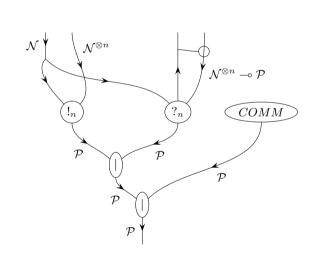
Casper Formalized Pt I

Applying π -calculus

to modeling the Casper protocol





Vlad Zamfir, L.G. Meredith

Aims and goals

Formal specification of Casper

Specification mapped to reference implementation

Specification mapped to simulation



Methodology

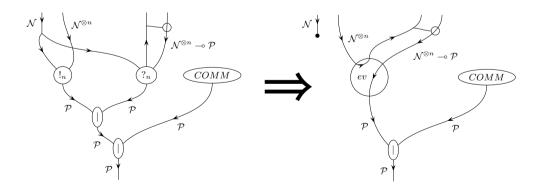
Bootstrap Casper spec using interaction diagrams

Map interaction diagrams to initial π -calculus specification

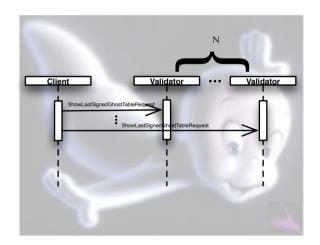
Refine specification

Map refined specification to SpecialK

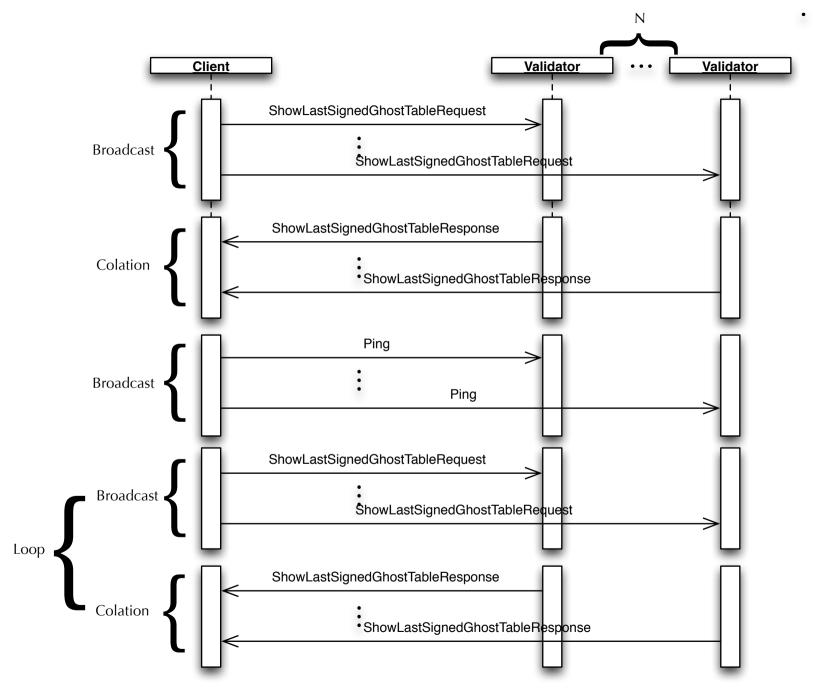
Map refined specification to Stochastic π -machine



