SEATS

Seasonal Adjustment With X-13ARIMA-SEATS

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census.gov

What is X-13ARIMA-SEATS?

X-13ARIMA-SEATS =

X-13A-S = X-12-ARIMA + SEATS

What is X-13ARIMA-SEATS?

- Produces model-based seasonal adjustments from the SEATS seasonal adjustment procedure (Gomez and Maravall 1996)

- Collaboration between the U. S. Census Bureau and the developers of SEATS, **Agustin Maravall** of the Bank of Spain and **Gianluca Caporello**.

SEATS

“Signal Extraction in ARIMA Time Series”

- Performs decomposition using a model based approach to compute the components
- Uses filters derived from an ARIMA model to describe the different components

- Uses the method of Hillmer and Tiao (1983)
  - Derives ARIMA models for the components of a seasonal adjustment from the ARIMA model of the original series
  - Provides total revision standard errors for the components

SEATS and X-11

- Ultimately, both X-11 and SEATS estimate the trend, seasonal and irregular components by passing moving average filters over the forecast extended series.

- X-11 Filters
    - Finite set of empirically developed moving average filters
    - Fixed filtering seen as easier to use (less statistical machinery)

- SEATS Filters
    - Specifies stochastic model for each unobserved component
    - Derives seasonal adjustment filters from these models
    - Infinite number of possible filter choices
    - Requires more statistical machinery

Two Important Assumptions

- The linearized series can be represented by an ARIMA model which captures the stochastic structure of the series
  - Linearized series = series with regression effects removed

- After differencing each with the ARIMA's differencing polynomial, the components are orthogonal (uncorrelated)

With this, we have:

- A regARIMA model $Y_t = \sum \beta_i x_{it} + Z_t$

- An ARIMA representation $\Phi(B)\, Z_t = \Theta(B)\, a_t$

- A decomposition model $Z_t = C_t + S_t + R_t + u_t$

  with C=trend S=seasonal

  R=transitory u=irregular

Transitory Component

- Captures short, erratic behavior that is not white noise.
  - Sometimes associated with awkward frequencies
- Modeling this separately is justified by:
  - Variance from transitory should not contaminate other components
  - Preserves a white noise residual
- In the final decomposition, the transitory and irregular are usually combined.
- SEATS does not always estimate a transitory component
  - Can only present when there is an AR component in the ARIMA model
  - Only produced when the roots of the AR polynomial have certain properties

SEATS Decomposition

- Factorizes the AR polynomial $\Phi(B)$ as in :

