Filters

Seasonal Adjustment With X-13ARIMA-SEATS

2019

Economic Statistical Methods Division

U.S. Census Bureau

U.S. Department of Commerce
Economics and Statistics Administration
U.S. CENSUS BUREAU
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Features of X-11 Seasonal Adjustment

- **Local smoothing**
  - Moving Averages
- Iterative refinement

Local Smoothing

- X-13ARIMA-SEATS (like its predecessors) uses finite moving average filters to estimate the trend and the seasonal effects
  - Trend filters average consecutive values
  - Seasonal filters average values within a month (or quarter)

Local Smoothing With Moving Averages

$$y_t \xrightarrow{F} x_t$$

$$x_t = \sum w_k y_{t+k}, \sum w_k = 1$$

Where

y_t is the original series

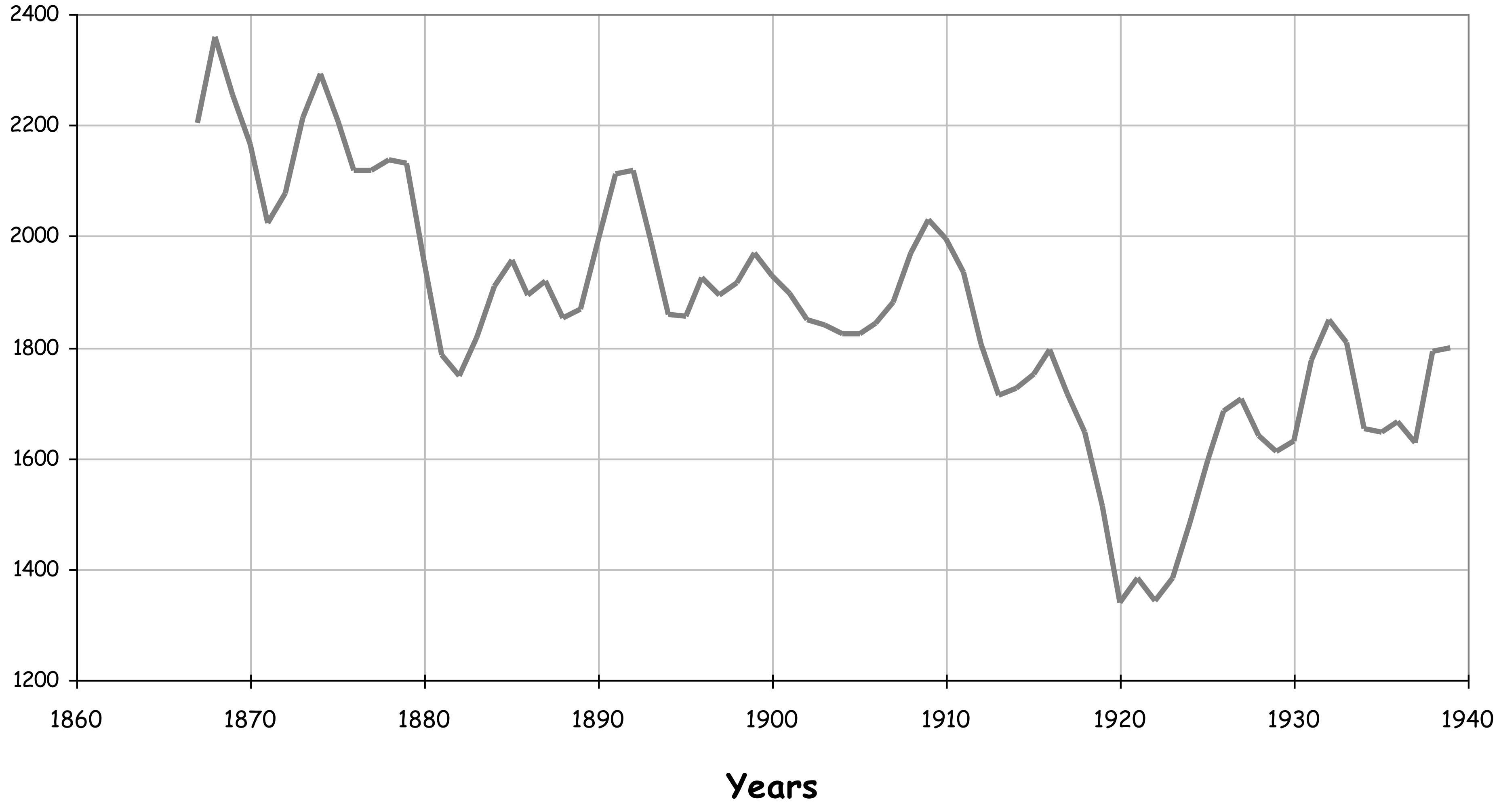
x_t is the smoothed series

weights (w_k) must sum to 1

Sheep Population in the UK 1867–1939

- Annual data – no seasonality
- From the book Time Series, 3rd Edition (1990) by Kendall and Ord, Oxford University Press: London
- In 10,000s

Annual Sheep Population in England and Wales



Simple 5-term Moving Average (Centered/Symmetric)

$$\frac{Y_{1867} + Y_{1868} + Y_{1869} + Y_{1870} + Y_{1871}}{5}$$

Recall

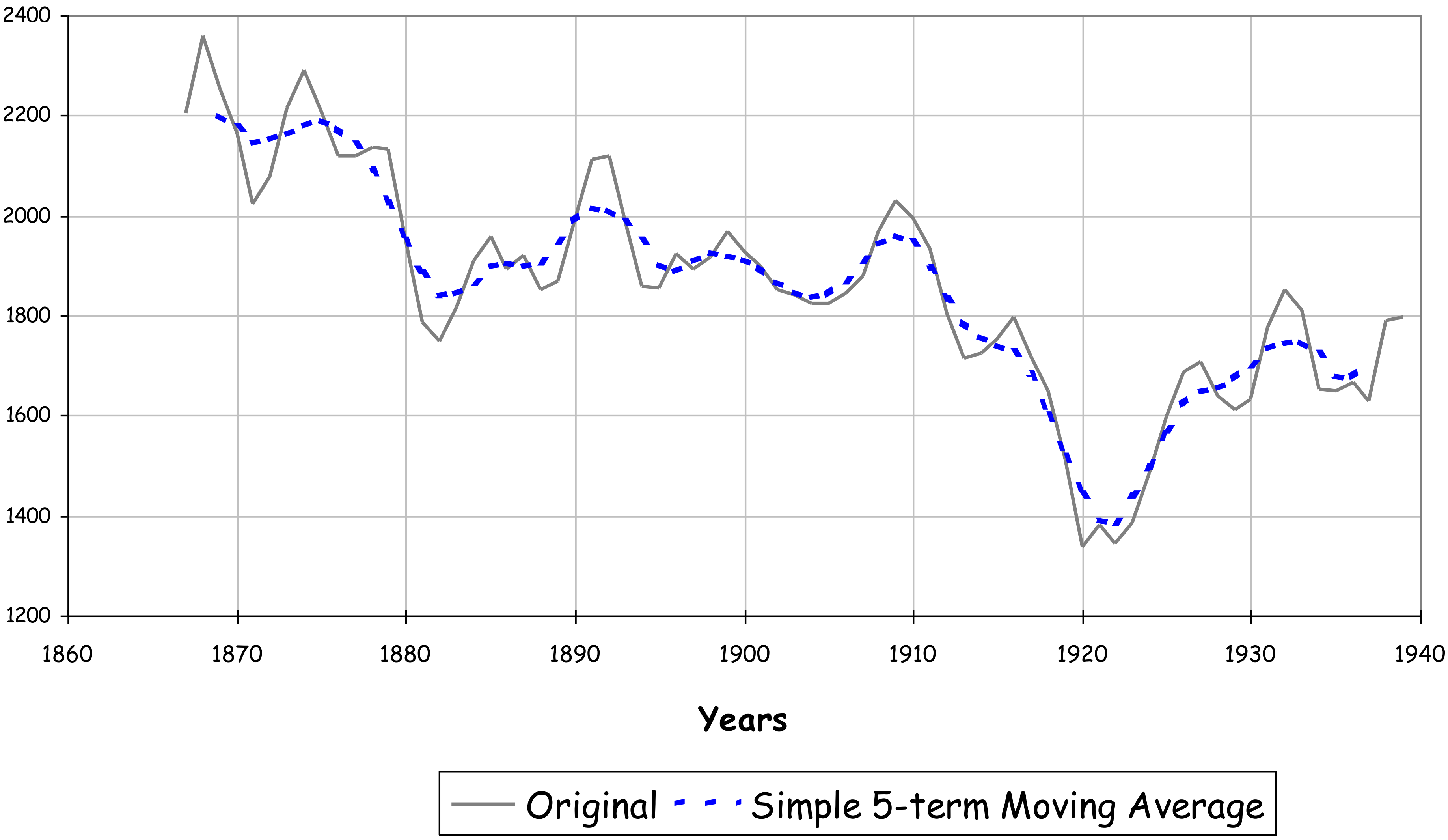
$$x_t = \sum w_k \, y_{t+k} , \quad \sum w_k = 1$$

$$x_t = 0.2\, y_{t-2} + 0.2\, y_{t-1} + 0.2\, y_t + 0.2\, y_{t+1} + 0.2\, y_{t+2}$$

