WATER TEMPERATURE MODELING

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Environmental Systems Modeling Team

DISCOVER | DEVELOP | DELIVER

Heat and Temperature

	Coffee	Constituent (e.g. salinity)	Heat Balance
Extensive	sugar	mass	heat
Intensive	sweetness	concentration	temperature
Relationship	lumps/cup	mass/volume	Heat/(density*specific heat capacity*volume)

Advection-Dispersion Equation Balance

$$\frac{\partial B\Phi}{\partial t}+\frac{\partial UB\Phi}{\partial x}+\frac{\partial WB\Phi}{\partial z}-\frac{\partial}{\partial x}\left[BD_x\frac{\partial \Phi}{\partial x}\right]-\frac{\partial}{\partial z}\left[BD_z\frac{\partial \Phi}{\partial z}\right]=q_\Phi B+S_\Phi B \qquad \textbf{(4-12)}$$

where:

- Φ = laterally averaged constituent concentration, $g\ m^{-3}$
- D_x = longitudinal temperature and constituent dispersion coefficient, $m^2\ sec^{-1}$
- D_z = vertical temperature and constituent dispersion coefficient, $m^2\ sec^{-1}$
- q_Φ = lateral inflow or outflow mass flow rate of constituent per unit volume, $g\ m^{-3}\ sec^{-1}$
- S_Φ = laterally averaged source/sink term, $g\ m^{-3}\ sec^{-1}$

Substitute H into the equation for Φ, which relates to temperature as:

$$T=\frac{H}{\rho C_p V}$$

$$H=\rho C_p V T$$

Units:

cgs system:

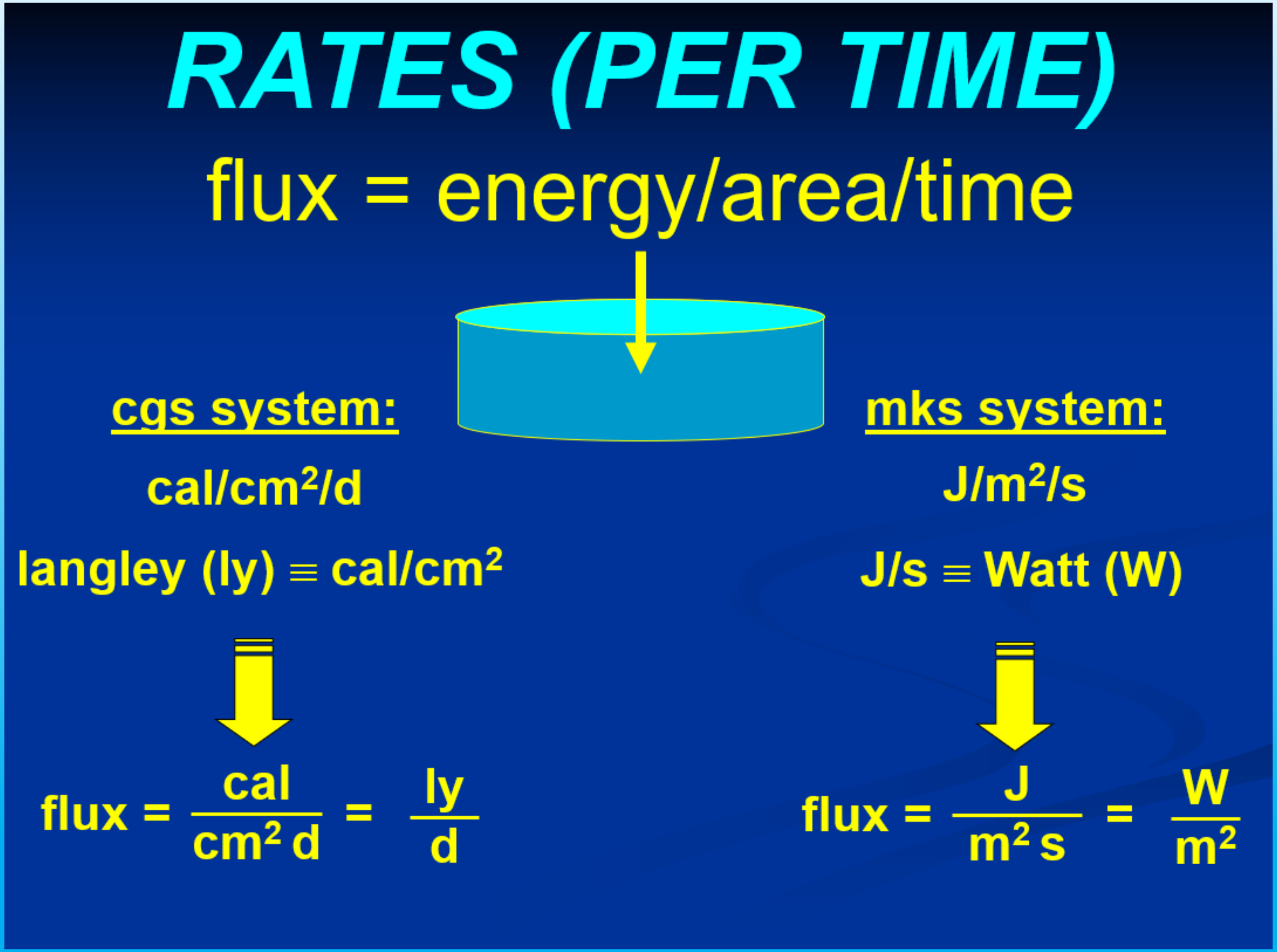
$$\frac{\cancel{cal}}{\frac{\cancel{g}}{\cancel{cm^3}}\ \frac{\cancel{cal}}{\cancel{g}\ ^\circ C}\ \cancel{cm^3}} \Rightarrow {}^\circ C$$

