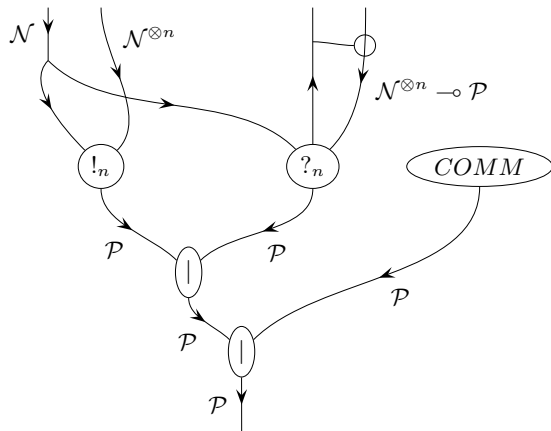


# Casper Formalized Pt I

Applying  $\pi$ -calculus  
to modeling the Casper protocol



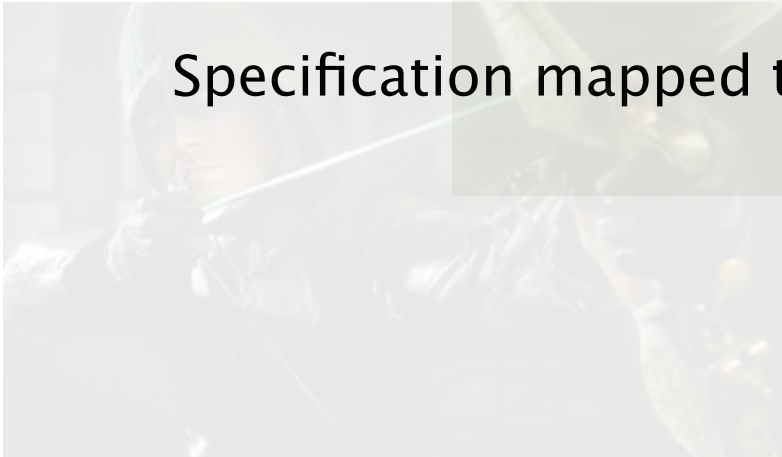
Vlad Zamfir, L.G. Meredith

# Aims and goals

Formal specification of Casper

Specification mapped to reference implementation

Specification mapped to simulation



# Methodology

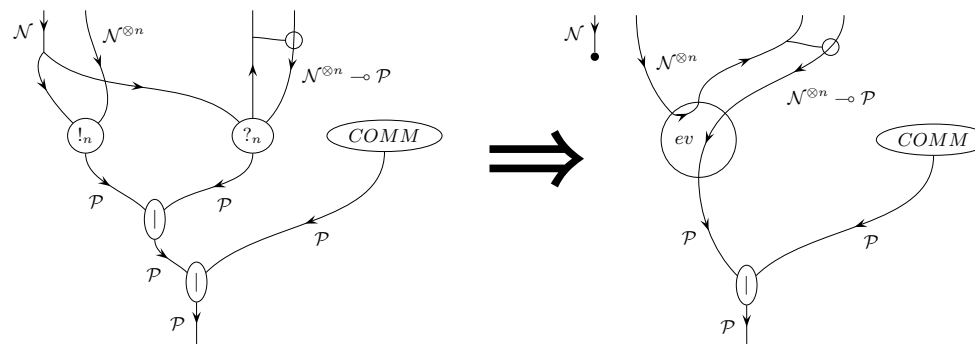
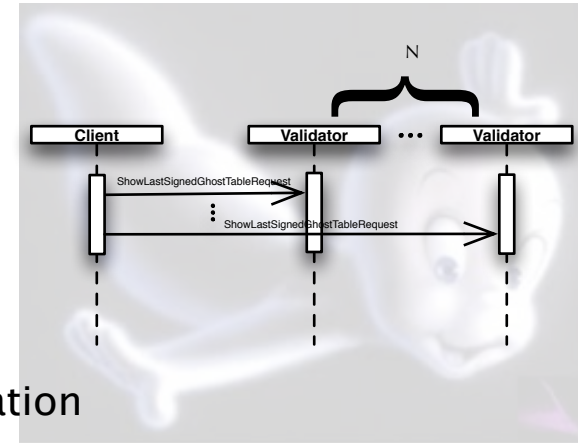
Bootstrap Casper spec using interaction diagrams

