Problem Ch.1

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January 9, 2022

2

$$y = \beta_0 + \beta_1 x + u \ s.t E(u) = \alpha_0 \neq 0$$

Show that the model can always be rewritten with the same slope, but a new intercept and error.

Let $e = u - \alpha_0$, then $E(e) = E(u - \alpha_0) = E(u) - E(\alpha_0) = 0$ The equation of the model is $y = \beta_0 = \beta_1 x + u$

$$\rightarrow y = (\alpha_0 + \beta_0) + \beta_1 x + u - \alpha_0$$
$$\rightarrow y = (\alpha_0 + \beta_0) + \beta_1 x + e, \ \because Assumption$$

3

1.

$$Let \ y = GPA, \ x = ACT$$
$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}, \ \hat{\beta}_1 = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$
$$\hat{\beta}_0 = 0.5681319, \ \hat{\beta}_1 = 0.1021978$$

2.

$$y = \hat{\beta}_0 + \hat{\beta}_1 x + u, \ \hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$$
$$y = \hat{y} + u, \ u = y - \hat{y}$$
$$\sum u = 4.440892e - 16$$

Table 1:

	id	fit	error
1	1	2.714	0.086
2	2	3.021	0.379
3	3	3.225	-0.225
4	4	3.327	0.173
5	5	3.532	0.068
6	6	3.123	-0.123
7	7	3.123	-0.423
8	8	3.634	0.066

3.

$$\widehat{GPA} = 0.5681319 + 0.1021978 * (ACT = 20)$$
$$\therefore \widehat{GPA} = 2.612088$$

4.
$$Variation\ in\ GPA = \sum(y - \hat{y}) = 8.881784e^{-16}$$

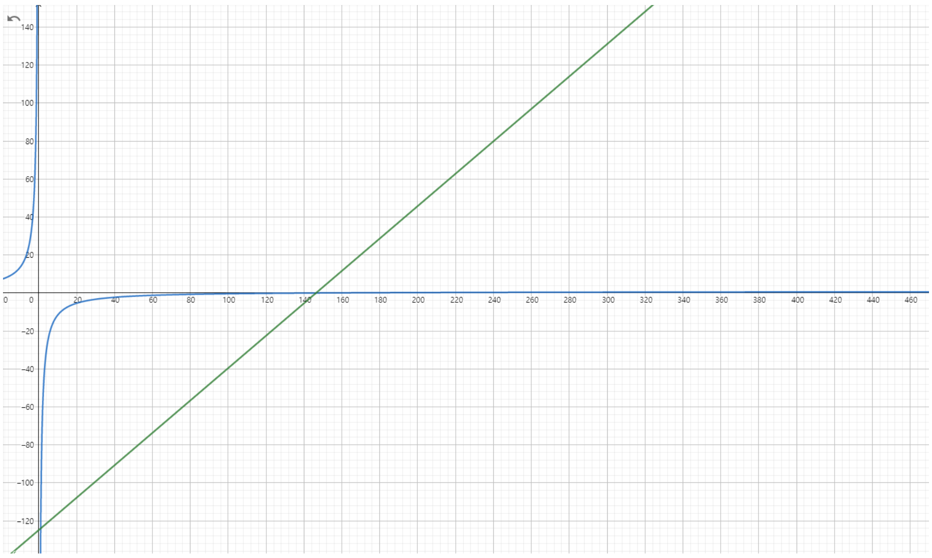
5

1. Intuitively, the consumption without income equals to $\widehat{cons} = \hat{\beta}_0 + \hat{\beta}_1(inc = 0)$.
   $\hat{\beta}_0$ implies the absolute consumption regardless of the income. Considering the economic theories, the intercept is the autonomous consumer expenditure.

2.
$$\widehat{cons} = -124.84 + 0.853(inc = \$30,000),\ \ \widehat{cons} = -99.25$$

3. The green line is MPC and APC for the blue line.

Figure 1: **MPC & APC**



6

1.
$$\frac{\Delta price/price}{\Delta dist/dist} = the\ elasticity\ of\ price\ w.r.t\ dist = 0.312$$

2. I'm not sure. There are many characteristics of influencing the price.

3. The distance from many amenities. It can be correlated with the distance from the incinerator.

7

1.
$$E(u \mid inc) = E(\sqrt{inc}e \mid inc) = \sqrt{inc}E(e \mid inc) = E(e) = 0\ (\because inc\&eareindependent)$$