mks system:

$$\frac{\cancel{J}}{\frac{\cancel{kg}}{\cancel{m^3}}\ \frac{\cancel{J}}{\cancel{kg}\ ^\circ C}\ \cancel{m^3}} \Rightarrow {}^\circ C$$

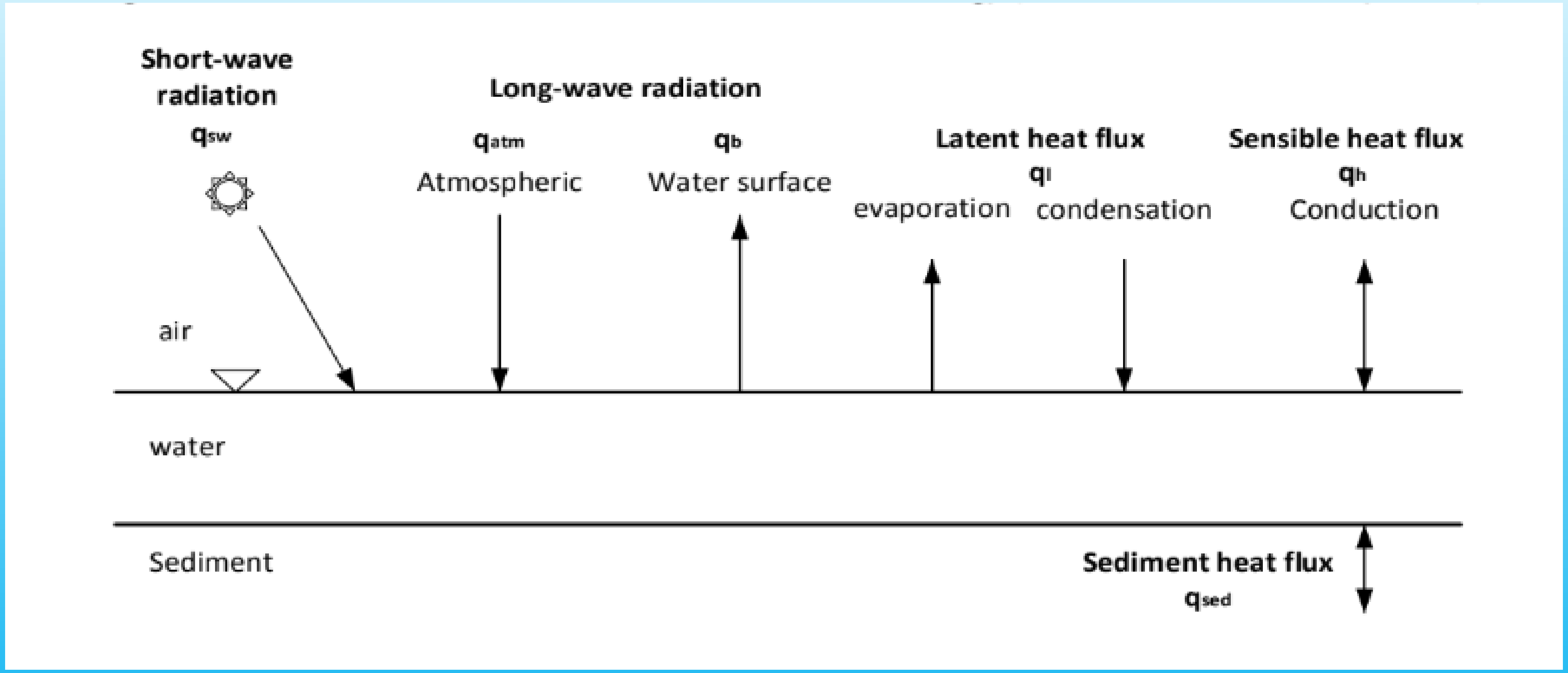
1 cal = 4.168 J

Rates



Blue slide images from Heat, Temperature, and Stratification lecture, Steve Chapra (2019)

Energy Balance



From ERDC-EL TR-16-1

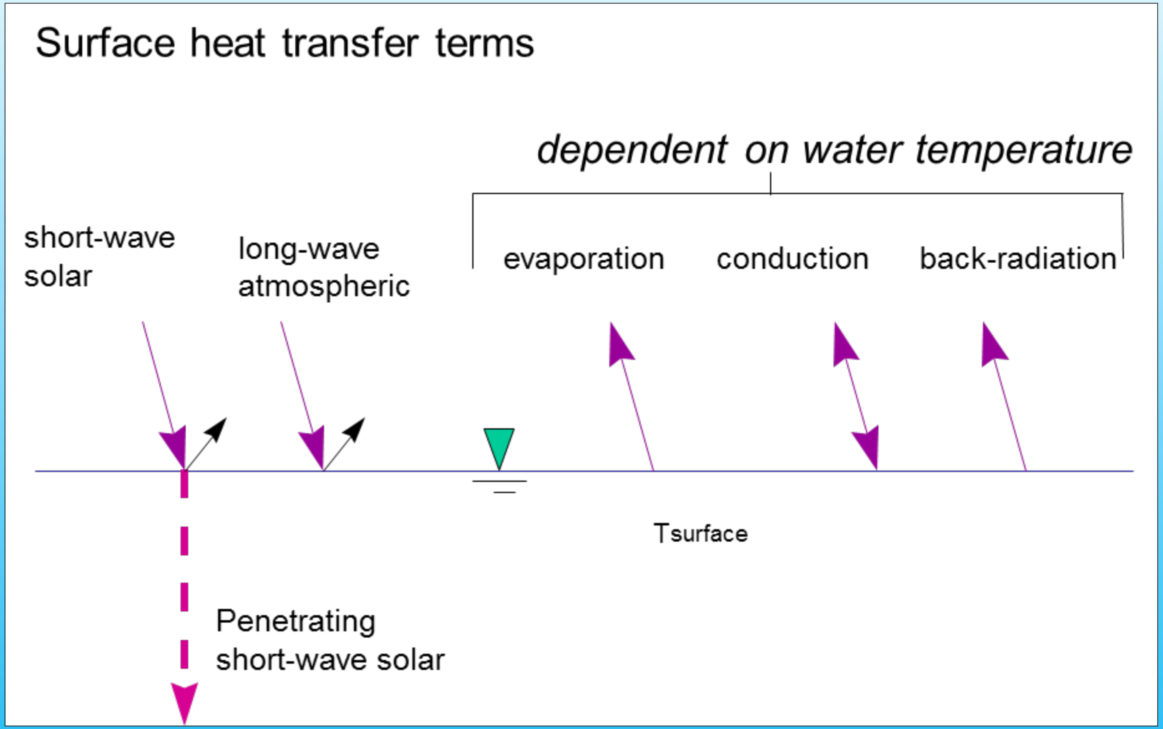
Sources and Sinks

Sources:

- Short-wave solar radiation
- Long-wave atmospheric radiation
- Conduction of heat from the atmosphere to the water
- Direct heat inputs

Sinks:

- Long-wave radiation emitted by the water
- Evaporation
- Conduction of heat from the water to the atmosphere



Source: CE-QUAL-W2 User's Manual

Radiation

Everything that has a temperature gives off electromagnetic radiation (light). Shortwave radiation contains higher amounts of energy, and longwave radiation contains a smaller amount of energy. Therefore, the sun gives off shortwave radiation, as it is extremely hot and has a lot of energy to give. On the other hand, Earth's energy is emitted as longwave radiation, as it is much cooler but still emits radiation.