Map interaction diagrams to initial  $\pi$ -calculus specification

Refine specification

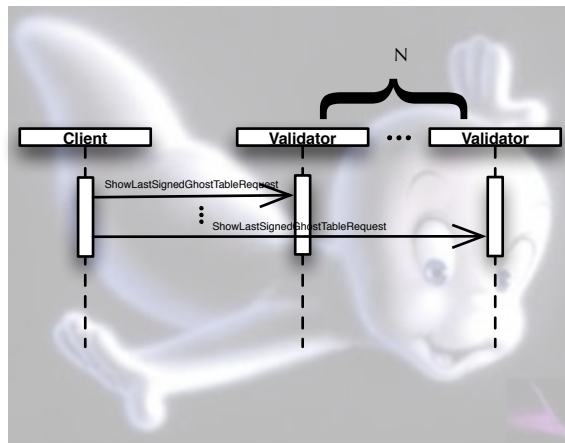
Map refined specification to SpecialK

Map refined specification to Stochastic  $\pi$ -machine



Casper in  $\pi$ -calculus

# Casper in interaction diagrams

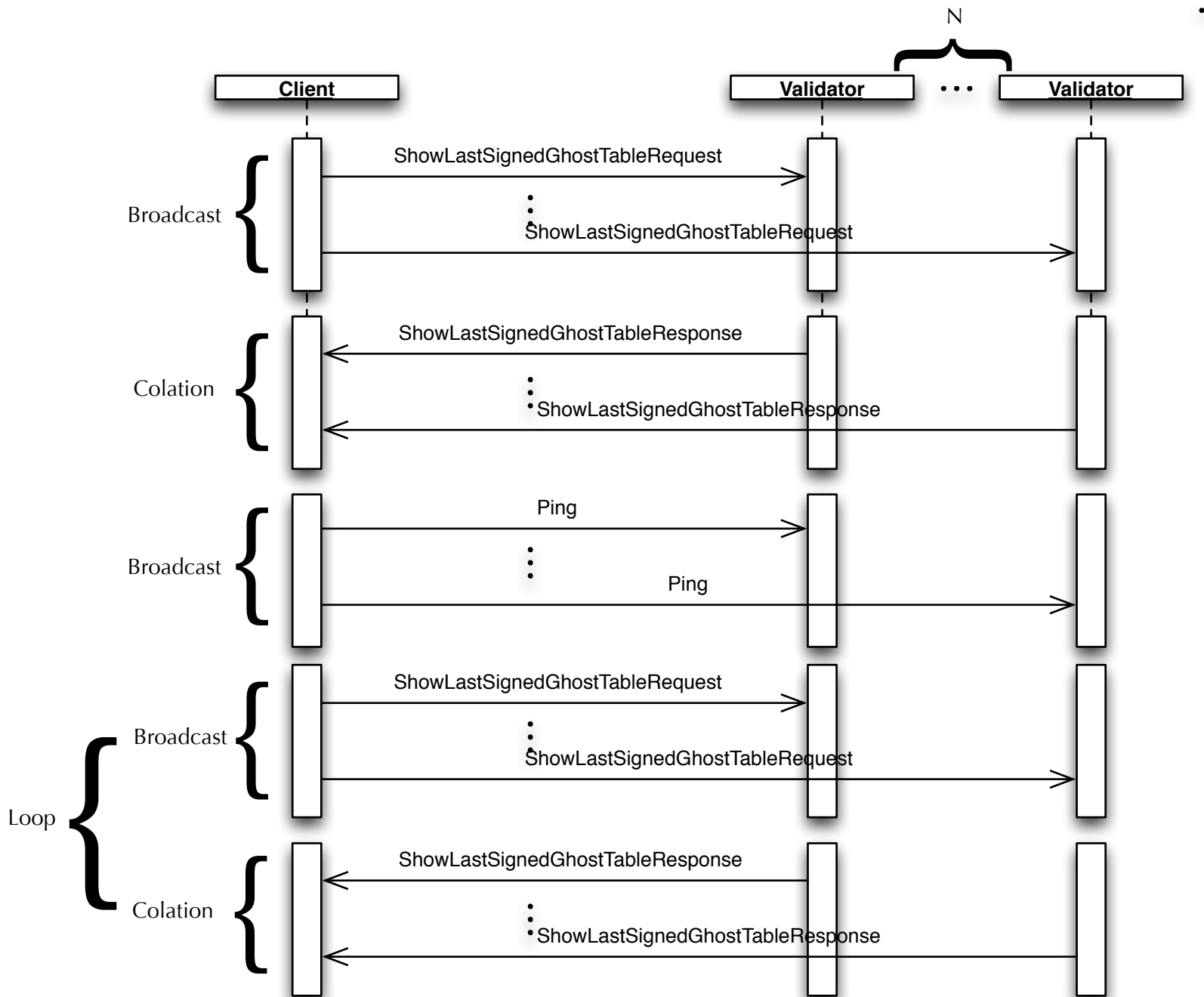


Casper in  $\pi$ -calculus

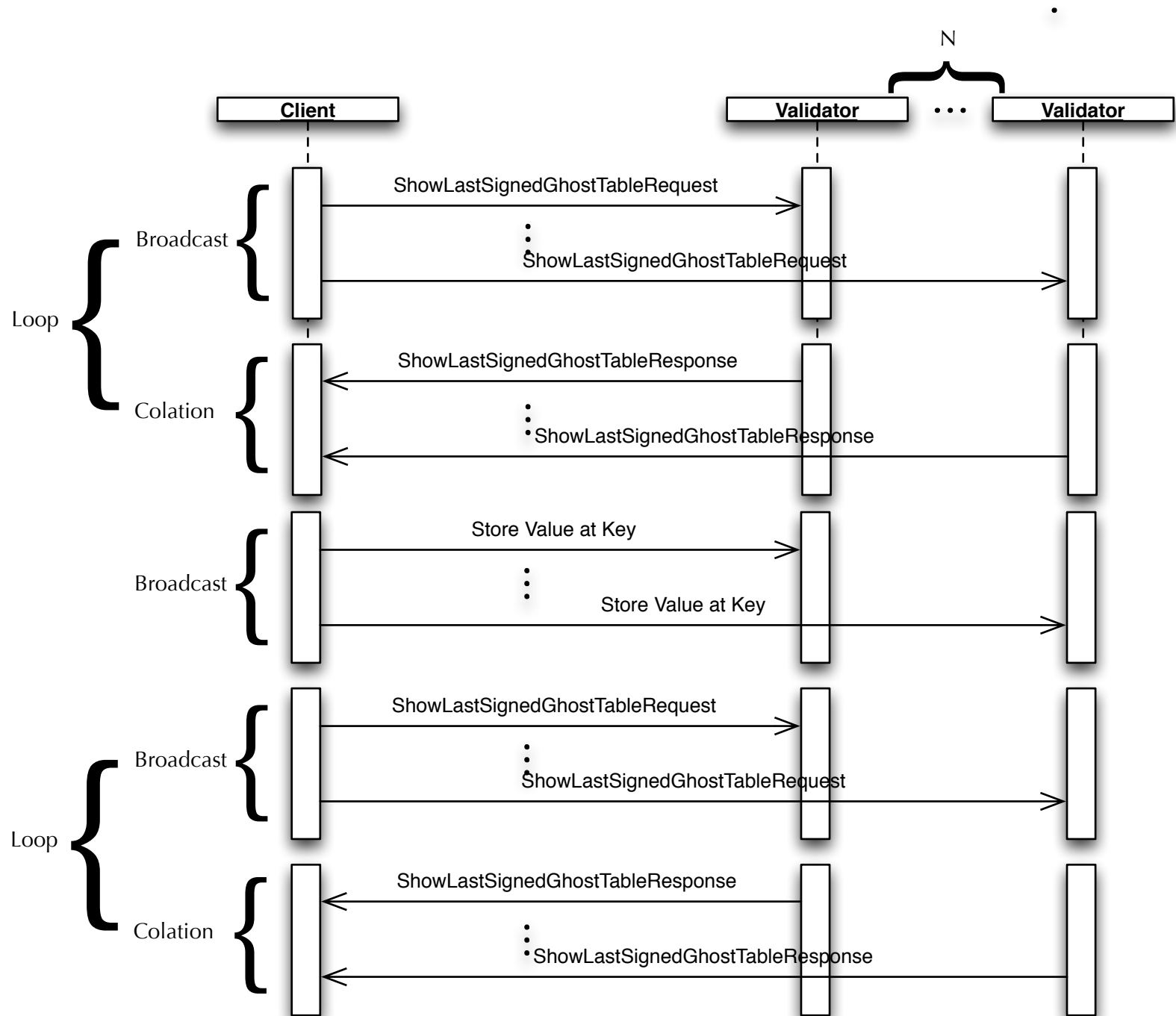
# Client $\leftrightarrow$ Validator interactions