2.
$$Var(u \mid inc) = Var(\sqrt{inc}e \mid inc) = incVar(e \mid inc) = incVar(e) = inc\ \sigma_e^2$$
$$Var(u \mid inc) = inc\ \sigma_e^2 \rightarrow "Heteroskedasticity"$$

3.

$$Var(sav \mid inc) = Var(\beta_0 + \beta_1 inc + u \mid inc) = Var(u \mid inc) = inc\ \sigma_e^2\ (\because \beta_0,\ \beta_1 inc\ are\ constants.)$$

모든 가계는 의식주 등 생활에 필수적인 지출 비용을 지불한다. 그런데 가계의 소득이 낮을수록 소득의 필수 지출 비중이 높아지고 이로 인하여 저축을 할 수 있는 여유 자금이 줄어든다. 즉, 소득이 줄어들수록 저축량의 변동성은 줄어들게 되므로 $Var(sav \mid inc) = inc\ \sigma_e^2$, 즉 "Heteroskedasticity"가 성립한다.

9

1.

$$y = \hat{\beta}_0 + \hat{\beta}_1 x + u,\ c_1 y = \tilde{\beta}_0 + \tilde{\beta}_1 c_2 x + u,\ where\ c_1\ is\ not\ zero$$

$$\tilde{\beta}_1 = \frac{\sum(c_2x - c_{\bar{2}}x)(c_1y - c_{\bar{1}}y)}{\sum(c_2x - c_{\bar{2}}x)^2}$$

$$\frac{\sum(c_2x - c_{\bar{2}}x)(c_1y - c_{\bar{1}}y)}{\sum(c_2x - c_{\bar{2}}x)^2} = \frac{c_2\sum(x-\bar{x})c_1(y-\bar{y})}{c_2^2\sum(x-\bar{x})^2}$$

$$\tilde{\beta}_1 = \frac{\sum(x-\bar{x})c_1(y-\bar{y})}{c_2\sum(x-\bar{x})^2} = \frac{c_1}{c_2}\hat{\beta}_1$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x} = \bar{y} - \frac{c_2}{c_1}\tilde{\beta}_1\bar{x},\ \because \tilde{\beta}_1 = \frac{c_1}{c_2}\hat{\beta}_1$$

$$\rightarrow \tilde{\beta}_1 = \frac{(\bar{y} - \hat{\beta}_0)c_1}{\bar{x}c_2}$$

$$\tilde{\beta}_0 = c_{\bar{1}}y - \tilde{\beta}_1 c_{\bar{2}}x = c_1\bar{y} - \frac{(\bar{y} - \hat{\beta}_0)c_1}{\bar{x}c_2}c_2\bar{x}$$

$$\tilde{\beta}_0 = c_1\hat{\beta}_0$$

$$\therefore\ \tilde{\beta}_0 = c_1\hat{\beta}_0,\ \tilde{\beta}_1 = \frac{c_1}{c_2}\hat{\beta}_1$$

2.

$$(y + c_1) = \tilde{\beta}_0 + \tilde{\beta}_1(x + c_2) + u,\ y = \hat{\beta}_0 + \hat{\beta}_1 x + u$$

$$\tilde{\beta}_1 = \frac{\sum((x+c_2) - x \bar{+} c_2)((y+c_1) - y \bar{+} c_1)}{\sum((x+c_2) - x \bar{+} c_2)^2} = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2} = \hat{\beta}_1$$

$$\tilde{\beta}_0 = y \bar{+} c_1 - \tilde{\beta}_1 x \bar{+} c_2 = y \bar{+} c_1 - be\hat{t}a_1 x \bar{+} c_2$$

$$\tilde{\beta}_0 = \bar{y} + c_1 - \hat{\beta}_1(c_2 + \bar{x})$$

$$\tilde{\beta}_0 = (\bar{y} - \hat{\beta}_1\bar{x}) + c_1 - \hat{\beta}_1 c_2 = \hat{\beta}_0 + c_1 - \hat{\beta}_1 c_2$$

$$\therefore\ \tilde{\beta}_0 = \hat{\beta}_0 + c_1 - \hat{\beta}_1 c_2,\ \tilde{\beta}_1 = \hat{\beta}_1$$

3. Hints: $log(cy) = log(c) + log(y)$, In part(ii), let c_2=0.

4. Hints: $log(cy) = log(c) + log(y)$, In part(ii), let c_1=0.

10

1.