$$\Phi(B) = \zeta_C(B) \cdot \zeta_S(B) \cdot \zeta_R(B)$$

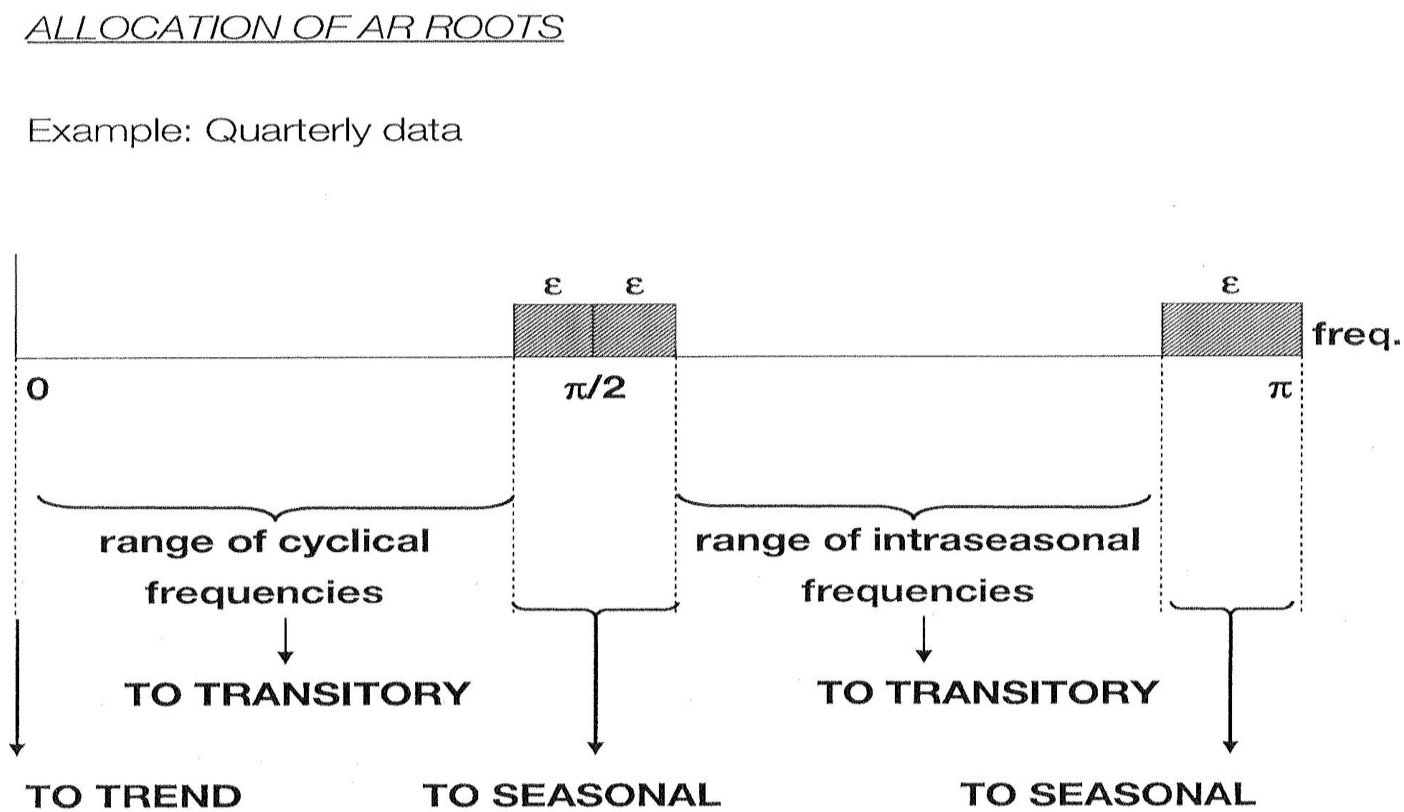
$$\zeta_C(B) = \text{trend-cycle roots}$$

$$\zeta_S(B) = \text{seasonal roots}$$

$$\zeta_R(B) = \text{transitory roots}$$

- where roots are assigned according to their associated frequency in the spectrum

Allocation of AR Roots



SEATS Decomposition

Recall: for specified AR and MA polynomials, that satisfy certain conditions (not discussed here), there is a unique Wold decomposition:

$$\Theta(B)\, Z_t = \Phi(B)\, a_t \quad \rightarrow \quad Z_t = \frac{\Phi(B)}{\Theta(B)}\, a_t$$

Z_t is now expressed as:

$$Z_t = \{\Theta(B)/\ \Phi(B)\}\ a_t =$$

$$\{\Theta_C (B)/\ \zeta_C(B)\ \}\ a_{C,t} +$$

$$\{\Theta_S (B)/\ \zeta_S(B)\ \}\ a_{S,t} +$$

$$\{\Theta_R (B)/\ \zeta_R(B)\ \}\ a_{R,t} + u_t$$

where u_t is white noise

Problem : Model Choice

- If the spectrum of all components is non-negative the decomposition is *admissible.*
  - In essence, SEATS finds admissible models for components C_t , S_t , R_t .

