Empirical Analysis of the Role of Energy in Economic Growth

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Abstract

*********** Add abstract ***********

Keywords: economic growth, energy, cobb-douglas, CES, LINEX

Caleb, put your LaTeX code here.

1. Cobb-Douglas Without Energy

Table 1: Cobb-Douglas parameters for 1980-2011 (US, UK, JP) or 1991-2011 (others). (Parameter estimates beneath symbol. 95% confidence bounds to left and right.)

		λ			α			β	
US	0.0087	0.0102	0.0116	0.21	0.27	0.34	0.66	0.73	0.79
UK	-0.0104	0.0097	0.0303	-0.25	0.44	1.12	-0.13	0.56	1.24
JP	0.0021	0.0052	0.0082	0.44	0.52	0.59	0.41	0.48	0.56
CN	-0.0405	0.0188	0.0779	0.11	0.71	1.32	-0.32	0.29	0.89
ZA	-0.0007	0.0008	0.0022	0.46	0.60	0.73	0.26	0.40	0.54
SA	-0.0159	-0.0123	-0.0087	0.21	0.45	0.68	0.32	0.55	0.78
IR	0.0032	0.0039	0.0045	0.49	0.60	0.70	0.30	0.40	0.51
TZ	-0.0039	0.0015	0.0068	0.50	0.73	0.95	0.05	0.27	0.50
ZM	0.0218	0.0249	0.0280	1.25	1.41	1.57	-0.57	-0.41	-0.25

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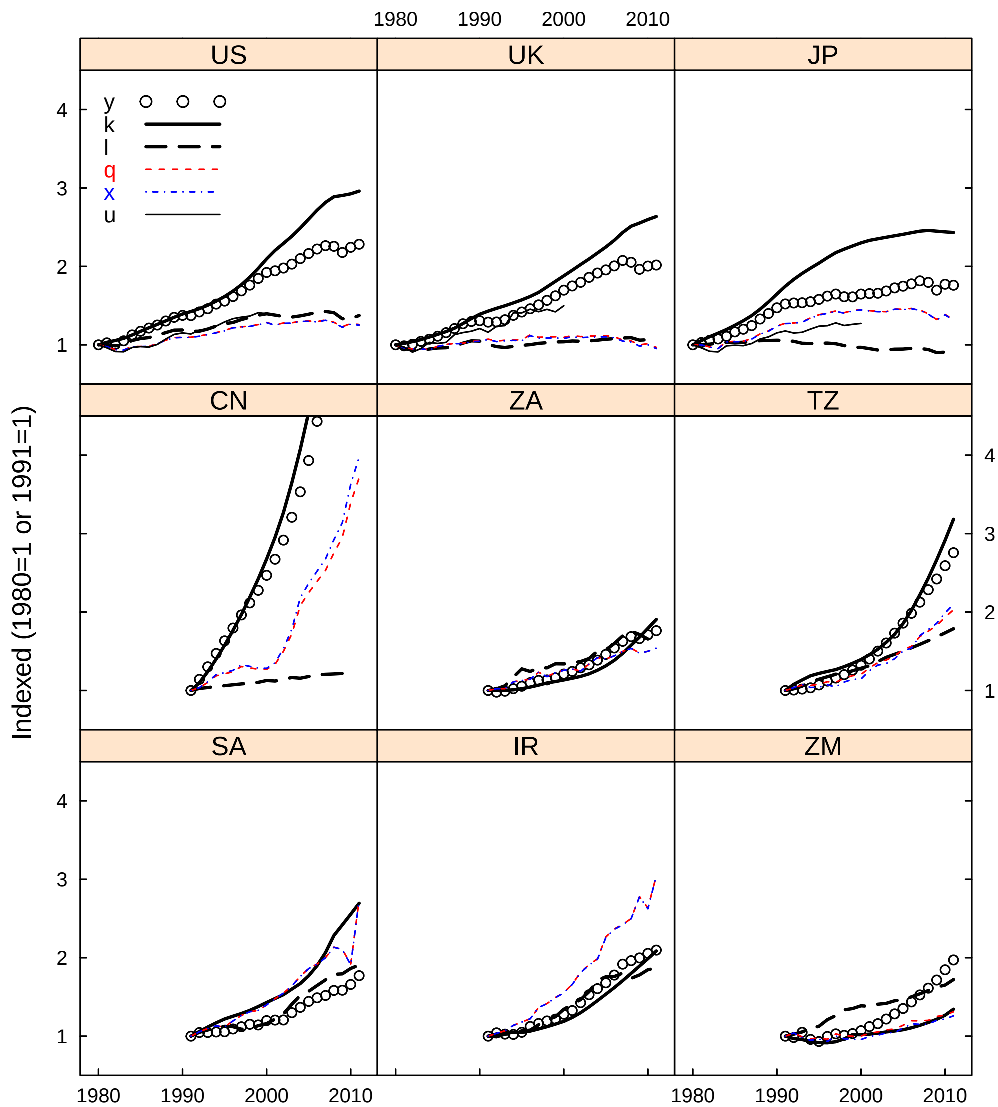