Source: North Carolina Climate Office

Latent vs. Sensible Heat

Sensible Heat:

Sensible heat is when energy is transferred as heat to an object, changing the temperature but not its state. If you can measure the temperature of the heat, it is sensible. A body (solid, liquid, or gas) of mass, m, and specific heat, c, is heated to change its temperature from T1 to T2 without changing its state. Indeed, the volume or the pressure of the body is unchanged. The energy received by the body responsible for its risen temperature is given by the relation:

- $Q = m*c*(T2 – T1)$, in joules
- $Q = m*c*(T2 – T1)*1055.06$, in BTU

Latent Heat:

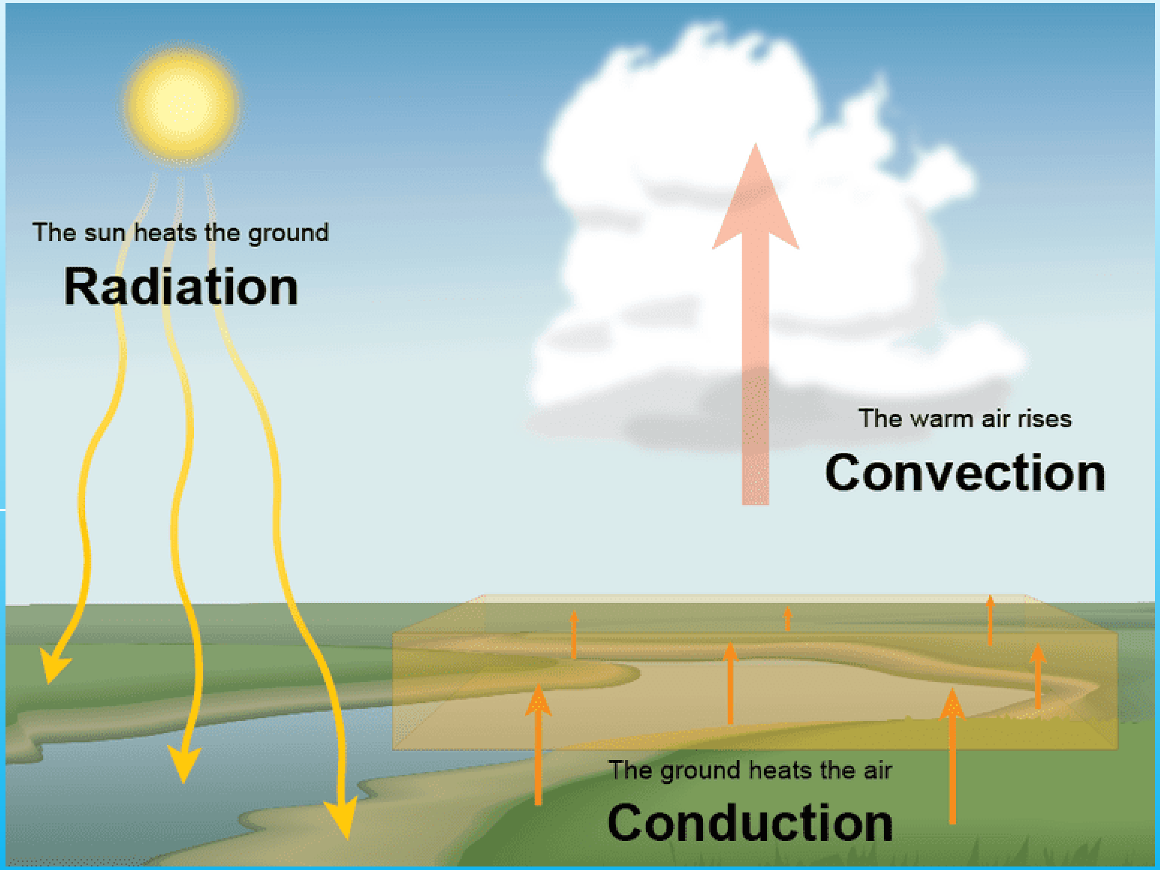
In contrast to sensible heat, latent heat is the energy released or absorbed that changes the state of a body during a constant temperature process. This process leaves temperature unaffected – it won't get higher or lower. The most common forms of latent heat are fusion at vaporization.