- **Problem:** Infinite number of models that yield the same aggregate
  - Differ in how white noise is allocated among the components

Solution

- SEATS uses the solution of Pierce, Box-Hillmer, Tiao and Burman:
  - Put all the white noise into the irregular components
  - Maximize the variance of the irregular
  - Minimizes the variance of the stationary transforms of the other components
- Called the “canonical decomposition”

SEATS filter Methodology

- Once we have our admissible models for each component, how do we find the moving-average filter that will ultimately be passed over the observed data to extract the trend, seasonal, and transitory components?
- Let the filter weights be $\{w_k\}$
- $\hat{S} = E[S_t|Z] = w_0 Z_t + \sum_{k=1}^{\infty} w_k (Z_{t-k} + Z_{t+k})$
- $\{w_k\}$ is the so-called Wiener-Kolmogorov filter
    - Convergent
    - Symmetric and centered
    - Adapts to the series

WK algorithm

$$S_t = \Psi_S(B) a_{S,t} \qquad X_t = \Psi(B) a_t$$

$$\hat{S}_t = \underbrace{\left[\frac{\Sigma_S}{\Sigma} \frac{\Psi_S(B)\Psi_S(F)}{\Psi(B)\Psi(F)} \right]}_{\text{WK filter weights}} X_t$$

- More than other coefficients the seasonal MA influences whether estimated seasonal factors change
  - Slowly over time (seasonal theta close to 1)
  - Rapidly over time (seasonal theta close to 0)

Example: Airline Model

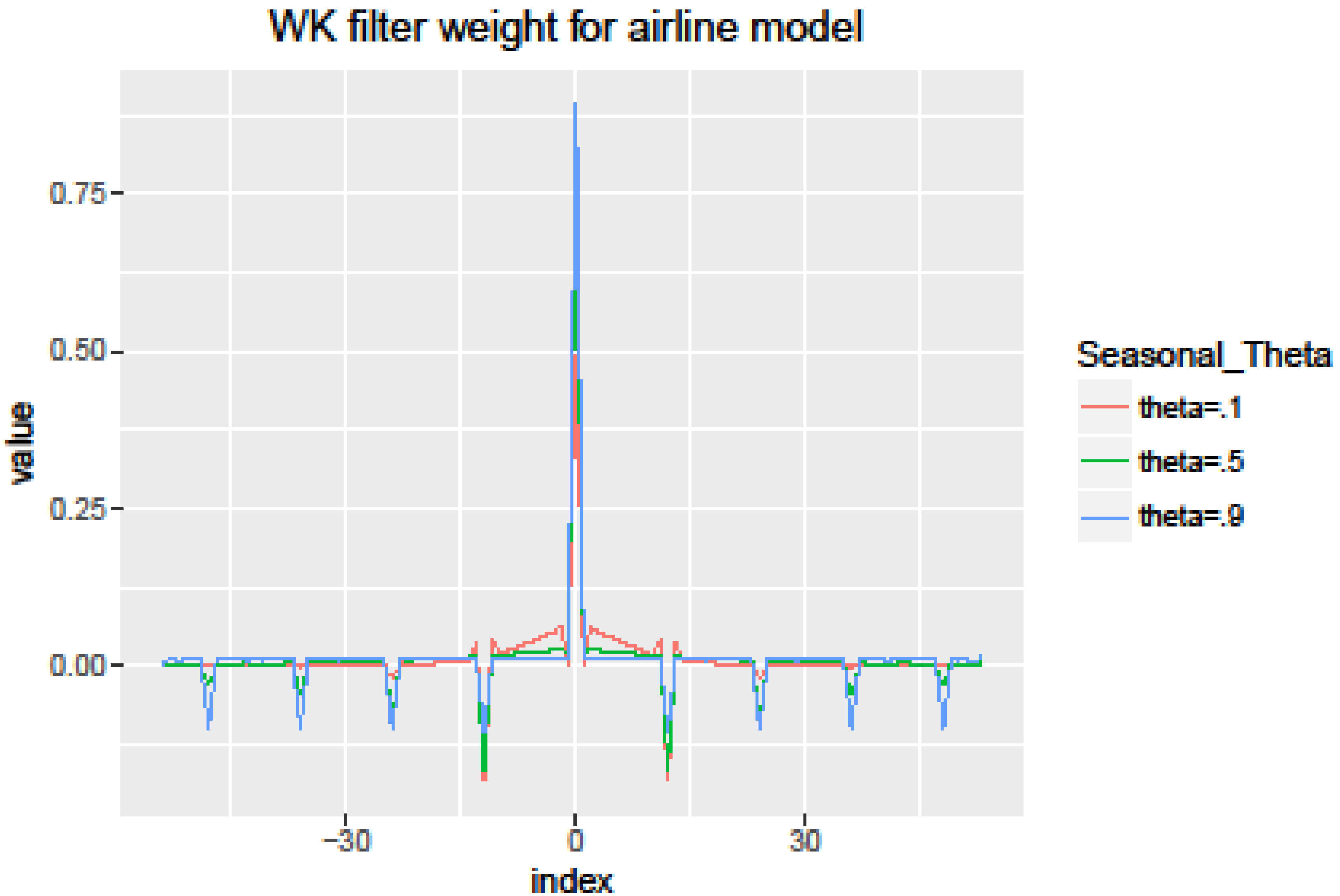
- Assume we are fitting an ``airline model'' to our observed series.
- The arima spec:
  - arima{ model = (0 1 1)(0 1 1)12 }

