Few notes on probability theory

flip the coin and compute the likelihood (am: 1h20 + 1h20 / pm 2h + 15min)

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Probability theory from scratch

Few notes about History (15 min - 7 slides)

- Few ideas on the underlying philosophy
- Le Chevalier De Méré, Pascal and Fermat: *the problem of points*
- Laplace *Théorie Analytique des Probabilités in 1812*
- Reverend Bayes *An Essay Towards Solving a Problem in the Doctrine of Chances*
- Kolmogorov / Shannon
- Importance of probabilities in Ecology
  - A few important ecology papers that use tricky proba results
  - First I have in mind in Allesina and Tang Science 2012

- Few basic concepts:
  1. Probability space $(\Omega, \mathcal{F}, \mathrm{P})$
  2. Trial, Outcome, sample, Events

- Determining the probability of events:
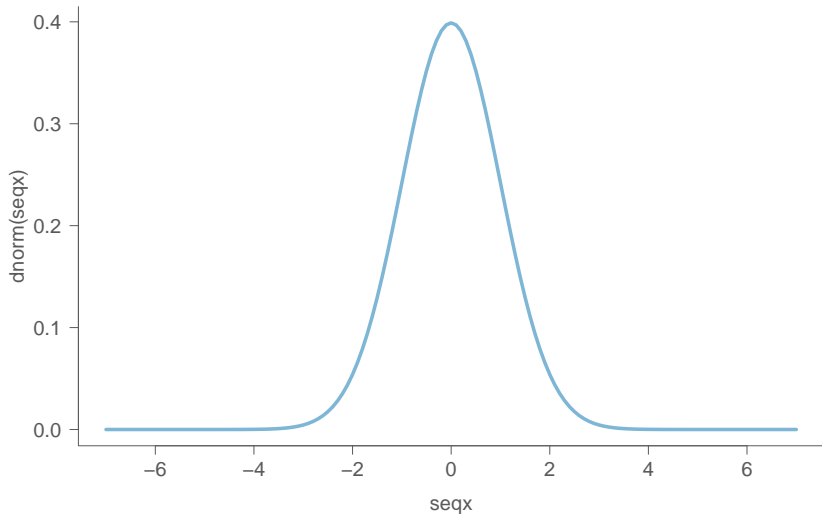 4. Cardinal, Combinatorics
  5. $\cap \cup$

Basic Combinatorics

- flip the coin and roll the dice
- I'll try to make it fun, *e.g.* how to simulate a dice using a coin?

1. random variables X, P(X)
2. discrete vs continuous
3. distributions pdf/cdf (ex: pdf next slides)

Example of a probability density function (pdf)

Random variables

- same kind of exercises as did previously but using the new formalism

Expectation / Variance / Moments / Quantiles (10 min - 4 slides)

- Definitions
- Basic examples
- König-Huygens theorem (it may be useful, I'll see if I keep it)

Be independent or not to be independent, that is the question.

- Definition
- This needs to be understood I'll therefore exemplify!

- What the reverend said: the original question he asked!
- Theorem (the one we learn)
- Implications

Let's recap

- 1 to 3 examples that would use *almost* everything mentioned so far! - I may use a classical example of conditional probability.

In part 3, I will:

1. Take some time to detail solutions of practice 3
2. explain the link between probability theory and statistics
3. based on this link, I'll propose exercises to calculate what students know very well: p-values and/or a CI!
4. The idea is to lead student towards the idea of likelihood that will be detailed by Dom.

Practice 3 - answers (15 min - 3 slides)

- I'll detail a couple of general ideas about probability and statistics link. I can't say too much, I want them to "discover" the concepts. To do so, I'll build 2 exercices.

1. I'll use a simple example: say we flip 100 times a given coin and record Heads and Tails, what can we learn form this data about the coin? Is is biased? How to proceed.
   - what is the probability of obtaining such results if the p (obtaining T) is 0? 1? 0.5? is 0.1?
   - compute $P(p \in [.4, .6])$
   - find a way to get a and b such as $P(p \in [a, b]) = .95$
   - Let's introduce a second coin and a new set of data, are the two coins similar?
2. Same kind of exercise with a normal distribution.
3. Find a way to find the best parameter values?

Let's take a step back (4 slides - 10 min)

- I'll take some time to explain what they actually did.

Examples of usage of conditional probabilities. TIB and co-occurrence

AlertBlock

Pass auf!