Source: hvacbrain.com

Convection vs. Conduction

Convection and conduction are both mechanisms of heat transfer. The main difference between them is:

- Convection: Heat is transferred through the flow of material.
- Conduction: Heat is transferred through collisions of particles that make up the material.



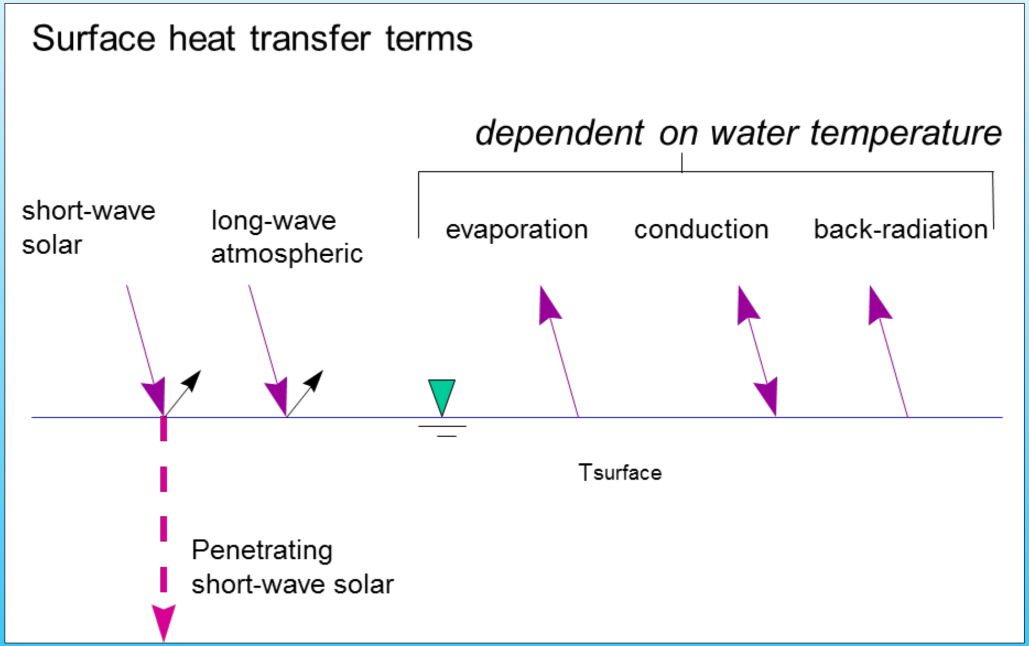
Source: University Corporation for Atmospheric Research (UCAR)

Surface Heat Exchange

The surface heat exchange is computed as:

$$\Phi_n = \Phi_s - \Phi_{sr} + \Phi_a - \Phi_{ar} - \Phi_{br} - \Phi_e - \Phi_c$$

ϕ_n = net surface heat flux, $W\ m^{-2}$
ϕ_s = incoming short-wave solar radiation, $W\ m^{-2}$
ϕ_{sr} = reflected short-wave solar radiation, $W\ m^{-2}$
ϕ_{sn} = net short-wave solar radiation, ϕ_s-ϕ_{sr}, $W\ m^{-2}$
ϕ_a = incoming long-wave atmospheric radiation, $W\ m^{-2}$
ϕ_{ar} = reflected atmospheric long-wave radiation, $W\ m^{-2}$
ϕ_{br} = back long-wave radiation, $W\ m^{-2}$
ϕ_e = evaporative heat loss, $W\ m^{-2}$
ϕ_c = conductive heat loss, $W\ m^{-2}$



Source: CE-QUAL-W2 User's Manual

Short-Wave Solar Radiation

Short-wave solar radiation is:

- Measured directly by a pyranograph or another instrument and contained in a data file with units W/m^2
  - Flawed because its hard to know all the details of how the data was collected… Does it account for reflection? Is it really measuring all radiation including long-wave?
- Computed based on sun angle, according to the following equation:

$$\boldsymbol{\phi_s = 24\left(2.044A_o + 0.1296A_o^2 - 1.941E - 3A_o^3 + 7.591E - 6A_o^4\right) * 0.1314}$$

- This is the EPA formulation based on empirical methods, which is similar to the Bird Clear Sky Model (https://www.nrel.gov/grid/solar-resource/clear-sky.html)
  - Calculates for radiation at sea-level. Elevation adjustments are made according to Annear and Wells 2007.
  - This equation is for clear day radiation, so we need to correct for cloud cover and shading from mountains or trees later.
  - Reflected short-wave atmospheric radiation is defined by the model user as a percentage.

