Tech Note for Updating Selective Withdrawal Algorithm and Computation in CE-QUAL-W2

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The updates and new features of the selective withdrawal algorithm and computation in the CE-QUAL-W2 V5.0 included:

1. Calculation of the Withdrawal Zone Limits

The limits of withdrawal zone are computed based on Froude number formulations developed by Bohan and Grace (1973) and Smith et al. (1987). The first case considers no interference of the withdrawal zone with a boundary, while the second one considers simultaneous surface and bottom interference of the withdrawal zone.

A horizontal withdrawal angle is included into the calculation of the withdrawal zone limits

The selective withdrawal algorithm determines the vertical zone limits based on the outflow, outlet geometry and in-pool densities. It then assigns flow for each layer within the identified withdrawal zone. This algorithm requires setting upper and lower boundary layers from which outflows do not occur. If inflow enters a stratified region, the thickness of the inflow zone depends upon the inflow rate and the existing density gradient over the zone of inflowing water. The velocity profile represents the distribution of velocity throughout the withdrawal zone. A typical velocity profile for a withdrawal is shown in Figure 1.

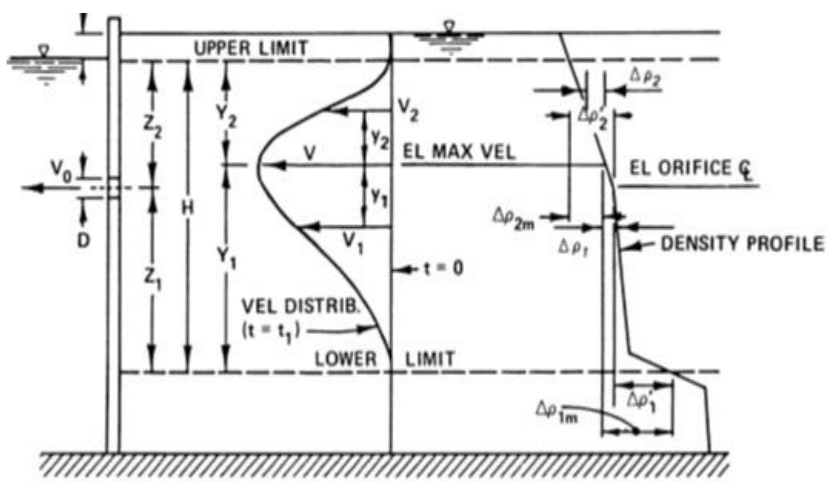

Figure 1. Schematic representation of the typical withdrawal profile

The physical significance of the effective angle of withdrawal (x-y plane) is observed as shown in Figure 2. Consequently, the inclusion of the effective angle of withdrawal has added a level of pseudo multidimensionality to the simulation of selective withdrawal. Smith et al. (1987) introduced the concept of the horizontal withdrawal angle of the outlet in the withdrawal equation to calculate distance between the port center line and the upper or lower withdrawal limit. This withdrawal angle is measured in radians on a horizontal plane with the vertex at the point sink and encompasses the geometry of the near field in the selective withdrawal structure. The horizontal withdrawal angle for a withdrawal port located on a wide dam with a vertical face would be π radians (180°).

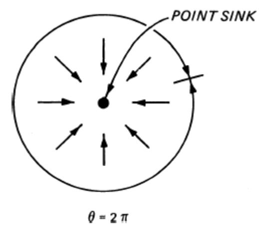

Figure 2. Plan view of withdrawal angles (θ)

The selective withdrawal equations are implemented in the W2 model to line or point sinks in their downstream and lateral withdrawal configurations. The characteristics of outflows to both point and line sinks are subject to quasi-linear stratification. The point sink approach assumes the port inflow is radial both longitudinally and vertically while the line sink approach assumes inflow approaches the outlet radially in the vertical. Regardless of whether interference exists at the upper or lower withdrawal boundaries, Bohan and Grace (1973) developed the following equations to calculate the withdrawal zone, including with withdrawal angle.