Figure 1: GDP (y), capital stock (k), labor (l), thermal energy (q), exergy (x), and useful work (u) for all economies. (China's indexed GDP and indexed capital stock rise to $y = 7.3$ and $k = 9.2$ in 2011.)

```
usModel <- cobbDouglasModel("US")
coefs <- coef(usModel)
# print(coef(usModel))
usPred <- predict(usModel) #See http://stackoverflow.com/questions/9918807/how-get
class(usPred)
```

```
[1] "numeric"
```

```
#print(usPred)
data.frame(usPred)
```

```
   usPred
1   1.000
2   1.021
3   1.026
4   1.057
5   1.119
6   1.162
7   1.196
8   1.244
9   1.296
10  1.348
11  1.375
12  1.383
13  1.408
14  1.458
15  1.520
16  1.579
17  1.628
18  1.702
19  1.772
20  1.844
21  1.912
22  1.941
23  1.963
24  1.997
25  2.058
26  2.128
```

```
27  2.204
28  2.260
29  2.281
30  2.215
31  2.241
32  2.323
```

```
allData <- loadData("All")
predictions <- cobbDouglasPredictionsColumn()
```

```
    predGDP
1    1.0000
2    1.0206
3    1.0260
4    1.0574
5    1.1188
6    1.1617
7    1.1960
8    1.2438
9    1.2959
10   1.3479
11   1.3747
12   1.3831
13   1.4078
14   1.4579
15   1.5203
16   1.5790
17   1.6283
18   1.7016
19   1.7716
20   1.8440
21   1.9118
22   1.9406
23   1.9632
24   1.9971
25   2.0579
26   2.1281
```

```
27   2.2039
28   2.2598
29   2.2806
30   2.2146
31   2.2410
32   2.3234
33   1.0000
34   0.9917
35   0.9988
36   1.0134
37   1.0516
38   1.0850
39   1.1125
40   1.1559
41   1.2165
42   1.2748
43   1.3089
44   1.3079
45   1.3146
46   1.3321
47   1.3702
48   1.4093
49   1.4477
50   1.4962
51   1.5457
52   1.5965
53   1.6418
54   1.6943
55   1.7365
56   1.7840
57   1.8393
58   1.8986
59   1.9551
60   2.0204
61   2.0744
62   2.0753
63   2.1175
```