$$\phi(B)X_t = \theta(B)a_t$$
$$(1-B)(1-B^{12})X_t = (1-\theta_1 B)(1-\theta_{12}B^{12})a_t$$
$$(1-B)(1-B)(1+B+B^2+\cdots+B^{11})X_t = (1-\theta_1 B)(1-\theta_{12}B^{12})a_t$$

Unobserved component models:

$$(1-B)^2 T_t = (1-\theta_1 B)a_{T,t}$$
$$(1+B+B^2+\cdots+B^{11})S_t = (1-\theta_{12}B^{12})a_{S,t}$$

Example: WK weights



SEATS Spec

- Controls the options for the SEATS model-based seasonal adjustment
- Most input arguments have same names as arguments in the SEATS program
- Cannot do X-11 and SEATS adjustment in the same run

Sample X-13A-S spec file

```
series{  title = "US Imports"
   format="datevalue"  file="m0.dat"  }
transform { function=log }
regression {  variables = ( td ao1997.oct )  }
arima{  model=(0 1 1)(0 1 1)  }
forecast{  maxlead = 24  }
#  x11 {  save = d11 }
seats {  save = s11 }
slidingspans {  savelog = pct }
history {  estimates = (sadj trend fcst)  }
```

`SEATS` Arguments

- Many SEATS arguments should be left at their defaults
- Arguments common to other X-13ARIMA-SEATS specs (**`print`**, **`save`**, **`savelog`**) are present in the **`seats`** spec

`qmax`

- Sets a limit for the Ljung-Box Q statistic
- Used to determine if the ARIMA model is of insufficient quality
  - If the Ljung-Box Q of the residuals is larger than this value, the model is rejected.
- Default: **`qmax = 50`**

hpcycle

- If **`hpcycle = yes`**, the program will decompose the trend-cycle into a long-term trend and a cycle component
  - uses the modified Hodrick-Prescott filter
  - Default is **`hpcycle = no`** in builds < 48, **`hpcycle = yes`** in later builds

More Information

- For more information on the Hodrick-Prescott filter, see

  Kaiser and Maravall (2001). *Measuring Business-Cycles in Economic Time Series*. New York: Springer Lecture Notes in Statistics, Vol. 154

tabtables

- A list of seasonal adjustment components and series to be stored in a separate file with extension .tbs or _tbs.html
- Components that can be added include:
  - “xo”: original series
  - “n”: seasonally adjusted series
  - “s”: seasonal factors
  - “p”: trend-cycle
  - “u”: irregular
  - “cy”: cycle
  - “ltp”: long term trend
- A listing of the possible settings for this argument can be found in Table 7.35 of the Reference Manual

tabtables

- Default : **`tabtables = all`**

- Note that this entry is a character string – no parenthesis are needed, as in the **`print`** argument