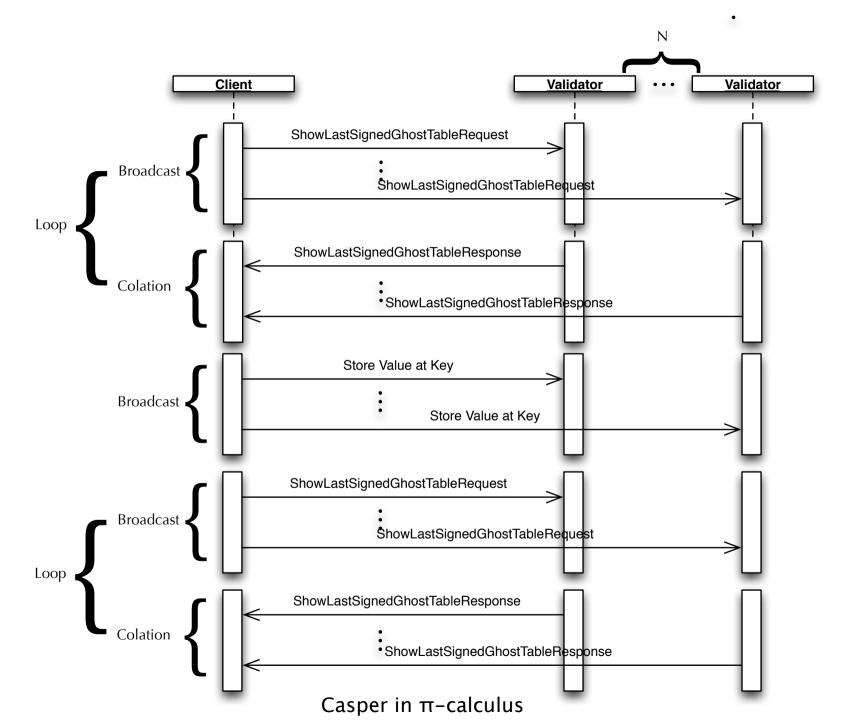
Casper in interaction diagrams



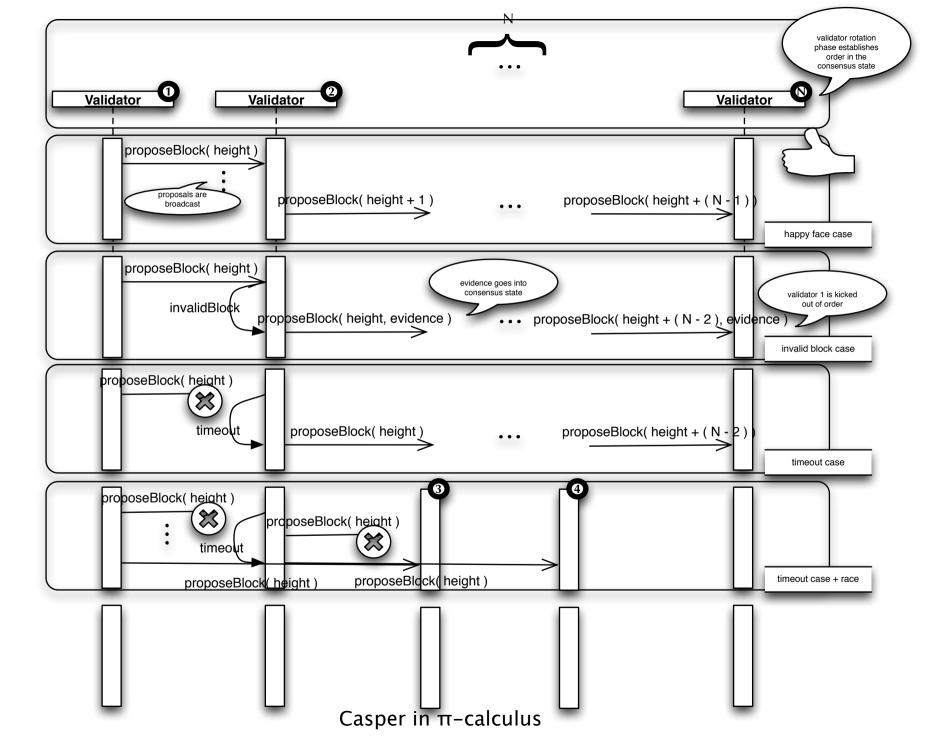


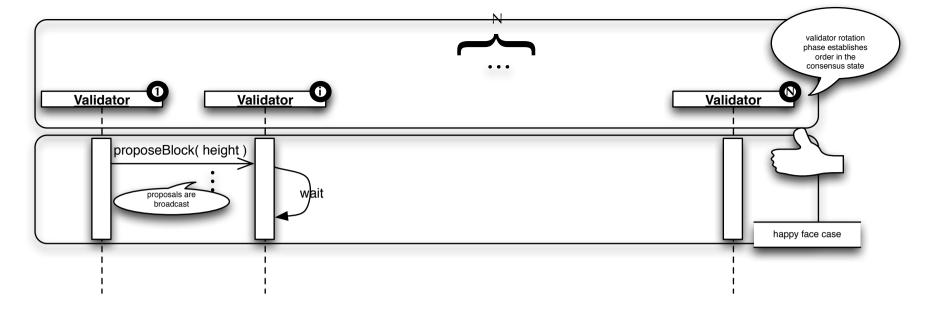


Casper in π -calculus



Consensus: Validator <-> Validator interactions



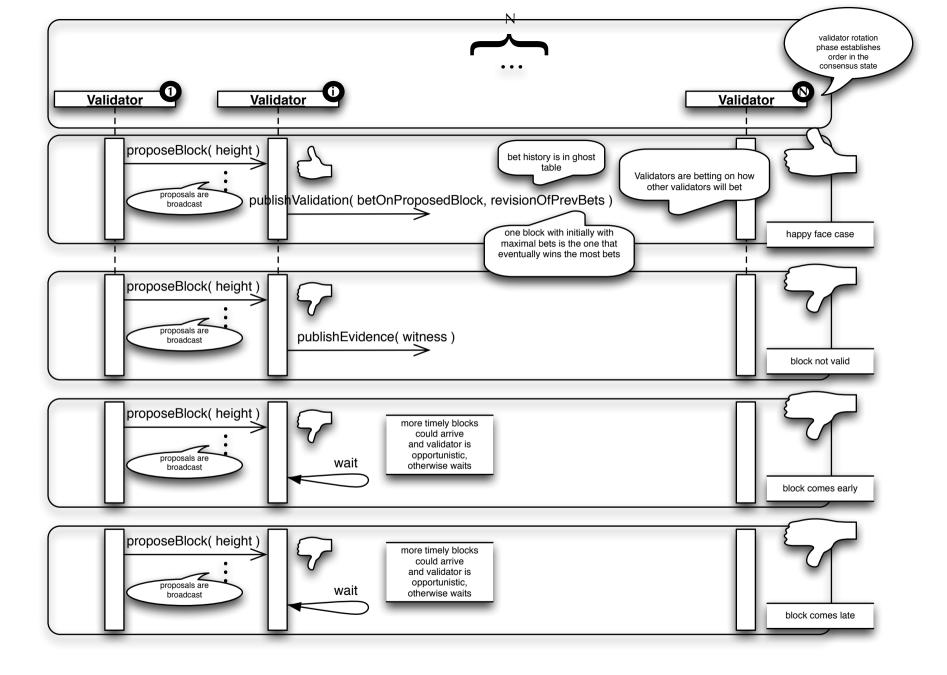


For validator i, v(i), if the proposal is from v(current), then v(i) can publish validation straightaway, otherwise wait for the proposer ==v(current), then publish validation.

Blocks contain validations of other blocks.

Publishing a validation is placing a bet that the validated block is the block that will be finalized at that height.

Validations also contain new bets on non-finalized previous blocks.



Casper in π -calculus

To do list

State transition function

Bonding/unbonding

Rejection of validators based on evaluation of evidence

Distribution of transaction fees and bonds

Execution, re-ordering, and finalizing of txns

Assume fn, current: time -> Z[N] where N is the number of validators. At end / beginning of bonding period current is recalculated.

current = current_1 || current_2 || ... || current_M

to get parallelism speed up opportunities

To do list

```
Block consists of
* ghost entries ( prev hash, next hash )
* reorg transitions
* new txns

valid( block ) ==
   stateX( block.ghostEntries._1 ) == block.ghostEntries._2
&& stateX( block.reorgX._1 ) == block.reorgX._2
```

