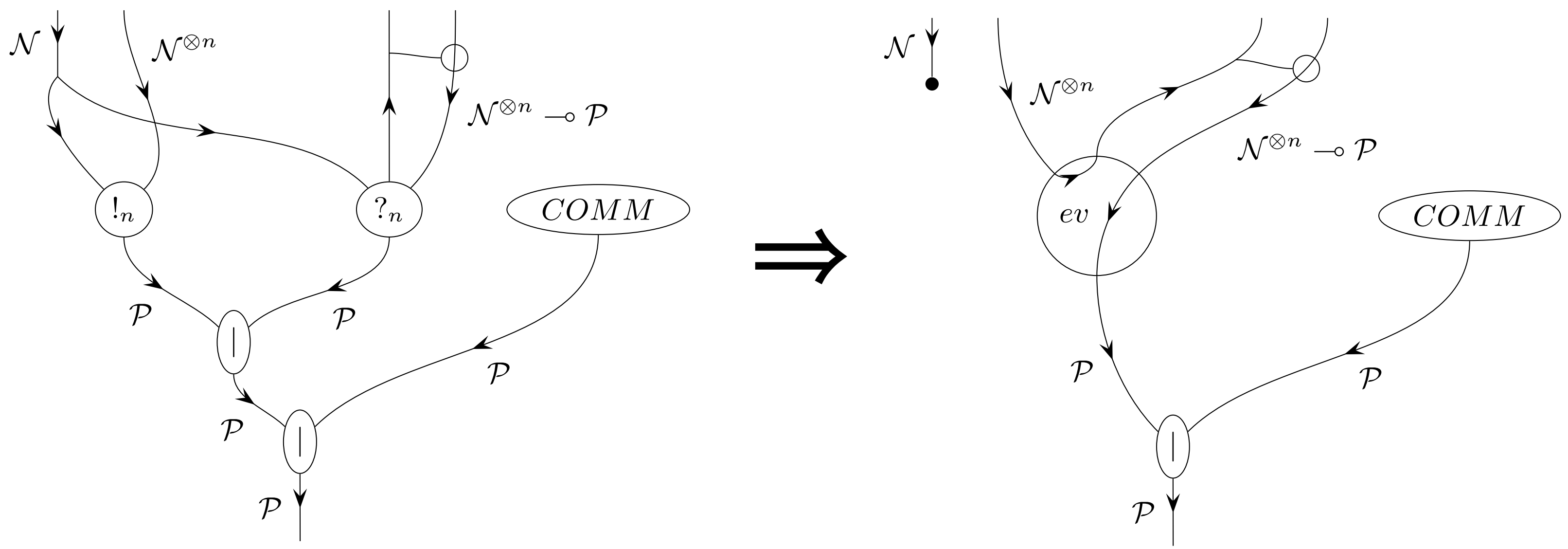
Higher category models of the π-calculus

The operational semantics and Curry-Howard



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HDRA 2015

Higher category models of the π-calculus

Modern presentations of computational calculi generalize generators and relations style presentations of universal algebra

They are typically given in terms of

computer science		universal algebra		category theory
a grammar	$\approx$	generators	$\approx$	a free functor
a structural equivalence	$\approx$	relations	$\approx$	an algebra of a monad
an operational semantics	$\approx$	no analog	$\approx$	2-cell