Sheep Data 5-term Smoothing

Year	Value	5-Term MA
1867	2203	NA
1868	2360	NA
1869	2254	2201.2
1870	2165	2176.2
1871	2024	2147.0
1872	2078	2154.6
1873	2214	2163.0

Annual Sheep Population in England and Wales

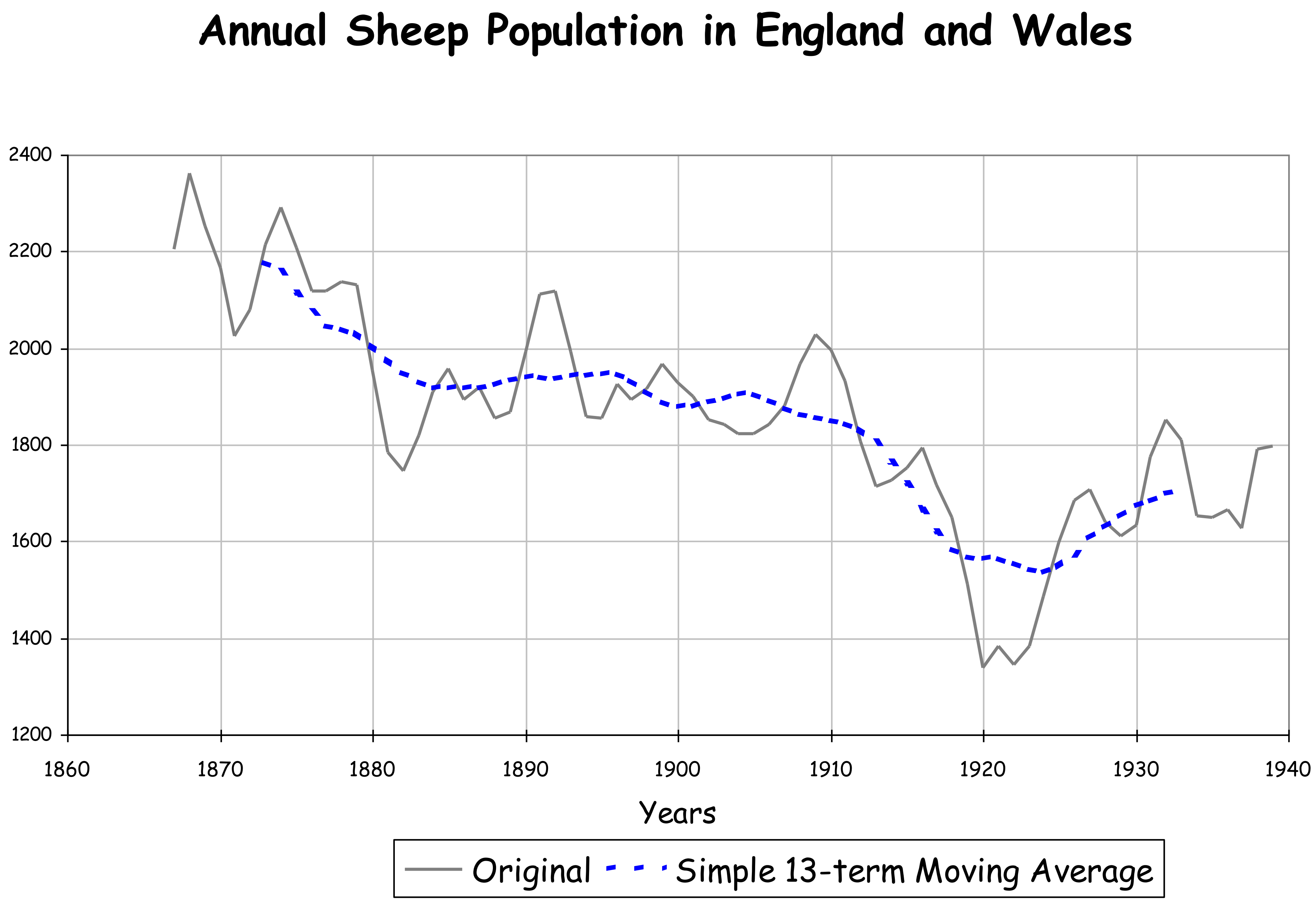


13-term Simple Filter

$$w_{t-6} = w_{t-5} = \ldots = w_t = \ldots = w_{t+5} = w_{t+6} = 1/13$$

Sheep Data 13-term Smoothing

Year	Value	13-Term MA
1867	2203	NA
. . .		
1872	2078	NA
1873	2214	2177.2
1874	2292	2158.2
1875	2207	2113.9
1876	2119	2074.9
1877	2119	2048.2



Annual Sheep Population in England and Wales
2400
2200
2000
1800
1600
1400
1200
1860
1870
1880
1890
1900
1910
1920
1930
1940
Years
Original
Simple 13-term Moving Average

3x11 Filter (for 1873)

$$\frac{\begin{array}{l} Y_{1867} + Y_{1868} + \ldots + Y_{1877} \\ \quad\quad\quad Y_{1868} + Y_{1869} + \ldots + Y_{1878} \\ \quad\quad\quad\quad\quad\quad Y_{1869} + Y_{1870} + \ldots + Y_{1879} \end{array}}{33}$$

(13 years – symmetric filter)

3x11 Filter Weights (for 1873)

$$\frac{1}{33}Y_{1867} + \frac{2}{33}Y_{1868} + \frac{3}{33}Y_{1869} + \frac{3}{33}Y_{1870} + \frac{3}{33}Y_{1871} +$$

$$\frac{3}{33}Y_{1872} + \frac{3}{33}Y_{1873} + \frac{3}{33}Y_{1874} + \frac{3}{33}Y_{1875} +$$

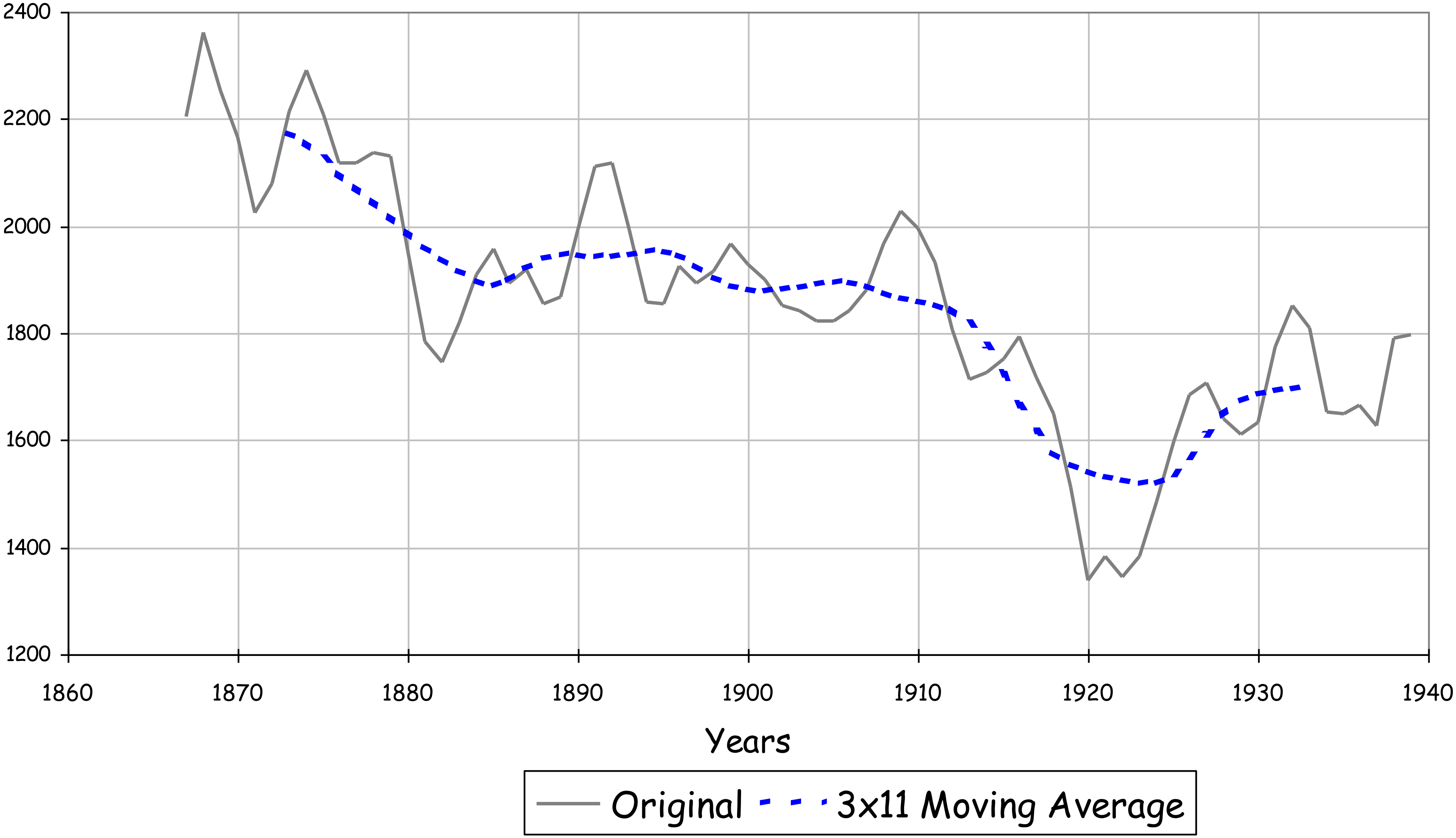
$$\frac{3}{33}Y_{1876} + \frac{3}{33}Y_{1877} + \frac{2}{33}Y_{1878} + \frac{1}{33}Y_{1879}$$