Casper in  $\pi$ -calculus

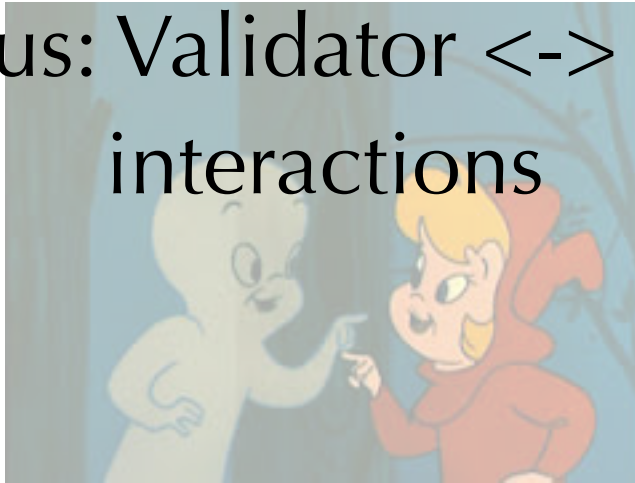


Casper in  $\pi$ -calculus

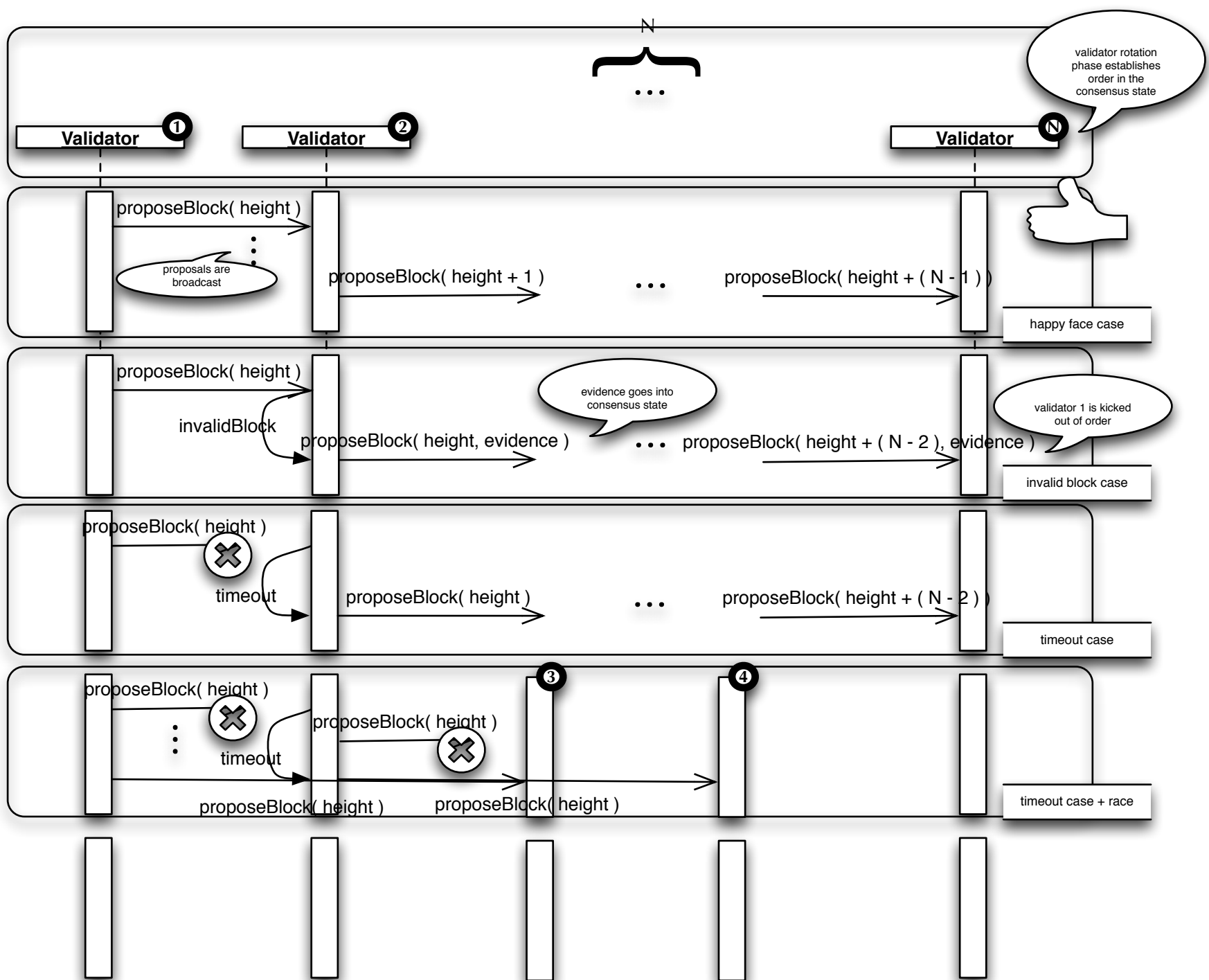


Casper in  $\pi$ -calculus

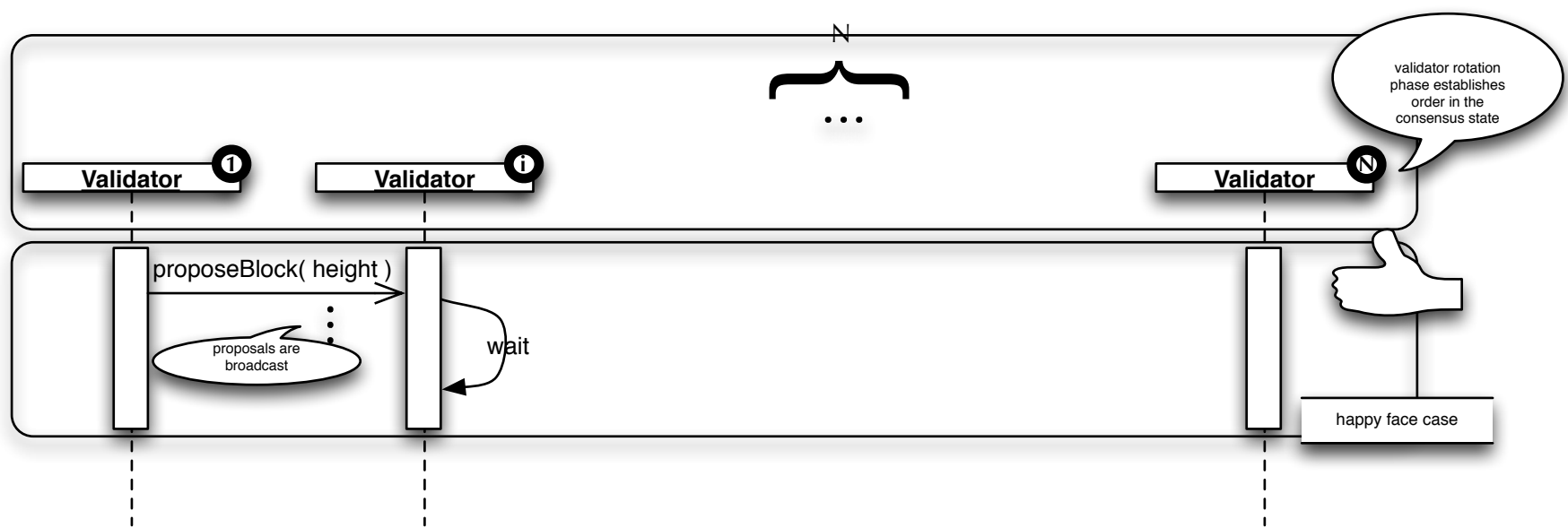
# Consensus: Validator $\leftrightarrow$ Validator interactions







