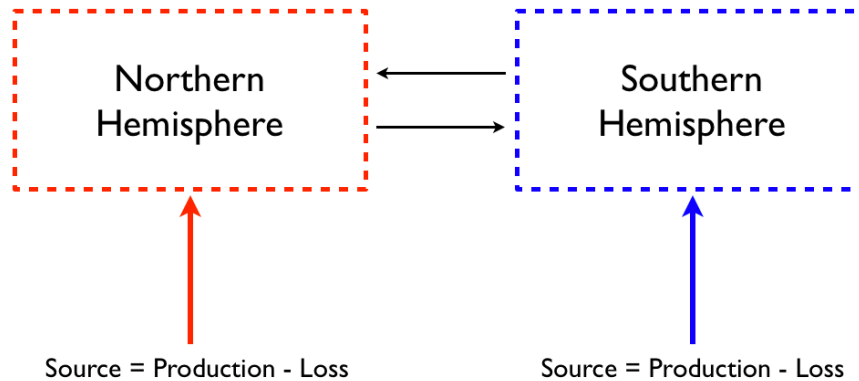


### Problem Set 4 – Inverting for Sources and Sinks of Carbon

The assignment explores the principal observations about the contemporary carbon cycle, and uses an atmospheric 2-box mixing model to infer the distribution of carbon sinks over the last few decades.



The 2-box model for atmospheric mixing:  $S_n$  and  $S_s$  are the net fluxes into each hemisphere = production (sources) – loss (sinks). There are two parameters:  $\tau$ , interhemispheric mixing time, and  $\gamma$ , the ratio of the surface hemispheric gradient to the column-averaged hemispheric gradient.

$$\frac{\partial M_n}{\partial t} = -\frac{M_n - M_s}{\tau} + S_n$$

$$\frac{\partial M_s}{\partial t} = \frac{M_n - M_s}{\tau} + S_s$$

$$S_s = P_s - L_s$$

$$S_n = P_n - L_n$$

We will use observations from Mauna Loa Observatory and South Pole Observatory to represent the mass of carbon in the Northern Hemisphere and Southern Hemisphere.  $\gamma$ , the ratio of surface measurement to the total column mass, will be multiplied by the observations to represent the total mass ( $M_s$  or  $M_n$ ).

We will also use the fossil fuel emissions dataset from the first problem set to estimate emissions.

Files needed for PS4:

- Fossil Fuel data is from the Global Carbon Project (same data as from PS1):  
<https://globalcarbonbudgetdata.org/latest-data.html>
  - Filename: National\_Fossil\_Carbon\_Emissions\_2023v1.0.xlsx

- CO<sub>2</sub> Flask data is from ESRL: <http://www.esrl.noaa.gov/gmd/dv/data/>
  - Filename for MLO: co2\_mlo\_surface-flask\_1\_ccgg\_month.txt
  - Filename for SPO: co2\_spo\_surface-flask\_1\_ccgg\_month.txt
- Python code is in a Jupyter notebook called: PS4\_Inversion.ipynb
  - Section 1 also loads Fossil Fuel and CO<sub>2</sub> flask data
    - ***You will define parameters at the top of Section 1***
  - Section 2 is a Forward integration of the 2-box model
  - Section 3 is the Inverse calculation
  - Section 4 has some additional calculations

The code performs a forward and inverse calculation of the sinks of CO<sub>2</sub> from 1981 to present. To start use  $\tau = 1$  year, and  $\gamma = 1$ . These are the defaults that are set - you will vary these parameters for questions 3 and 4 by changing them at the top of Section 1. NOTE: In the forward integration of the model (Section 2 in the code) we are assuming that FF is 100% airborne.

The filenames for plots will be automatically named with your choice of tau and gamma, so you will get new saved figures each time you change a parameter.

-----PS 4 Questions-----

1. In Section (2) of the code – Forward Integration of a 2-Box Model. In the forward integration of the code (Section 2) we are assuming that Fossil Fuel remains 100% airborne. The plot created in this section compares the modeled surface hemispheric difference in CO<sub>2</sub> to the observed difference (MLO-SPO). What does the difference between the modeled result and the observations imply about the sinks ( $S_n$ ,  $S_s$ ,  $L_n$  and  $L_s$ )? Why is this a good or bad assumption? Answer should be qualitative and possibly include equations (not primarily numeric).
2. In Section (3) of the code – “Inverse calculation”. In the inverse calculation the code calculates the NH and SH sinks ( $L_n$  and  $L_s$ ) from the observed MLO-SPO difference. See section 4 of the code for “additional calculations”. Note the following numbers (and come back to these in questions 3 and 4).
  - a. What is the cumulative total sink over this time period (in PgC)?
  - b. What is the Cumulative sink expressed as a percent of the cumulative FF emission over this time period? How does this compare to what we learned in PS1 about the airborne fraction?
  - c. What is the NH fraction of the total sink, averaged over the period? Does this number match your prior expectations?
3. An uncertainty in the determination of  $L_n$  and  $L_s$  is  $\gamma$ , an index of the vertical gradient of CO<sub>2</sub>.
  - a. Why is there a vertical gradient in CO<sub>2</sub>?
  - b. What is the NH fraction of the total sink, for  $\gamma = 1.0, 1.2, 1.5, 2?$

- c. Why does NH sink vary as it does in (3b)?
4. The interhemispheric exchange time  $\tau$  is determined using mid-latitude industrial emissions.
- a. What would a shift of emissions to China and India do to  $\tau$ ?
  - b. What is the NH fraction of the total sink, for  $\tau = 0.8, 1, 1.2, 1.5, 2$ ?
  - c. Can you guess why our estimate of NH fraction of the sink changes as it does in (4b)?

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