(13 years – symmetric filter)

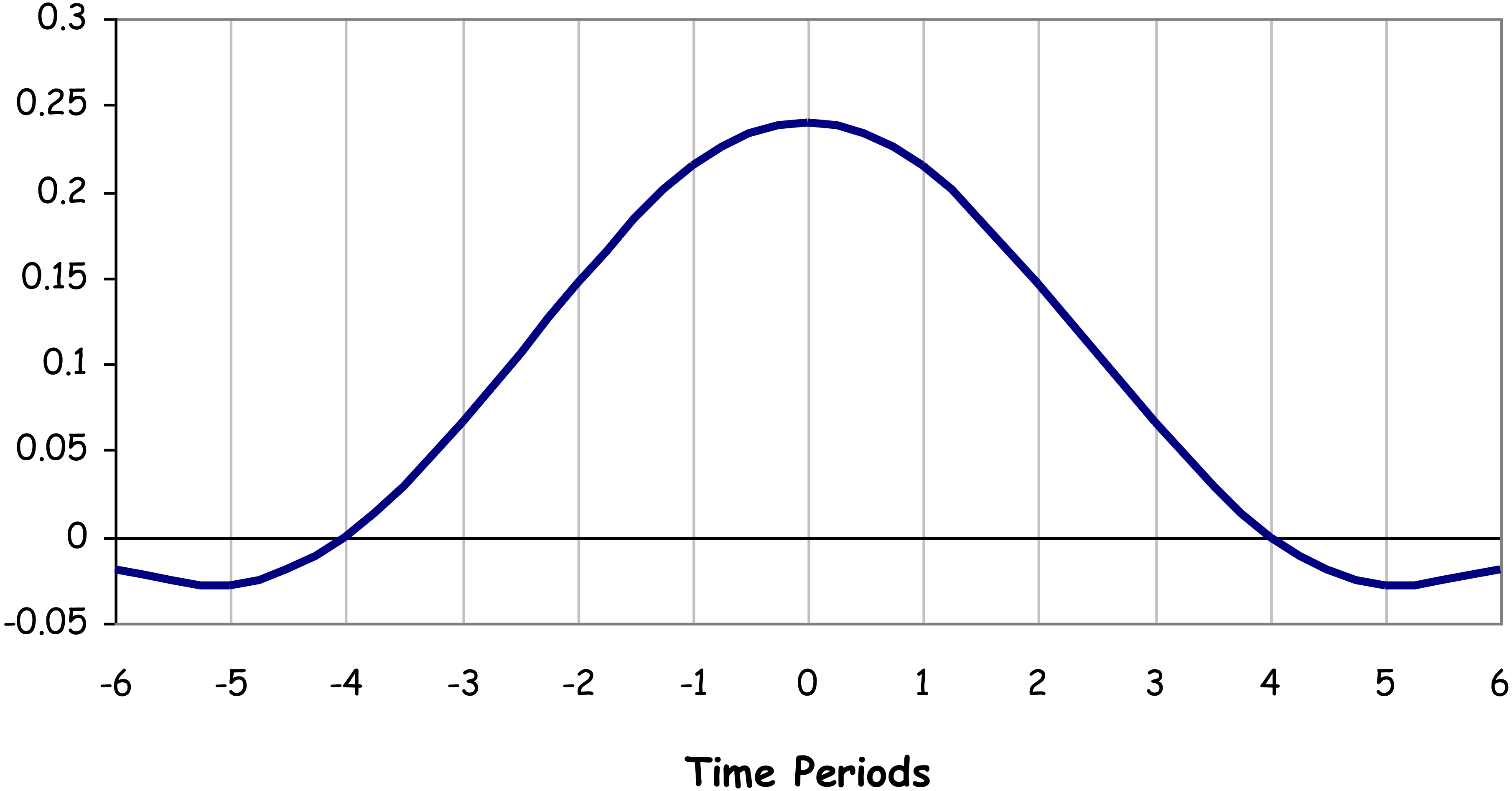
Sheep Data 3x11 Smoothing

Year	Value	13-Term MA
1867	2203	NA
. . .		
1872	2078	NA
1873	2214	2174.1
1874	2292	2156.1
1875	2207	2128.6
1876	2119	2099.7
1877	2119	2071.9

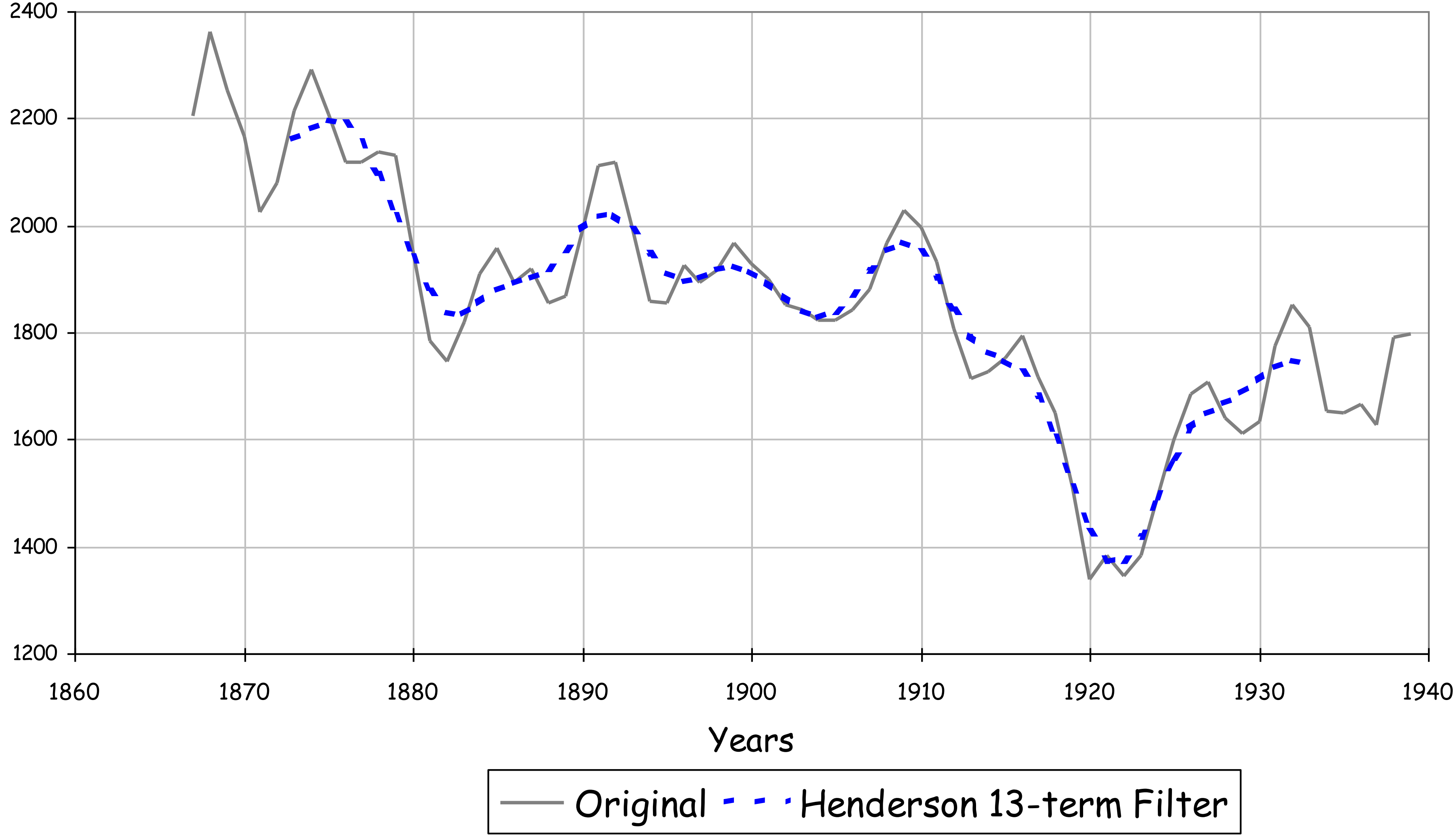
Annual Sheep Population in England and Wales