64	2.1507
65	1.0000
66	1.0341
67	1.0675
68	1.1023
69	1.1362
70	1.1669
71	1.2030
72	1.2440
73	1.2962
74	1.3463
75	1.4004
76	1.4489
77	1.4862
78	1.5076
79	1.5401
80	1.5772
81	1.6141
82	1.6410
83	1.6477
84	1.6528
85	1.6774
86	1.6836
87	1.6845
88	1.7023
89	1.7261
90	1.7432
91	1.7679
92	1.7838
93	1.7840
94	1.7526
95	1.7641
96	1.7440
97	1.0000
98	1.0975
99	1.2235
100	1.3647

```
101  1.5167
102  1.6811
103  1.8553
104  2.0466
105  2.2459
106  2.4723
107  2.6961
108  2.9711
109  3.2927
110  3.6207
111  4.0117
112  4.4483
113  4.9207
114  5.4276
115  6.0583
116  6.7711
117  1.0000
118  1.0099
119  1.0264
120  1.0732
121  1.1213
122  1.1238
123  1.1582
124  1.1761
125  1.2075
126  1.2210
127  1.2402
128  1.2634
129  1.2996
130  1.3643
131  1.4082
132  1.4839
133  1.5783
134  1.6682
135  1.7175
136  1.7616
137  1.0000
```

```
138  1.0411
139  1.0691
140  1.0932
141  1.1105
142  1.1084
143  1.0905
144  1.0967
145  1.1211
146  1.1406
147  1.1748
148  1.2125
149  1.2863
150  1.3501
151  1.3953
152  1.4590
153  1.5323
154  1.6186
155  1.6435
156  1.6990
157  1.7406
158  1.0000
159  1.0178
160  1.0454
161  1.0538
162  1.0717
163  1.0899
164  1.1390
165  1.1857
166  1.2278
167  1.2926
168  1.3538
169  1.4234
170  1.5165
171  1.6318
172  1.7109
173  1.7735
174  1.8569
```

```
175  1.8944
176  1.9799
177  2.0761
178  2.1548
179  1.0000
180  1.0639
181  1.1155
182  1.1713
183  1.2042
184  1.2333
185  1.2610
186  1.2943
187  1.3360
188  1.3816
189  1.4377
190  1.5055
191  1.5879
192  1.6755
193  1.7869
194  1.9203
195  2.0749
196  2.2377
197  2.4150
198  2.6023
199  2.8013
200  1.0000
201  0.9708
202  0.9669
203  0.9380
204  0.9327
205  0.9256
206  0.9514
207  1.0057
208  1.1091
209  1.1221
210  1.1656
211  1.2052
```

```
212  1.2585
213  1.2987
214  1.3548
215  1.4198
216  1.4976
217  1.5886
218  1.6883
219  1.8248
220  1.9983
```

```
allData <- cbind(allData, predictions)
```

```
Error:  arguments imply differing number of rows:  222, 220
```

```
print(allData)
```

```
    Year iYear   iGDP iLabor iCapStk     iQ     iX     iU Country
1   1980     0 1.0000 1.0000  1.0000 1.0000 1.0000 1.0000      US
2   1981     1 1.0254 1.0021  1.0322 0.9768 0.9764 0.9617      US
3   1982     2 1.0055 0.9872  1.0551 0.9389 0.9379 0.9148      US
4   1983     3 1.0509 1.0049  1.0829 0.9337 0.9327 0.9116      US
5   1984     4 1.1264 1.0556  1.1253 0.9799 0.9790 0.9657      US
6   1985     5 1.1730 1.0797  1.1718 0.9790 0.9782 0.9782      US
7   1986     6 1.2137 1.0924  1.2172 0.9818 0.9808 0.9712      US
8   1987     7 1.2525 1.1220  1.2608 1.0134 1.0126 1.0051      US
9   1988     8 1.3040 1.1555  1.3054 1.0590 1.0582 1.0732      US
10  1989     9 1.3506 1.1874  1.3511 1.0915 1.0897 1.1328      US
11  1990    10 1.3759 1.1894  1.3927 1.0973 1.0948 1.1516      US
12  1991    11 1.3727 1.1726  1.4246 1.0970 1.0941 1.1389      US
13  1992    12 1.4193 1.1736  1.4612 1.1126 1.1098 1.1874      US
14  1993    13 1.4598 1.2010  1.5048 1.1340 1.1312 1.1996      US
15  1994    14 1.5192 1.2386  1.5570 1.1561 1.1532 1.2387      US
16  1995    15 1.5574 1.2691  1.6155 1.1816 1.1779 1.2920      US
17  1996    16 1.6157 1.2850  1.6848 1.2200 1.2164 1.3335      US
18  1997    17 1.6877 1.3226  1.7662 1.2303 1.2271 1.3523      US
19  1998    18 1.7612 1.3511  1.8634 1.2384 1.2353 1.3669      US
20  1999    19 1.8462 1.3775  1.9745 1.2584 1.2550 1.4091      US
```

