X-11 Seasonal Adjustment

Seasonal Adjustment With X-13ARIMA-SEATS

2019

Economic Statistical Methods Division

U.S. Census Bureau

United States Census Bureau
U.S. Department of Commerce
Economics and Statistics Administration
U.S. CENSUS BUREAU
census.gov

Objectives

- **At the end of this unit, you should understand –**
  - Basics of the X-11 seasonal adjustment method

Two Parts of X-11 Adjustment
(1) RegARIMA Modeling

- Prior adjusts the series before seasonal adjustment
  - Trading day, moving holiday adjustments
  - Outlier adjustments
  - Forecasts

  - Parametric: model with coefficients

Two Parts of X-11 Adjustment
(2) X-11

- Seasonally adjusts the series
  - Nonparametric: uses filters but no model, no coefficients

Tables of X-13A-S Output

- A: Prior adjustments (regARIMA model) before any seasonal adjustment
- B: X-11's preliminary estimation of seasonal, trend, and extreme value weights
- C: X-11's intermediate estimation of seasonal and trend, plus final estimation of extreme value weights
- **D: X-11's final estimation of components**
- **S: SEATS seasonal adjustment estimation**

More Output Tables

- E: Additional X-11 tables
- F: X-11's seasonal adjustment **diagnostics**

- G: Spectral **diagnostics**
- R: Revisions history **diagnostics**
- S: Sliding spans **diagnostics**

- Yes, S denotes SEATS and sliding spans tables, but the table numbers do not overlap, and they appear in separate parts of the output

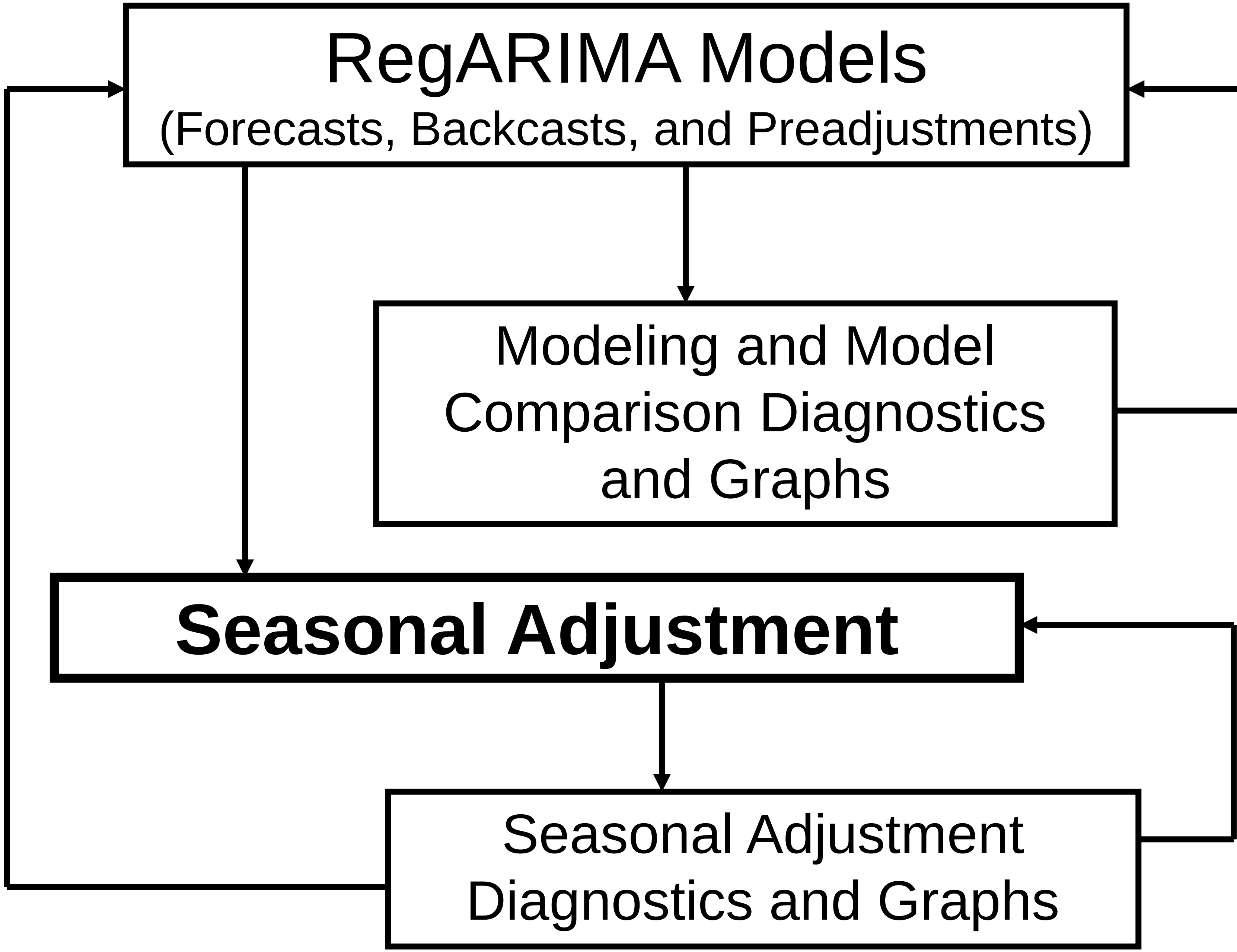
Before X-11-ARIMA

- Before Statistics Canada released X-11-ARIMA, the program was just X-11 – just seasonal adjustment, although the software included
  - Methods for trading day and moving holiday adjustments
  - Treatment of extreme values

X-11 Approach to Seasonal Adjustment

- Use moving average filters to estimate and remove the seasonal pattern

- The **`X11`** spec corresponds to the X-11 method


RegARIMA Models
(Forecasts, Backcasts, and Preadjustments)
Modeling and Model Comparison Diagnostics and Graphs
Seasonal Adjustment
Seasonal Adjustment Diagnostics and Graphs

Step-by-Step Through X-11

Seasonal Adjustment With the X-11 Method

by Dominique Ladiray and Benoît Quenneville (2001)

Springer-Verlag Lecture Notes in Statistics 158

[Download in Spanish: https://www.census.gov/ts/papers/x11_spanish.pdf](https://www.census.gov/ts/papers/x11_spanish.pdf)

[Download in French: https://www.census.gov/ts/papers/x11_french.pdf](https://www.census.gov/ts/papers/x11_french.pdf)

Shows intermediate steps not available in output tables

Additional Necessity of Local Smoothing

- X-11 local smoothing with filters requires additional protection against extreme values (different from outliers)
  - Outliers are prior-adjusted out of the series using the regARIMA model
  - Extreme values are replaced within the X-11 procedure

Extreme Values in Final Seasonally Adjusted Series

- Extreme values are assigned to the irregular component and are included in the seasonally adjusted series
  - Like some outliers (additive outliers, temporary changes)

Extreme Value Replacement – Background

- Y = C x S x I (or Y = C + S + I)
- Detrend the original series = *SI ratio*
  - Y/C = S × I (or Y – C = S + I)
  - "SI Difference" for additive adjustments, in some papers
  - NOT the Moving Seasonality Ratio (MSR), which is I/S!
  - Rough estimate of the seasonal component
- Replace the SI ratios that are "extreme"
  - Based on 1964 procedure from the Bureau of Labor Statistics
- Use the new SI ratios to estimate the seasonal component

Extreme Value Replacement Procedure

- Estimate a standard error from the irregular component
  - Sigma (σ)
  - (Estimate over moving 5-year subspans)
- Reduce or eliminate the influence of points with large irregular compared to that standard error

- By default, points with irregular
  - Less than 1.5 σ are not replaced ("fully weighted," weight = 1)
  - Greater than 2.5 σ a are replaced entirely (weight = 0)
  - Between 1.5 σ and 2.5 σ are partially replaced (weight is assigned linearly, between 0 and 1)

Extreme Value Weight Calculations for Multiplicative Adjustments

Set weights, $0 \leq W_t \leq 1$, for I_t from σ_t as

$$|I_t - 1| \leq 1.5\,\sigma_t \quad \Rightarrow \quad W_t = 1$$

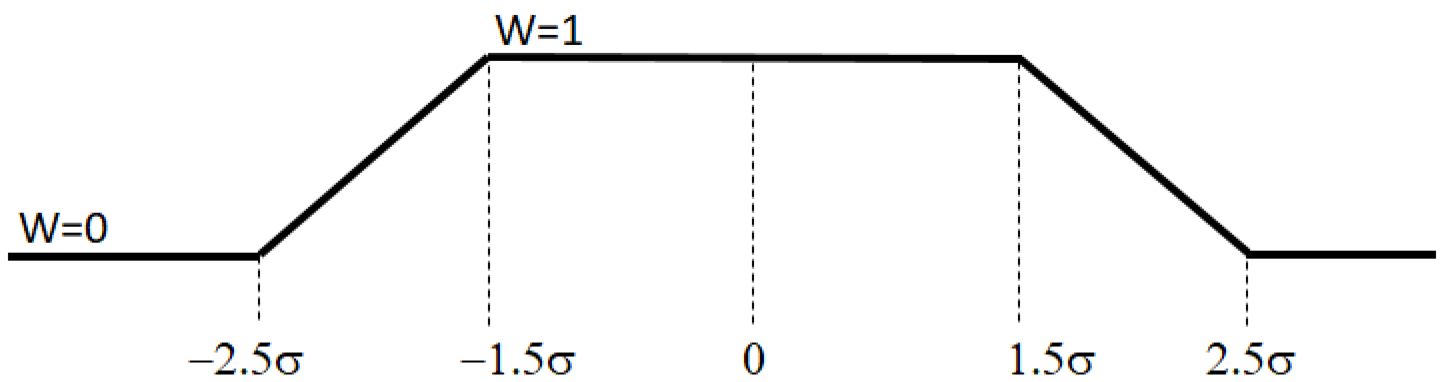
$$1.5\,\sigma_t < |I_t - 1| < 2.5\,\sigma_t \quad \Rightarrow$$

$$W_t = (2.5\,\sigma_t - |I_t - 1|) \,/\, (2.5\,\sigma_t - 1.5\,\sigma_t)$$

$$2.5\,\sigma_t \leq |I_t - 1| \quad \Rightarrow \quad W_t = 0$$

Extreme Value Weight Function

Assign partial weights (0 < W < 1) linearly between 1.5σ and 2.5σ



Sigma Limits

- Default values: $1.5\sigma - 2.5\sigma$
- If changed, usually raised
  - For example, $1.8\sigma - 2.8\sigma$ or $1.8\sigma - 3.0\sigma$
    - **`sigmalim = (1.8 2.8)`**
  - Fewer extreme values
  - Higher partial weights

Why Change Sigma Limits?