$$y = \beta_0 + \beta_1 x + u,\ d = x - \bar{x},\ SST_x = \sum(x_i - \bar{x}),\ w = d/SST_x$$

$$\hat{\beta_1} = \frac{\sum(x-\bar{x})(y-\bar{y})}{(x-\bar{x})^2} = \frac{\sum(x-\bar{x})y}{(x-\bar{x})^2},\ \because AppendixA$$

$$\frac{\sum(x-\bar{x})y}{(x-\bar{x})^2} = \frac{\sum(x-\bar{x})(\beta_0+\beta_1 x+u)}{(x-\bar{x})^2},\ \because y = \beta_0 + \beta_1 x + u$$

$$\frac{\sum(x-\bar{x})(\beta_0+\beta_1 x+u)}{(x-\bar{x})^2} = \frac{\sum(x-\bar{x})(\beta_0+\beta_1 x+u)}{SST_x},\ \because SST_x = (x-\bar{x})^2$$

$$\frac{\sum(x-\bar{x})(\beta_0+\beta_1 x+u)}{SST_x} = \frac{1}{SST_x}\sum(x-\bar{x})\beta_0 + \sum(x-\bar{x})x\beta_1 + \sum(x-\bar{x})u)$$

$$\beta_1 + \sum\frac{u}{x-\bar{x}},\ \because \sum(x-\bar{x})x = \sum(x-\bar{x})$$

$$\beta_1 + \sum\frac{u}{x-\bar{x}}\frac{x-\bar{x}}{x-\bar{x}} = \beta_1 + \sum\frac{u(x-\bar{x})}{(x-\bar{x})^2} = \beta_1 + \sum\frac{u\ d}{SST_x},\ \because d = x-\bar{x}$$

$$\therefore \hat{\beta_1} = \beta_1 + \sum\frac{u\ d}{SST_x} = \beta_1 + \sum wd$$

2.

$$Cov(\beta_1, \bar{u}) = E(\beta_1\bar{u}) = E((\hat{\beta_1}-\beta_1)\bar{u}),\ \because \beta_1\ is\ a\ constant$$

$$\hat{\beta_1} = \beta_1 + \sum wu \rightarrow \hat{\beta_1} - \beta_1 = \sum wu$$

$$E((\hat{\beta_1}-\beta_1)\bar{u}) = E(\sum wu * \bar{u}) = \sum wE(u\bar{u})$$

$$E(u\bar{u}) = E(\frac{u^2}{n}) = \frac{\sigma^2}{n}$$

$$\sum wE(u\bar{u}) = \sum w\frac{\sigma^2}{n} = 0$$

$$\therefore Cov(\beta_1, \bar{u}) = \sum w\frac{\sigma^2}{n} = 0$$

3.

$$\bar{y} = \beta_0 + \beta_1\bar{x} + \bar{u},\ \hat{\beta_0} = \hat{y} - \hat{\beta_1}\bar{x}$$

$$\hat{\beta_0} = (\beta_0 + \beta_1\bar{x} + \bar{u}) - \hat{\beta_1}\bar{x}$$

$$\therefore\ \hat{\beta_0} = \beta_0 + \bar{u} - (\hat{\beta_1} - \beta_1)\bar{x}$$

4.

$$Var(\hat{\beta_0}) = Var(\beta_0 + \bar{u} - (\hat{\beta_1} - \beta_1)\bar{x})$$

$$Var(\hat{\beta_0}) = Var(\bar{u}) + \bar{x}^2 Var(\hat{\beta_1}),\ \because Cov(\beta_1, \bar{u}) = 0$$

$$Var(\hat{\beta_0}) = \frac{\sigma^2}{n} + (\bar{x})^2(\frac{\sigma}{SST_x}\ \because iid\ assumption\ and\ Var(\hat{\beta_1}) = \frac{\sigma_2}{\sum(x-\bar{x})^2}$$

$$\therefore Var(\hat{\beta_0}) = \frac{\sigma^2}{n} + (\bar{x})^2\frac{\sigma^2}{SST_x}$$

5.

$$Var(\hat{\beta_0}) = \sigma^2((SST_x/n) + \bar{x}^2)/SST_x$$

$$= \sigma^2((n^{-1}\sum x^2 - \bar{x}^2) + \bar{x}^2)SST_x$$

$$= \sigma^2(n^{-1}\sum x^2)/SST_x$$