Point sink withdrawal limits:

$$d = \left(\frac{\pi c_{bi} Q}{\theta N}\right)^{1/3} \quad \textbf{(1a)}$$

Line sink withdrawal limits:

$$d = \left(\frac{2 c_{bi} q}{\theta N}\right)^{0.5} \quad \textbf{(1b)}$$

For a point sink, the relationship states that the half-thickness of the withdrawal zone is a function of the ratio of the flow rate to the buoyancy frequency of the fluid raised to the one-third power. In contrast, the corresponding relationship for a line sink equates the half thickness of the withdrawal zone to the square root of the unit discharge divided by the buoyancy frequency. The buoyancy frequency (*N*) is calculated from the density profile data reported by Wunderlich and Elder (1973).

$$N = \sqrt{\frac{g}{\rho}\frac{\Delta\rho}{\Delta z}} \quad \textbf{(1c)}$$

or

$$N = \sqrt{\frac{g}{\rho}\frac{\Delta\rho}{d}} \quad \textbf{(1d)}$$

where

θ = effective angle of withdrawal, radians
d = withdrawal zone half height, *m*
Q = total outflow, $m^3\ s^{-1}$
N = internal buoyancy frequency, *Hz,*
q = Q/W, outflow per unit width, $m^2\ s^{-1}$
c_{bi} = boundary interference coefficient
W = port width of the line sink
$\Delta\rho$ = density difference between the center line of the port and the upper or lower withdrawal limit
ρ = density at the center line of the port.

Smith et al. (1987) presented that boundary interference coefficient (c_{bi}) = 8 for intermediate withdrawal, 2 for bottom or top withdrawal.

Boundary interference is included into the calculation of the withdrawal zone limits

The vertical withdrawal zones and vertical distribution of layer-specific outflow rates play a critical role in determining the temperature and water quality characteristics of the total withdrawal outflow. The withdrawal zone is generally hydrodynamically established within a thickness of one withdrawal zone near the reservoir. However, the development of the withdrawal zone may be affected if there are physical obstructions, such as the reservoir floor. Figure 3 shows the potential velocity profile when the withdrawal port elevation is located close to the water surface elevation or the reservoir bottom. In such cases, the physical boundary conditions can interfere with the velocity profile.

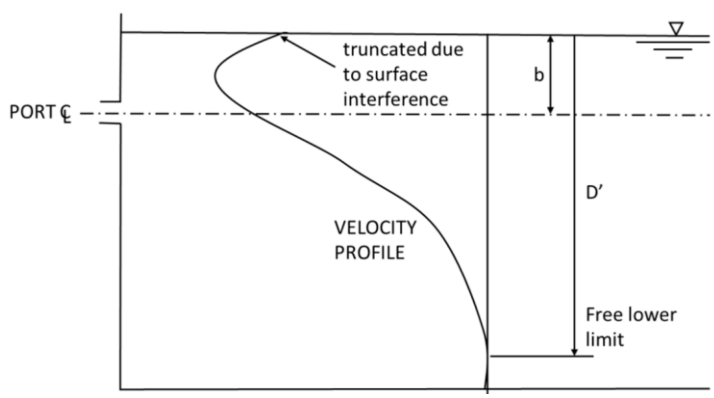

Figure 3. Withdrawal profile with a surface boundary interference.

Eq. 1 only has only been applied to port withdrawal zones that do not intersect either the water surface (upper boundary) or the reservoir bottom (lower boundary). If boundary interference is encountered, the withdrawal zone limits must be updated within the W2 model. Smith et al. (1987) developed an analytical solution for Eq. 1 to account for these obstructions. Boundary interference is determined by substituting the distance from the outlet to the physical boundary for Z and the density difference between the port centerline and the surface or bottom layer for $\Delta\rho$.