- "Too many" extreme values at the default level?
  - Fewer extreme values mean fewer replacements
- More stability in the adjustment with higher limits?
  - Stability diagnostics discussion coming soon

Replacement Values in Preliminary Tables (B4, B9)

- In general, the replacement value comes from an average with the two nearest preceding SI ratios and two nearest following SI ratios of the same month/quarter ***that are fully weighted*** together with the partially-weighted original value
  - That is, for an extreme value in 2008 Q3, use fully-weighted Q3 SI ratios from 2006, 2007, 2009, and 2010

- Use fully-weighted points from further out if there are not enough in the closest years

Preliminary Replacement Value (Monthly)

- Calculate replacement values
  - If $W_t = 1$, then do not replace the value
  - If $W_t < 1$ and $W_{t \pm 12} = W_{t \pm 24} = 1$, replace SI_t with

$$SI_t^{(rep)} = \frac{1}{4 + W_t} \left[SI_{t-24} + SI_{t-12} + W_t SI_t + SI_{t+12} + SI_{t+24}\right]$$

(For quarterly series, use $SI_{t \pm 4} = SI_{t \pm 8}$)

- $SI_t^{(rep)}$ replaces the extreme SI_t in the computations

Final Extreme Value Adjustments (B20, C20)

- Remember Y = S * C * I or Y = S + C + I
- For extreme values, I is considered too big, so an adjustment value (V_t) is calculated based on the estimated irregular (I_t) and weight (W_t):
  - $V_t = I_t / (1 + W_t * (I_t - 1))$ (multiplicative)
  - $V_t = I_t * (1 - W_t)$ (additive)
- The extreme value adjustments are divided from the prior-adjusted series (C1, D1)
  - Note if a point is zero-weighted, $V_t = I_t$ so the entire estimate of the irregular is removed from the value for seasonal factor calculation

Tables in the Output That Show Replacements

- Table C17 shows the final weights for points determined to be extreme
- Table C20 is the replacement adjustment
- Table D1 is the prior-adjusted series, with an adjustment for the points found to be extreme –B1/C20 (B1 – C20)
  - Start of the D iteration
- Table D7 is a preliminary Henderson trend estimate
- Table D8 are the unmodified SI ratios – B1 / D7 (B1 – D7)
- Table D9 shows the replacement SI ratios for points which are extreme -- D1/D7 (D1 – D7)

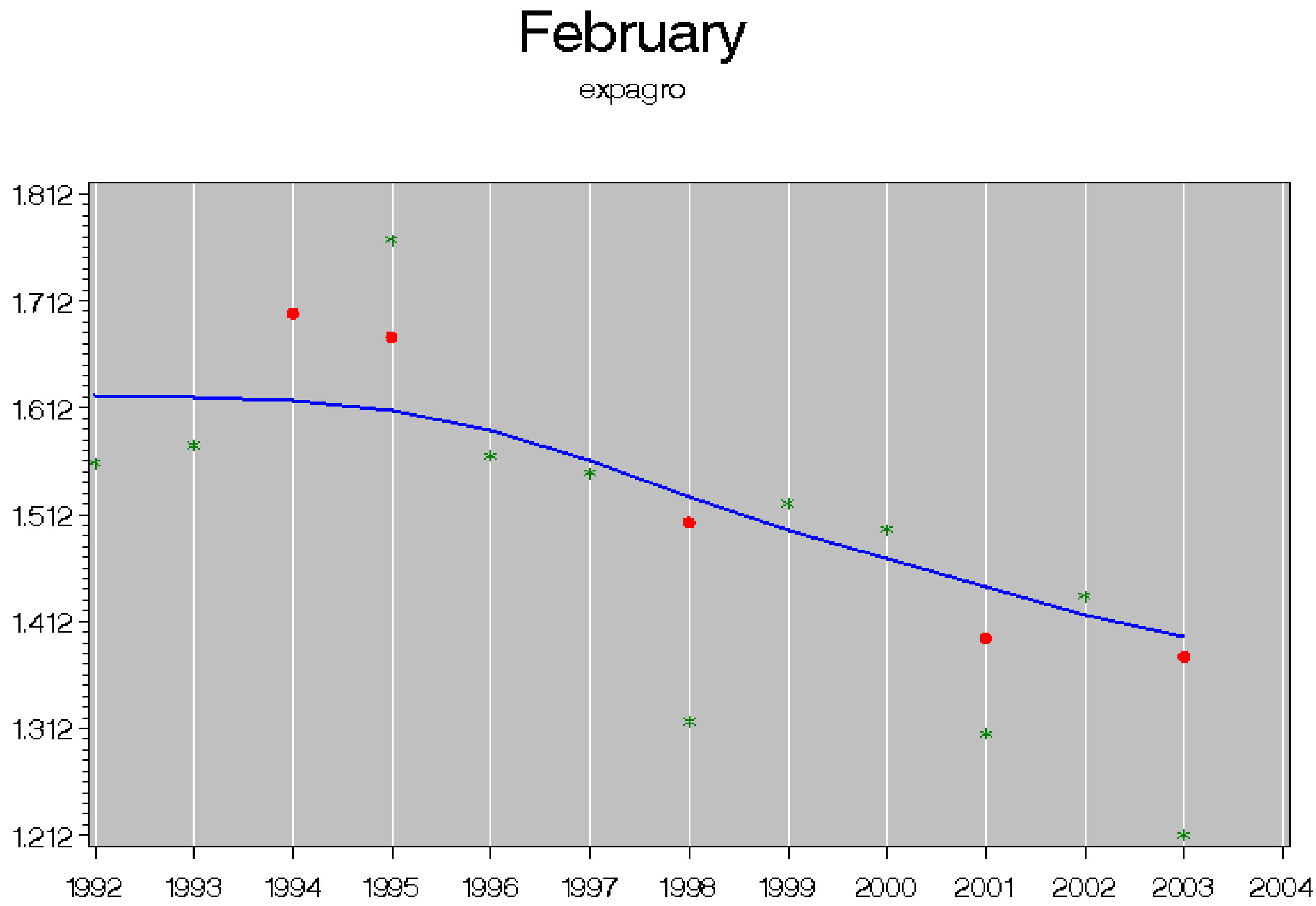
Table C17 (Final Weights)

```
C 17  Final weights for irregular component
 From  1992.Jan to 2003.Jun
 Lower sigma limit    1.50        Upper sigma limit    2.50
-------------------------------------------------------------
          Jan      Feb      Mar      Apr      May      Jun
          Jul      Aug      Sep      Oct      Nov      Dec
-------------------------------------------------------------
 1992    100.0    100.0     23.6      0.0    100.0    100.0
         100.0    100.0    100.0    100.0    100.0    100.0

 1993    100.0    100.0    100.0    100.0    100.0    100.0
         100.0    100.0    100.0    100.0    100.0    100.0

 1994     81.3     27.6    100.0    100.0    100.0    100.0
         100.0    100.0    100.0    100.0    100.0     91.2
```

Table D 9

D 9 Final replacement values for SI ratios
From 1992.Jan to 2003.Jun
Observations 138

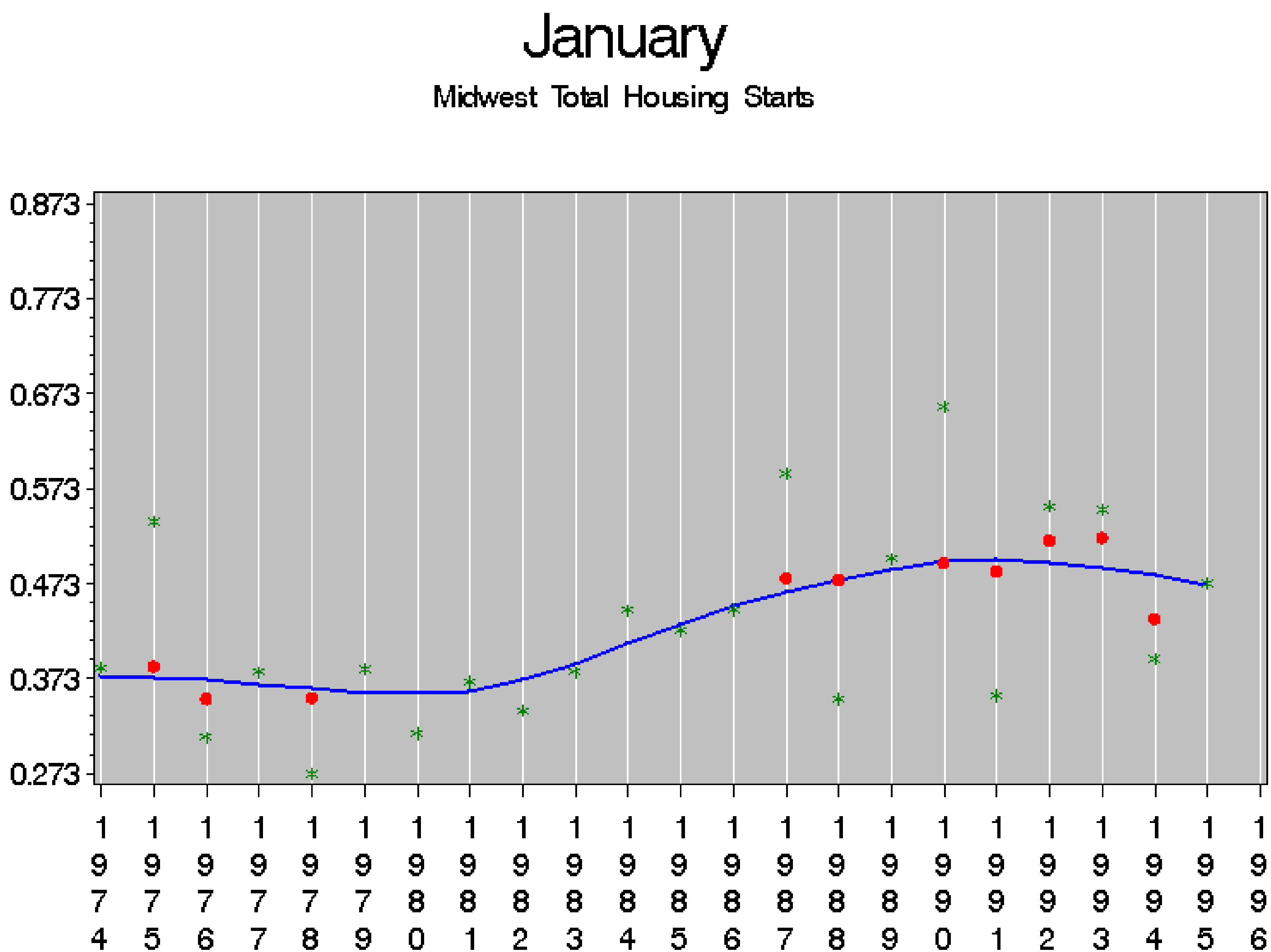
	Jan / Jul	Feb / Aug	Mar / Sep	Apr / Oct	May / Nov	Jun / Dec
1992	********	********	170.1	127.1	********	********
	********	********	********	********	********	********
1993	********	********	********	********	********	********
	********	********	********	********	********	********
1994	125.4	170.1	********	********	********	********
	********	********	********	********	********	95.1



SI Ratio plot – Seasonal factors (solid line, Table D10) plotted against the Unadjusted SI ratios (green stars, Table D 8) and the Replacement Values (red dots, Table D 9)

Extreme Value Replacement

- Especially at the end of the series, extreme value replacements can strongly affect the seasonal adjustment
- New nonextreme values can greatly change the irregular weights
- Extreme value choices can change from one month to the next (with new or revised values)


January
Midwest Total Housing Starts
0.873
0.773
0.673
0.573
0.473
0.373
0.273
1974
1975
1976
1977
1978
1979
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996

Note on Extreme Values

- Replacements are for individual series points, and the extreme value procedure does not account well for more complicated outlier effects, like level shifts, temporary changes, ramps, etc.