- Tables can only be added – cannot delete files as in the **`print`** argument

- Example: **`tabtables = "xo n s p u"`**

Usimp.spc

```
series{  title = "US Imports"
   format=datevalue  file="m0.dat"
   name="m0"   }
transform { function=log }
regression { variables = ( td ao1997.oct ) }
arima{  model=(0 1 1)(0 1 1)   }
forecast{  maxlead = 24   }
seats {  hpcycle = yes
         qmax = 75
         save = (s11 s12 s10)   }
```

Usimp.spc

```
series{  title = "US Imports"
   format=datevalue  file="m0.dat"  }
transform { function=log }
regression { variables = ( td ao1997.oct ) }
arima{  model=(0 1 1)(0 1 1)   }
forecast{  maxlead = 24  }
seats {  tabtables = "n p s"
         save = (s11 s12 s10}
```

Using SEATS in X-13A-S

- Model limitations
- Model span
- Number of forecasts
- File extensions
- Additional output files
- “Final seasonal” in the SEATS output
- Viewing the seasonal and trend filters
- Model implications
- Notation

Model Limitations

- SEATS does not accept missing lag models
  - Acceptable : (0 1 3)(0 1 1)
  - Unacceptable : (0 1 [1 3])(0 1 1)
- AR and MA orders (p and q) cannot be greater than 3

Model Limitations (2)

- Sometimes, the estimated values of coefficients make it impossible to estimate components from the estimated ARIMA models
  - In this case, SEATS will usually change the model and re-estimate it in order to get an admissible decomposition
    - For example, an seasonal AR(1) term is sometimes changed to a seasonal IMA(1,1)
  - In many cases, the airline model is used as a replacement model
    - Usually gives acceptable results for a broad range of series

Stable Seasonal Regressors

- A SEATS seasonal adjustment is not performed for models with stable seasonal regressors
  - **`variables = seasonal`** in the regression spec

- Seasonal regression factors provide the seasonal factors
- SEATS will generate a trend (and possibly a transitive) component from the model adjusted by the seasonal regressors

Model Span

- It can be difficult to find a regARIMA model for the entire series that fits well
- Often, it is easier to find a model for a shorter span of the series
  - Since SEATS derives the adjustment from the values of the model parameters, one can get quite different adjustments with the same model by choosing a different model span

Number of Forecasts

- SEATS needs the series to be extended by a number of forecasts in order to adjust the ends of the series.
- Recent versions of SEATS in X-13ARIMA-SEATS need at least 3 years of forecasts
  - If you specify less than that, the program will change the value automatically

File Extensions

- The **`seats`** and **`x11`** specs use a different set of extensions to save
  - the seasonally adjusted series,
  - the seasonal factors,
  - the trend-cycle component, and
  - the irregular component.

Table Names/File Extensions

Table description	X11 spec	Seats spec
Seasonally adjusted series	D11	S11
Seasonal factors	D10	S10
Trend-cycle component	D12	S12
Irregular component	D13	S13
Combined adjustment factors	D16	S16
Combined adjustment ratios	E18	S18

Additional Output Files

- In addition to the component matrix file (.tbs) controlled by **`tabtables`**, SEATS produces two other output files
  - Summary file (.sum, _sum.html): Contains summary information and diagnostics from SEATS adjustment, including model information and coefficients
  - Rates of growth statistics file (.rog, _rog.html): Stores selected statistics related to growth rates

“Final seasonal” in SEATS Output

- The table labelled “Final seasonal” in the SEATS output of X-13ARIMA-SEATS is actually a combined adjustment factor
  - Seasonal, trading day, and holiday factors together
  - When you save the seasonal factors (s10), you are only saving the seasonal factors

Viewing the Seasonal and Trend Filters

- By default SEATS filter and diagnostic output are obtained from infinite Wiener-Kolmogorov filters.
- Set **`finite = yes`** to produce these using finite quantities