Casper in  $\pi$ -calculus

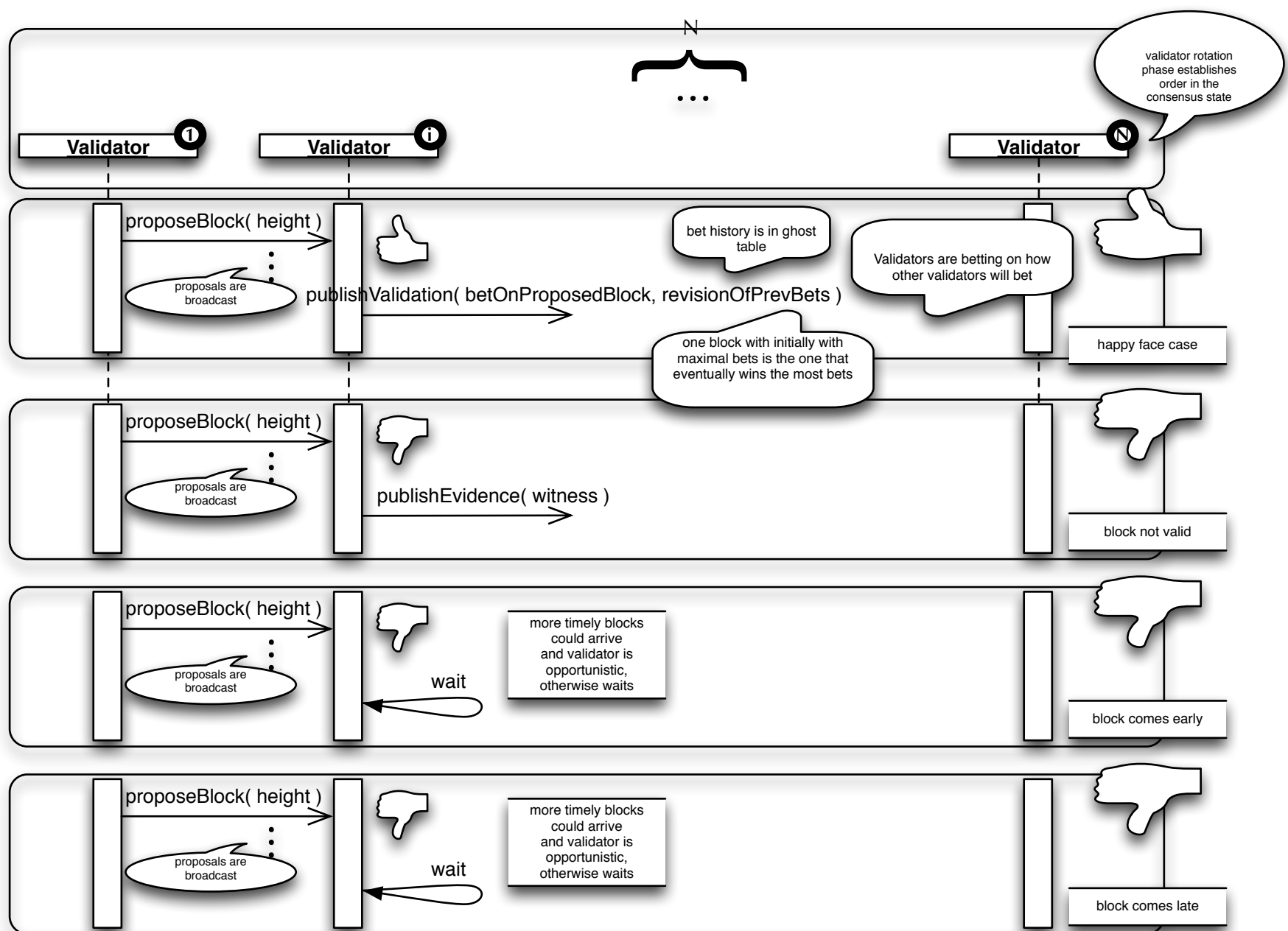


For validator  $i$ ,  $v(i)$ , if the proposal is from  $v(\text{current})$ , then  $v(i)$  can publish validation straightaway, otherwise wait for the proposer  $== v(\text{current})$ , then publish validation.

Blocks contain validations of other blocks.

Publishing a validation is placing a bet that the validated block is the block that will be finalized at that height.

Validations also contain new bets on non-finalized previous blocks.



# To do list

State transition function

Bonding/unbonding

Rejection of validators based on evaluation of evidence

Distribution of transaction fees and bonds

Execution, re-ordering, and finalizing of txns

Assume  $fn, current : time \rightarrow Z[N]$  where  $N$  is the number of validators. At end / beginning of bonding period current is recalculated.