$$\frac{Q}{D'^3 N} - \frac{0.125\phi}{X^3}\frac{\theta}{\pi} = 0 \qquad \textbf{(2a)}$$

$$\phi = \frac{1}{2}\left[1 + \frac{1}{\pi}\sin\left(\frac{\frac{b}{D'}}{1-\frac{b}{D'}}\pi\right) + \frac{\frac{b}{D'}}{1-\frac{b}{D'}}\right] \quad \textbf{(2b)}$$

$$X = \frac{1}{2}\left[1 + \frac{\frac{b}{D'}}{1-\frac{b}{D'}}\right] \quad \textbf{(2c)}$$

$$N = \sqrt{\frac{g}{\rho}\frac{\Delta\rho}{D'}} \quad \textbf{(2d)}$$

where

Q = discharge flow rate from the withdrawal zone, m^3/sec
D' = distance between the boundary of interference and the free limit of withdrawal, m
N - buoyancy frequency, sec^{-1}
b = distance between the centerline and the interference boundary, m
$\Delta\rho$ = absolute value of the density difference between the surface layer and lower free boundary (for surface interference) or between the bottom layer and upper free boundary (for bottom interference).

Eq. 1 is used when there is no boundary interference, while Eq. 2 is applied to address the boundary interference problem. Both equations incorporate the effective angle of the withdrawal zone. Eq. 1 and 2 can be further rearranged to yield:

$$Q - \frac{\theta N}{\pi c_{bi}} D'^3 = 0 \quad \textbf{(3a)}$$

$$Q - D'^3 N \frac{\theta}{2\pi} \frac{\left(1+\frac{1}{\pi}sin\left(\frac{\frac{b}{D'}}{1-\frac{b}{D'}}\pi\right)+\frac{\frac{b}{D'}}{1-\frac{b}{D'}}\right)}{\left(1+\frac{\frac{b}{D'}}{1-\frac{b}{D'}}\right)^3} = 0 \quad \textbf{(3b)}$$

With b and Q known, location of the free limit (D') is computed by applying the bisection root-finding algorithm to Eq. 3b.

2. Calculation of the Withdrawal Zone Velocity Profile

The calculation of the normalized velocity profile is conducted with incorporating boundary interference and the shifted location of maximum velocity within the profile and velocity profile.

In the current version of the W2 model, once the limits of the withdrawal zone are determined, the normalized velocity profile is computed at the centerline of the outlet port for the withdrawal zone. However, as reported by Bohan and Grace (1973), the maximum velocity occurs at the centerline only if the withdrawal zone is distributed symmetrically around the outlet port. Otherwise, depending on the velocity profile (Figure 1), the location of maximum velocity may be either above or below the centerline of the outlet.

The profile of velocities of outflowing waters is assumed to be approximately parabolic within the withdrawal zone limits. The location (and the layer) of maximum velocity is calculated using Eq. 4 provided by Bohan and Gloriod (1973).

$$Y_1 = H\left[\sin\left(1.57\frac{Z_1}{H}\right)\right]^2 \quad \textbf{(4)}$$

where

Y_1 = vertical distance from the elevation of the maximum velocity to the lower limit of the zone of withdrawal (ft)
H = total thickness of the withdrawal zone ($Z_1 + Z_2$) (ft) or ($Y_1 + Y_2$) (ft)
Z_1 = vertical distance from the outlet centerline to the lower limit of the withdrawal zone (ft)

With the knowledge of the layer containing the maximum profile velocity, the velocity distribution can be determined. Bohan and Grace (1973) developed Equation 5 to calculate the normalized velocity profile. The normalized velocities for the portion of the outflow experiencing interference are calculated based on the vertical location of the maximum profile velocity within the withdrawal zone.

$$v_k = \left(1 - \frac{y_i}{Y_1}\frac{\Delta\rho}{\Delta\rho_m}\right)^2 \quad \textbf{(5)}$$

where

v_k= normalized velocity within a given layer of the withdrawal zone, fps
y_i = vertical distance from the elevation of maximum velocity to that of the given layer, ft
$\Delta\rho$ = difference in density between that at the elevation of maximum velocity and that at the given layer
$\Delta\rho_m$ = difference in density between that at the elevation of maximum velocity and that at the elevation of the upper or lower limit as appropriate.