- With **`finite = yes`**, X-13 can also save the
  - concurrent and symmetric seasonal adjustment filters (fac and faf)
  - concurrent and symmetric trend filters (ftc and ftf)
  - squared gain for concurrent and symmetric seas adj filters (gac and gaf)
  - squared gain for concurrent and symmetric trend filters (gtc and gtf)
  - time shift of the concurrent seas adj and trend filters (tac and ttc)

  - These are all automatically saved as graphics files, and can be graphed with X-13-Graph

Influence of Seasonal MA on SEATS adjustments

- More than other coefficients of the ARIMA model, the seasonal MA (Θ) influences whether estimated seasonal factors change
  - slowly over time (when Θ is close to 1)
  - or rapidly over time (when Θ close to 0),
  - or something in-between.

Influence of Seasonal MA

- Rapidly changing seasonal factors generally
  - provide considerable smoothing of the series,
  - but also lead to large revisions when adding additional data
  - These revisions last for a small number of years
- Slowly changing seasonal factors generally
  - provide a less smooth adjustment,
  - with relatively small revisions,
  - but the revisions do not become negligible for a number of years.

Notation

- Model coefficients in most of the X-13ARIMA-SEATS are expressed in Box-Jenkins notation
  - $(1-B)(1-B^{12})y_t = (1-B\theta)(1-B^{12}\Theta)a_t$
- Models coefficients in SEATS output of X-13ARIMA-SEATS are expressed differently
  - $(1-B)(1-B^{12})y_t = (1+B\theta)(1+B^{12}\Theta)a_t$
  - The opposite sign as those listed in the X-13A-S modeling output

Comparing X-11 and SEATS Adjustments

- In our experience,
  - X-11 and SEATS seasonal adjustments are very similar for many series
  - SEATS adjustments are often smoother than X-11 seasonal adjustments

Comparing X-11 and SEATS

- This is an issue of interest for several agencies
- We'll concentrate on the experiences of two in particular
  - U. S. Bureau of Labor Statistics
  - German Bundesbank

BLS 2007 Study

- The Bureau of Labor Statistics formed a group to do a comparison study between X-11 and model-based seasonal adjustments
- Examined a cross section of 87 BLS series with X-11, SEATS, and STAMP
  - Used spectral, revisions history, model adequacy and sliding spans diagnostics

Summary of BLS Results

- SEATS is more flexible in estimating a seasonal component
  - It is capable of selecting a slowly evolving seasonal or a rapidly-varying one
  - X-11 has a finite set of seasonal and trend filters

- SEATS' seasonal factors are usually more stable than X-11's
  - Then the seasonally adjusted series is less smooth
- X-11's trend component is usually more stable than SEATS'
  - Tends to put more variation in the irregular component

Summary of BLS Results (2)

- Among seasonal series, residual seasonality almost never appears in the seasonally adjusted series from either method
  - There was seasonality in SEATS' model residuals in a small number of cases, reflecting model inadequacy for the full span of data
  - Use a shorter data span (if possible).

- For a few series, SEATS has difficulty in identifying a usable model for decomposition and falls back on the "airline model"
  - Even in such cases, SEATS' seasonal adjustment is usually reasonable.

Summary of BLS Results (3)

- For series with little or no evidence of seasonality, both methods tend to give appropriate signals:
  - X-11 signals with failing diagnostics;
  - SEATS selects a nonseasonal model.