- Outlier adjustments are good for improving the model's estimation of forecasts and calendar effects, especially for these complicated outliers, but the extreme value replacements are needed for the local smoothing of X-11

Features of X-11 Seasonal Adjustment

- Local smoothing
- **Iterative refinement**
  - Each iteration improves the estimates of the next iteration

Estimation Difficulty

- Estimating trend in the presence of seasonal movements is difficult
- Estimating seasonal movements is difficult in the presence of a trend-cycle

X-11's Solution

- Iterate between trend and seasonal estimation to get successively more refined estimates of the seasonal and trend-cycle

Iterative Refinement in X-11

- Step 1. Estimate the trend
- Step 2. Detrend the series
- Step 3. Estimate the seasonal
- Repeat Steps 1-3
- **Repeat that procedure twice**

- Estimate the final trend and final irregular

Part A (RegARIMA)

- Calculations before X-11, already done
- Choice of multiplicative or additive adjustment
  - Log or no log
- Adjustments for outliers, trading day and moving holiday effects, other prior adjustments
  - Generally, adjustments are from regression estimates

Different Modes of Adjustment

- Multiplicative adjustments have nice characteristics, more intuitive, when appropriate for the series
  - Differences can be thought of in terms of percent changes
  - Seasonal factors centered on 1, always positive (nonzero)
    - Also true of the Irregular component
- Additive adjustments
  - Series with zero or negative values require additive adjustment (cannot take log transformation)
  - Seasonal factors centered on 0, positive and negative (can be zero)
    - Also true of the Irregular component

Tied to the transformation choice (log or none)

Seasonal Adjustment Modes

Multiplicative (Log Transformation)

$Y = S \times C \times I$ $A = Y / S = C \times I$

Additive (No Transformation)

$Y = S + C + I$ $A = Y - S = C + I$

Y = Original series

A = Adjusted series

Part B (B Iteration) – Purpose

- Preliminary estimation of the extreme values
- Feed the preliminary replacements for extreme values to the C iteration

Part C (C Iteration) – Purpose

- Final estimation of the extreme values
- Feed the final replacements for the extreme values to the D iteration

Part D (D Iteration) – Purpose

- Final estimation of
  - Seasonally adjusted series (A)
  - Seasonal factors (S)
    - Combined factors incorporate the trading day and moving holiday regression effects estimated in Part A
  - Trend-cycle (C)
  - Irregular (I)

- Publish the final seasonally adjusted series (and maybe the combined factors as well)

Assumptions for the Formulas

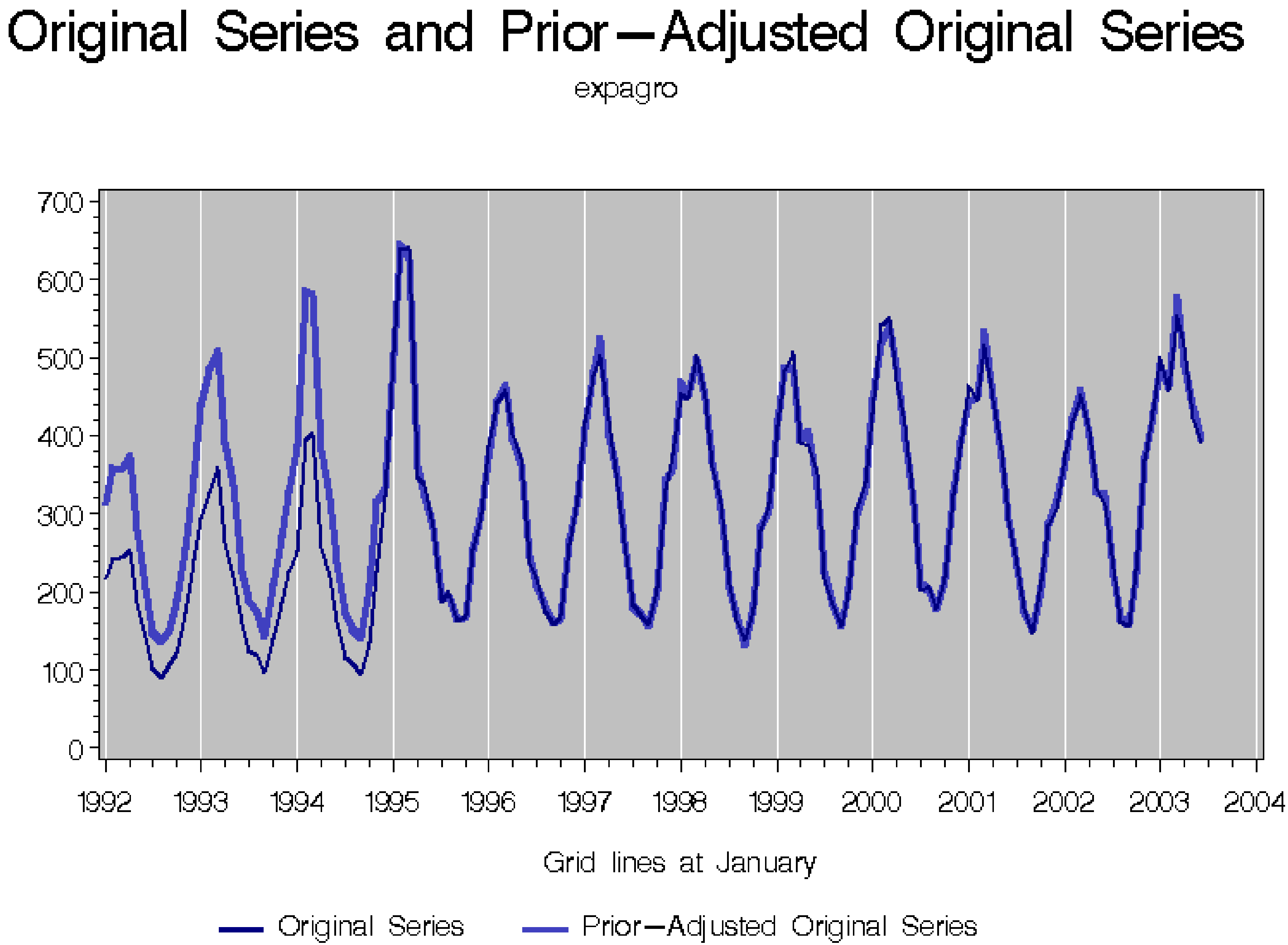
- Monthly series (period is 12)
- Multiplicative seasonal adjustment

$$(Y_t = C_t \times S_t \times I_t)$$

- Primarily default (automatic) settings

Starting Point for X-11 Method

- X-11's starting point, which is the B iteration, is Table B1 – the prior-adjusted original series
  - Starting point for the C iteration is Table C1, the prior-adjusted series also adjusted for *preliminary* extreme value weights
  - Starting point for the D iteration is Table D1, the prior-adjusted series also adjusted for *final* extreme value weights


Original Series and Prior-Adjusted Original Series
expagro
700
600
500
400
300
200
100
0
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
Grid lines at January
Original Series
Prior-Adjusted Original Series

Iteration Steps

- Each iteration generally has the same steps – differences include
  - Starting version of the series
  - Extreme value weights and replacements
  - Filter choices (possibly)
- Can see *most* tables using **`print=all`** in the **`X11`** spec

Repeated Method

The method always is

- Step 1. Estimate the trend
- Step 2. Detrend the series
- Step 3. Estimate the seasonal*

Filter choices may differ; starting series will differ

*The estimation always normalizes the seasonal factors with a 2x12 (or 2x4) filter

Estimate Crude Trend (Tables B2, C2, D2)

- Compute a centered 12-term (2×12) moving average as a first trend estimate
  - It has 13 terms

$$C_t = C_t^{2x12}$$

$$= \frac{1}{24}Y_{t-6} + \frac{1}{12}Y_{t-5} + \cdots + \frac{1}{12}Y_t + \cdots + \frac{1}{12}Y_{t+5} + \frac{1}{24}Y_{t+6}$$

Detrend the Series (Tables B3, C4, D4)

SI Ratio: Ratio of the starting series to the estimated trend

(starting series depends on the iteration)

$$SI_t = \frac{Y_t}{C_t}$$

B iteration: B3 = B1/B2

C iteration: C4=C1/C2

D iteration: D4=D1/D2

Find and Replace Preliminary Extreme Values (B4)

- In the B iteration only
  - Estimate the seasonal factors
  - Estimate the irregular
  - Calculate a moving standard deviation
  - Detect the extremes according to the sigma limits
  - Adjust the extreme SI ratios with a linear combination of four nearest surrounding fully weighted SI ratios

Estimate Preliminary Seasonal Factors From the SI Ratios (B5, C5, D5)

- Preliminary estimates of the seasonal factors
- Automatic filter choice is 3x3
  - If moving average is set, use the chosen filter
- Average of SI ratios, using replacements for extreme values wherever necessary

$$S_t^{3x3} = \frac{1}{9}SI_{t-24} + \frac{2}{9}SI_{t-12} + \frac{1}{3}SI_t + \frac{2}{9}SI_{t+12} + \frac{1}{9}SI_{t+24}$$

Estimate Seasonally Adjusted Series (Tables B6, C6, D6)

$$A_t = \frac{Y_t}{S_t}$$

B iteration: B6 = B1/B5

C iteration: C6 = C1/C5

D iteration: D6 = D1/D5

Now, Repeat

Estimate Trend (B7, C7, D7)

- Estimate the trend by applying a Henderson trend filter to the preliminary seasonally adjusted series (B6, C6, D6)
- By default, the series components determine the Henderson filter length, 9, 13, or 23 (or user can set length 2H+1)

Trend Filter Choices

- Henderson trend filter choices are based on noise-to-signal ratios, labeled I/C in the output file
  - 9-term filter ($H = 4$) when $I/C < 1.0$
  - 13-term filter ($H = 6$) when $1.0 \leq I/C < 3.5$
  - 23-term filter ($H = 11$) when $3.5 \leq I/C$

  - For B7, choice is between 9-term and 13-term only

Now: Look at Remaining Tables in the B and C Iterations
Later: Look at D Tables

Detrend the Series (B8 & B9, C9)

- Get SI ratios by dividing the starting series by the Henderson trend
  - B8 = B1/B7; C9 = C1/C7

- C9 is detrended & extreme adjusted (C1 has adjustments for extreme values)
- B8 has no adjustments for extreme values
- B9 – repeats steps of B4 using B8 values to get detrended & extreme adjusted series

Estimate Seasonal Factors (B10, C10)

- Automatic filter choice is 3x5
  - If seasonal moving average is set, use the chosen filter
  - Apply filter to B8 corrected for extremes (B9) and to C9

Estimate Seasonally Adjusted Series (B11, C11)

- B11 = B1/B10; C11 = B1/C10

Note: Neither B11 nor C11 have extreme value adjustments. Final extreme values for the tables are estimated from these series.