Henderson-13 Filter Weights



Annual Sheep Population in England and Wales

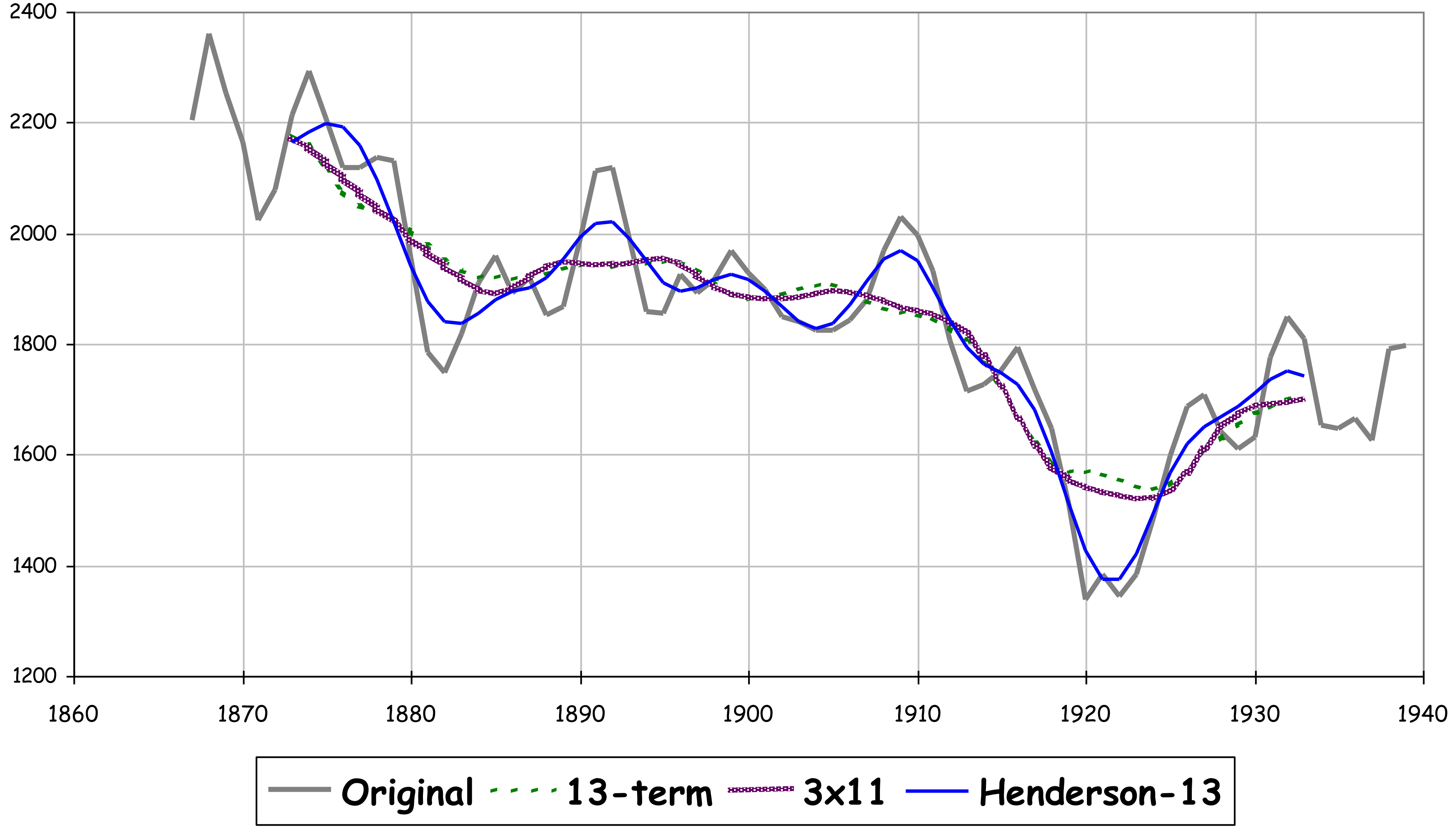


Filter Comparison

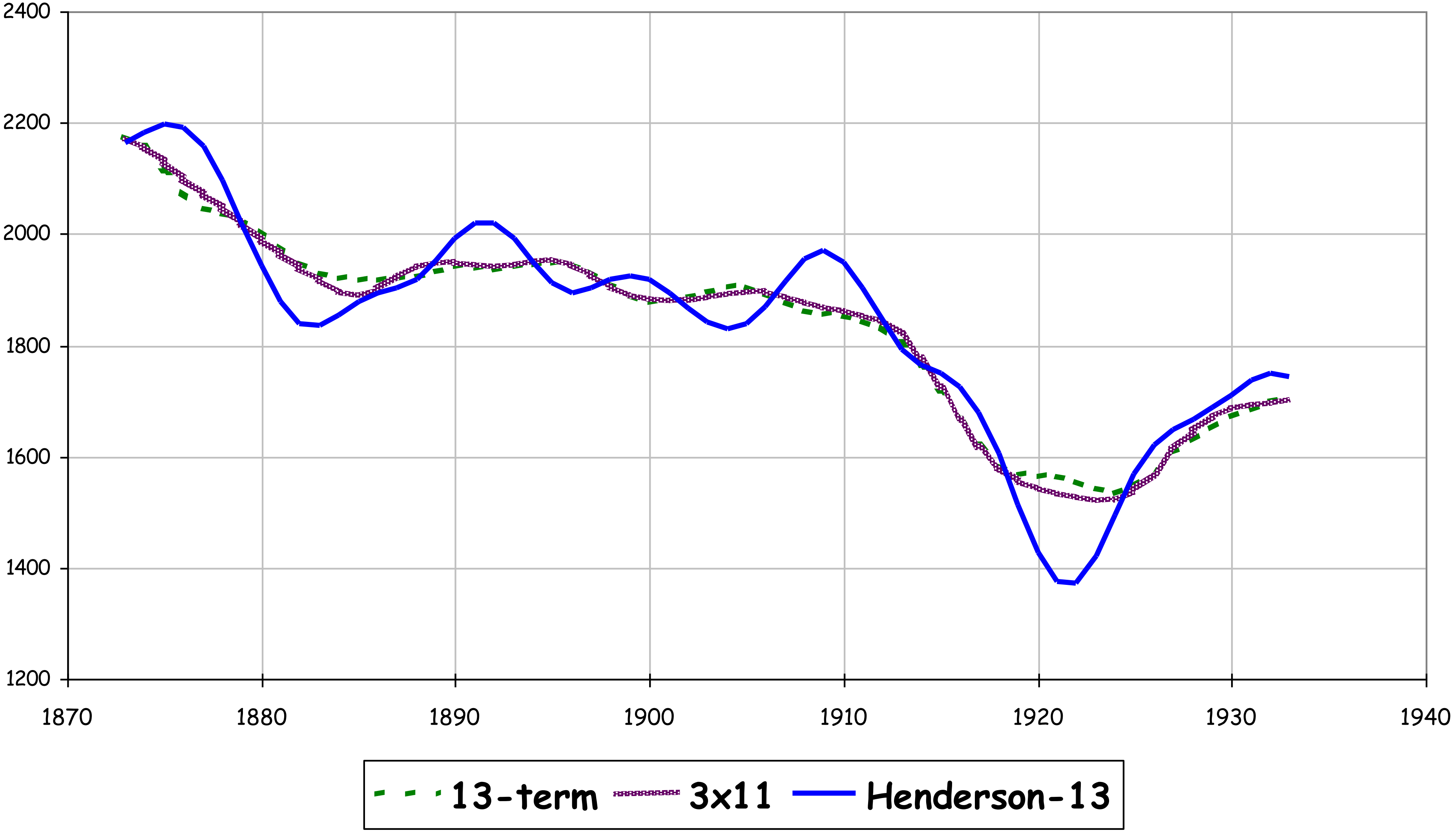
- 13-term
- 3x11
- Henderson-13

- Which filter would you prefer for this series?