uni-directional rewrites

bi-directional rewrites

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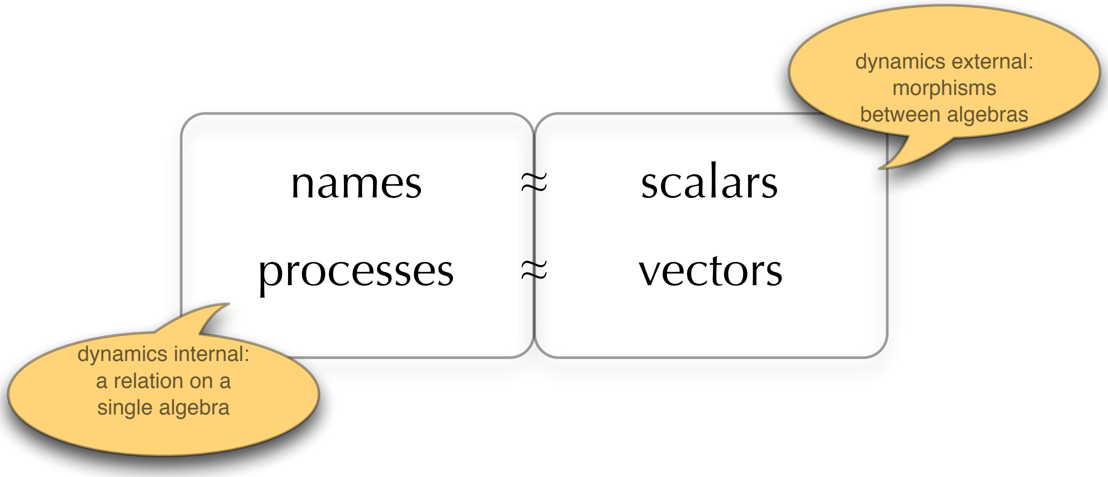
Process calculi, like vector spaces, are two sorted algebraic structures



A morphism from one process calculus to another ***preserves*** computational dynamics

A morphism from one vector space to another ***is*** computational dynamics

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Syntax

$$
\begin{aligned}
P ::= {} & 0 && \text{stopped process} \\
\mid {} & x?(y_1, \ldots, y_n) \Rightarrow P && \text{input} \\
\mid {} & x!(y_1, \ldots, y_n) && \text{output} \\
\mid {} & (\mathsf{new}\ x)P && \text{new channel} \\
\mid {} & P \mid Q && \text{parallel}
\end{aligned}
$$

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Free and bound names

$$\mathcal{FN}(0) \coloneqq \emptyset$$

$$\mathcal{FN}(x?(y_1, \ldots, y_n) \Rightarrow P) \coloneqq$$
$$\quad \{x\} \cup (\mathcal{FN}(P) \setminus \{y_1, \ldots y_n\})$$

$$\mathcal{FN}(x!(y_1, \ldots, y_n)) \coloneqq \{x, y_1, \ldots, y_n\}$$

$$\mathcal{FN}((\mathsf{new}\ x)P) \coloneqq \mathcal{FN}(P) \setminus \{x\}$$

$$\mathcal{FN}(P \mid Q) \coloneqq \mathcal{FN}(P) \cup \mathcal{FN}(Q)$$

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Structural congruence

The *structural congruence* of processes, noted $\equiv$, is the least congruence containing α-equivalence, $\equiv_\alpha$, making $(P, |, 0)$ into commutative monoids and satisfying

$$(\mathsf{new}\ x)(\mathsf{new}\ x)P \equiv (\mathsf{new}\ x)P$$

$$(\mathsf{new}\ x)(\mathsf{new}\ y)P \equiv (\mathsf{new}\ y)(\mathsf{new}\ x)P$$

$$((\mathsf{new}\ x)P) \mid Q \equiv (\mathsf{new}\ x)(P \mid Q)$$

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Operational semantics

$$\frac{|\vec{y}| = |\vec{z}|}{x?(\vec{y}) \Rightarrow P \mid x!(\vec{z}) \to P\{\vec{z}/\vec{y}\}} \qquad (\textsc{Comm})$$

$$\frac{P \to P'}{P \mid Q \to P' \mid Q} \qquad (\textsc{Par})$$

$$\frac{P \to P'}{(\mathsf{new}\ x)P \to (\mathsf{new}\ x)P'} \qquad (\textsc{New})$$

$$\frac{P \equiv P' \qquad P' \to Q' \qquad Q' \equiv Q}{P \to Q} \qquad (\textsc{Equiv})$$