$current = current\_1 \parallel current\_2 \parallel \dots \parallel current\_M$

to get parallelism speed up opportunities

# To do list

Block consists of

- \* ghost entries ( prev hash, next hash )
- \* reorg transitions
- \* new txns

valid( block ) ==

stateX( block.ghostEntries.\_1 ) == block.ghostEntries.\_2  
&& stateX( block.reorgX.\_1 ) == block.reorgX.\_2

# Blocks and ghost table structure

$\text{block} ::= (\text{entry}^*, \text{signature})$

$\text{entry} ::= \text{ghost} \mid \text{reorg} \mid \text{txn}$

$\text{ghost} ::= (\text{prev}, (\text{block} \mid \text{validation} \mid \text{evidence}), \text{post})$

$\text{evidence} ::= (\text{address}, \text{bet}^*)$

$\text{reorg} ::= (\text{prev}, \text{txn}^*, \text{post})$

$\text{txn} ::= (\text{prev}, (\text{receiver}, \text{data}, \text{signature}), \text{post})$

$\text{validation} ::= ((\text{hash}(\text{block}), \text{bet})^*, \text{signature})$

$\text{state} ::= (\text{cmgrState}, \text{appState})$

$\text{cmgrState} ::= (\text{ghostTable}, \text{validator}^*, \text{evidenceChecker}, \text{txnFees})$

# Blocks and ghost table structure

$\text{appState} ::= \text{state of a deterministic automaton}$

$\text{history} : \text{ghostTable} \rightarrow \text{txn}^*$

$\text{stateFn} : ( \text{appState}, \text{txn} ) \rightarrow \text{appState}$

$\text{appState}$

$==$

$( ( \text{initialState} ) \setminus \text{history} ) ( ( \text{txn}, \text{state} ) \Rightarrow \text{stateFn}( \text{txn}, \text{state} ) )$

$\text{stateHash} ::= \text{hash}( ( \text{hash}( \text{cmgr} ), \text{hash}( \text{appState} ) ) )$

$\text{prev} ::= \text{stateHash}$

$\text{post} ::= \text{stateHash}$

$\text{ghostTable} ::= \text{height} \rightarrow ( \text{block} \rightarrow ( \text{bet}, \text{validator} )^* )^*$

# Blocks and ghost table structure

$\text{validator} ::= (\text{address}, \text{deposit})$

$\text{evidenceChecker} ::= (\text{ghostTable}, \text{address}) \rightarrow \text{boolean}$   
 $\quad \quad \quad | \text{evidence} \rightarrow \text{boolean}$

$\text{txnFees} ::= \text{height} \rightarrow \text{double}$



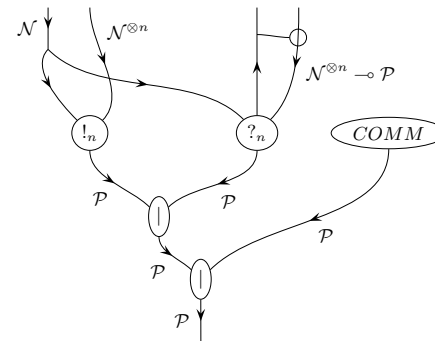
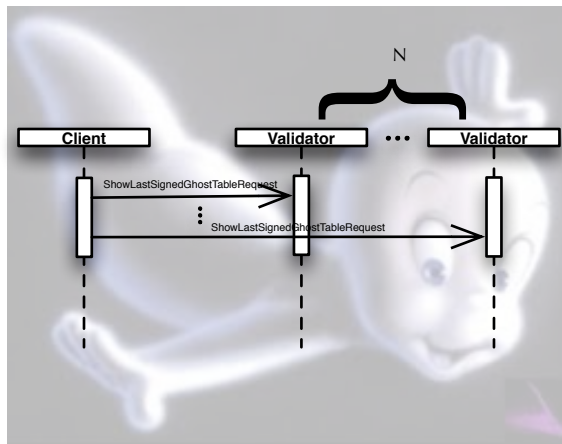
# To do list

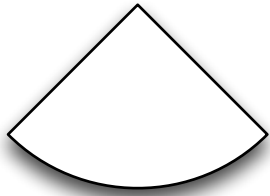
Assume  $fn, current : time \rightarrow Z[N]$  where  $N$  is the number of validators. At end / beginning of bonding period current is recalculated.

$current = current\_1 \parallel current\_2 \parallel \dots \parallel current\_M$

to get parallelism speed up opportunities

# How to turn interaction diagrams into $\pi$ -calculus specs





# programming model



Claimant



Verifier



Relying party

allow Abed  
to check  
claim C



for(

?

e1 <- chan?( C2V )( allowCheck( ) ) ;

Verify  
Troy's  
claim C

?

e2 <- chan?( R2V )( verify( ) )

) {

confirm  
Troy's  
claim C

!

chan!( V2R )( confirm( ) )

}

verifier behavior

Just walk this line to calculate the specification of the verifier's  
behavior and derive code

•  
Casper in  $\pi$ -calculus



allow Abed  
to check  
claim C

chan!( C2V )( allowCheck( ) , ... )

claim: C &  
Dean will  
verify

chan!( C2V )( claim( ), ... )



Claimant

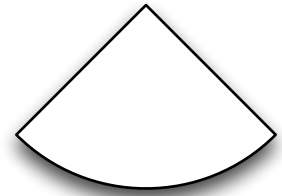


Verifier



Relying party

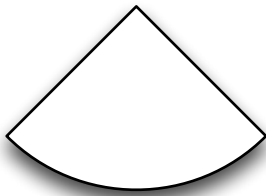
claimant behavior



programming  
model

and this line to calculate the specification of the claimant's  
behavior and derive code

Casper in  $\pi$ -calculus



# programming model



Claimant



Verifier



Relying party

relying party behavior

for(

e1 <- chan?( C2R )( claim( ) )

) {

for(

confirm  
Troy's  
claim C

) {

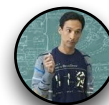
...