Sheep Population 1867-1939 (10,000s)



Sheep Population Moving Averages Only



Moving average filters in X-11

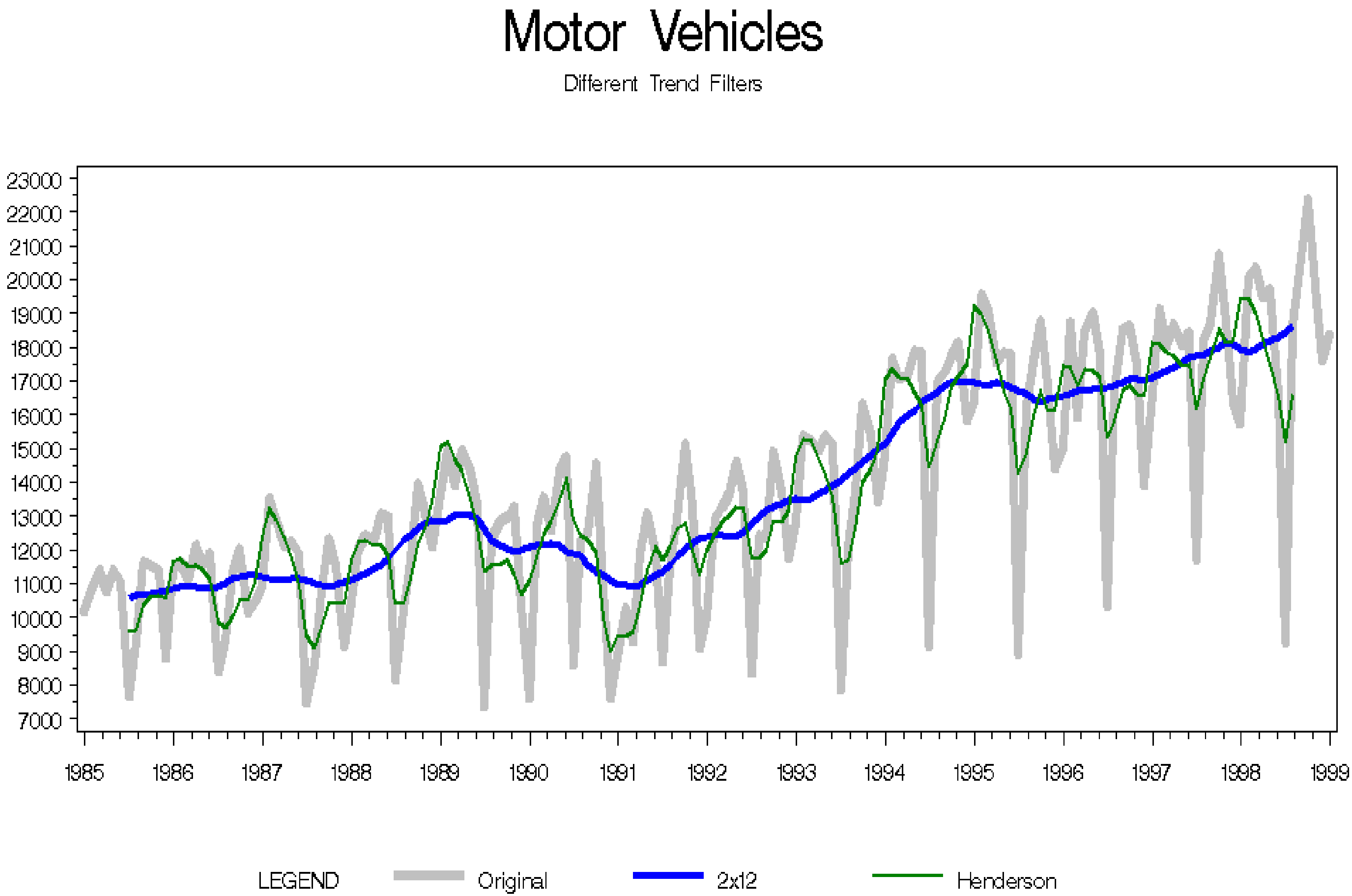
Local Smoothing in X-11

- The X-11 method in X-13ARIMA-SEATS uses a variety of moving average filters to smooth the data
  - Filters for trend and seasonality
  - Some filters chosen automatically

Seasonal Series & Trend Filters (1)

- We apply a crude trend (2x12) filter and a Henderson-13 filter to a seasonal series
  - 2x12 filter: A 13-year filter where $w_{t-5} = \cdots = w_t = \cdots = w_{t+5} = 1/12$ and $w_{t\pm 6} = 1/24$
  - Henderson-13: see previous slide.

Seasonal Series & Trend Filters (2)



Seasonal Series & Trend Filters (3)

- Often a Henderson filter applied to a seasonal series results in a seasonal series.

- In X-11:
  - We use a crude trend filter on the seasonal series to calculate the first estimate of the trend.
  - After the series has been deseasonalized, we use a Henderson filter to better capture the local level of the series.

2 x 12 Trend Filter, for July

1	1	1	1	1	1	1	1	1	1	1	1	
	1	1	1	1	1	1	1	1	1	1	1	1
Jan	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**<u>Jul</u>**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**	**Jan**
-6	-5	-4	-3	-2	-1	t=0	1	2	3	4	5	6
$\frac{1}{24}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{24}$

Rewritten for Quarterly Series

Example for quarterly series,

2x4 trend filter for 1st Quarter 1990

$$\frac{\begin{array}{l}1989.3 + 1989.4 + 1990.1 + 1990.2 \\ \qquad\quad + 1989.4 + 1990.1 + 1990.2 + 1990.3\end{array}}{8}$$

What About Seasonal Filters?

- We can also use local smoothing to estimate the seasonal effect
  - For seasonal filters, we average values for a particular month (or quarter) (averages of Januaries, Februaries, etc.)

Example: 3x3 Seasonal Filter

3 x 3 seasonal filter for 1st Quarter 1990

(or January 1990)

$$\frac{\begin{array}{l} 1988.1 + 1989.1 + 1990.1 \\ \qquad\quad + 1989.1 + 1990.1 + 1991.1 \\ \qquad\qquad\qquad + 1990.1 + 1991.1 + 1992.1 \end{array}}{9}$$