Estimate the Irregular (B13, C13)

- Remove the latest estimate of the trend from the seasonally adjusted series
  - B13 = B11/B7; C13 = C11/C7

Extreme Value Weights, Preliminary (B17) and Final (C17)

- Using the irregular component (B13, C13), find the moving standard deviation, sigma
- Find the extreme values using the sigma limits specified
  - Default limits are 1.5 σ – 2.5 σ
- Find the weights for each extreme value

Adjustment Values for Extreme Values (B20, C20)

- For each partially- or zero- weighted point
  - B20 = B13/ [1 + B17 × (B13 – 1)] (multiplicative)
  - Or B20 = B13 × (1 – B17) (additive)
  - Recall
    - B13 is an estimate of the irregular
      - Multiplicative irregular is centered on 1
      - Additive irregular is centered on 0
    - B17 is the extreme-value weight
- Similarly for C20

Starting Points for the C and D Iterations (C1, D1)

- For each point identified as extreme in the previous iteration, divide the prior-adjusted series by the extreme value adjustment; that is,
  C1 = B1/B20 if B17 < 1, else C1=B1

  D1 = B1/C20 if C17 < 1, else D1=B1

- Recall, B17 and C17 are the extreme value weights
- B20 and C20 are the adjustment values for extreme values

C1 & D1 Extreme Value Adjustment

- Series is $S \times C \times I$
- Fully weighted points (not extremes) stay the same
- Zero-weighted points have adjustment value equal to B13|C13, the iteration's final irregular, so C1|D1 = $(S \times C \times I) / I = S \times C$
- Otherwise the adjustment depends on the weight and the size of the irregular

D Iteration

- The final steps of the D iteration are slightly different
- Start with the estimated trend, D7

Unmodified and Modified SI Ratios (D8, D9)

- To see the SI ratios before modification, divide the original series by the trend: D8 = B1/D7

- To see the SI ratios with extreme value replacement, divide the original series adjusted for extremes by the trend: D9 = D1/D7

Estimate Seasonal Factors (D10)

- Automatic method computes the final seasonal factors with a seasonal moving average determined by the Global Moving Seasonality Ratio (GMSR)
  - 3x3 when GMSR $\leq$ 2.5
  - 3x5 when 3.5 < GMSR $\leq$ 5.5
  - 3x9 when GMSR > 6.5
- Apply the weights to D8, which has D9 replacements for extremes

Seasonal Filter Choice

- When seasonal moving average filter is set to a specific length, it is the same for all iterations and all steps
- (We strongly recommend setting seasonal filters for production runs, not using automatic selection)

How the Outliers Get Back Into the Seasonally Adjusted Series

- X-11 starts with Table B1, with outlier (and other) effects removed from the series

- Only in D Iteration, once X-11 calculates the final seasonal factors, it calculates the final seasonally adjusted series using the original series including outlier effects (but adjusted for trading day and moving holiday effects – not Table A1, not Table B1)

Final Seasonally Adjusted Series (D11)

- The final seasonally adjusted series is the original series, modified by calendar effects only, divided by the final seasonal factors
    - For additive adjustments, subtract the final seasonal factors

Final Henderson Trend (D12)

- D11 (final seasonally adjusted series) modified for extreme values and smoothed with the Henderson trend filter chosen on basis of the I/C ratio
  - Or user can set the Henderson trend filter

Final Irregular Component (D13)

- Compute irregular factors from the ratio of the seasonally adjusted series to the Henderson trend

- D13 = D11/D12

Final Combined Factors (D16)

- The final seasonally adjusted series

  D11 = A1/D16 (multiplicative)

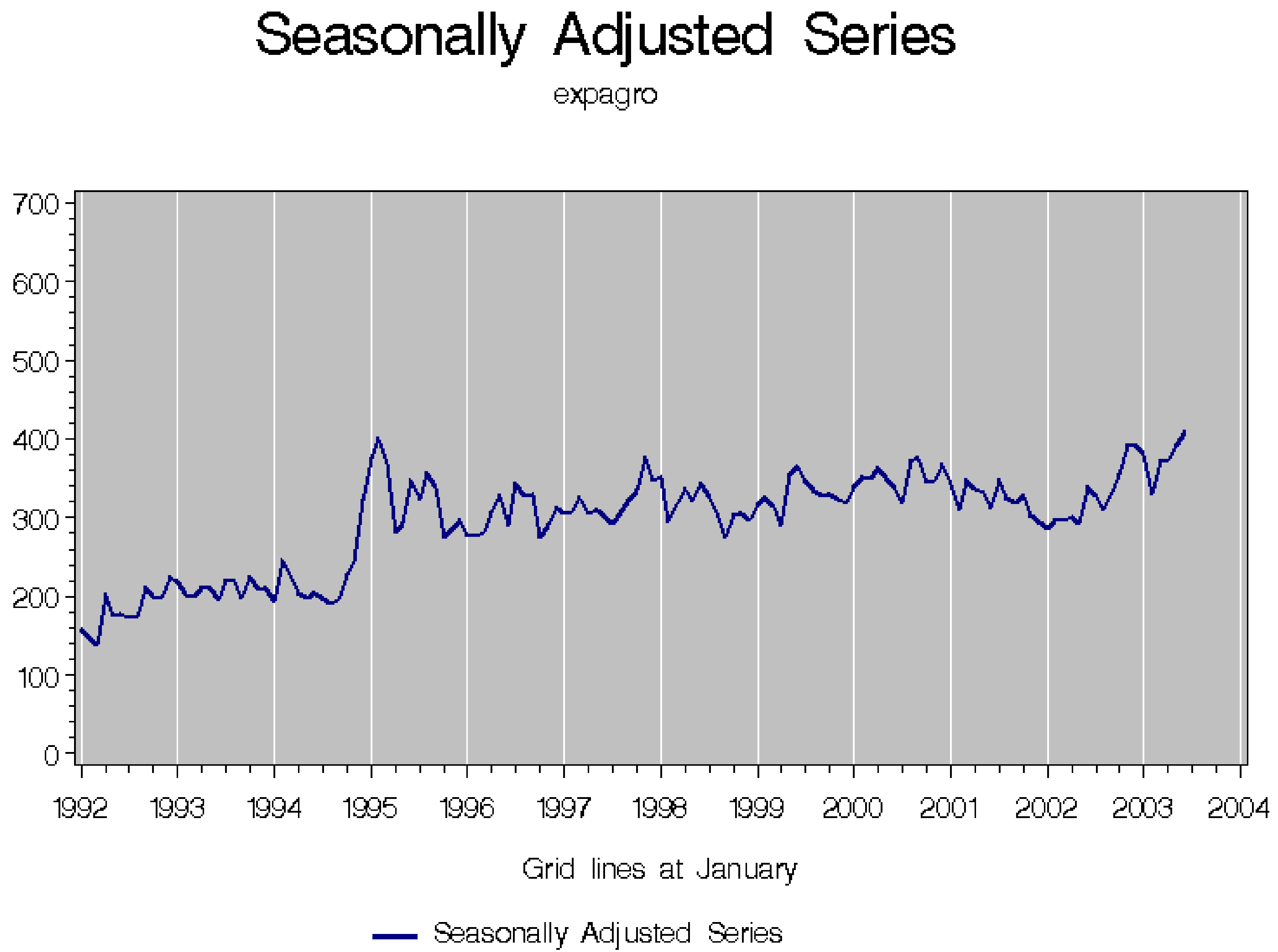
  D11 = A1 – D16 (additive)

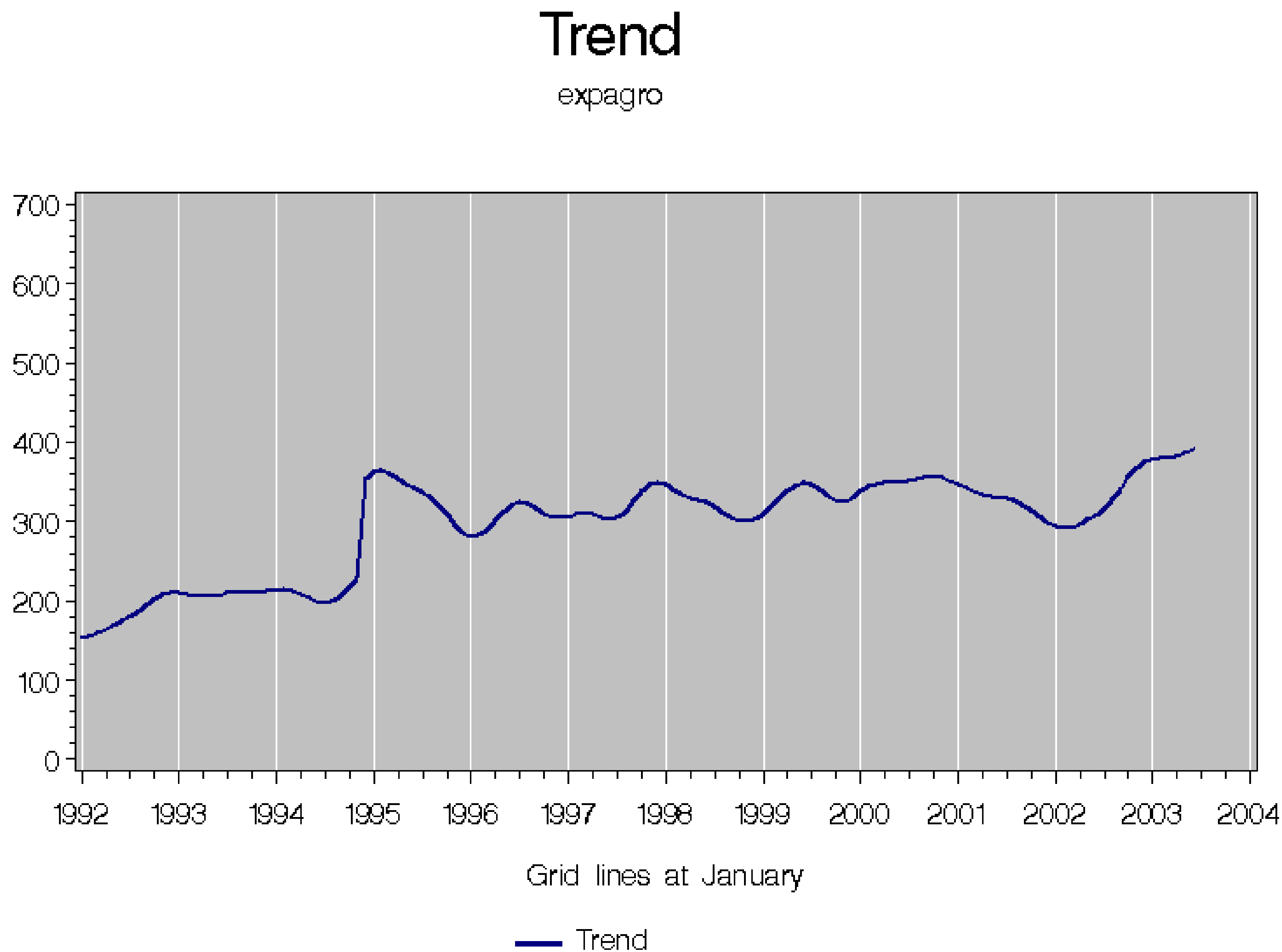
  - (Usually)
- D16 is the combined seasonal, holiday, and trading day factors (includes any user-defined holiday effects – as well as other types of user-defined effects if set as permanent)