BLS Study

- In their sample of 87 series, the BLS study compared
  - the frequency of the implied X-11 seasonal filter closest to the SEATS filter
  - the frequency of the actual seasonal filter lengths chosen by X11
- They found that the SEATS filter often implied a more slowly evolving seasonal component than the filter chosen by X-11

Depoutot and Planas (1998)

- Found a close correspondence between adjustment filters from the airline model in SEATS and the adjustment filters in X-11
- Generated a table between the level of the seasonal MA parameter and the seasonal filter from X-11 that matches the adjustment filter best

Depoutot and Planas (1998) Results

Seasonal MA(12) interval	X-11 seasonal filter
0.88 to 1.00	3x15 seasonal filter
0.75 to 0.87	3x9 seasonal filter
0.51 to 0.74	3x5 seasonal filter
0.0 to 0.5	3x3 seasonal filter

Chu, Tiao, and Bell (2012)

- A more extensive study can be found in the paper "A Mean Squared Error Criterion for Comparing X-12-ARIMA and Model-Based Seasonal Adjustment Filters" by Chu, Tiao, and Bell (2012) www.census.gov/ts/papers/chutiaobelltefp.pdf

- Compares X-11 filters with model based filters using MSE
    - Suggests using MSE results as a guide in choosing X-11 filters
    - Developed canonical filters for airline models with $\theta = (.1, .3, .5, .7, .9)$ and $\Theta = (.1, .2, .3, .4, .5, .6, .7, .8, .9)$.
    - Found seasonal MA best predictor of best X-11 filter.

Chu, Tiao, and Bell (2012) Results

Seasonal MA(12)	0.1-0.2	0.3-0.4	0.5-0.6	0.7	0.8-0.9
Best Choice of Moving Average (1st iteration, 2nd iteration)	3x1, 3x1	3x3, 3x3	3x3, 3x5	3x3, 3x9	3x15, 3x15

Note: Chu, Tiao and Bell state that the best filter is in some cases not the only choice – for example for seasonal MA = 0.8 the (3x3, 3x9) is almost as good as the (3x15, 3x15).

Distribution of BLS series

Seasonal MA(12) interval	% of seasonal series	X-11 seasonal filter	% of all series
0.88 to 1.00	28.0	3x15	0.0
0.75 to 0.87	18.7	3x9	2.4
0.51 to 0.74	36.0	3x5	79.3
0.0 to 0.5	17.3	3x3	18.3

Webel (2013)

- Develops criteria for choosing between X-11 and SEATS seasonal adjustment methods
  - A decision tree that allows analysts to choose between the two methods

1) Is seasonality present in the original series?

- Diagnostics used
  - Seasonal F-test of original series
  - Kruskal-Wallis test
  - Spectral test
- If not seasonal, do not perform seasonal adjustment

2) Presence of Seasonal Heteroskedasticity

- For some series, the variance of the series can be different based on the month or seasonal
  - For example, U.S. Housing Starts is more variable in the winter months than in the summer due to the differences in warm and cold winters

- ARIMA model-based seasonal adjustment does not handle this situation very well
  - Assumes a constant variance
- X-11 has extreme value procedures for this situation
  - Need to determine if the degree of seasonal heteroskedasticity is significant enough
  - Can also use monthly moving seasonality ratios (MSR) table in D9A of X-13A-S to choose calendar-month specific seasonal filters

2) Seasonal Heteroskedasticity, cont.

