Practical

Start by loading usefull libraries:

```
library(dplyr)
library(INLA)
library(ggplot2)
library(patchwork)
library(inlabru)
# load some libraries to generate nice map plots
library(scico)
```

Aim of this practical: In this first practical we are going to look at some simple models

1. A Gaussian model with simulated data
2. A GLM model with random effects
3. A disease mapping model

we are going to learn:

- How to fit a simple model with `inlabru`
- How to explore the results
- How to change the prior distributions
- How to get predictions for missing data points

Linear Model

As our first example we consider a simple linear regression model with Gaussian observations $y_i \sim \mathcal{N}(\mu_i, \sigma^2)$, $i = 1, \dots, N$ where σ^2 is the observation error, and the mean parameter μ_i is linked to the linear predictor through an identity function:

$$\eta_i = \mu_i = \beta_0 + \beta_1 x_i$$

where x_i is a covariate and β_0, β_1 are parameters to be estimated.

To finalize the Bayesian model we need to assign a Gamma(a, b) prior to the precision parameter $\tau = 1/\sigma^2$ and two independent Gaussian priors with mean 0 and precision τ_β to the regression parameters β_0 and β_1.

Question

What is the dimension of the hyperparameter vector and latent Gaussian field?
Answer
The hyperparameter vector has dimension 1, $\boldsymbol{\theta} = (\tau)$ while the latent Gaussian field $\boldsymbol{u} = (\beta_0, \beta_1)$ has dimension 2, 0 mean, and sparse precision matrix:

$$\boldsymbol{Q} = \tau_\beta \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Note

We can write the linear predictor vector $\boldsymbol{\eta} = (\eta_i, \dots, \eta_N)$ as

$$\boldsymbol{\eta} = \boldsymbol{A}\boldsymbol{u} = \boldsymbol{A}_1\boldsymbol{u}_1 + \boldsymbol{A}_2\boldsymbol{u}_2 = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \beta_0 + \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} \beta_1$$

Our linear predictor consists then of two components.

Simulate example data

In this practical we will use simulated Gaussian data to get familiar with the `inlabru` workflow. Moreover, we will see how to change the prior distributions both for the fixed effects β_0 and β_1 and for the hyperparameter $\tau = 1/\sigma^2$. First, we simulate data from the model

$$y_i \sim \mathcal{N}(\eta_i, 0.1^2), \; i = 1, \dots, 100$$

with

$$\eta_i = \beta_0 + \beta_1 x_i$$

where $\beta_0 = 2$,$\beta_1 = 0.5$ and the values of the covariate x are generated from an Uniform(0,1) distribution. The simulated response and covariate data are then saved in a `data.frame` object.

```r
beta = c(1,1)
sd_error = 1

n = 100
x = rnorm(n)
y = beta[1] + beta[2] * x + rnorm(n, sd = sd_error)

df = data.frame(y = y, x = x)
```

Fitting a linear regression model with `inlabru`

Defining model components

The model has two parameters to be estimated β_1 and β_2. We need to define the two corresponding model components:

```r
cmp =  ~ Intercept(1) + beta_1(x, model = "linear")
```

The `cmp` object is here used to define model components. We can give them any useful names we like

> **Note**
>
> Note that `Intercept()` is one of `inlabru` special names and it is used to define a global intercept. You should explicitly exclude automatic intercept when not using the special `Intercept` name, e.g.
>
> ```r
> cmp =  ~ -1 + myIntercept(1) + beta_1(x, model = "linear")
> ```

Observation model construction

The next step is to construct the observation model by defining the model likelihood. The most important inputs here are the `formula`, the `family` and the `data`.

The `formula` defines how the components should be combined in order to define the model predictor.

```r
formula = y ~ Intercept + beta_1
```

> **Note**
>
> In this case we can also use the shortcut `formula = y ~ ..` This will tell `inlarbu` that the model is linear and that it is not necessary to linearize the model and assess convergence.