Final Combined Factors (D16)
But in E18

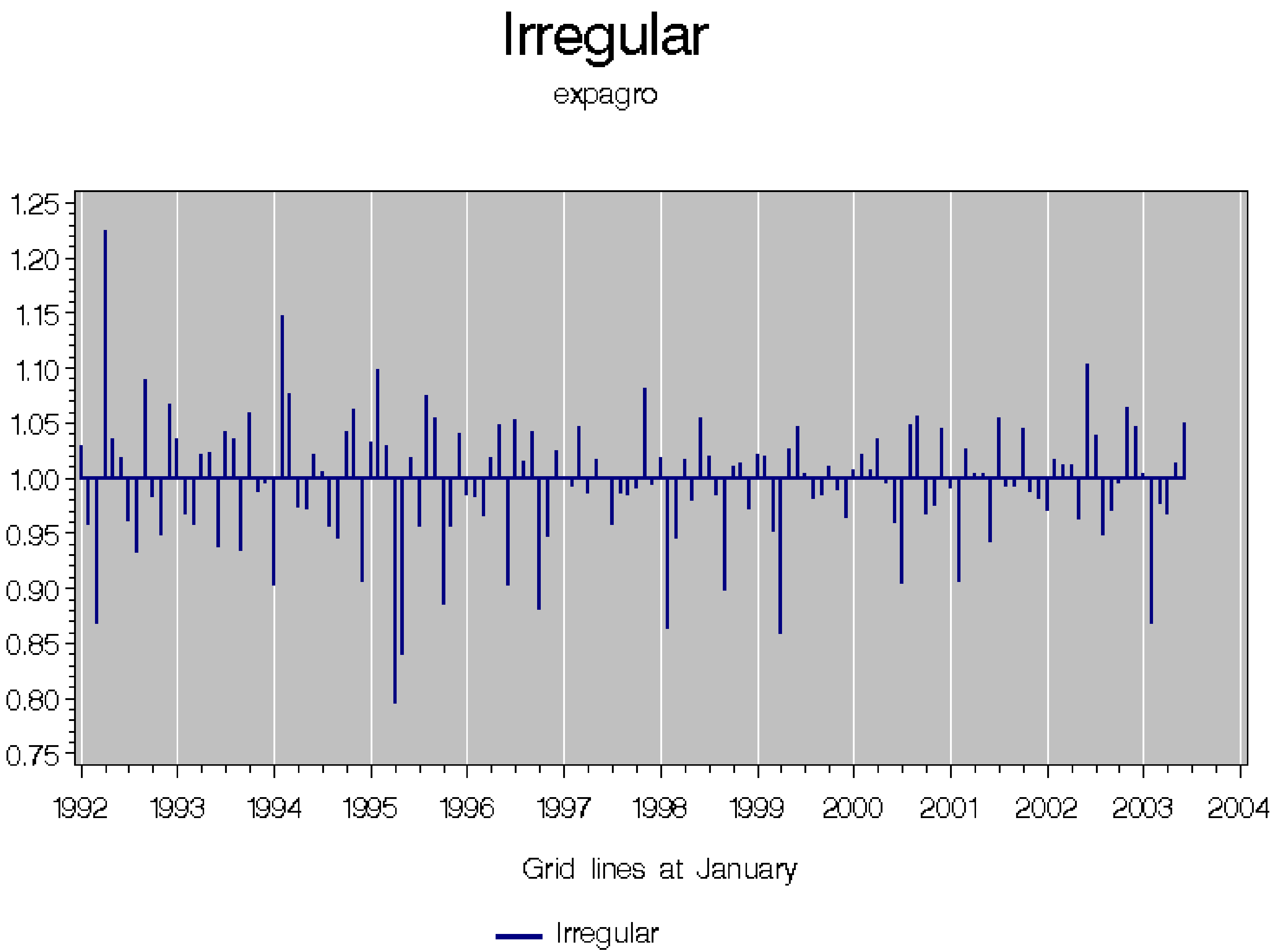
- Final combined factors
  - Seasonal x TD x Holiday
- For additive adjustments
  - Seasonal + TD + Holiday

- Final adjustment ratio is Table E18
  - E18 = A1 / D11
  - For multiplicative OR additive, as long as all values of A1 are > 0

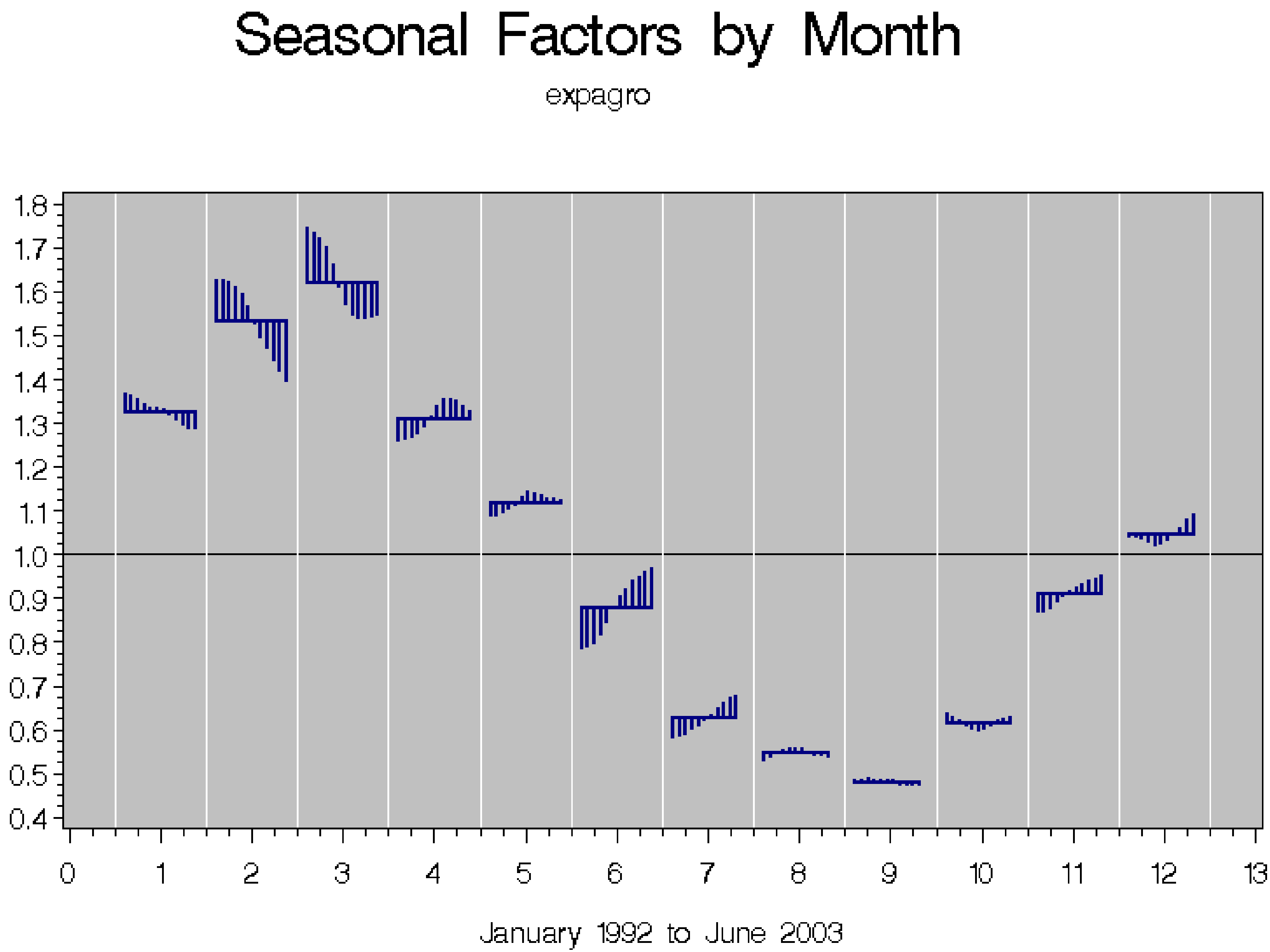

Seasonally Adjusted Series
expagro
700
600
500
400
300
200
100
0
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
Grid lines at January
Seasonally Adjusted Series



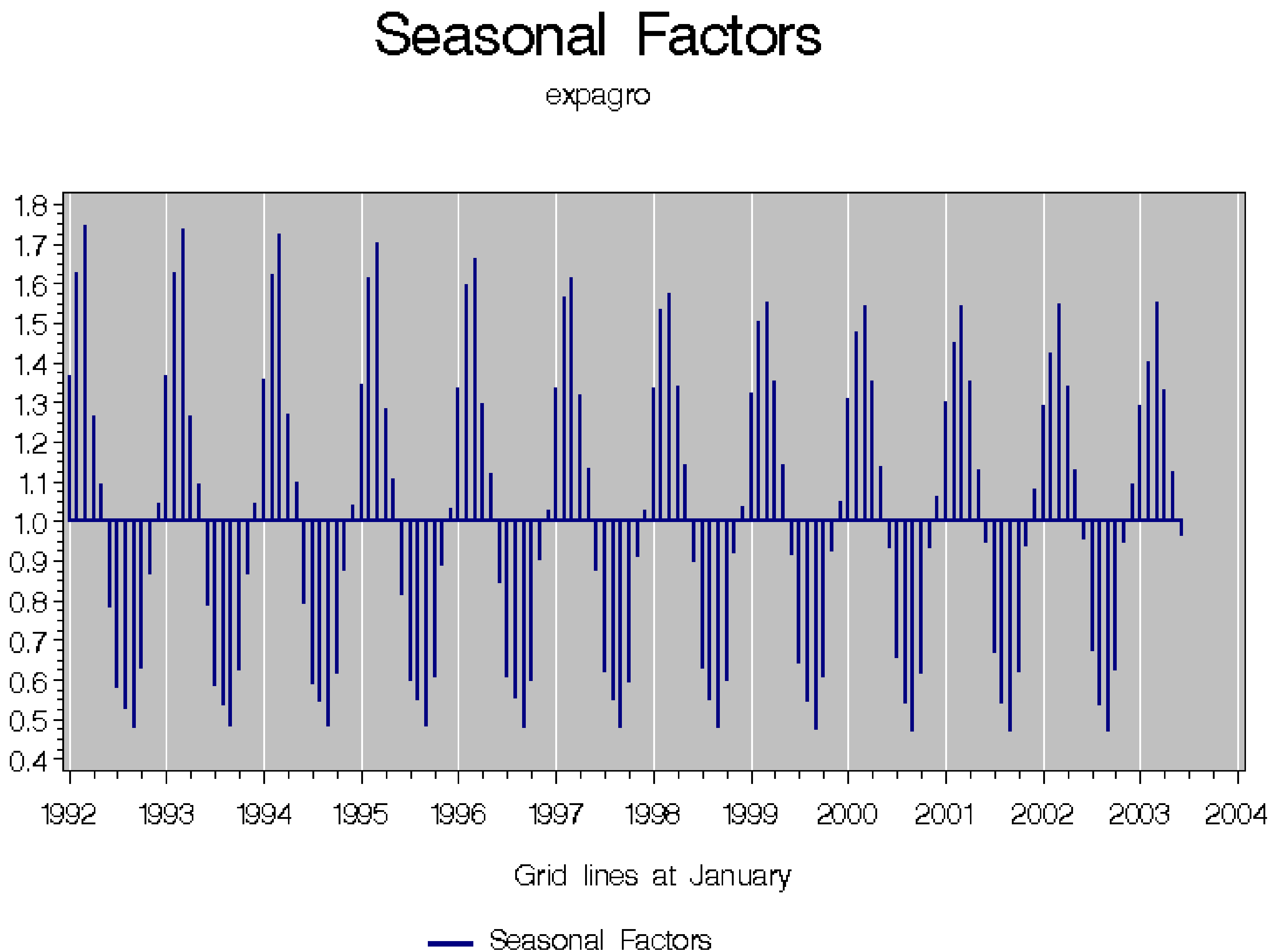
Trend
expagro
700
600
500
400
300
200
100
0
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
Grid lines at January
Trend



Irregular
expagro
1.25
1.20
1.15
1.10
1.05
1.00
0.95
0.90
0.85
0.80
0.75
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
Grid lines at January
Irregular



Seasonal Factors by Month
expagro
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0 1 2 3 4 5 6 7 8 9 10 11 12 13
January 1992 to June 2003