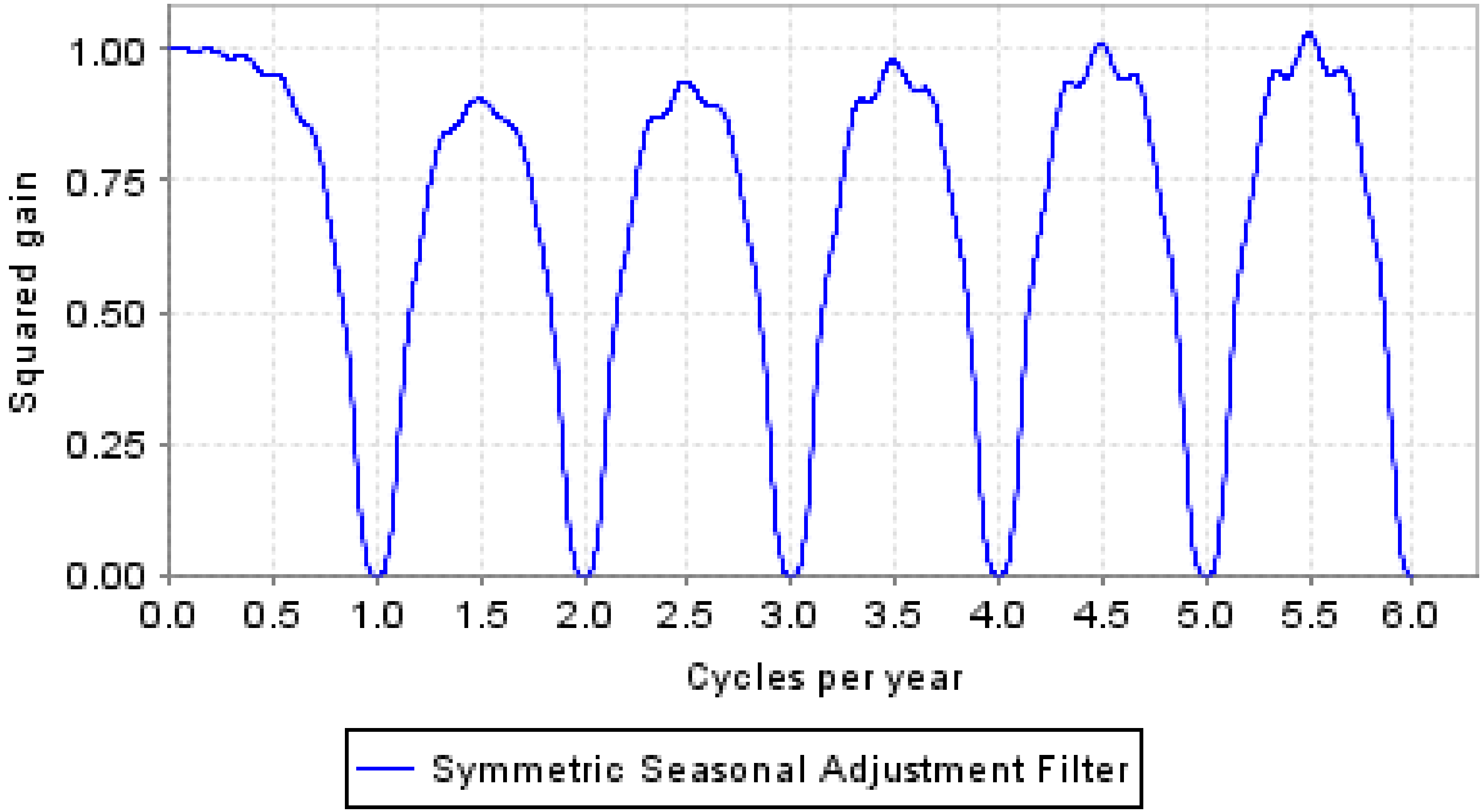
- Diagnostics:
  - Cochran Test, Visual Examination
- If significance is high, use X-11
- If not, are there period-specific causes of seasonality?
  - Use visual examination, knowledge of the process
  - If yes, use X-11

3) Squared gain of SEATS SA filter

- Examine a graph of the squared gain of the SEATS seasonal adjustment filter (run with **`finite = yes`**)
- Needs to show:
  - Troughs at seasonal frequencies
  - Approximately equal to one at non-seasonal frequencies
- If plot does not show these qualities, use X-11

Squared Gains of the Symmetric Seasonal Adjustment Filter

International Airline Passengers Data from Box and Jenkins



4) Revisions

- Diagnostic:
  - Revisions history of seasonally adjusted series
  - Revisions history of month-to-month changes
- Choose method with lowest revisions
  - Note: revisions for model based method can depend on the value of model coefficients, especially the seasonal MA parameter

For more details, see

- "Data-driven selection criteria for X-13ARIMA-SEATS seasonal adjustment algorithms" by Karsten Webel, Proceedings of the 2013 Joint Statistical Meetings.