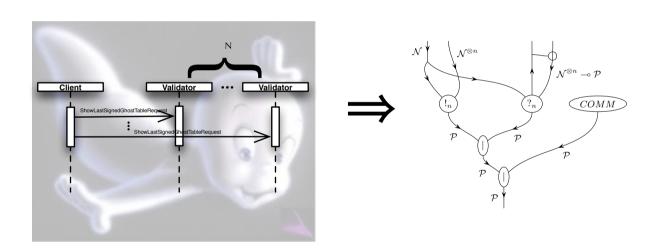
To do list

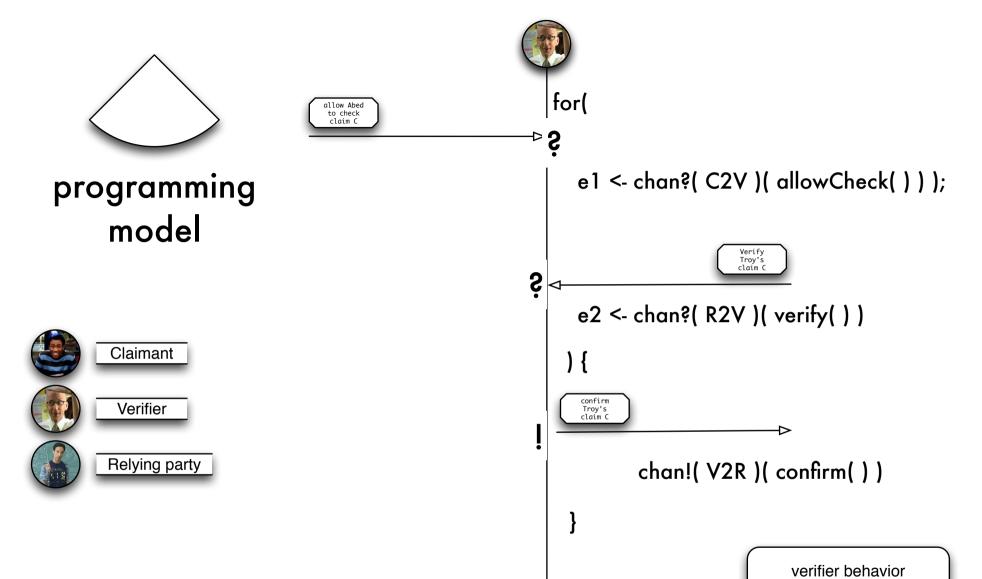
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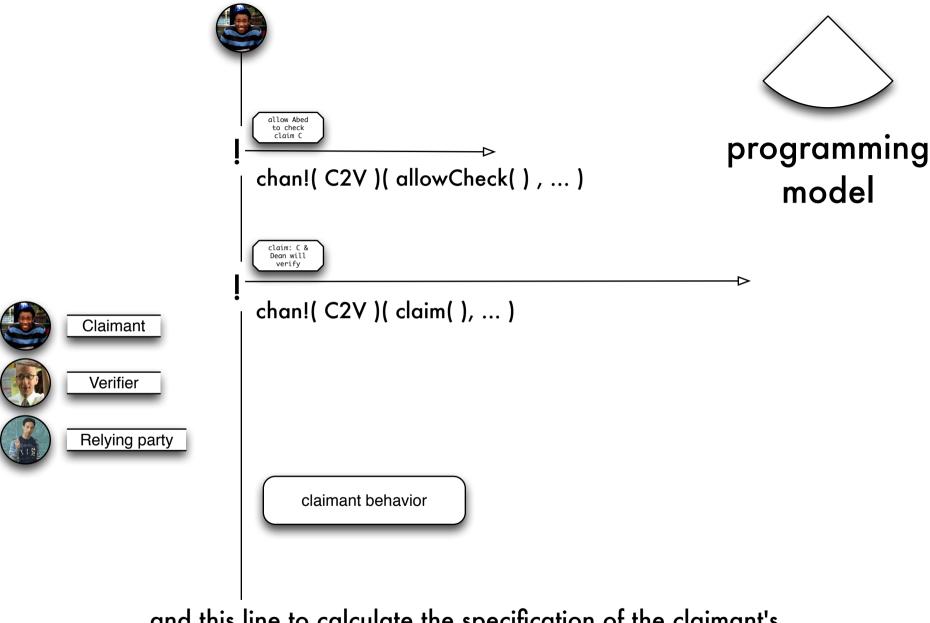
to get parallelism speed up opportunities