The likelihood is defined using the `bru_obs()` function as follows:

```
lik =  bru_obs(formula = y ~.,
               family = "gaussian",
               data = df)
```

Fit the model

We fit the model using the `bru()` functions which takes as input the components and the observation model:

```
fit.lm = bru(cmp, lik)
```

The `summary()` function will give access to some basic information about model fit and estimates

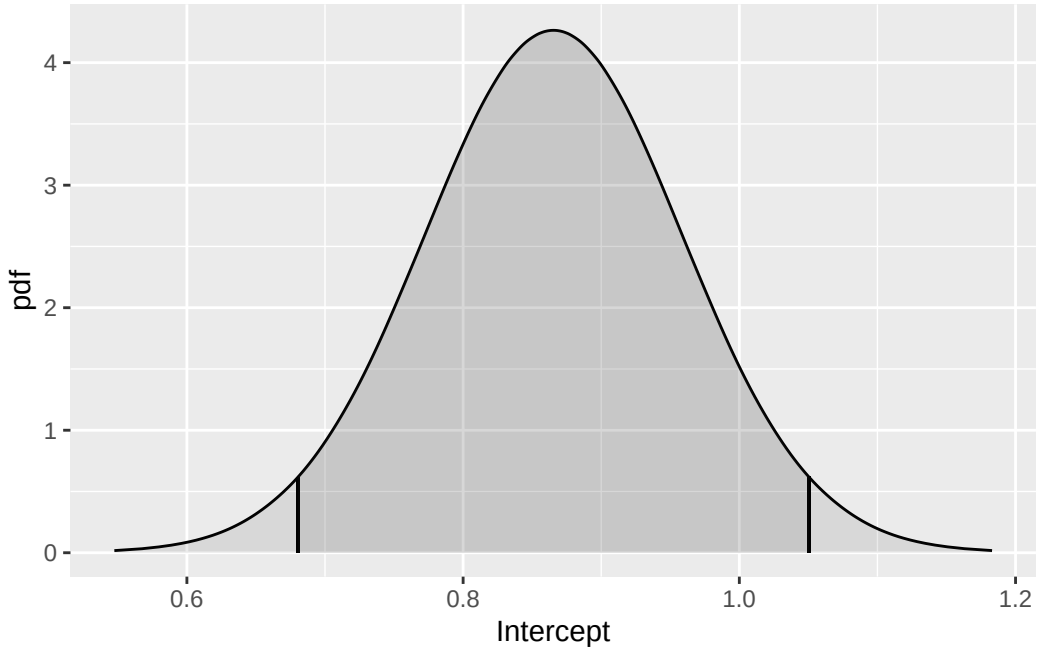
```
summary(fit.lm)
```

```
inlabru version: 2.12.0
INLA version: 25.02.10
Components:
Intercept: main = linear(1), group = exchangeable(1L), replicate = iid(1L), NULL
beta_1: main = linear(x), group = exchangeable(1L), replicate = iid(1L), NULL
Likelihoods:
  Family: 'gaussian'
    Tag: ''
    Data class: 'data.frame'
    Response class: 'numeric'
    Predictor: y ~ .
    Used components: effects[Intercept, beta_1], latent[]
Time used:
    Pre = 0.376, Running = 0.182, Post = 0.0744, Total = 0.633
Fixed effects:
           mean    sd 0.025quant 0.5quant 0.975quant  mode kld
Intercept 0.865 0.095      0.679    0.865      1.051 0.865   0
beta_1    0.967 0.088      0.795    0.967      1.139 0.967   0
```

```
Model hyperparameters:
                                          mean    sd 0.025quant 0.5quant
Precision for the Gaussian observations 1.14 0.161      0.848     1.13
                                          0.975quant mode
Precision for the Gaussian observations       1.48 1.12

Deviance Information Criterion (DIC) ...............: 276.50
Deviance Information Criterion (DIC, saturated) ....: 105.46
Effective number of parameters .....................: 3.00

Watanabe-Akaike information criterion (WAIC) ...: 276.56
Effective number of parameters .................: 2.95

Marginal log-Likelihood:  -157.78
 is computed
Posterior summaries for the linear predictor and the fitted values are computed
(Posterior marginals needs also 'control.compute=list(return.marginals.predictor=TRUE)')
```

We can see that both the intercept and slope and the error precision are correctly estimated. We can then plot the marginal posterior for β_0 as follows:

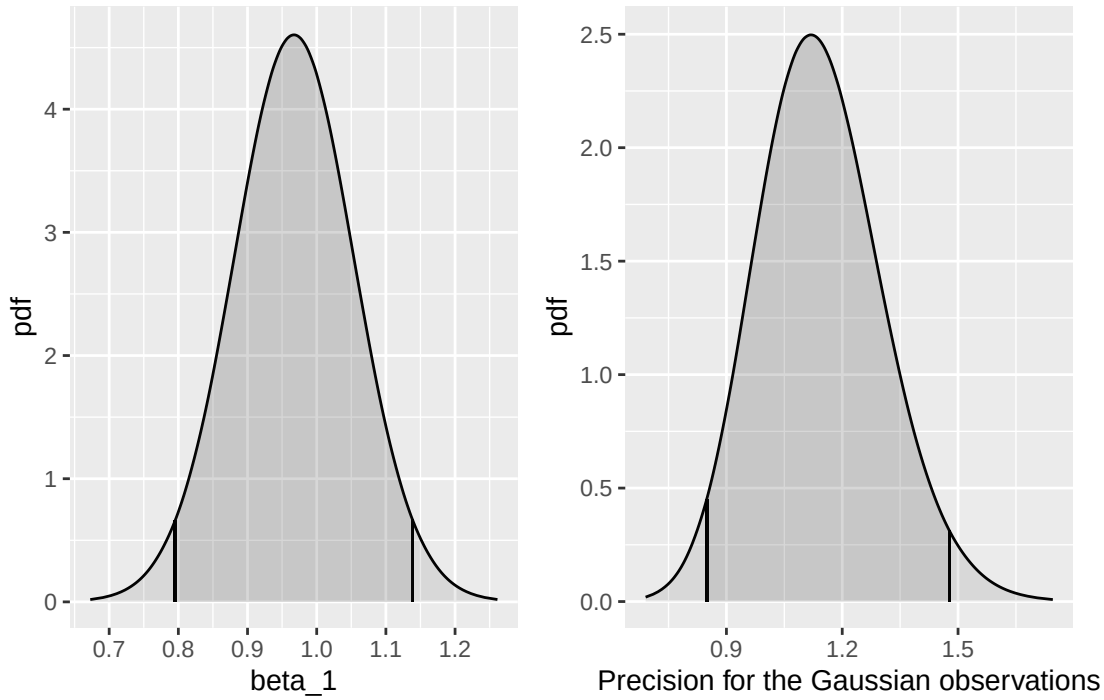
```
plot(fit.lm, "Intercept")
```



Task

Plot the posterior marginals for β_1 and for the precision of the observation error $\pi(\tau|y)$
Take hint
See the `summary()` output to check the names for the different model components.
Click here to see the solution

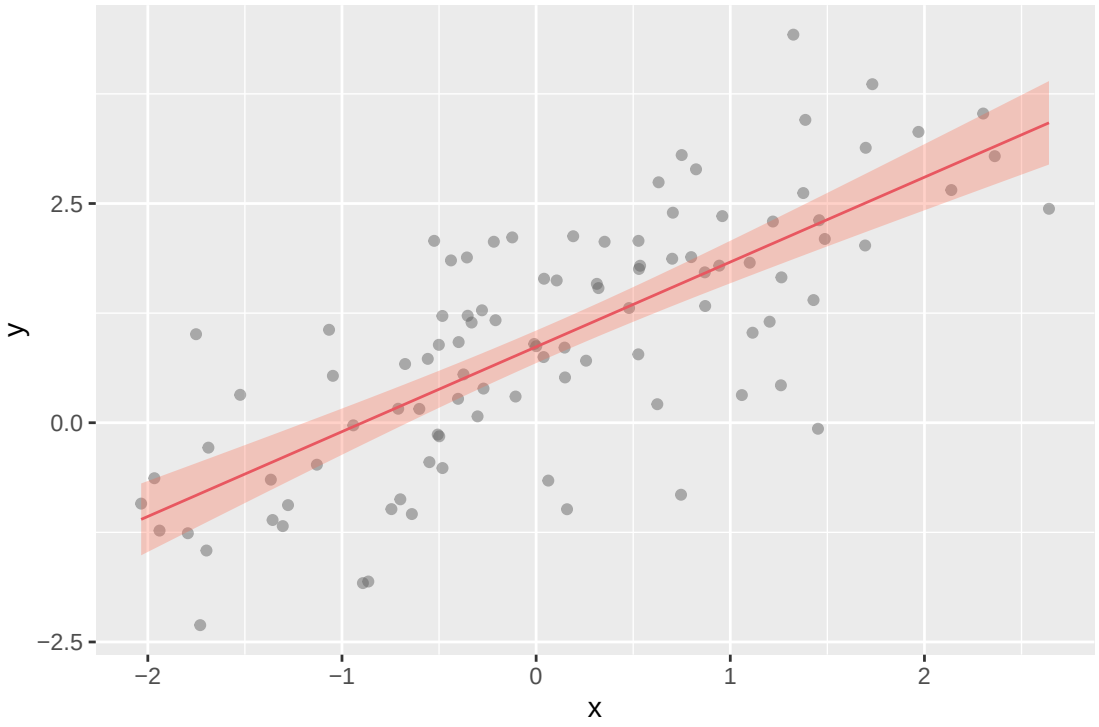
```
plot(fit.lm, "beta_1") +
plot(fit.lm, "Precision for the Gaussian observations")
```



