Model Comparison

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Douter de tout ou tout croire sont deux solutions également commodes, qui nous dispensent de réfléchir.

–Henri Poincaré

Introduction to Model Comparison

Why compare models?

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Why compare models?

- All models are imperfect
- How good is our model *given the modelling goals?*

Comparing models

Before beginning, evaluate the goals of the comparison

- Predictive performance
- Hypothesis testing
- Reduction of overfitting

If you are asking yourself, "should I use A/B/DIC?"

Remember Betteridge's law...

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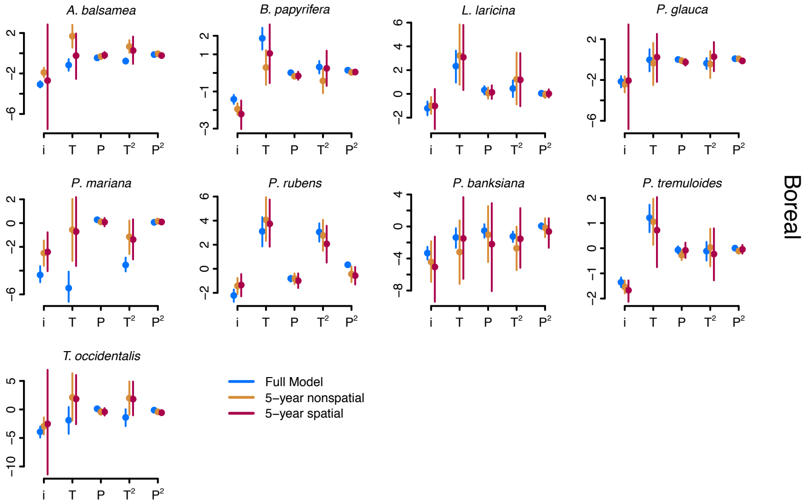
> *Any headline that ends in a question mark can be answered with the word "NO"*

Informal model comparison



Comparison through evaluation

If the goal is predictive performance, evaluate directly.

- Cross-validation
- k-fold cross validation

Cost: can be computationally intensive (especially for Bayesian). But you are already paying this cost (you ARE evaluating your models, right?)

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Requires selecting an evaluation score

- ROC/TSS (classification)
- RMSE (continuous)
- Goodness of fit
- ...

Bayesian predictive performance

Consider a regression model

$$
\begin{aligned}
pr(\theta|y, x) &\propto pr(y, x, |\theta)pr(\theta) \\
y &\sim \mathcal{N}(\alpha + \beta x, \sigma)
\end{aligned}
$$

From a new value $\hat{x}$ we can compute a posterior prediction $\hat{y} = \alpha + \beta x$

Bayesian predictive performance

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Where is the prior?

We want to summarize lppd taking into account:

- an entire set of prediction points $\hat{x} = \{x_1, x_2, \ldots x_n\}$
- the entire posterior distribution of θ
    - (or, realistically, a set of S draws from the posterior distribution)

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To compare two competing models θ_1 and θ_2, simply compute lppd_{θ_1} and lppd_{θ_2}, the “better” model (for prediction) is the one with a larger lppd.

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Thus, we require a method for penalizing the larger (or more generally, more flexible) model to avoid simply overfitting, especially when validation data are unavailable.

$$\mathrm{AIC} = 2k - 2\log \mathrm{pr}(x|\hat{\theta})$$

- $\mathrm{pr}(x|\hat{\theta}) = \max(\mathrm{pr}(x|\theta))$ and k is the number of parameters.
- AIC increases as the model gets worse or the number of parameters gets larger
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AIC

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What is the number of parameters in a hierarchical model?

DIC

$$D(\theta) = -2\log(\mathrm{pr}(x|\theta))$$

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We still penalize the model based on complexity, but we must estimate how many *effective* parameters there are:

$$p_D = \mathbb{E}[D(\theta)] - D(\mathbb{E}[\theta])$$

DIC

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$$p_D = \mathbb{E}[D(\theta)] - D(\mathbb{E}[\theta])$$

$$\mathrm{DIC} = D(\mathbb{E}[\theta]) + 2p_D$$

DIC

Pros:

- Easy to estimate
- Widely used and understood
- Effective for a variety of models regardless of nestedness or model size

Cons

- Not Bayesian
- Assume $\theta \sim \mathcal{MN}$
- Modest computational cost

Consider two competing models θ_1 and θ_2

In classical likelihood statistics, we can compute the likelihood ratio:

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A fully Bayesian approach is to take into account the entire posterior distribution of both models:

$$K = \frac{\mathrm{pr}(\theta_1|X)}{\mathrm{pr}(\theta_2|X)}$$

Bayes factor

For a single posterior estimate of each model:

$$
\begin{aligned}
K &= \frac{\mathrm{pr}(\theta_1|X)}{\mathrm{pr}(\theta_2|X)} \\
&= \frac{\mathrm{pr}(X|\theta_1)\mathrm{pr}(\theta_1)}{\mathrm{pr}(X|\theta_2)\mathrm{pr}(\theta_2)}
\end{aligned}
$$

To account for the entire distribution:

$$
\begin{aligned}
K &= \frac{\int pr(\theta_1|X)d\theta_1}{\int pr(\theta_2|X)d\theta_2} \\
&= \frac{\int pr(X|\theta_1)pr(\theta_1)d\theta_1}{\int pr(X|\theta_2)pr(\theta_2)d\theta_2}
\end{aligned}
$$

And others

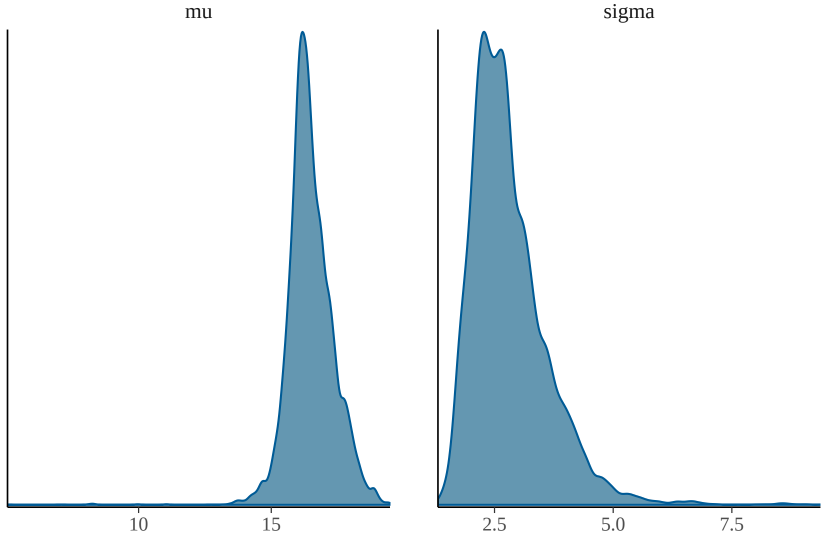
- Bayesian model averaging
- Reversible jump MCMC

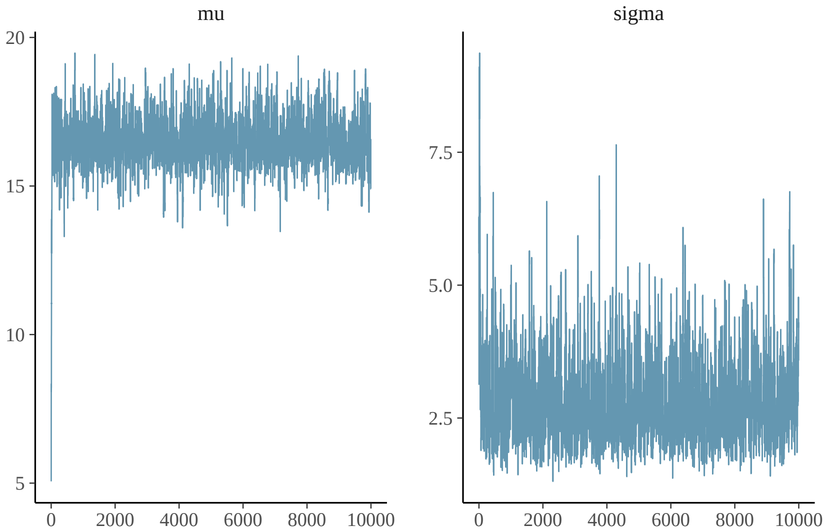
```r
library(mcmc)
suppressMessages(library(bayesplot))


logposterior <- function(params, dat)
{
  if(params[2] <= 0)
    return(-Inf)

  mu <- params[1]
  sig <- params[2]

  lp <- sum(dnorm(dat, mu, sig, log=TRUE)) +
      dnorm(mu, 16, 0.4) + dgamma(sig, 1, 0.1)
  return(lp)
}
```

Software

```
X <- c(15, 19.59, 15.06, 15.71, 14.65, 21.4, 17.64, 18.31,
    15.12, 14.40)
inits <- c(5, 2)
tuning <- c(8, 2.5)

model <- metrop(logposterior, initial = inits,
    nbatch = 10000, dat = X)
model$accept
```

```
## [1] 0.4035
```


mu
sigma
10
15
2.5
5.0
7.5


mu
20
15
10
5
0
2000
4000
6000
8000
10000
sigma
7.5
5.0
2.5
0
2000
4000
6000
8000
10000

Other software

- mcmc
- LaplacesDemon
- JAGS
- Stan