How to turn interaction diagrams into π -calculus specs

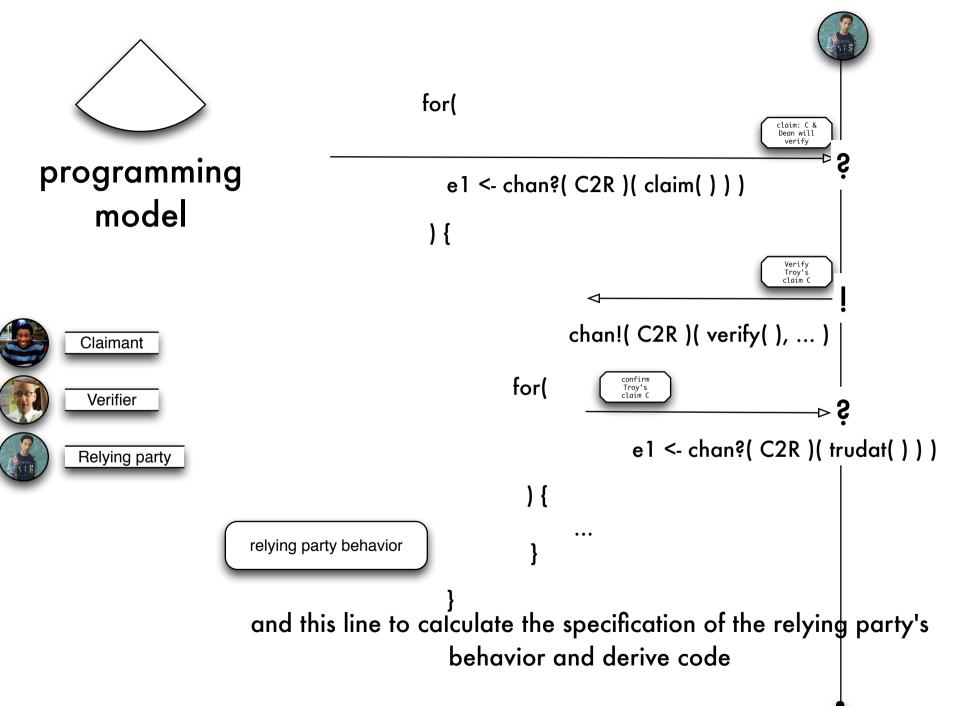




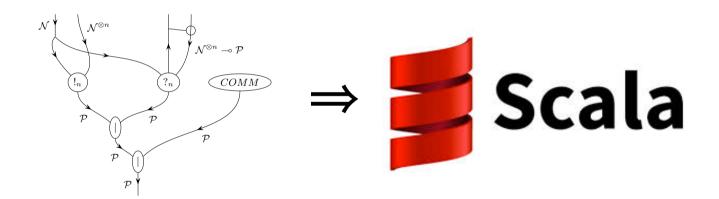
Just walk this line to calculate the specification of the verifier's behavior and derive code



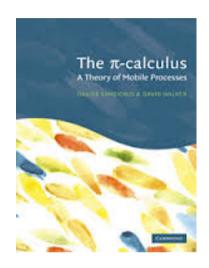
and this line to calculate the specification of the claimant's behavior and derive code