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Bisimulation

$$\frac{}{x!(\vec{y}) \downarrow x}$$

$$\frac{P \downarrow x \;\; or \; Q \downarrow x}{P \mid Q \downarrow x}$$

DEFINITION 2.1.2. *An* barbed bisimulation, *is a symmetric binary relation $\mathcal{S}$ between agents such that $P \; \mathcal{S} \; Q$ implies:*

1. *If $P \rightarrow P'$ then $Q \rightarrow Q'$ and $P' \; \mathcal{S} \; Q'$.*
2. *If $P \downarrow x$, then $Q \downarrow x$.*

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Bisimulation

P *is barbed bisimilar to* Q, *written* $P \stackrel{\bullet}{\approx} Q$, *if* $P \mathrel{\mathcal{S}} Q$ *for some barbed bisimulation* $\mathcal{S}$.

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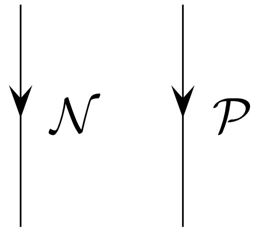
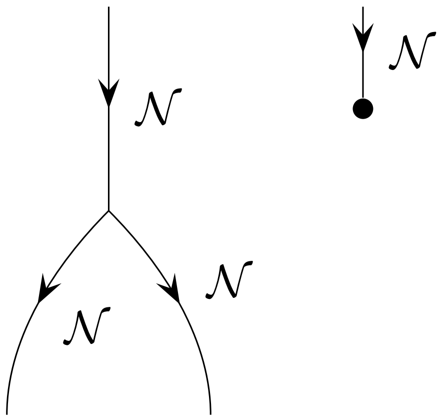
The categorical machinery

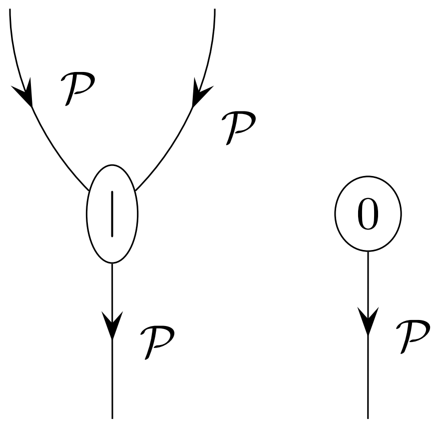
objects $\mathcal{N}$ for names and $\mathcal{P}$ for processes



1-morphisms $\Delta\colon \mathcal{N} \to \mathcal{N} \otimes \mathcal{N}$ and $\delta\colon \mathcal{N} \to I$



1-morphisms $|\colon \mathcal{P} \otimes \mathcal{P} \to \mathcal{P}$ and $0\colon I \to \mathcal{P}$,



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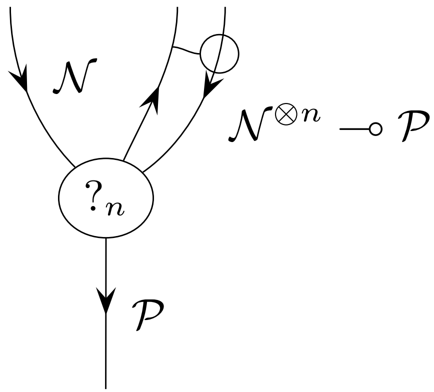
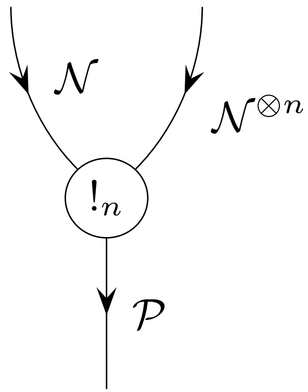
The categorical machinery

1-morphism $?_n : \mathcal{N} \otimes (\mathcal{N}^{\otimes n} \multimap \mathcal{P}) \to \mathcal{P}$ and $!_n : \mathcal{N} \otimes \mathcal{N}^{\otimes n} \to \mathcal{P}$ for each natural number $n \geq 0$,