Multiple outflows are allocated to layers assuming superposition. For the case where both upper and lower boundaries experience interference, theoretical withdrawal limits are calculated for both.

The calculation of the flow rate profile is conducted with two options, 1) Existing method, 2) Davis (1987) with respect to the use of a weighting factor based on the reservoir.

Once the normalized velocity distribution is determined for a port, the withdrawal flow profile is calculated for each layer based on the total flow rate through the port. The flow value for each layer is scaled to sum to the magnitude of the total outflow over the port. The equation used in the current version of W2 model to compute the withdrawal profile is

$$q_k = \frac{v_k H_k W_k}{\sum(v_k H_k W_k)} Q \qquad \textbf{(6)}$$

The W2 model weights the flow rate for a given layer by the layer's thickness and the reservoir width at that layer. The model assumes that the entire reservoir width contributes to the withdrawal. For example, if the reservoir width is 200m at the top of the withdrawal zone and 100m at the bottom of the withdrawal zone, then the flow rate profile at the top of the withdrawal zone would have a weighting factor of twice that of the bottom of the withdrawal zone. This assumption could result in a flow rate profile that has an excessively larger outflow in the upper portion of the withdrawal zone.

Davis et al. (1987) developed a different equation to calculate the flow rate profile over the withdrawal zone. This profile is normalized in the sense that the individual point velocities are divided by the total profile velocity and subsequently scaled such that would yield the discharge producing them.

$$q_k = \frac{v_k}{\sum v_k} Q \qquad \textbf{(7)}$$

where

q_k = flow from the k^{th} layer
v_k = normalized velocity at the k^{th} layer
H_k = thickness of the k^{th} layer
W_k = width of the k^{th} layer
Q = total flow out of the port

The updated withdrawal computation gives the user a choice of applying Eq. 6 or Eq. 7. The effective angle of withdrawal, which is used in the calculation of withdrawal zone limits, has been added to the W2 control file and defined at the end of the structures section of the control file.

	SINKC3 - Sink type used in the selective withdrawal algorithm, LINE or POIN	POINT		
	SINKC4 - Sink type used in the selective withdrawal algorithm, LINE or POIN	POINT		
Increase # of rows if have more than 5 structures	SINKC5 - Sink type used in the selective withdrawal algorithm, LINE or POIN			
	ESTR1-Centerline elevation of structure 1, m	104		
	ESTR2-Centerline elevation of structure 2, m	110		
	ESTR3-Centerline elevation of structure 3, m	107		
	ESTR4-Centerline elevation of structure 4, m	72		
Increase # of rows if have more than 5 structures	ESTR5-Centerline elevation of structure 5, m			
	WSTR1 - Structure 1 width if "LINE" chosen, Width of structure (line sink), m	0		
	WSTR2- Structure 2 width if "LINE" chosen, Width of structure (line sink), m	0		
	WSTR3- Structure 3 width if "LINE" chosen, Width of structure (line sink), m	0		
	WSTR4- Structure 4 width if "LINE" chosen, Width of structure (line sink), m	0		
Increase # of rows if have more than 5 structures	WSTR5- Structure 5 width if "LINE" chosen, Width of structure (line sink), m			
	Theta_STRT1 - Structure 1 Effective Angle of Withdrawal, radians	3.14		
	Theta_STRT2 - Structure 2 Effective Angle of Withdrawal, radians	3.14		
	Theta_STRT3 - Structure 3 Effective Angle of Withdrawal, radians	3.14		
	Theta_STRT4 - Structure 4 Effective Angle of Withdrawal, radians	3.14		
Increase # of rows if have more than 5 structures	Theta_STRT5 - Structure 5 Effective Angle of Withdrawal, radians			
	PIPES	**PIPE1**	PIPE2	PIPI
	IUPI - Upstream segment number			
	IDPI - Downstream segment number			
	EUPI - Elevation upstream invert, m			
	EDPI - Elevation downstream invert, m			