21	2000	20	1.9226	1.3959	2.0955	1.2848	1.2816	1.3964	US
22	2001	21	1.9434	1.3788	2.2033	1.2548	1.2516	NA	US
23	2002	22	1.9786	1.3609	2.2927	1.2779	1.2739	NA	US
24	2003	23	2.0289	1.3538	2.3844	1.2806	1.2769	NA	US
25	2004	24	2.0992	1.3690	2.4885	1.3018	1.2981	NA	US
26	2005	25	2.1637	1.3901	2.6029	1.3036	1.2997	NA	US
27	2006	26	2.2212	1.4155	2.7162	1.2960	1.2916	NA	US
28	2007	27	2.2637	1.4253	2.8158	1.3178	1.3132	NA	US
29	2008	28	2.2561	1.4099	2.8877	1.2874	1.2819	NA	US
30	2009	29	2.1774	1.3323	2.9041	1.2307	1.2239	NA	US
31	2010	30	2.2434	1.3318	2.9249	1.2710	1.2647	NA	US
32	2011	31	2.2823	1.3742	2.9597	1.2586	1.2508	NA	US
33	1980	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	UK
34	1981	1	0.9868	0.9541	1.0184	0.9607	0.9599	0.9725	UK
35	1982	2	1.0074	0.9334	1.0407	0.9489	0.9473	0.9069	UK
36	1983	3	1.0439	0.9234	1.0663	0.9531	0.9509	0.9521	UK
37	1984	4	1.0718	0.9466	1.0990	0.9374	0.9331	1.0317	UK
38	1985	5	1.1104	0.9598	1.1339	0.9824	0.9789	1.0175	UK
39	1986	6	1.1549	0.9630	1.1687	1.0053	1.0024	1.0315	UK
40	1987	7	1.2076	0.9848	1.2120	1.0173	1.0145	1.1298	UK
41	1988	8	1.2684	1.0214	1.2709	1.0199	1.0161	1.1561	UK
42	1989	9	1.2974	1.0503	1.3343	1.0447	1.0404	1.1731	UK
43	1990	10	1.3075	1.0475	1.3900	1.0496	1.0448	1.2083	UK
44	1991	11	1.2893	1.0051	1.4296	1.0726	1.0676	1.1603	UK
45	1992	12	1.2911	0.9774	1.4653	1.0474	1.0410	1.2362	UK
46	1993	13	1.3198	0.9663	1.4981	1.0593	1.0509	1.2455	UK
47	1994	14	1.3763	0.9798	1.5348	1.0655	1.0563	1.4084	UK
48	1995	15	1.4183	0.9931	1.5732	1.0623	1.0519	1.4071	UK
49	1996	16	1.4593	1.0021	1.6167	1.1283	1.1167	1.4547	UK
50	1997	17	1.5092	1.0190	1.6683	1.0963	1.0838	1.4218	UK
51	1998	18	1.5672	1.0273	1.7385	1.1004	1.0872	1.4527	UK
52	1999	19	1.6244	1.0365	1.8091	1.1026	1.0885	1.4201	UK
53	2000	20	1.6969	1.0389	1.8797	1.0955	1.0815	1.4983	UK
54	2001	21	1.7503	1.0489	1.9506	1.1173	1.1032	NA	UK
55	2002	22	1.7968	1.0460	2.0239	1.1051	1.0904	NA	UK
56	2003	23	1.8602	1.0499	2.0944	1.1122	1.0982	NA	UK
57	2004	24	1.9151	1.0593	2.1707	1.1144	1.1008	NA	UK