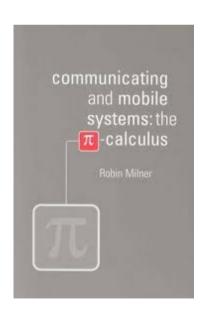
How to turn π -calculus specs into scala code



```
{ }
P,Q ::= 0
                                        [| a |](m) ![ [| v1 |](m), ..., [| vn |](m) ]
     a![ v1, ..., vn ]
                                        for([x1, ..., xn] \leftarrow [|a|](m)){
     a?( x1, ..., xn )P
                                             [|P|](m)(x1, ..., xn)
     P | Q
                                         spawn{ [| P |](m) }; spawn{ [| Q |](m) }
                                         { val q = new Queue(); [| P |](m[a <- q])}
     (new a)P
     ( def X( x1, ..., xn ) = P )[v1, ..., vn]
                                         object X {
                                            def apply(x1, ..., xn) = {
                                                [|P|](m) (x1, ..., xn)
     X[v1, ..., vn]
                                        X([| v1 |](m), ..., [| vn |](m))
            [|-|](-): (\pi-calculus, Map[Symbol, Queue]) -> Scala
```

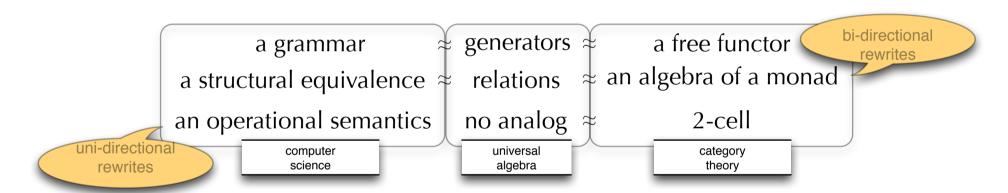


applied π -calculus



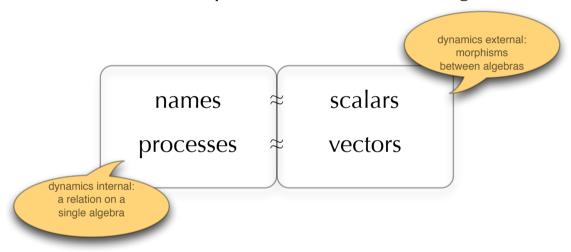
How to specify a computational calculus

Modern presentations of computational calculi generalize generators and relations style presentations of universal algebra They are typically given in terms of



How to specify a computational calculus

Process calculi, like vector spaces, are two sorted algebraic structures



A morphism from one process calculus to another *preserves* computational dynamics

A morphism from one vector space to another *is* computational dynamics

Syntax

```
P,Q ::= 0

a![t]Q

a?(t)P

P | Q

(new a)P

( def X(t) = P)[u]

X[t]
```

```
t,u ::= val | var | fn(t1, ..., tN)
val ::= bool | int | string | double | ...
var ::= _ | X | Y | Z | ...
fn ::= x | y | z | ...
```

Structural congruence

The structural congruence of processes, noted \equiv , is the least congruence containing α -equivalence, \equiv_{α} , making (P, |, 0) into commutative monoids and satisfying

$$(\operatorname{new} x)(\operatorname{new} x)P \equiv (\operatorname{new} x)P$$

$$(\operatorname{new} x)(\operatorname{new} y)P \equiv (\operatorname{new} y)(\operatorname{new} x)P$$

$$((\operatorname{new} x)P) \mid Q \equiv (\operatorname{new} x)(P \mid Q)$$

Reduction rules

unify(t, u, s)
$$a?(t)P \mid a![u]Q \rightarrow Ps \mid Qs$$

$$P \rightarrow P'$$

$$P \mid Q \rightarrow P' \mid Q$$

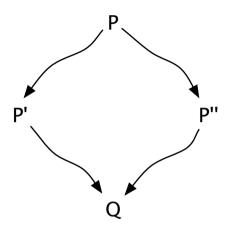
$$P \rightarrow P'$$

$$(new a)P \rightarrow (new a)P'$$

$$P \equiv P' \quad P' \rightarrow Q' \quad Q' \equiv Q$$

$$P \rightarrow Q$$

Important properties of computational calculi



Confluence

lambda calculus is confluent

 π -calculus is **not** confluent

a![u1]Q1 | a?(t)P | a![u2]Q2 typical race condition