3. Single Wet Well Withdrawal with Multi-Level Ports

A multi-level port structure allows the release of flows at various elevations in the water column. Simultaneous multi-level withdrawal from stratified reservoirs involves in the interaction between overlapping withdrawal zones in the pool. For single wet well structure where the total flow rate is known but the flow rate through individual ports is unknown, the method developed by Howington (1990) can be applied to estimate flows through the individual ports. The updated withdrawal algorithm includes this method to estimate port flows in single wet well structures.

Calculate the vertical shift for overlapping withdrawal zones

If withdrawal zones from different ports overlap vertically, the velocities of one of the withdrawal profiles can influence those of the other by reducing the shear force in any horizontal layer (Figure 5). The result of this influence is an increase in velocities for both withdrawal layers within this zone. Shifting the inner withdrawal limits to increase the depths of both zones can lead to an increase in velocities in the overlapping zone. Since the predicted velocity profile given by Eq. 4 for each individual outlet does not account for this apparent vertical shift, the superposition of the individual profiles will not address it either. The limits of withdrawal zones when operating multi-level ports need to be recalculated in W2. This adjustment is necessary to correct the difference between the simple superposition of the predicted withdrawal zones for each outlet and the actual zone as shown in Figure 4.

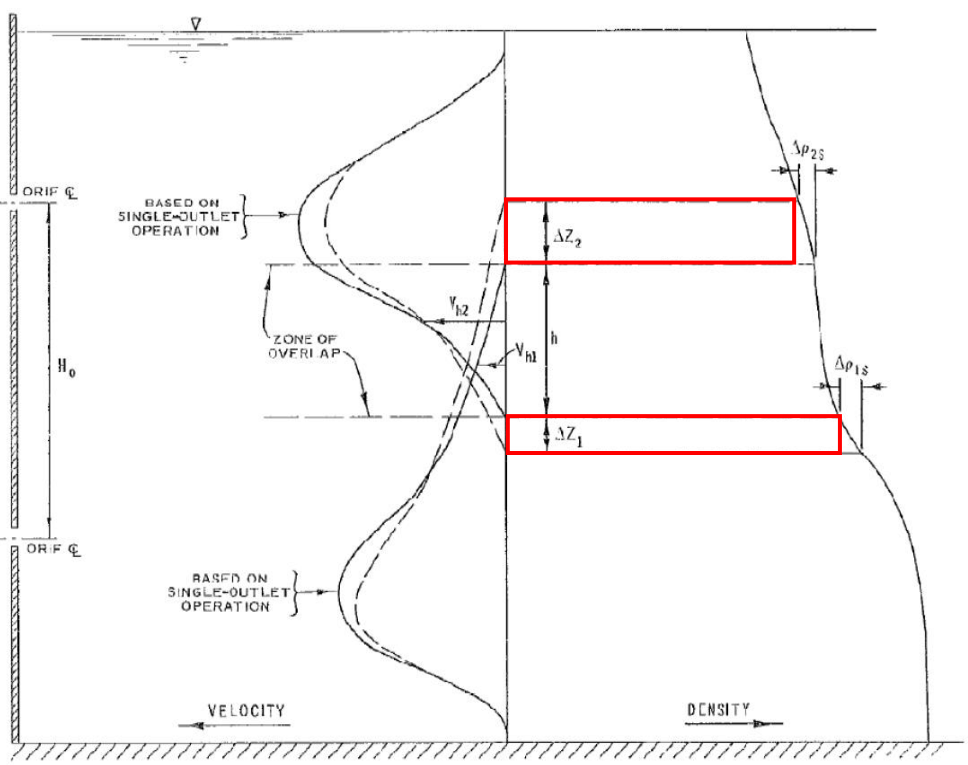

Figure 4. Multi-port withdrawal profiles with overlapping withdrawal zones

Bohan and Gloried (1972) modified the velocity distribution to produce individual withdrawal zones whose superposition closely approximated the actual zone. Bohan and Gloried (1973) determined that the adjustment was a function of the amount of overlap of the two individual zones, the vertical spacing between the outlets, the density distribution of the reservoir, and the average velocity of each withdrawal zone in the region of overlap. The shift equation is given as follows.