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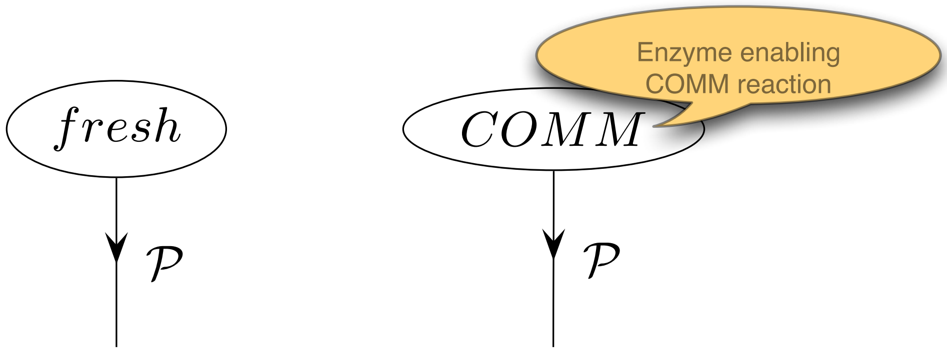
The categorical machinery

1-morphisms $fresh\colon I \to \mathcal{P}$ and $COMM\colon I \to \mathcal{P}$



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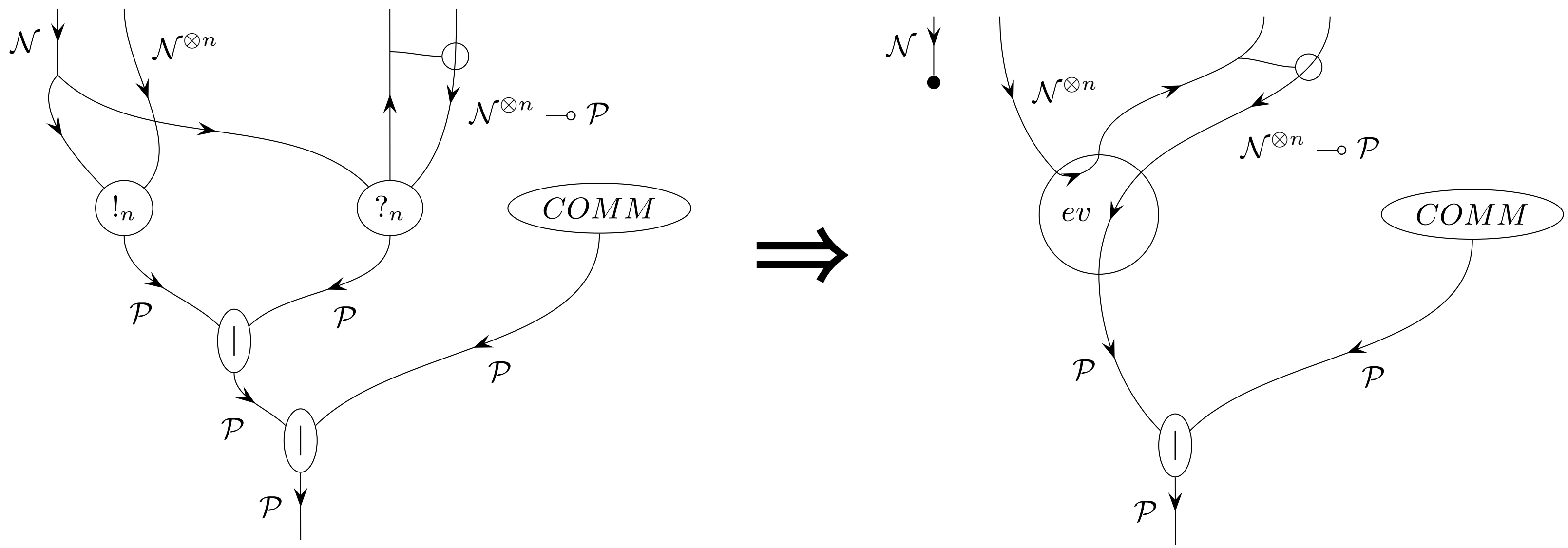
The categorical machinery

a 2-morphism $comm_n$ encoding the COMM rule for each natural number $n \geq 0$.



