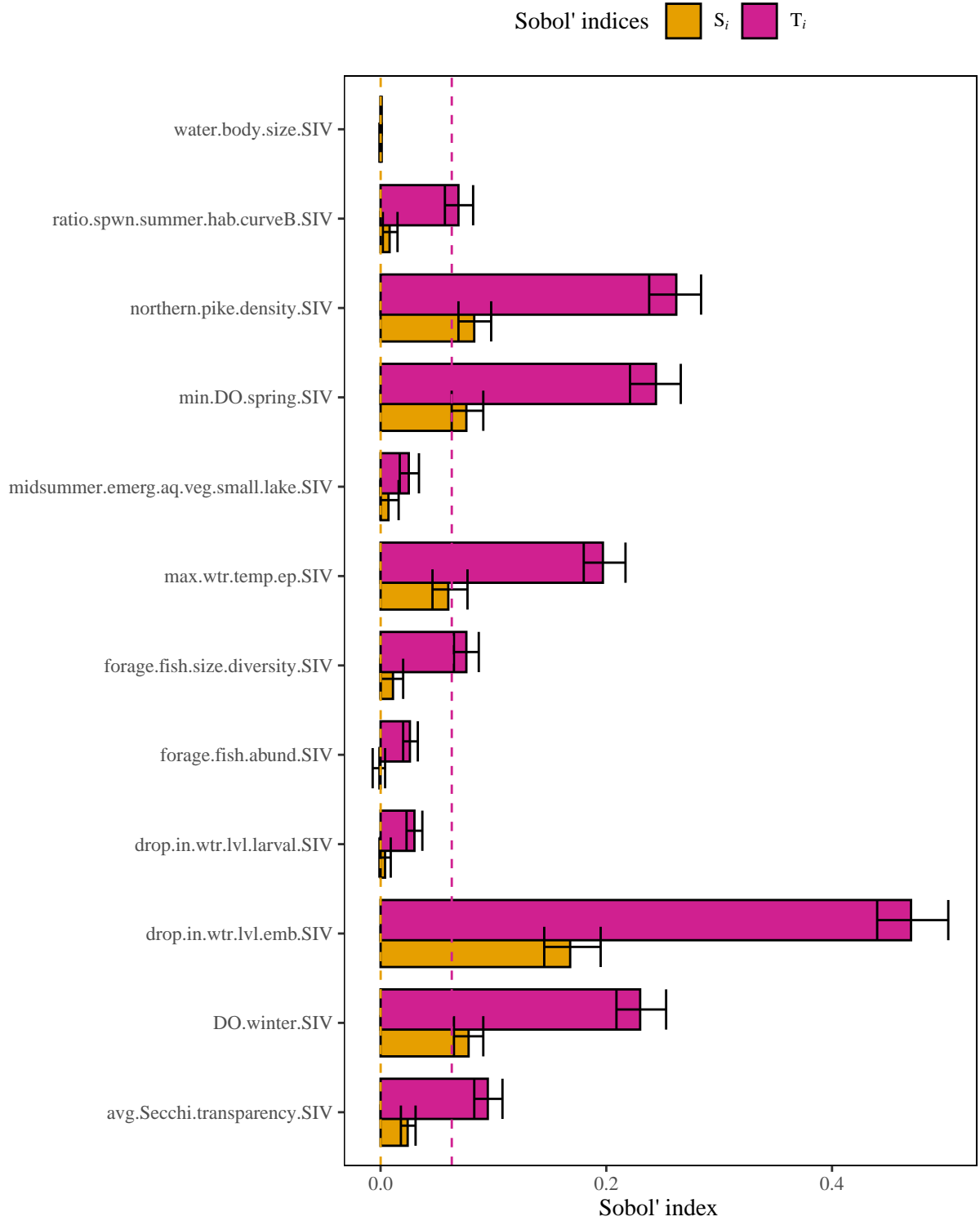


## Geometric mean model



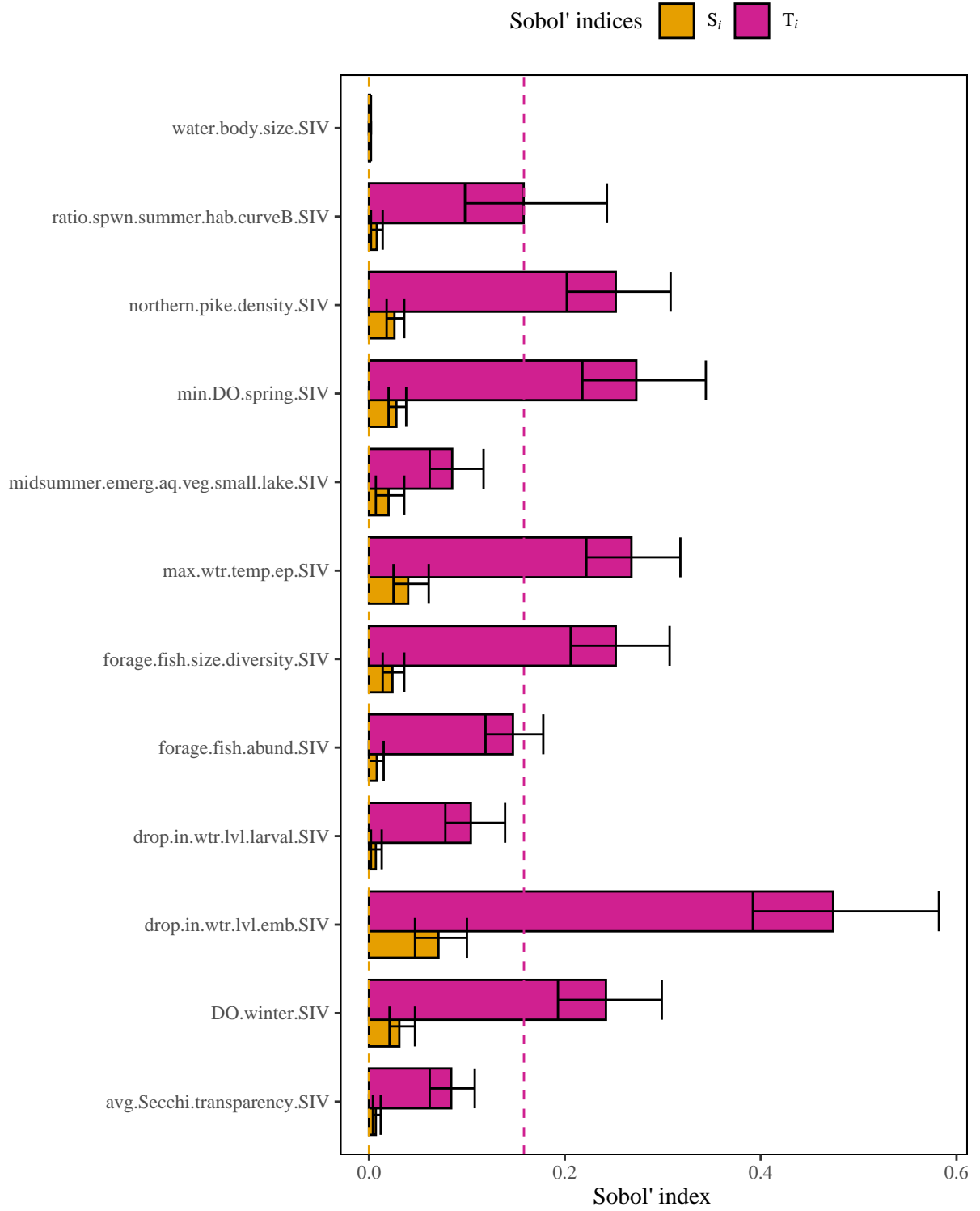
**Figure 5.** Results of a sensitivity analysis for the muskellungeSmallLakeSpwnVegCurveB model based on a geometric mean structure. Dashed lines represent baseline numerical approximation error for  $S_i$  and  $T_i$  (*i.e.*, dummy variables).