Task

Plot the fitted values with 95% Credible intervals.
Take hint
`bru` objects information about the linear predictor can be accessed through `fit.lm$summary.fitted.values`.
Click here to see the solution

```
df %>% mutate(post_mean = fit.lm$summary.fitted.values[1:100,"mean"],
              q25 = fit.lm$summary.fitted.values[1:100,"0.025quant"],
              q975 = fit.lm$summary.fitted.values[1:100,"0.975quant"])%>%
  ggplot()+geom_point(aes(x=x,y=y),alpha=0.5,color="grey40")+
  geom_line(aes(x=x,y=post_mean),col=2)+
  geom_ribbon(aes(x = x, ymax = q975, ymin = q25),fill="tomato", alpha = 0.3)
```



Generate model predictions

Now we can take the fitted `bru` object and use the `predict` function to produce predictions given a new set of values for the model covariates or the original values used for the model fit

```
new_data = data.frame(x = c(df$x, runif(10)),
                      y = c(df$y, rep(NA,10)))
pred = predict(fit.lm, new_data, ~ Intercept + beta_1)
```

Plot

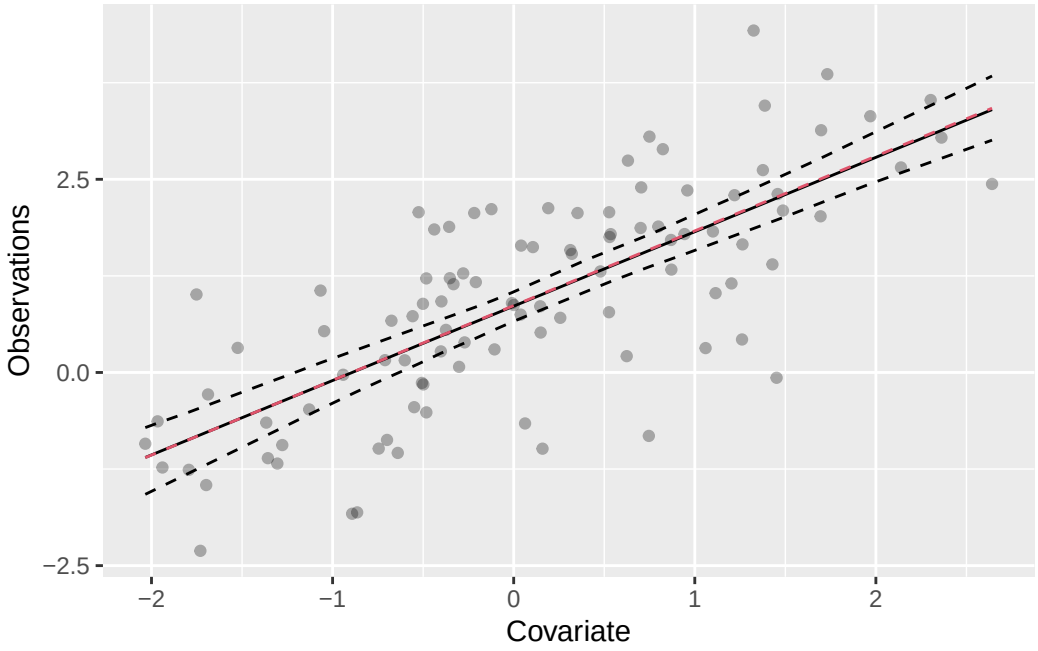


Figure 1: Data and 95% credible intervals

R Code

```
pred %>% ggplot() +
  geom_point(aes(x,y), alpha = 0.3) +
  geom_line(aes(x,mean)) +
  geom_line(aes(x, q0.025), linetype = "dashed")+
  geom_line(aes(x, q0.975), linetype = "dashed")+
  xlab("Covariate") + ylab("Observations")
```