Seasonal Factors
expagro
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
1992
1993
1994
1995
1996
1997
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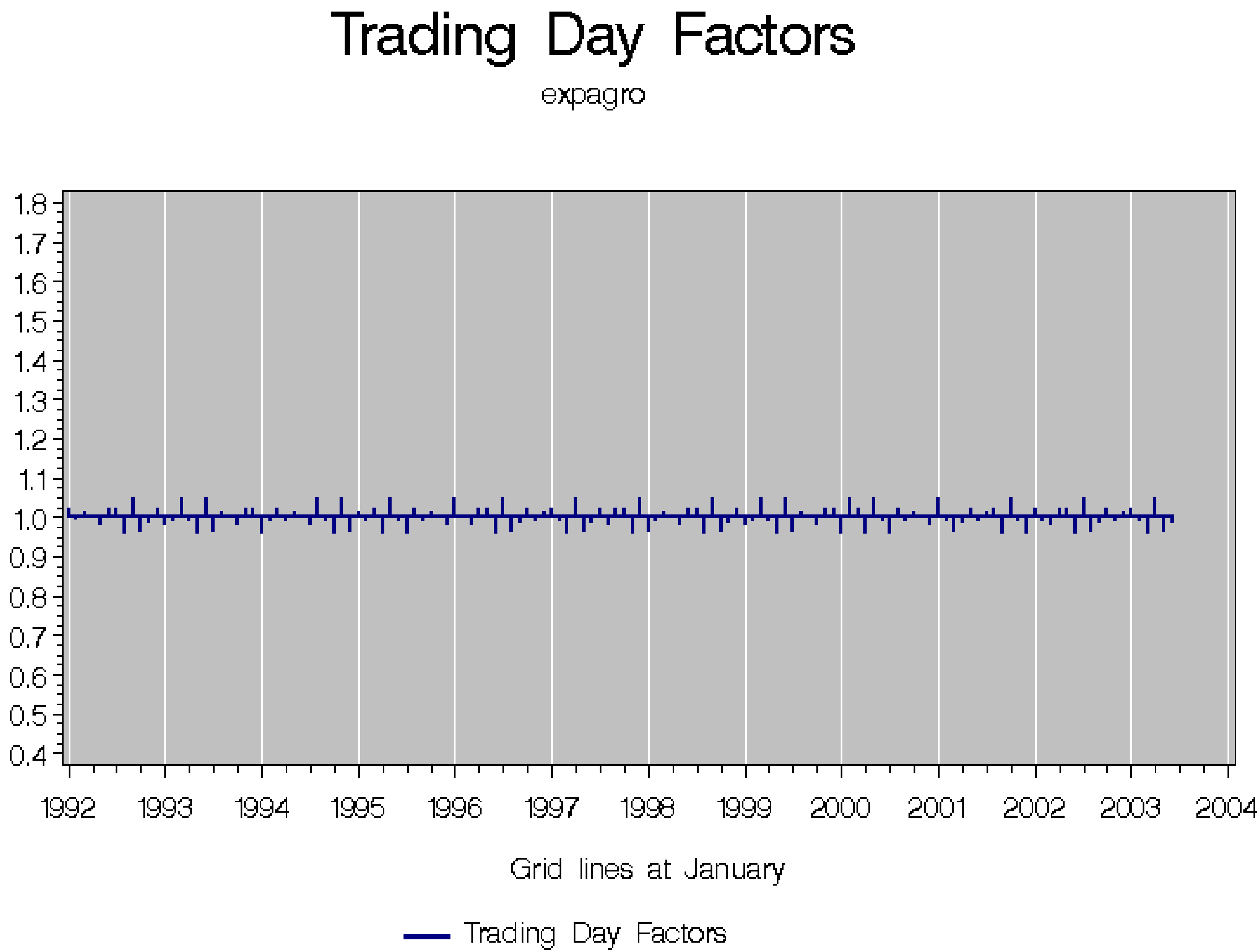
2000
2001
2002
2003
2004
Grid lines at January
Seasonal Factors


Trading Day Factors
expagro
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
1992
1993
1994
1995
1996
1997
1998
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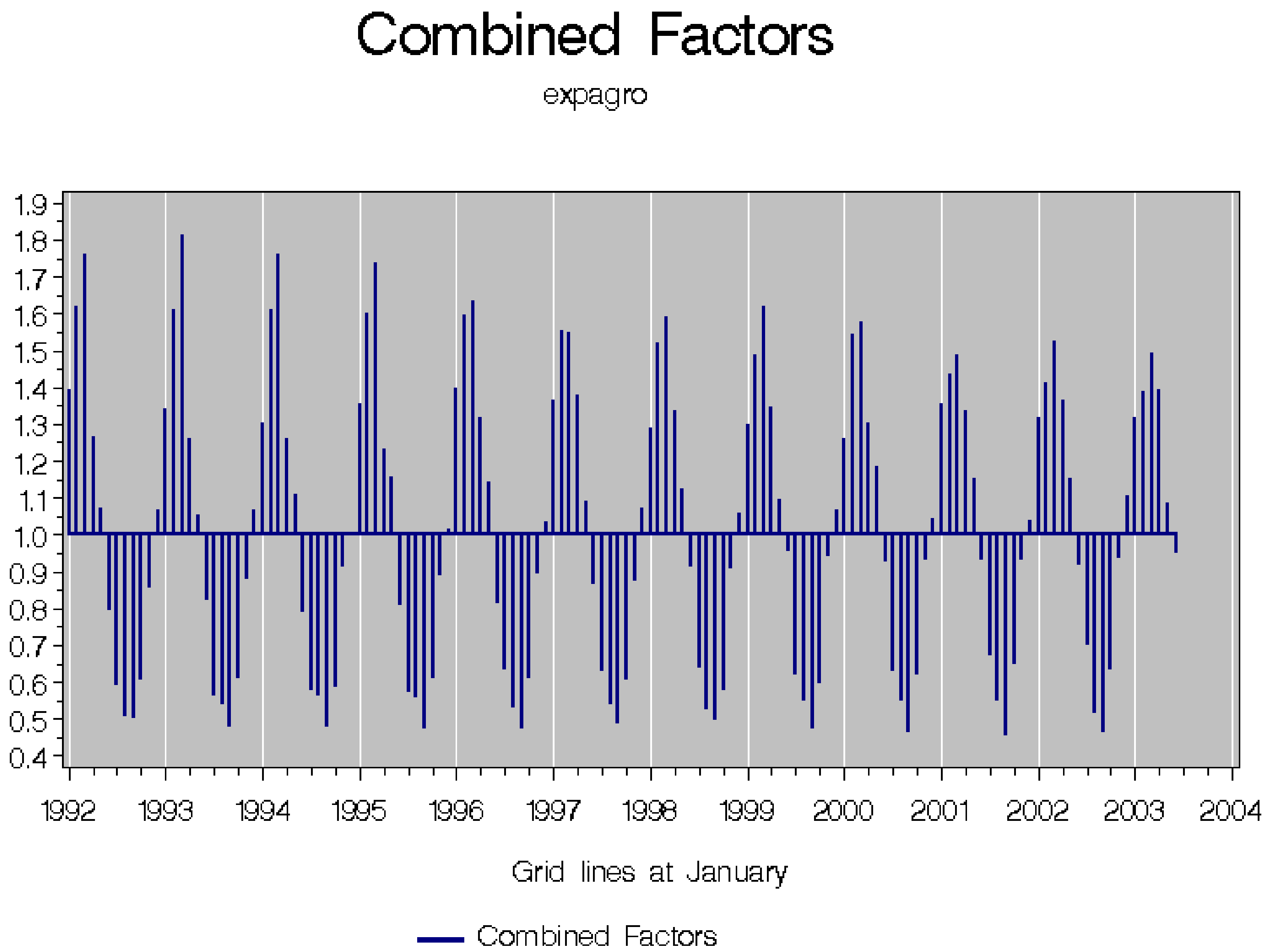
2000
2001
2002
2003
2004
Grid lines at January
Trading Day Factors


Combined Factors
expagro
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
Grid lines at January
Combined Factors

X-11 B, C, and D Iterations

Raw or prior-adjusted series	B1	C1	D1
Trend estimation #1: C(1)	B2	C2	D2
SI estimation #1: SI(1)	B3	C4	D4
Replacement values #1: RSI(1)	B4	---	---
Seasonal estimation #1: S(1)	B5	C5	D5
SA series #1: A(1)	B6	C6	D6

B, C, and D Iterations (p. 2)

Trend estimation #2: C(2)	B7	C7	D7
SI estimation #2: SI(2)	B8	C9	**D8**
Replacement values #2: RSI(2)	B9	---	**D9**
Moving Seasonality Ratios	---	---	**D9A**
Seasonal estimation #2: S(2)	B10	C10	**D10**
SA series #2: A(2)	B11	C11	**D11**

B, C, and D Iterations (p. 3)

Irregular estimation: I(2)	B13	C13	---
Trend estimation #3: C(3)	**---**	**---**	**D12**
Irregular estimation: I(3)	**---**	**---**	**D13**

where I(2) = A(2) / C(2) and I(3) = A(2) / C(3)

B, C, and D Iterations (p. 4)

Weights for irregular: W	B17	**C17**	---
Adjustment values: V	B20	C20	---

where $V = I(2) / [1 + W (I(2) - 1)]$

B, C, and D Iterations (p. 5)

	B	C	D
Combined seasonal and TD factors	B18	C18	**D16**
Ratio of Original Series to Final Adjusted Series	**---**	**---**	**E18**

Saving the Final Adjustment

- D11 is the final seasonally adjusted series
  - **`save=d11`**
- Some users also want to save factors
  - **`save=(d11 e18)`**

    E18 is the final adjustment ratio

`X11` Spec

- Allows the user to set seasonal adjustment options such as
  - Seasonal filters
  - (Trend filters)
  - Sigma limits for the extreme value identification and adjustment
  - Seasonal adjustment decomposition mode (although determined by the transformation)
  - Whether or not to seasonally adjust
    - **`type=trend`**

Simple **`X11`** Spec

In addition to **`series`** and **`transform`**, all we need to get a "default X-11" seasonal adjustment –

```
x11 { }
```

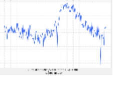
Default is not good for production

X11 Spec – Frequently Used Options

```
x11{
 seasonalma = s3x5
 sigmalim = (1.8 2.8)
#  mode = mult
 savelog = all
 save = (d11 e18)
 print = See Manual/Quick Reference
}
```

Summary – X-11 Algorithm

- The X-11 algorithm iterates between estimating trend and estimating seasonal effects
- Trend filters average consecutive values
- Seasonal filters average within a type of month (quarter)
- X-11 procedure adjusts for extreme values (different from outliers)



Revisit: Choosing Seasonal Filters

- Avoid using stability diagnostics alone to choose filters
  - Tend to favor longer filters
- Use Global MSR and MSR by month (quarter) as guides
  - Table D9.A
- Make sure no residual seasonal effects remain after seasonal adjustment
  - Spectrum diagnostic for monthly series
  - QS
  - Test significance of seasonal regressors for the adjusted series

Example of Seasonal Filter Choice
Housing Starts: Midwest Single Family

Automatic Seasonal Filter Choice: 3x5

The final Moving Seasonality Ratio (I/S)
from Table D10: 6.24

(Gray Zone)

Table D 9.A MSRs: January through April

D 9.A Moving seasonality ratio

	Jan	Feb	Mar	Apr
I	8.444	6.736	8.459	5.155
S	1.248	1.209	0.638	0.782
RATIO	6.766	5.571	13.254	6.591

