Keywords
higher category theory, concurrency, message-passing, types, Curry-Howard

ABSTRACT
We present an approach to modeling computational calculi using higher category theory. While the paper focuses on applications to the mobile process calculi, and more specifically, the π-calculus, because they provide unique challenges for categorical models, the approach extends smoothly to a variety of other computational calculi, including important milestones such as the λ-calculus. One of the key contributions is a method of restricting rewrites to specific contexts inspired by catalysis in chemical reactions.

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Higher category models of mobile process calculi

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1. INTRODUCTION

TBD

1.0.1 Organization of the rest of the paper

TBD

2. THE CALCULUS

Some examples of process expressions.

2.1 Our running process calculus

2.1.1 Syntax

$M, N ::= 0$	stopped process
$\mid x?(y_1, \ldots, y_N) \Rightarrow P$	input
$\mid x!(y_1, \ldots, y_N)$	output
$\mid M+N$	choice
$P, Q ::= M$	include IO processes
$\mid P \mid Q$	parallel

2.1.2 Free and bound names

$$\mathcal{FN}(0) \coloneqq \emptyset$$
$$\mathcal{FN}(x?(y_1, \ldots, y_N) \Rightarrow P) \coloneqq$$
$$\{x\} \cup (\mathcal{FN}(P) \setminus \{y_1, \ldots y_N\})$$
$$\mathcal{FN}(x!(y_1, \ldots, y_N)) \coloneqq \{x, y_1, \ldots, y_N\}$$
$$\mathcal{FN}(P \mid Q) \coloneqq \mathcal{FN}(P) \cup \mathcal{FN}(Q)$$

An occurrence of x in a process P is *bound* if it is not free. The set of names occurring in a process (bound or free) is denoted by $\mathcal{N}(P)$.

2.1.3 Structural congruence

The *structural congruence* of processes, noted $\equiv$, is the least congruence containing α-equivalence, $\equiv_\alpha$, making $(P, |, 0)$ and $(P, +0)$ commutative monoids.

2.1.4 Operational Semantics

$$\frac{|\vec{y}| = |\vec{z}|}{P_1 + x_0?(\vec{y}) \Rightarrow P \mid x_1!(\vec{z}) + P_2 \to P\{@\vec{z}/\vec{y}\}} \quad (\textsc{Comm})$$

In addition, we have the following context rules:

$$\frac{P \to P'}{P \mid Q \to P' \mid Q} \quad (\textsc{Par})$$

$$\frac{P \equiv P' \qquad P' \to Q' \qquad Q' \equiv Q}{P \to Q} \quad (\textsc{Equiv})$$

2.1.5 Bisimulation

Definition 2.1.1. *An* observation relation, *$\downarrow_\mathcal{N}$, over a set of names, $\mathcal{N}$, is the smallest relation satisfying the rules below.*

$$\frac{x \in \mathcal{N}}{x!(\vec{y}) \downarrow_\mathcal{N} x} \quad (\textsc{Out-barb})$$

$$\frac{P \downarrow_\mathcal{N} x \text{ or } Q \downarrow_\mathcal{N} x}{P \mid Q \downarrow_\mathcal{N} x} \quad (\textsc{Par-barb})$$

We write $P \Downarrow_\mathcal{N} x$ if there is Q such that $P \Rightarrow Q$ and $Q \downarrow_\mathcal{N} x$.

Notice that $x?(y) \Rightarrow P$ has no barb. Indeed, in RHO-calculus as well as other asynchronous calculi, an observer has no direct means to detect if a sent message has been received or not.

DEFINITION 2.1.2. *An* $\mathcal{N}$-barbed bisimulation *over a set of names,* $\mathcal{N}$*, is a symmetric binary relation* $\mathcal{S}_{\mathcal{N}}$ *between agents such that* $P \, \mathcal{S}_{\mathcal{N}} Q$ *implies:*

1. *If* $P \rightarrow P'$ *then* $Q \Rightarrow Q'$ *and* $P' \, \mathcal{S}_{\mathcal{N}} Q'$.
2. *If* $P \downarrow_{\mathcal{N}} x$*, then* $Q \Downarrow_{\mathcal{N}} x$.

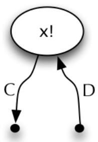
P *is* $\mathcal{N}$*-barbed bisimilar to* Q*, written* $P \stackrel{\cdot}{\approx}_{\mathcal{N}} Q$*, if* $P \, \mathcal{S}_{\mathcal{N}} Q$ *for some* $\mathcal{N}$*-barbed bisimulation* $\mathcal{S}_{\mathcal{N}}$.

3. CATEGORICAL MACHINERY

TBD

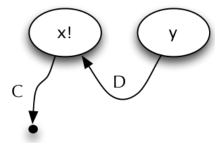
4. THE INTERPRETATION

TBD



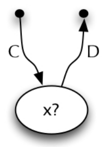
Interpreting names a morphisms, x : J -> D* ⊠ C

Figure 1: Interpretation of output



That means we can interpret output, x!(y), as connecting a source to the input of the morphism.

Figure 2: Interpretation of output - again



And the adjoint morphism, x : D* ⊠ C -> J, corresponds to input

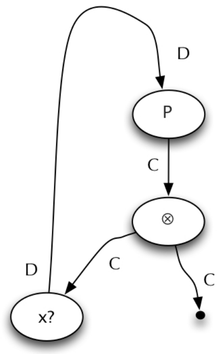
Figure 3: Interpretation of input

5. CONCLUSIONS AND FUTURE WORK

TBD

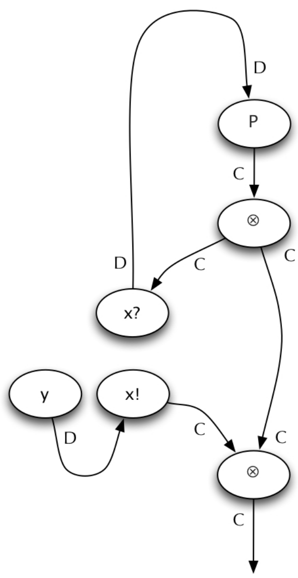
Acknowledgments.. TBD

6. REFERENCES



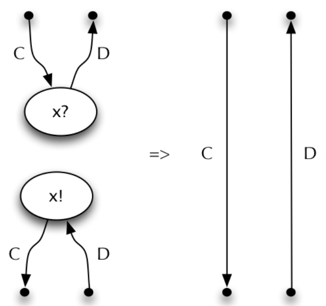
This provides the interpretation of x?(y)P.

Figure 4: Interpretation of input guarded process



This provides the interpretation of x?(z)P | x!(y).

Figure 5: Interpretation of basic π-calculus redex



The co-unit of the adjunction provides a mechanism for synchronization and data flow.

Figure 6: Interpretation of adjunction-based rewrite template