$$v_h - 0.7\left(\frac{h}{H_0}\right)^{1.25}\sqrt{\frac{\Delta\rho_s}{\rho_s}g\Delta Z} = 0 \qquad \textbf{(8a)}$$

or

$$\Delta Z = \frac{(v_h)^2}{0.49\left(\frac{h}{H_0}\right)^{2.5}\frac{\Delta\rho_s}{\rho_s}g} \qquad \textbf{(8b)}$$

where

Δz = vertical shift of the withdrawal limit, ft
v_h = average velocity in the zone of overlap or lower withdrawal layer, ft/sec
$\Delta\rho_s$ = density difference of fluid between the elevations of the original withdrawal limit and the shifted withdrawal limit, g/cm^3
ρ_s = density of fluid at the elevation of the original withdrawal limit, g/cm^3
h = vertical distance of overlap of the velocity profiles, ft
Ho = vertical distance between centerlines of ports, ft

The important variables in Eq. 8 are the size of the outlet, the velocity through the outlet, the density profile, and the vertical location of the outlet relative to the density profile.

Calculate flow distribution and outflow for simultaneous, multi-level ports

Outflow through ports depends upon the discharge, stratification, geometry of the port, bathymetry of the impoundment, and any in-reservoir circulation. For simultaneous, multiple-level withdrawal through a single flow control as shown in Figure 5, the stratified-flow-distribution algorithm developed by Howington (1990) has been found to predict adequately the flow distribution among open ports, given the appropriate input.

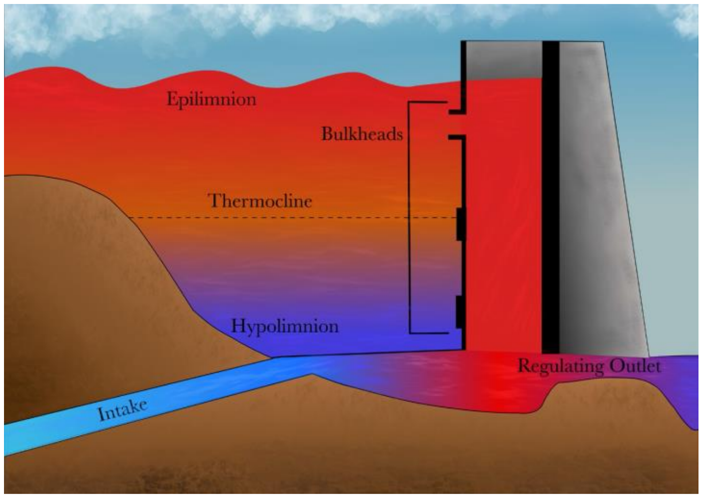

Figure 5. Multi-port withdrawal through a single wet well.

When two or more selective withdrawal zones overlap, superposition is used to get the total velocity profile. Howington (1990) developed the following equation for estimating flows through multiple ports operating simultaneously.

$$QT = \sum_i^n \sqrt{\frac{2gA_i^2}{k_i}(BH_i + \Delta H)} \qquad \textbf{(9)}$$

where

QT - total structure discharge, m^3/sec
n - number of open port levels
A_i = area of port level, m^2
k_i = head-loss coefficient for port level
BH_i = buoyancy head for port level, m
ΔH = water surface differential between the pool and the well, m