Table D 9.A MSRs: May through August

D 9.A Moving seasonality ratio

	May	Jun	Jul	Aug
I	4.664	6.841	6.733	5.476
S	0.995	0.795	1.214	1.009
RATIO	4.688	8.604	5.544	5.425

Table D 9.A MSRs: September through December

D 9.A Moving seasonality ratio

	Sep	Oct	Nov	Dec
I	6.191	7.700	6.568	7.386
S	1.029	1.230	1.348	1.398
RATIO				
	6.015	6.260	4.872	5.282

Comparing Filters

- Planas and Depoutot (2002)
  - Performed ARIMA model-based seasonal adjustment (airline model) and X-11 adjustment
  - Table of model coefficients that matched most closely with given Henderson trend filters and seasonal filters
- Chu, Tiao, and Bell (2006)
  - Further filter comparisons

Table 1 From Planas and Depoutot (2002)

Filter	Seasonal MA
3x3	0.364 – 0.400
X11default	0.546 – 0.556
3x5	0.543 – 0.563
3x9	0.723 – 0.732
3x15	0.824 – 0.828

Look at ARIMA Model Seasonal MA Coefficient

- Even if it is not an airline model, look at the seasonal MA coefficient
- For higher values of the seasonal MA, probably choose 3x9 even if the moving seasonality ratio choice is shorter (assuming you have confidence regarding your model fit)

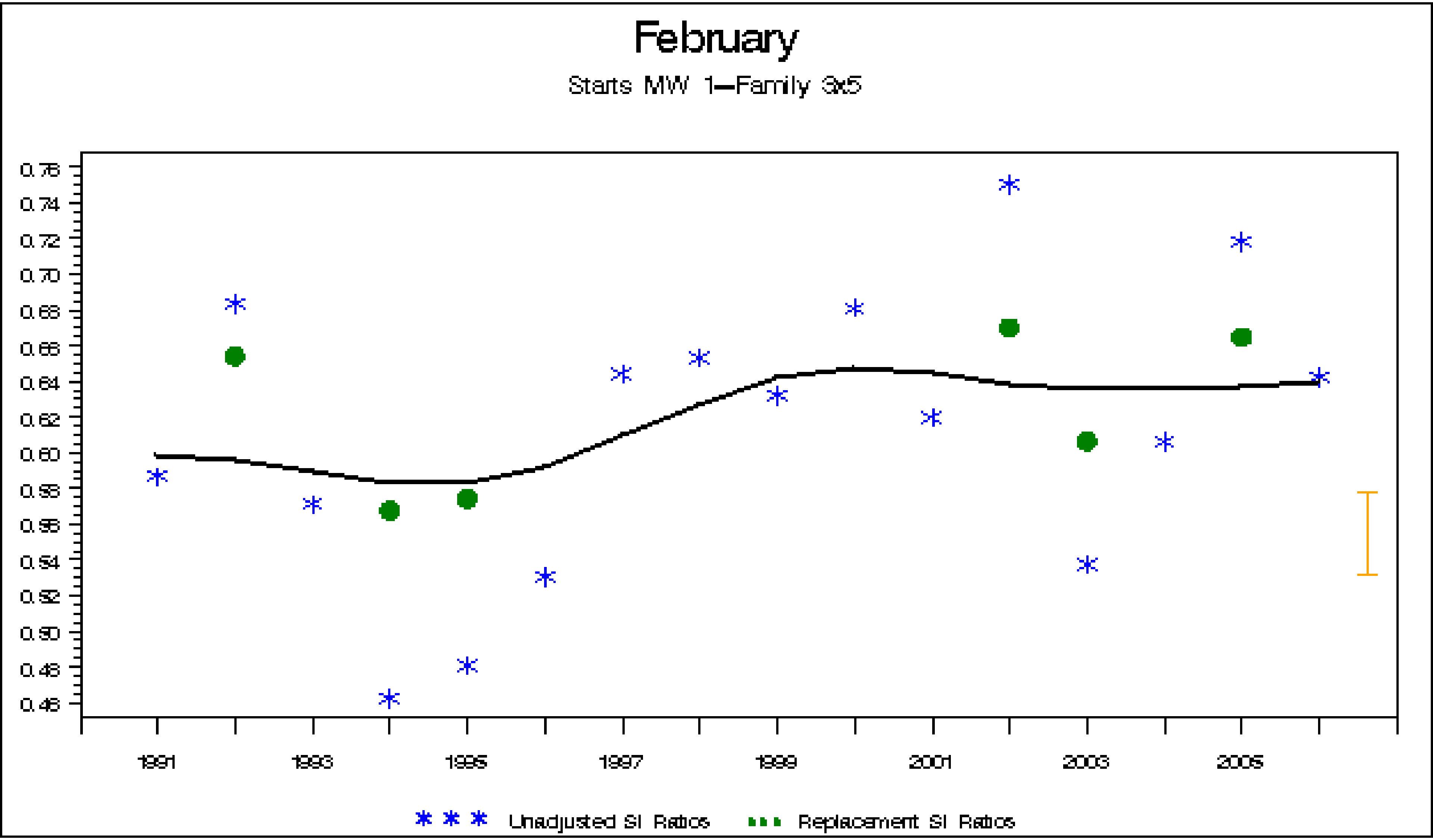
ARIMA Coefficient Indicates Longer Filter

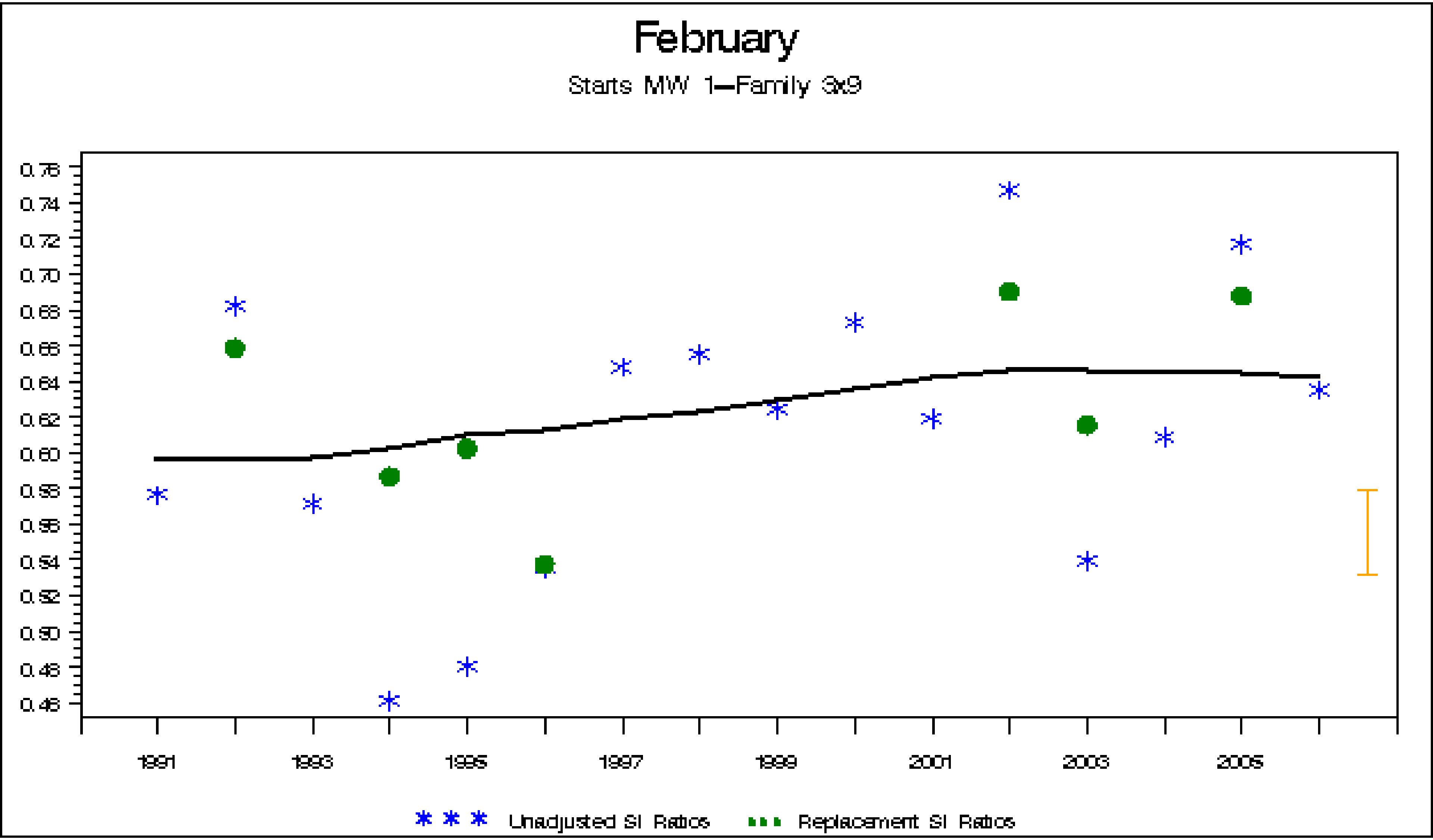
```
ARIMA Model:  (0 1 1)(0 1 1)
                               Standard
Parameter      Estimate         Errors
-----------------------------------
Nonseasonal MA
   Lag  1         0.6389        0.05463
Seasonal MA
   Lag 12         0.8369        0.04755
```