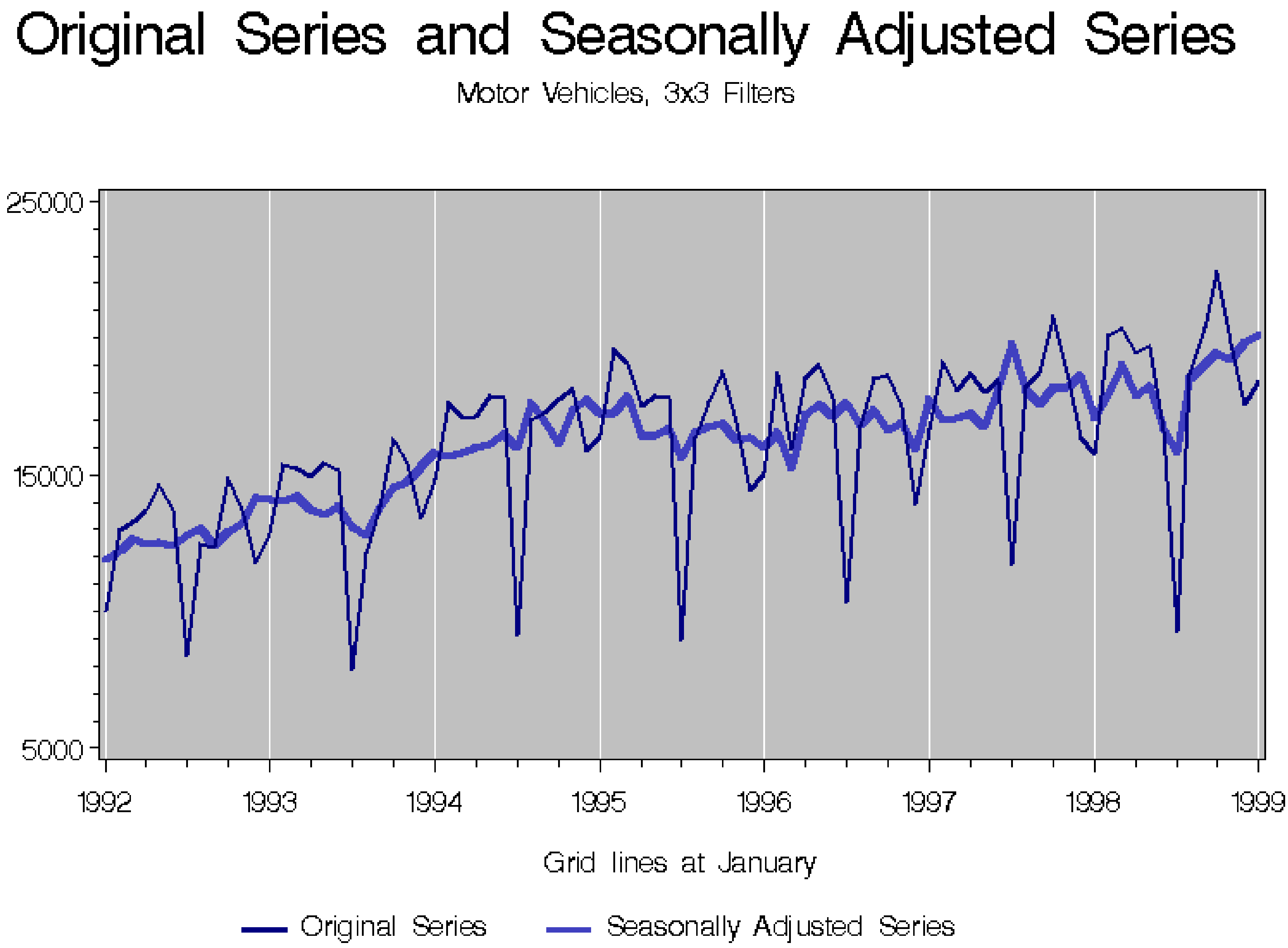
Weights for 3x3 Filter Centered at t=0

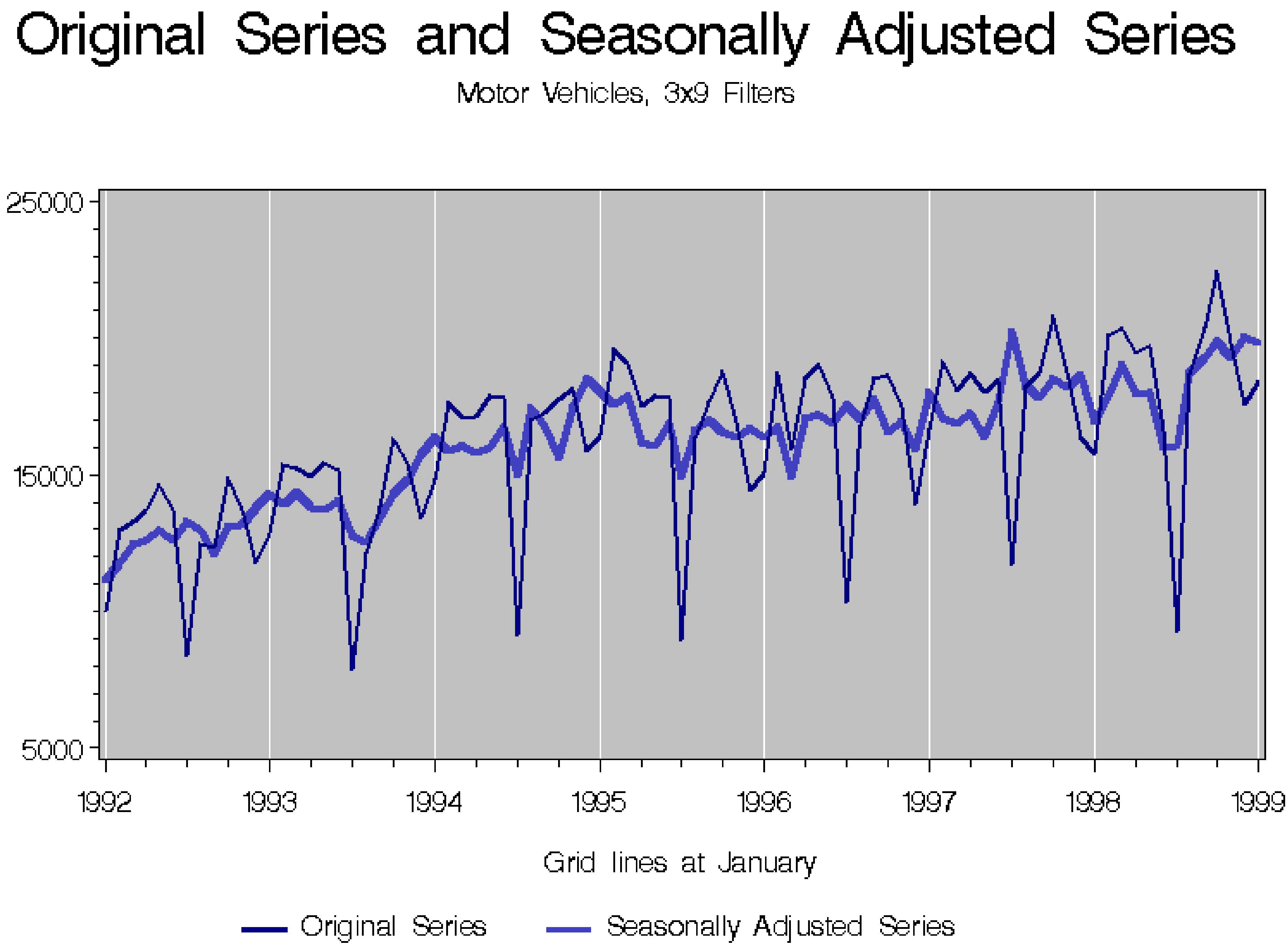
1	1	1		
	1	1	1	
		1	1	1
		9		
-2yrs	-1yr	t=0	+1yr	+2yrs
Weights				
.11	.22	.33	.22	.11

Common X11 Seasonal Filters

- 3x3 – 5 year filter
- 3x5 – 7 year filter
- 3x9 – 11 year filter

- Look at the difference between the seasonal adjustment using 3x3 filters and the adjustment using 3x9 filters. (We make no case as to which is better at this time; but note that the filter choice can have a large impact on the adjustment.):


Original Series and Seasonally Adjusted Series
Motor Vehicles, 3x3 Filters
25000
15000
5000
1992
1993
1994
1995
1996
1997
1998
1999
Grid lines at January
Original Series
Seasonally Adjusted Series


Original Series and Seasonally Adjusted Series
Motor Vehicles, 3x9 Filters
25000
15000
5000
1992
1993
1994
1995
1996
1997
1998
1999
Grid lines at January
Original Series
Seasonally Adjusted Series

X-11 Default Seasonal Moving Average Filter

3x3 for preliminary seasonal estimate

3x5 for second estimate

- (B Iteration – all about iterations just a little later)

3x3 again

3x5 again

(C Iteration)

. . . continued

X-11 Default Seasonal Moving Average Filter (continued)

. . .

