$$p(S, Y) = \prod_{n=1}^{N} \left(p\left(s_1^n\right) \prod_{t=2}^{T_n} p\left(s_t^n | s_{t-1}^n\right) \right) \left(\prod_{t=1}^{T_n} p\left(\mathbf{y}_t^n | s_t^n\right) \right)$$