The buoyancy head values (BH) for port level are dependent on the density of fluid entering the ports, the density in the reservoir at the port elevation is used in Eq. 9. The buoyancy head (BH) is estimated by:

$$BH_i = BH_{i-1} + \frac{1}{\rho_i}\int_{Z_{i-1}}^{Z_i}\left[\rho(Z) - \frac{\sum_{j=1}^{i-1} Q_j\rho_j}{\sum_{j=1}^{i-1} Q_j}\right]dz \qquad \textbf{(10)}$$

where

ρ_i = water density at port i
Q_j = flow entering port level j
ρ_j = water density at port j

$\rho(Z) =$ water density at elevation z
Z = distance from the port to the upper (lower) withdrawal limit, m

To determine the flow rate through multiple ports given the total flow rate *QT*, the water surface differential between the pool and the well (ΔH) is solved using the bisection root-finding algorithm:

$$QT - \sum_i^n \sqrt{\frac{2gA_i^2}{k_i} HL_i} = 0 \quad \textbf{(11)}$$

The head loss HL_1 across the top port i=1 is:

$$HL_1 = \Delta H \quad \textbf{(12)}$$

and for i >1 the head loss is calculated with

$$HL_i = \frac{1}{\rho_i} \int_i^{i-1} [\rho(z) - \rho_{i-1}]\, dz + \frac{\rho_{i-1}}{\rho_i} HL_{i-1} \quad \textbf{(13)}$$

where z is elevation.

Once ΔH has been solved for, the flow through each port can be calculated using

$$Q_i = \sqrt{\frac{2gA_i^2}{k_i} HL_i} \quad \textbf{(14)}$$

In this approach it is assumed that the density of water in the reservoir at the center line elevation of the port is the density of water that will be withdrawn through the port.

While the algorithm above provides a method for estimating the flows through multiple ports operating simultaneously, there are several parameters that must be provided by the user: area and head-loss coefficient (*ki*) for each port. As Howington (1990) indicated, the head-loss coefficients used in Eq. 9 are dependent upon many variables and are difficult to define accurate values for them. A new input file named "***w2_singlewetwellwd.csv***" is required be in the model directory if the single wet well feature is to be applied. This file is a comma delimited text file as shown below. These parameters are used in addition to other structural withdrawal parameters in the control file.

	A	B	C	D	E
1	**$W2 V5 Updated selective with drawal computation**				
2	Single wet well withdrawal computation algorithm referenced from Howington (1990)				
3	'WD Calculation for Single Wet Well Outflow'	OFF	OFF	OFF	OFF
4	**Shape of intake port (1 = circular 2 = rectangular)'**	BR1	BR2	BR3	BR4
5	STR1	1	1	1	1
6	STR2	1	1	1	1
7	STR3	1	1	1	1
8	STR4	1	1	1	1
9	STR5	1	1	1	1
10	**Cross-sectional area coefficient for intake port (meter) (diameter if port shape=1 port height if port shape=**	BR1	BR2	BR3	BR4
11	STR1	3.3	3.3	3.3	3.3
12	STR2	3.3	3.3	3.3	3.3
13	STR3	3.3	3.3	3.3	3.3
14	STR4	3.3	3.3	3.3	3.3
15	STR5	3.3	3.3	3.3	3.3
16	**'Cross-sectional area coefficient for intake port (meter) if port shape=2'**	BR1	BR2	BR3	BR4
17	STR1	6	6	6	6
18	STR2	6	6	6	6
19	STR3	6	6	6	6
20	STR4	6	6	6	6
21	STR5	6	6	6	6
22	**'Intake port loss coefficient'**	BR1	BR2	BR3	BR4
23	STR1	0.2	0.2	0.2	0.2
24	STR2	0.2	0.2	0.2	0.2
25	STR3	0.2	0.2	0.2	0.2
26	STR4	0.2	0.2	0.2	0.2
27	STR5	0.2	0.2	0.2	0.2
28	**Calculating vertical flow rate profile (1 =Original, 2 =Davis1987)'**	BR1	BR2	BR3	BR4
29	STR1	1	1	1	1
30	STR2	1	1	1	1
31	STR3	2	2	2	2
32	STR4	2	2	2	2
33	STR5	2	2	2	2

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