3x3 again

3x3 or 3x5 or 3x9 very last time

- (D Iteration)
- Choice based on something called the "Global Moving Seasonality Ratio"

Moving Seasonality Ratios (MSRs)

For a multiplicative adjustment, calculate the mean annual changes for each component for month (quarter) *I*

$$\bar{S}_i = \frac{1}{N_i - 1} \sum_{t=2}^{N_i} \left| \frac{S_{i,t}}{S_{i,t-1}} - 1 \right|$$

and

$$\bar{I}_i = \frac{1}{N_i - 1} \sum_{t=2}^{N_i} \left| \frac{I_{i,t}}{I_{i,t-1}} - 1 \right|$$

where N_i = the number of observations for month (quarter) *i*

MSR for Each Month (Quarter)

MSR for month (quarter) *i*

$$MSR_i = \frac{\overline{I_i}}{\overline{\overline{S_i}}}$$

Global MSR (GMSR)

Calculate the GMSR from the monthly (quarterly) MSRs

$$\text{GMSR} = \frac{\sum_i N_i \times \overline{I_i}}{\sum_i N_i \times \overline{S_i}}$$

Automatic Seasonal Filter Choices for Very Last Choice

- MSR is ${}^{\bar{I}}/_{\bar{S}}$
  - Average change in irregular divided by average change in seasonal pattern
- X-13ARIMA-SEATS chooses (for the second pass in the D iteration – very last time)
  - 3x3 when GMSR is small
  - 3x9 when GMSR is large
  - 3x5 otherwise

Final Seasonal Filter Choices (D Iteration, Second Pass)

GMSR $\leq$ 2.5	Use 3x3
3.5 < GMSR $\leq$ 5.5	Use 3x5
GMSR > 6.5	Use 3x9

"Gray" zones: 2.5 – 3.5 and 5.5 – 6.5

"Gray" Zones

- If the GMSR is in one of the gray zones, X-13ARIMA-SEATS removes one year of values **from the end of the series** and recalculates the GMSR
- If the GMSR is still in the gray zone, the process repeats (removing up to five years) until the GMSR isn't in the gray zone
- If GMSR remains in the gray zone, the program sets the filter to 3x5

Choosing Seasonal Filters

- 3x5 is most common choice
- Use 3x3 when seasonal pattern is changing rapidly
- Use 3x9 when seasonal pattern isn't changing much or when irregular component is large
  - Extremes affect the averages less than with 3x5

Additional Seasonal Filters (Rarely Used)

- 3x1
  - Simple 3-term moving average
- 3x15
  - 17-year filter
- Stable
  - All years included
  - Simple moving average

Additional Seasonal Filters (Not-so-rarely used)

- x11default
  - 3x3 filter used for preliminary seasonal estimate, and
  - 3x5 filter used for second seasonal estimate
  - (All iterations)

Different Months (Quarters)

- Table D9.A may show large differences in MSRs for different months (quarters)

- May be appropriate to use different filters for different months (quarters)

`Seasonalma` Argument

- Use **`seasonalma`** to set the seasonal filter. If it is not included, the MSR filter is used.
- If **`seasonalma`** is set to a certain length, it will be set to that length for every iteration
  - Applies to the options **`s3x1`**, **`s3x3`**, **`s3x5`**, **`s3x9`**, **`s3x15`**, and **`stable`** (filter arguments cannot start with a number, hence **`s3x3`** and not 3x3); **`x11default`** alternates between 3x3 & 3x5

X11 Spec, Seasonal Filter

```
x11{
. . .
  seasonalma = s3x9
. . .
}
```

X11 Spec, Different Filters for Different Quarters

```
x11{
. . .
 seasonalma=(s3x9 s3x5 s3x5 s3x5)
. . .
}
```

Real Life: Choosing Seasonal Filters

- Use Global MSR and MSR by month (quarter) as guides
  - Table D9.A
  - Output shows Global MSR as I/S
  - MSR (or I/S) is NOT the "SI ratio"!
- Make sure no residual seasonal effects remain after seasonal adjustment
  - Spectrum diagnostic & QS – coming up later
- More information coming up

Henderson Trend Filters

- Trends are estimated with Henderson filters multiple times in X-11.
- You can set the filter length with the **`trendma`** argument; this will use the supplied filter each time
- Otherwise, the program will select the filter length each time using the I/C ratio

Automatic Henderson Trend Filter Choices

- Monthly series
  - 9-term filter when $I / C < 1.0$
  - 13-term filter when $1.0 \leq I / C < 3.5$
  - 23-term filter when $3.5 \leq I / C$
- Quarterly series
  - 5-term when $I/C < 1.0$
  - 7-term otherwise

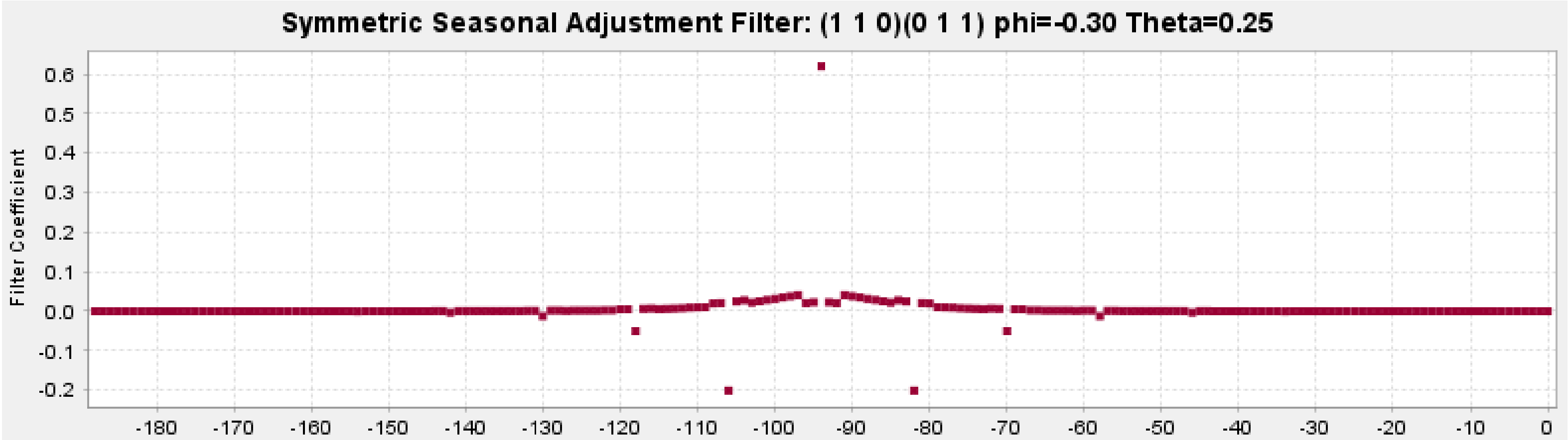
Real Life: Choosing Trend Filters

- Usually allow X-13ARIMA-SEATS to choose trend filters each time
  - Result rarely changes
- Can set trend filter if any concern about change