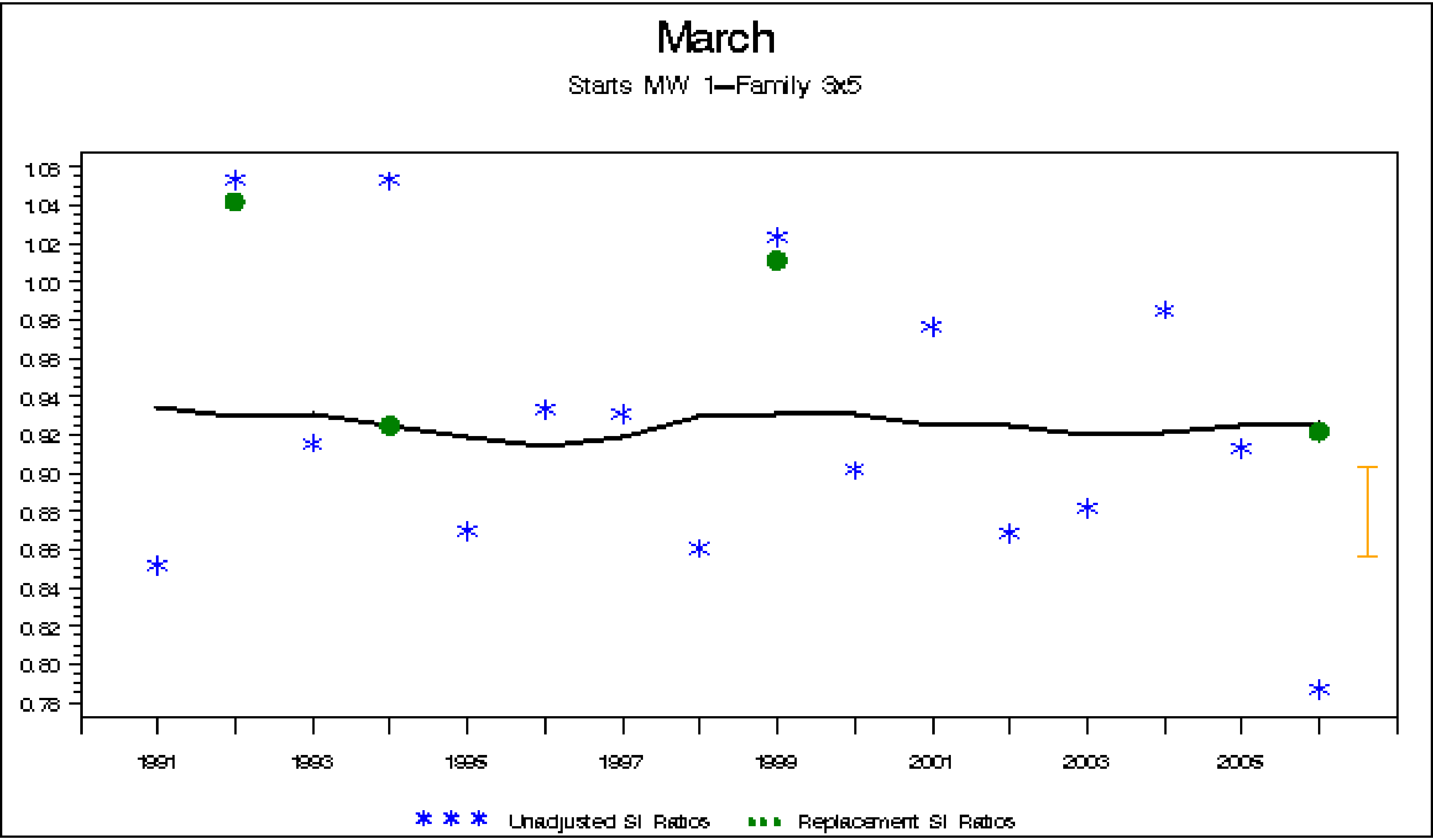
Extreme Values and Filters

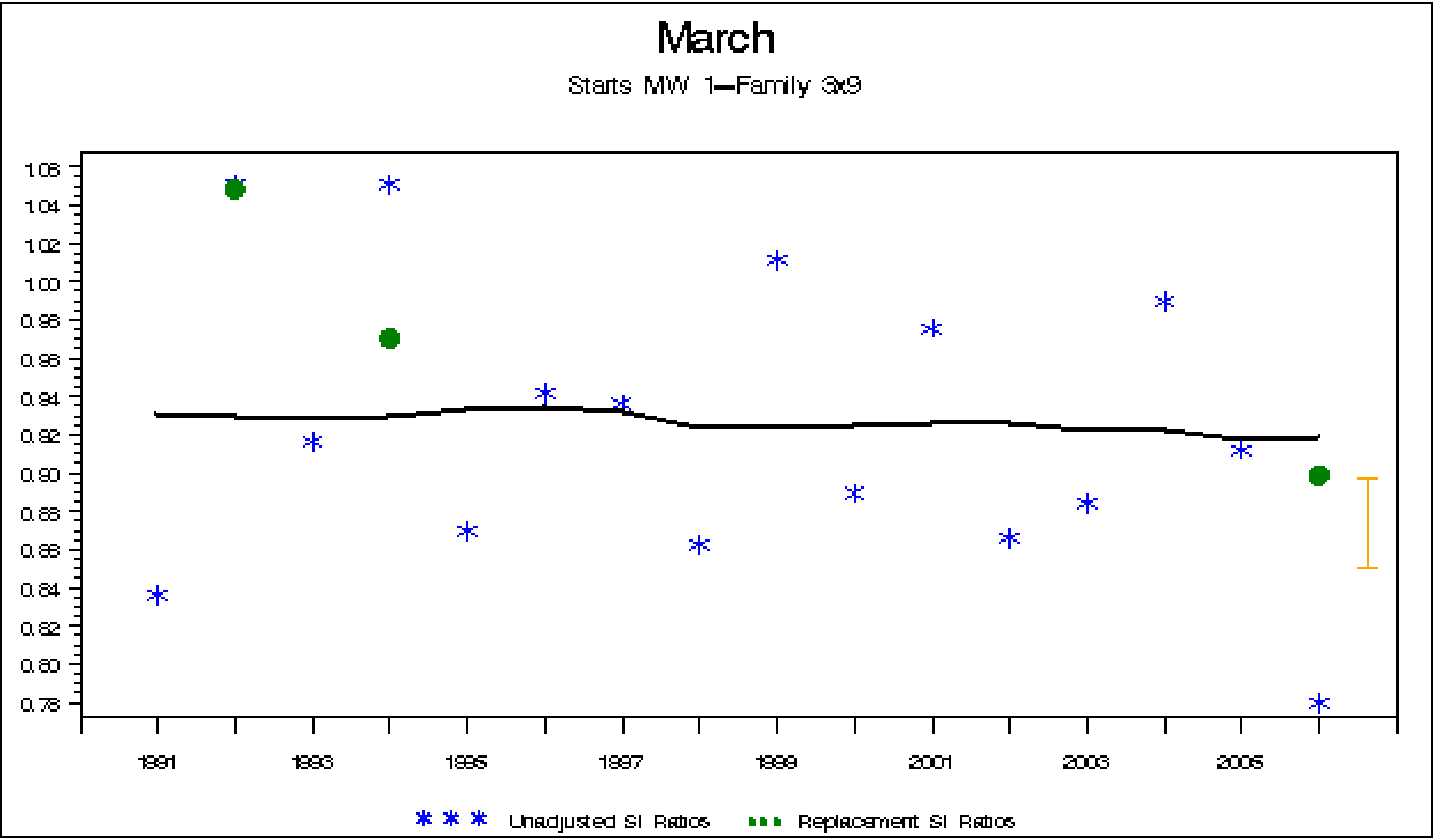
- Changing filters can affect which values are extreme
- Changing sigma limits changes the extreme values and can affect what filter seems most appropriate

* Experiment with changing filters and sigma limits for your series to see the results

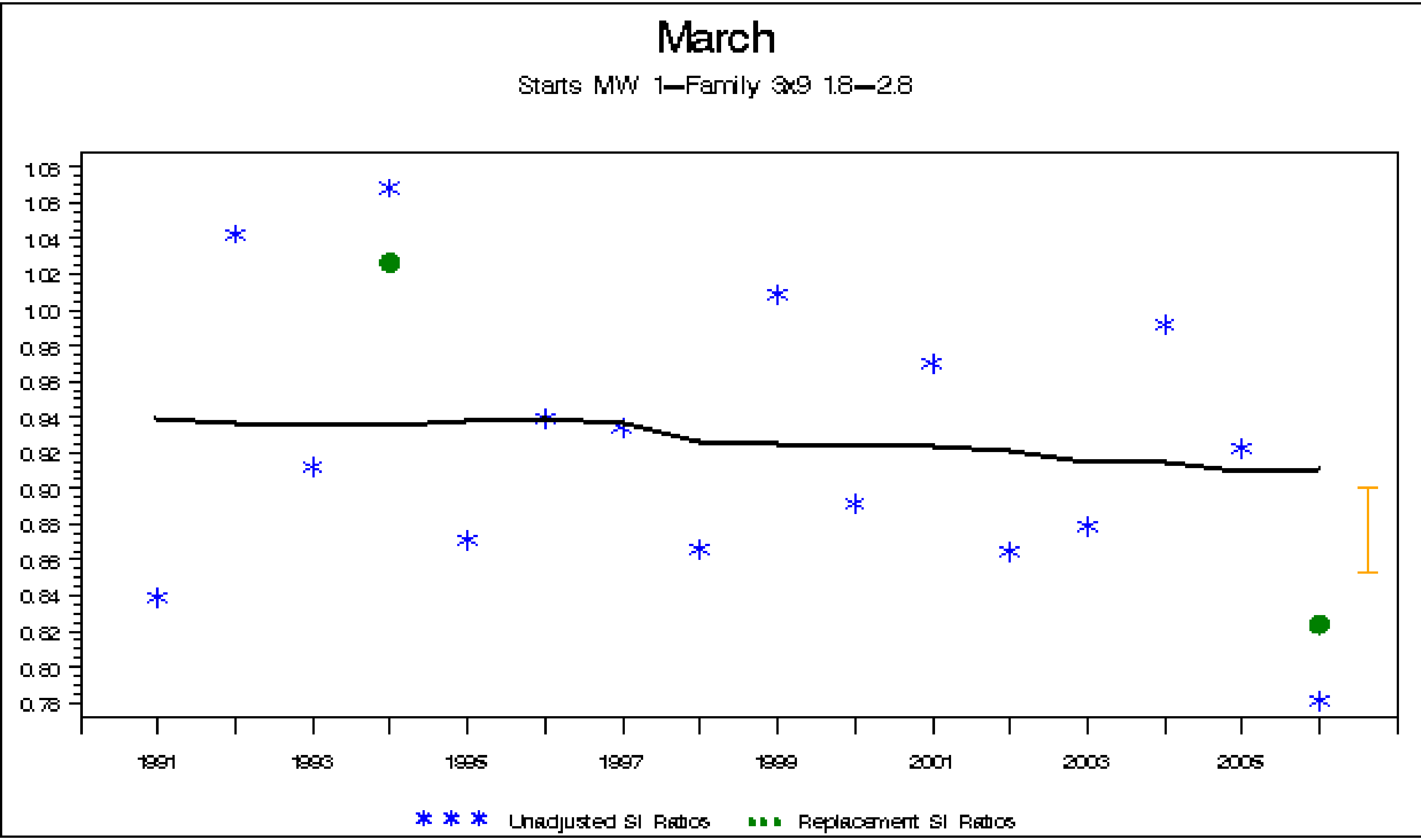

February
Starts MW 1-Family 3x5
0.76
0.74
0.72
0.70
0.68
0.66
0.64
0.62
0.60
0.58
0.56
0.54
0.52
0.50
0.48
0.46
1991
1993
1995
1997
1999
2001
2003
2005
Unadjusted SI Ratios
Replacement SI Ratios


February
Starts MW 1-Family 3x9
0.76
0.74
0.72
0.70
0.68
0.66
0.64
0.62
0.60
0.58
0.56
0.54
0.52
0.50
0.48
0.46
1991
1993
1995
1997
1999
2001
2003
2005
Unadjusted SI Ratios
Replacement SI Ratios


March
Starts MW 1—Family 3x5
1.06
1.04
1.02
1.00
0.98
0.96
0.94
0.92
0.90
0.88
0.86
0.84
0.82
0.80
0.78
1991
1993
1995
1997
1999
2001
2003
2005
Unadjusted SI Ratios
Replacement SI Ratios



March
Starts MW 1—Family 3x9
1.06
1.04
1.02
1.00
0.98
0.96
0.94
0.92
0.90
0.88
0.86
0.84
0.82
0.80
0.78
1991
1993
1995
1997
1999
2001
2003
2005
Unadjusted SI Ratios
Replacement SI Ratios



March
Starts MW 1-Family 3x9 1.8-2.8
1.08
1.06
1.04
1.02
1.00
0.98
0.96
0.94
0.92
0.90
0.88
0.86
0.84
0.82
0.80
0.78
1991
1993
1995
1997
1999
2001
2003
2005
Unadjusted SI Ratios
Replacement SI Ratios

Choosing Filters: Bottom Line

- Compare the adjustments to see if a change really makes a difference
- Always check adjustments for residual seasonality
  - Spectrum, QS, F test within X-11, F test for seasonal regressors fit to a subset of the seasonally adjusted series
- Use filters of similar length in adjacent months or quarters
  - Ideally use only one or two different filters
- Set seasonal filters for production so they cannot change just because of an added time point