}

}

and this line to calculate the specification of the relying party's  
behavior and derive code

Casper in  $\pi$ -calculus



claim: C &  
Dean will  
verify

?

Verify  
Troy's  
claim C

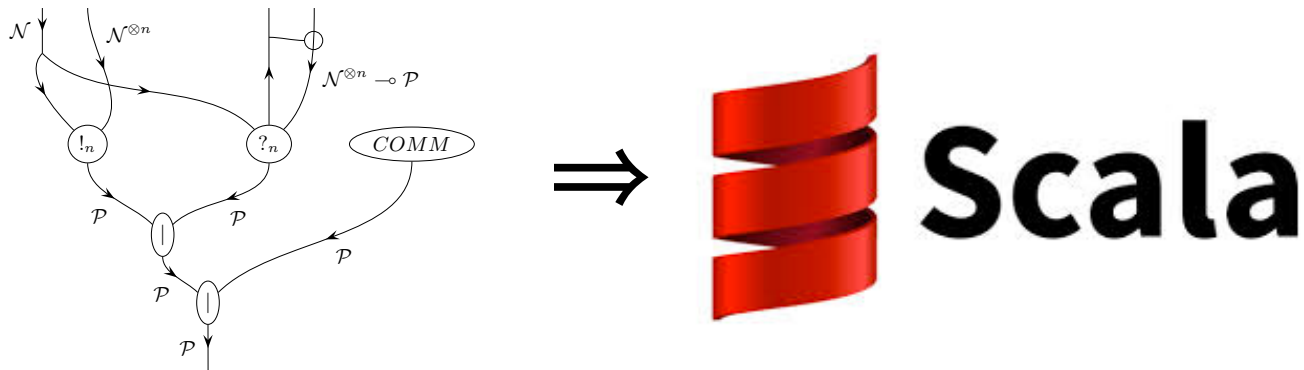
!

chan!( C2R )( verify( ), ... )

?

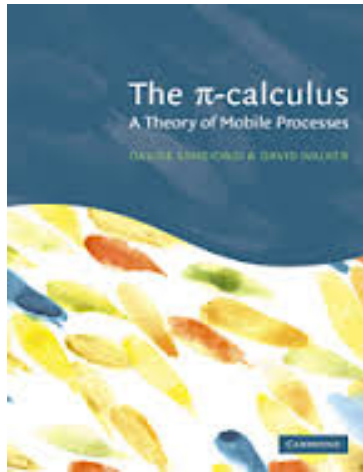
e1 <- chan?( C2R )( truat( ) )

# How to turn $\pi$ -calculus specs into scala code

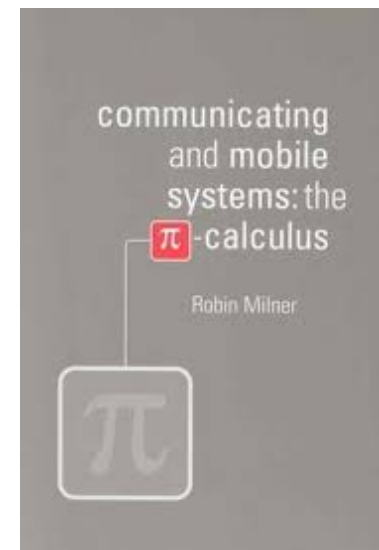


$P, Q ::= 0$	<code>{ }</code>
$a![v_1, \dots, v_n]$	<code>[  a  ](m) ![   v1  ](m), ..., [  vn  ](m) ]</code>
$a?(x_1, \dots, x_n)P$	<code>for( [  x1, ..., xn ] &lt;- [  a  ](m) ){     [  P  ](m)( x1, ..., xn ) }</code>
$P \mid Q$	<code>spawn{ [  P  ](m) };spawn{ [  Q  ](m) }</code>
$(\text{new } a)P$	<code>{ val q = new Queue(); [  P  ](m[ a &lt;- q ] ) }</code>
$(\text{def } X(x_1, \dots, x_n) = P)[v_1, \dots, v_n]$	<code>object X {     def apply( x1, ..., xn ) = {         [  P  ](m) ( x1, ..., xn )     } }</code>
$X[v_1, \dots, v_n]$	<code>X( [  v1  ](m), ..., [  vn  ](m) )</code>