Terms:

- **Solar Altitude (A_o)**
- **Local Hour Angle (H)**
- **Equation of time – difference between true solar time and mean solar time (EQT)**
- **Local hour and lat/long terms**
- **Solar Declination Angle (δ)**
- **Angular Fraction of the Year (τ_d)**

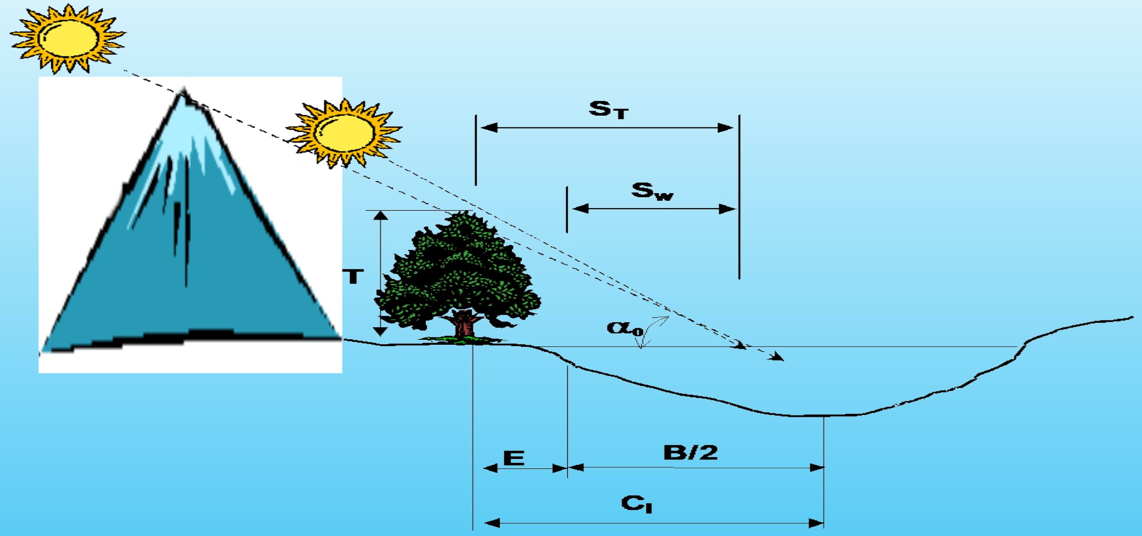
Source: CE-QUAL-W2 User's Manual, Chapter 4

Shading

- Shading of the waterbody can occur from cloud cover or nearby physical features, such as trees, mountains, or buildings.
- The W2 model uses the following equation:

$$\phi_{s\_net} = \phi_{s\_clearsky}(1 - 0.65C^2)$$

- Where C is defined in the met.npt file as 'cloud'.
- Shading from physical features on the ground is defined in the shade.npt file as 'dynshade'.



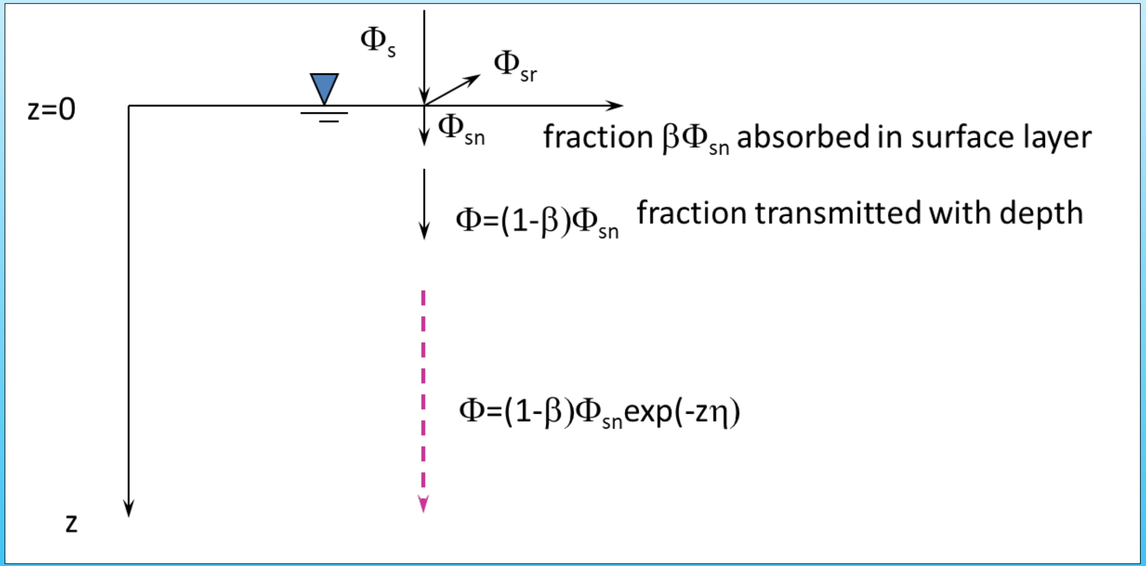
Short-Wave Solar Radiation Penetration Depth