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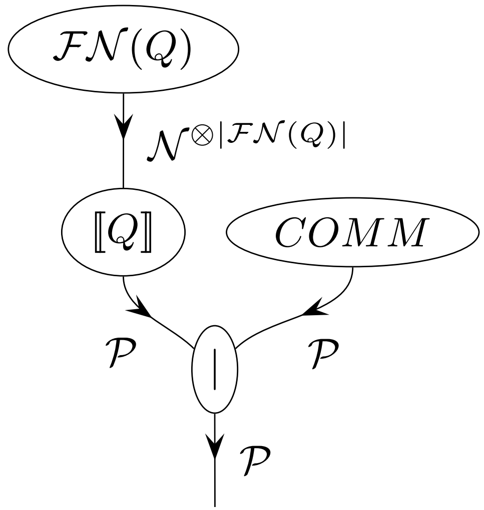
The semantics

$[\![Q]\!]_{top} :=$



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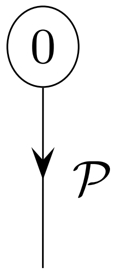
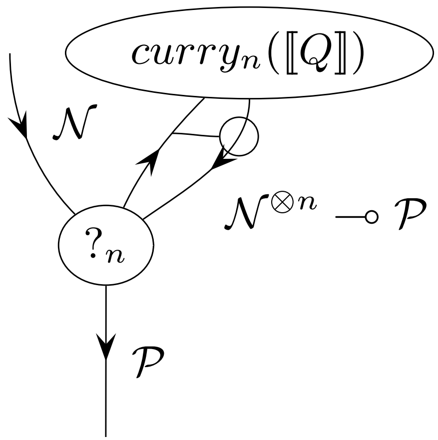
The semantics

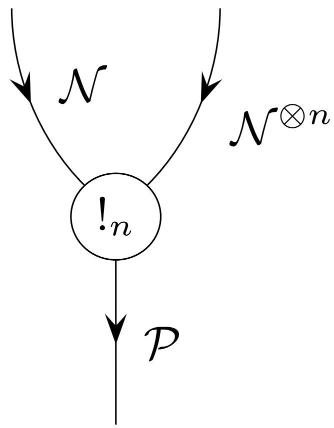
$$[\![0]\!] \coloneqq$$



$$[\![x?(y_1,\ldots,y_n) \Rightarrow Q]\!] \coloneqq$$



$$[\![x!(y_1,\ldots,y_n)]\!] \coloneqq$$



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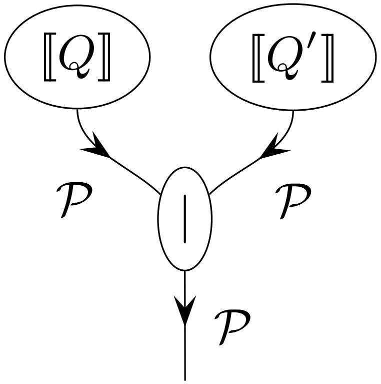
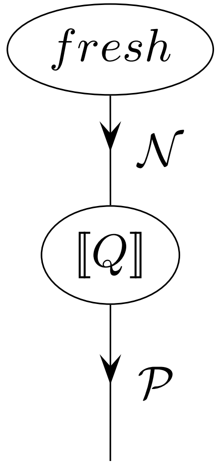
The semantics