`[| - |]( - ) : (  $\pi$ -calculus, Map[Symbol,Queue] ) -> Scala`



# applied $\pi$ -calculus

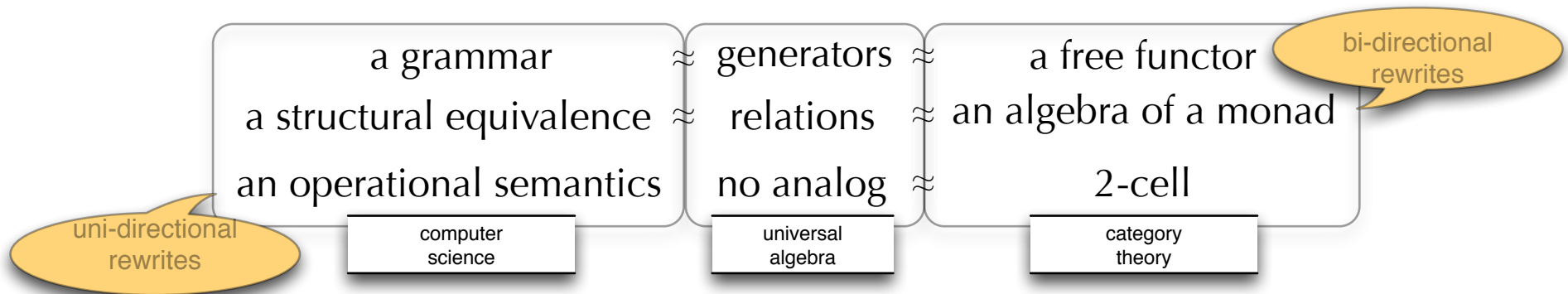


Casper in  $\pi$ -calculus



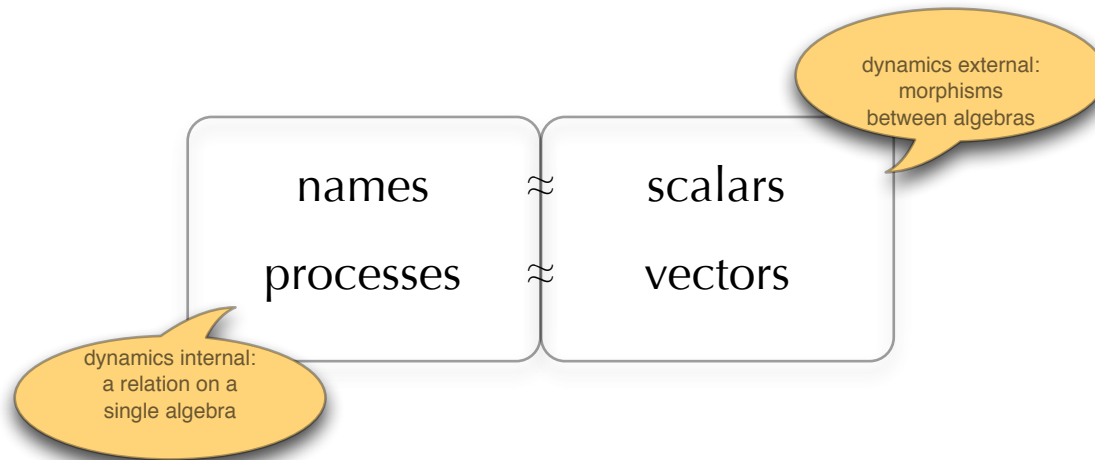
# How to specify a computational calculus

Modern presentations of computational calculi generalize  
generators and relations style presentations of universal algebra  
They are typically given in terms of



# How to specify a computational calculus

Process calculi, like vector spaces, are two sorted algebraic structures



A morphism from one process calculus to another **preserves** computational dynamics

A morphism from one vector space to another **is** computational dynamics

# Syntax

$P, Q ::= 0$

$a![t]Q$

$a?(t)P$

$P \mid Q$

$(\text{new } a)P$

$(\text{def } X(t) = P)[u]$

$X[t]$

$t, u ::= \text{val} \mid \text{var} \mid \text{fn}(t_1, \dots, t_N)$

$\text{val} ::= \text{bool} \mid \text{int} \mid \text{string} \mid \text{double} \mid \dots$

$\text{var} ::= \_ \mid X \mid Y \mid Z \mid \dots$

$\text{fn} ::= x \mid y \mid z \mid \dots$

## Structural congruence

The *structural congruence* of processes, noted  $\equiv$ , is the least congruence containing  $\alpha$ -equivalence,  $\equiv_\alpha$ , making  $(P, |, 0)$  into commutative monoids and satisfying

$$(\text{new } x)(\text{new } x)P \equiv (\text{new } x)P$$

$$(\text{new } x)(\text{new } y)P \equiv (\text{new } y)(\text{new } x)P$$

$$((\text{new } x)P) \mid Q \equiv (\text{new } x)(P \mid Q)$$

# Reduction rules

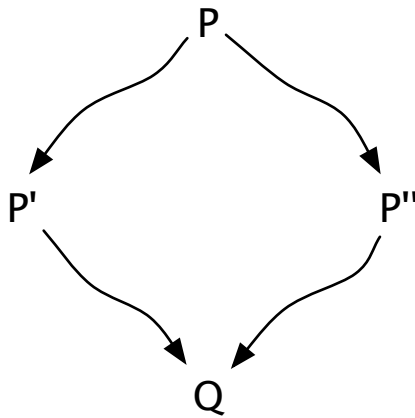
$$\frac{\text{unify}(t, u, s)}{a?(t)P \mid a![u]Q \rightarrow Ps \mid Qs}$$

$$\frac{P \rightarrow P'}{P \mid Q \rightarrow P' \mid Q}$$

$$\frac{P \rightarrow P'}{(\text{new } a)P \rightarrow (\text{new } a)P'}$$

$$\frac{P \equiv P' \quad P' \rightarrow Q' \quad Q' \equiv Q}{P \rightarrow Q}$$

# Important properties of computational calculi



Confluence

lambda calculus is confluent

$\pi$ -calculus is **not** confluent

$a![u_1]Q_1 \mid a?(t)P \mid a![u_2]Q_2$

typical race condition

Casper in  $\pi$ -calculus