## Limiting factor model



**Figure 6.** Results of a sensitivity analysis for the muskellungeSmallLakeSpwnVegCurveB model based on a limiting factor structure. Dashed lines represent baseline numerical approximation error for  $S_i$  and  $T_i$  (*i.e.*, dummy variables).

## Multiplicative model



**Figure 7.** Results of a sensitivity analysis for the muskellungeSmallLakeSpwnVegCurveB model based on a multiplicative mean structure. Dashed lines represent baseline numerical approximation error for  $S_i$  and  $T_i$  (*i.e.*, dummy variables).

## Summary of influential variables

**Original model** In the **original muskellungeSmallLakeSpwnVegCurveB** model, **8 of 12** variables are influential and **drop.in.wtr.lvl.emb.SIV** has the highest first order sensitivity. In addition, **drop.in.wtr.lvl.emb.SIV** has the highest total order sensitivity.

### Un-influential variables in original model:

forage.fish.abund.SIV  
forage.fish.size.diversity.SIV  
midsummer.emerg.aq.veg.small.lake.SIV  
water.body.size.SIV

**Arithmetic mean model** In the **arithmetic mean muskellungeSmallLakeSpwnVegCurveB** model, **6 of 12** variables are influential and **northern.pike.density.SIV** has the highest first order sensitivity. In addition, **northern.pike.density.SIV** has the highest total order sensitivity.

### Un-influential variables in arithmetic mean model:

forage.fish.abund.SIV  
midsummer.emerg.aq.veg.small.lake.SIV  
drop.in.wtr.lvl.emb.SIV  
drop.in.wtr.lvl.larval.SIV  
ratio.spwn.summer.hab.curveB.SIV  
water.body.size.SIV

**Geometric mean model** In the **geometric mean muskellungeSmallLakeSpwnVegCurveB** model, **11 of 12** variables are influential and **DO.winter.SIV** has the highest first order sensitivity. In addition, **DO.winter.SIV** has the highest total order sensitivity.

### Un-influential variables in geometric mean model:

water.body.size.SIV

**Limiting factor model** In the **limiting factor muskellungeSmallLakeSpwnVegCurveB** model, **8 of 12** variables are influential and **drop.in.wtr.lvl.emb.SIV** has the highest first order sensitivity. In addition, **drop.in.wtr.lvl.emb.SIV** has the highest total order sensitivity.

### Un-influential variables in limiting factor mean model:

forage.fish.abund.SIV  
midsummer.emerg.aq.veg.small.lake.SIV  
drop.in.wtr.lvl.larval.SIV  
water.body.size.SIV

**Multiplicative model** In the **multiplicative mean muskellungeSmallLakeSpwnVegCurveB** model, **10 of 12** variables are influential and **drop.in.wtr.lvl.emb.SIV** has the highest first order sensitivity. In addition, **drop.in.wtr.lvl.emb.SIV** has the highest total order sensitivity.

### Un-influential variables in multiplicative model:

forage.fish.abund.SIV  
water.body.size.SIV

## References

1. Cook, MF, and RC Solomon. 1987. Habitat suitability index models: Muskellunge. U.S. Fish Wildl. Serv. Biol. Rep. 82(10.148). 33 pp.
2. McKay S, D Hernandez-Abrams, and K Cushway. 2024. ecoREST: conducts analyses informing ecosystem restoration decisions. R package version 2.0.0, <https://CRAN.R-project.org/package=ecoREST>.
3. Puy, A, S Lo Piano, A Saltelli, and SA Levin. 2022. sensobol: an R package to compute variance based sensitivity indices. *Journal of Statistical Software* 102(5):1-37. doi: 10.18637/jss.v102.i05