Short wave solar radiation penetrates the surface and decays exponentially with depth according to Beer's Law:

$$\phi_s(z) = (1-\beta)\phi_{sn}e^{-\eta z}$$

where:

$\phi_s(z)$ = short wave radiation at depth z, W m^{-2}
β = fraction absorbed at the water surface
η = extinction coefficient, m^{-1}
ϕ_{sn} = net short-wave radiation reaching the surface, W m^{-2}



Long-Wave Atmospheric Radiation

The long wave atmospheric radiation is computed from air temperature and cloud cover or air vapor pressure using Brunt's formula. The right-hand terms are all water surface temperature dependent.

$$DLR = \sigma \varepsilon_a {T_a}^4 \tag{26}$$

where σ is the Stefan–Boltzmann constant. ε_a and $T_a(K)$ are the clear-sky air emissivity and the screen level (approximately 2 m above the surface) temperature, respectively. In meteorological parameter-based models, the air emissivity ε_a is a function of the screen-level water vapor pressure e and the screen-level temperature T_a or of T_a alone. Thus, the surface DLR can be estimated using the screen-level temperature and humidity by these models. The detailed information of the eight selected models is presented in **Table 11**.

Table 11. Eight meteorological parameter-based methods for estimating the surface DLR

Model source	Algorithm
Angstrom (1918)	$\varepsilon_a = 0.83 - 0.18 \times 10^{(-0.067e)}$
Brunt (1932)	$\varepsilon_a = 0.605 + 0.048e^{0.5}$

Source:
S. Liang, D. Wang, X. Tao, J. Cheng, Y. Yao, X. Zhang, T. He, 2.12 - Methodologies for Integrating Multiple High-Level Remotely Sensed Land Products, Editor(s): Shunlin Liang, *Comprehensive Remote Sensing*, Elsevier, 2018

Water-Emitted Long-Wave Radiation

Water surface back-radiation is computed as:

$$\phi_{br} = \varepsilon\sigma(T_s + 273.15)^4$$

where:

ε = emissivity of water, 0.97
σ = Stephan-Boltzmann constant, *5.67 x 10*$^{-8}$ *W* m^{-2} ^{o}K^{-4}
T_s = water surface temperature, ^{o}C

Evaporative Heat Loss

Also called latent heat loss, will not change the temperature, but the heat content will change, because of the energy it takes to change phases.

Evaporative heat loss is computed as:

$$\phi_e = f(W_z)(e_s - e_a)$$

where:

$f(W_z)$ = evaporative wind speed function at wind height of z, $W\ m^{-2}\ mm\ Hg^{-1}$
e_s = saturation vapor pressure at the water surface, mm Hg
e_a = atmospheric vapor pressure, mm Hg

Conductive/Convective Heat Loss

Surface heat conduction is computed as:

$$\phi_c = C_c f(W_z)(T_s - T_a)$$

where:

C_c = Bowen's coefficient, *0.47* mm Hg $^{o}C^{-1}$
T_a = air temperature, ^{o}C

Wind Function

The model user can include different evaporation formulations using an evaporation wind speed formula of the form:

$$f(W_z) = a + bW_z^c$$

where:

$f(W_z)$ = wind speed function with wind measured at height z, $W\ m^{-2}\ mm\ Hg^{-1}$
a = empirical coefficient, default = 9.2
b = empirical coefficient, 0.46 default = 0.46
c = empirical coefficient, default = 2
W_z = wind speed measured at 2 m above the ground, $m\ s^{-1}$

Questions?