A triple $\mathcal{C}(O, M, \bullet)$ where

- $\forall o \in O, \exists id_o \in M$
- $\bullet$ is defined for all connected objects
- $\bullet(A \to B, B \to C) = A \to C$
- $(f \bullet g) \bullet h == f \bullet (h \bullet g)$
- $id_x \bullet f = f \bullet id_y$

Category Theory

Category

Monoid

Action

Free Monoid

An action of a $\mathcal{M}(M, id_0, \star)$ over a set S of states is a function

$$M \times S \to S$$

It is equivalent to an automata

A free monoid of M is just a monoid $\mathcal{M}(List[M], [], ++)$.

A monoid $\mathcal{M}(M, id_0, \star)$ is just a category $\mathcal{C}(O, M, \star)$ where O = { o_1 }