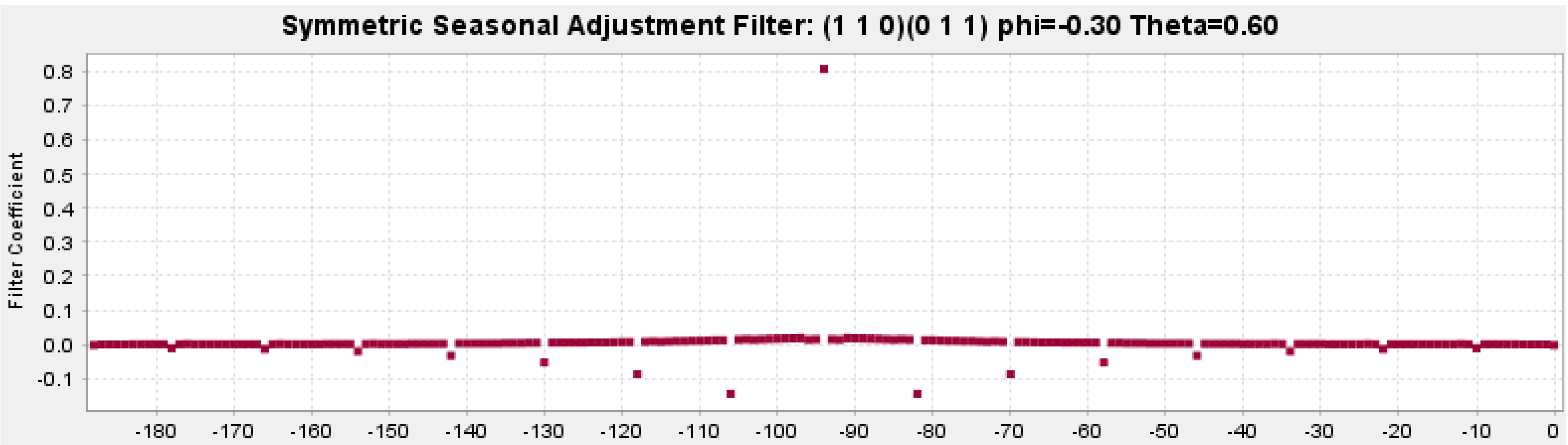
Moving average filters in SEATS

Moving Average Filters in SEATS

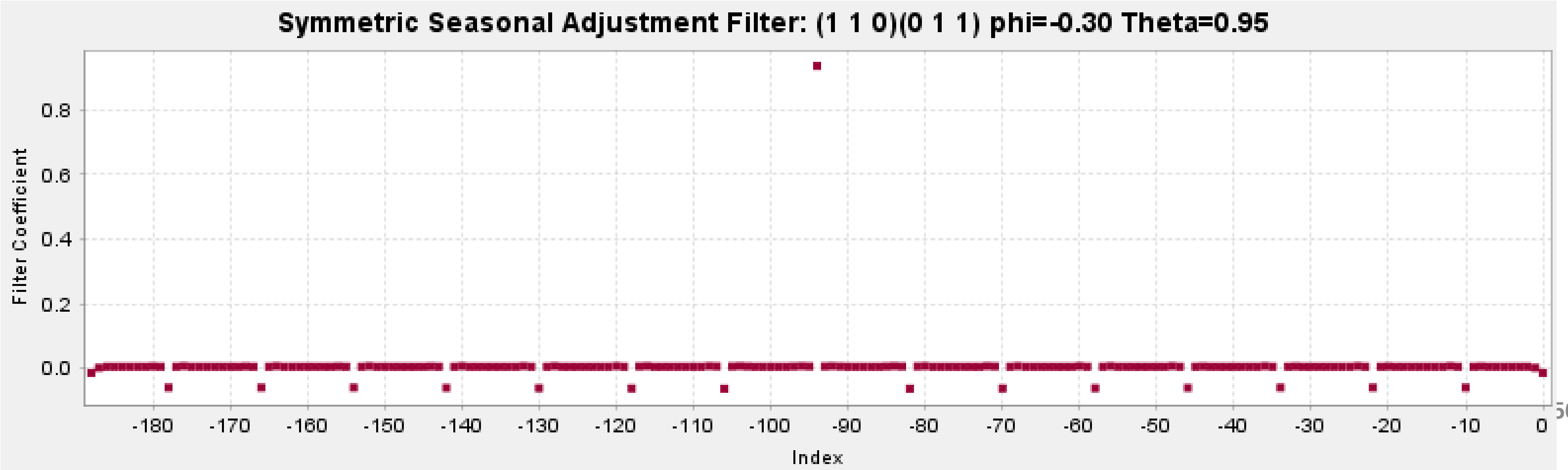
- The SEATS procedure creates a seasonal and trend filter for a series based on the ARIMA model



Symmetric Seasonal Adjustment Filter: (1 1 0)(0 1 1) phi=-0.30 Theta=0.25
Filter Coefficient



Symmetric Seasonal Adjustment Filter: (1 1 0)(0 1 1) phi=-0.30 Theta=0.60
Filter Coefficient



Symmetric Seasonal Adjustment Filter: (1 1 0)(0 1 1) phi=-0.30 Theta=0.95
Filter Coefficient
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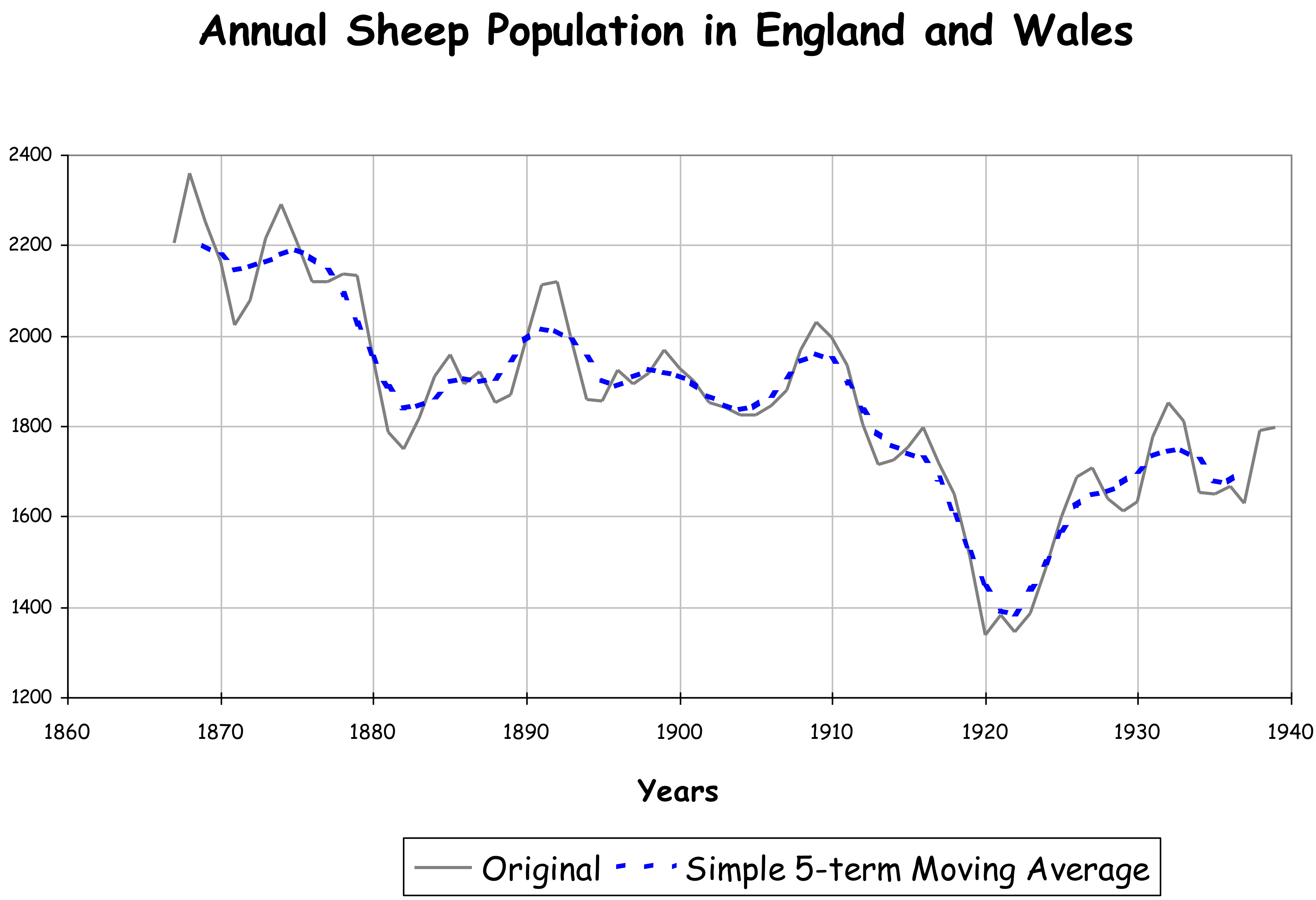
What About the Ends of the Series?

- X-13ARIMA-SEATS applies asymmetric filters at the ends of series
  - These filters don't perform as well
- What values are of most interest to data users and the media?
  - Values at the end of the series, the ones generated by the asymmetric filters, generate most media coverage



Annual Sheep Population in England and Wales
2400
2200
2000
1800
1600
1400
1200
1860
1870
1880
1890
1900
1910
1920
1930
1940
Years
Original
Simple 5-term Moving Average

Sheep Series

Year	Value	5-Term MA
1932	1850	1743.8
1933	1809	1747.0
1934	1653	1725.0
1935	1648	1680.4
1936	1665	1676.8
1937	1627	1705.6
1938	1791	NA
1939	1797	NA

Asymmetric Filter

Simple "asymmetric 5-term" moving average for 1939

$$\frac{Y_{1937} + Y_{1938} + Y_{1939}}{3}$$

$z_t = 0.333\, y_{t-2} + 0.333\, y_{t-1} + 0.333\, y_t$

(1939's value becomes 1738.3)

Alternate Asymmetric Filter

Simple "asymmetric 5-term" moving average for 1939

$$\frac{Y_{1937} + Y_{1938}}{4} + \frac{Y_{1939}}{2}$$

$z_t = 0.25\, y_{t-2} + 0.25\, y_{t-1} + 0.5\, y_t$

(1939's value is 1753)

Forecast Extension

- In the 1980s Statistics Canada showed that forecasting the series and treating the forecasts as real values at the end of the series ***reduces revisions***
  - Use symmetric X-11 filters
  - Technically, the filters still are asymmetric because we do not have future values
  - (Also can backcast the series)
  - X-11-ARIMA, new program

Changing From Default Filters?

- Trend Filters
  - First trend estimation always is 2x12 (2x4 for quarterly series)
  - Users can set Henderson trend filters
- Users can (should) set seasonal filters
- RegARIMA model changes will change the forecasts, changing the filter results