$[\![Q \mid Q']\!] \coloneqq$ 

$[\![(\mathsf{new}\ x)Q]\!] \coloneqq$ 

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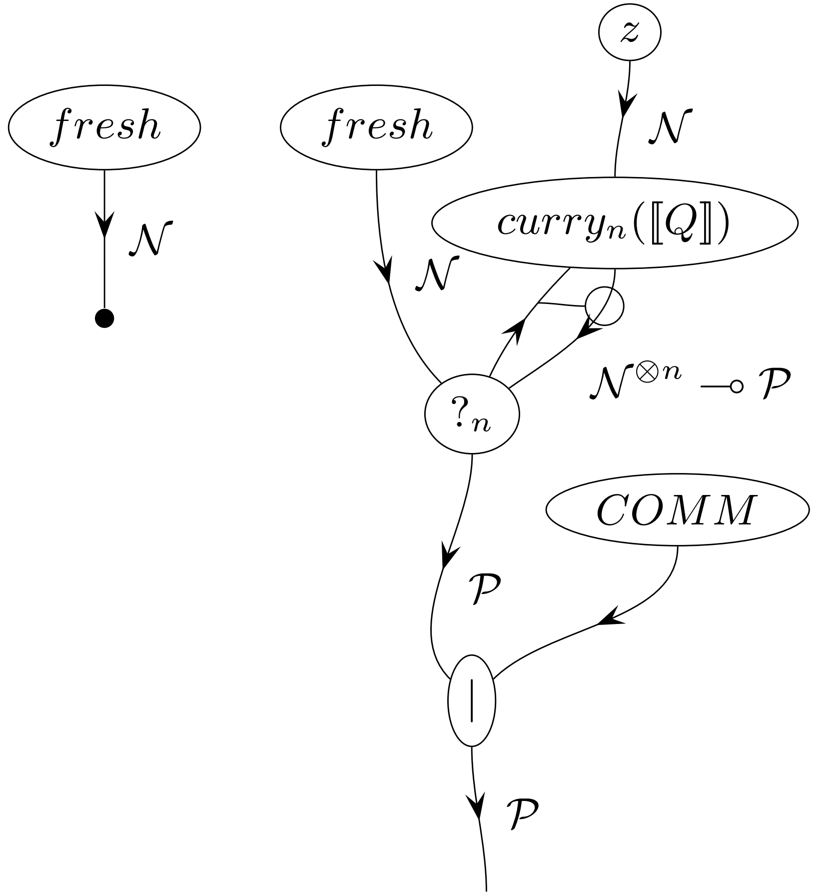
The semantics

$[\![(\mathsf{new}\ y)(\mathsf{new}\ x)x?(y_1,\ldots,y_n) \Rightarrow Q]\!]_{top}$



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The semantics

$$[\![x!(y_1,\ldots,y_n)]\!] \Downarrow [\![x]\!]$$

$$[\![P]\!] \Downarrow [\![x]\!] \text{ or } [\![Q]\!] \Downarrow [\![x]\!] \text{ implies } [\![P]\!] \mid [\![Q]\!] \Downarrow [\![x]\!]$$

Taken together these two notions provide an immediate lifting of the syntactic notion of bisimulation to a corresponding semantic notion, which we write, $\approx$.

THEOREM 4.1.1 (FULL ABSTRACTION).

$$P \mathrel{\dot{\approx}} Q \quad \iff \quad [\![P]\!] \approx [\![Q]\!]$$

Higher category models of the π-calculus

Conclusions and future work

Mellies and Zeilberger types

Explicit computational resources

True concurrency

P,Q ::= 0

```
{ }
```

a![v1, …, vn]

```
[| a |]( m ) ![ [| v1 |]( m ), …, [| vn |]( m ) ]
```

a?(x1, …, xn)P

```
for( [ x1, …, xn ] <- [| a |]( m ) ){
    [| P |]( m )( x1, …, xn )
}
```

P | Q

```
spawn{ [| P |]( m ) };spawn{ [| Q |]( m ) }
```

(new a)P

```
{ val q = new Queue(); [| P |]( m[ a <- q ] ) }
```

(def X(x1, …, xn) = P)[v1, …, vn]

```
object X {
   def apply( x1, …, xn ) = {
      [| P |]( m ) ( x1, …, xn )
   }
}
```

X[v1, …, vn]

```
X( [| v1 |]( m ), …, [| vn |]( m ) )
```

```
[| - |]( - ) : ( π-calculus, Map[Symbol,Queue] ) -> Scala
```