58	2005	25	1.9551	1.0720	2.2471	1.1153	1.1017	NA	UK
59	2006	26	2.0061	1.0778	2.3327	1.1029	1.0899	NA	UK
60	2007	27	2.0756	1.0868	2.4320	1.0609	1.0479	NA	UK
61	2008	28	2.0527	1.0914	2.5117	1.0494	1.0360	NA	UK
62	2009	29	1.9629	1.0597	2.5523	0.9990	0.9838	NA	UK
63	2010	30	2.0040	1.0649	2.5969	1.0153	1.0003	NA	UK
64	2011	31	2.0171	1.0634	2.6358	0.9669	0.9518	NA	UK
65	1980	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	JP
66	1981	1	1.0293	1.0030	1.0536	1.0001	0.9998	0.9624	JP
67	1982	2	1.0578	1.0087	1.1034	0.9730	0.9710	0.9176	JP
68	1983	3	1.0748	1.0222	1.1481	0.9547	0.9510	0.9101	JP
69	1984	4	1.1084	1.0319	1.1949	1.0393	1.0367	0.9869	JP
70	1985	5	1.1647	1.0290	1.2489	1.0439	1.0397	0.9950	JP
71	1986	6	1.1992	1.0332	1.3068	1.0464	1.0415	0.9891	JP
72	1987	7	1.2447	1.0400	1.3722	1.0787	1.0734	1.0168	JP
73	1988	8	1.3289	1.0531	1.4539	1.1391	1.1351	1.0744	JP
74	1989	9	1.3992	1.0554	1.5462	1.1774	1.1737	1.1009	JP
75	1990	10	1.4719	1.0587	1.6473	1.2402	1.2371	1.1523	JP
76	1991	11	1.5209	1.0559	1.7466	1.2748	1.2710	1.1769	JP
77	1992	12	1.5333	1.0451	1.8342	1.2783	1.2745	1.1505	JP
78	1993	13	1.5360	1.0200	1.9101	1.2933	1.2881	1.1599	JP
79	1994	14	1.5492	1.0163	1.9775	1.3434	1.3398	1.2008	JP
80	1995	15	1.5791	1.0208	2.0417	1.3836	1.3792	1.2375	JP
81	1996	16	1.6211	1.0223	2.1111	1.4058	1.4013	1.2441	JP
82	1997	17	1.6472	1.0137	2.1754	1.4318	1.4265	1.2796	JP
83	1998	18	1.6125	0.9899	2.2197	1.4101	1.4036	1.2463	JP
84	1999	19	1.6112	0.9669	2.2603	1.4364	1.4317	1.2612	JP
85	2000	20	1.6472	0.9683	2.2994	1.4497	1.4454	1.2732	JP
86	2001	21	1.6530	0.9513	2.3313	1.4360	1.4316	NA	JP
87	2002	22	1.6577	0.9336	2.3517	1.4262	1.4227	NA	JP
88	2003	23	1.6862	0.9356	2.3713	1.4231	1.4213	NA	JP
89	2004	24	1.7256	0.9447	2.3900	1.4564	1.4541	NA	JP
90	2005	25	1.7478	0.9459	2.4089	1.4525	1.4494	NA	JP
91	2006	26	1.7772	0.9547	2.4294	1.4642	1.4602	NA	JP
92	2007	27	1.8157	0.9541	2.4489	1.4436	1.4407	NA	JP
93	2008	28	1.7962	0.9401	2.4587	1.3966	1.3927	NA	JP
94	2009	29	1.6969	0.9005	2.4489	1.3225	1.3155	NA	JP

95	2010	30	1.7726	0.9067	2.4397	1.3873	1.3815	NA	JP
96	2011	31	1.7598	NA	2.4320	1.3182	1.3156	NA	JP
97	1991	0	1.0000	1.0000	1.0000	1.0000	1.0000	NA	CN
98	1992	1	1.1420	1.0258	1.0985	1.0356	1.0366	NA	CN
99	1993	2	1.3019	1.0377	1.2404	1.1039	1.1109	NA	CN
100	1994	3	1.4724	1.0501	1.4015	1.1878	1.2025	NA	CN
101	1995	4	1.6329	1.0615	1.5763	1.2123	1.2276	NA	CN
102	1996	5	1.7962	1.0727	1.7664	1.2396	1.2565	NA	CN
103	1997	6	1.9633	1.0838	1.9675	1.3058	1.3298	NA	CN
104	1998	7	2.1164	1.0944	2.1906	1.2877	1.3073	NA	CN
105	1999	8	2.2772	1.1039	2.4224	1.2716	1.2873	NA	CN
106	2000	9	2.4685	1.1277	2.6767	1.2703	1.2822	NA	CN
107	2001	10	2.6734	1.1206	2.9517	1.3336	1.3521	NA	CN
108	2002	11	2.9167	1.1431	3.2685	1.4993	1.5349	NA	CN
109	2003	12	3.2084	1.1654	3.6489	1.7220	1.7815	NA	CN
110	2004	13	3.5324	1.1563	4.0735	2.0815	2.1830	NA	CN
111	2005	14	3.9316	1.1782	4.5471	2.2471	2.3645	NA	CN
112	2006	15	4.4309	1.2000	5.0820	2.3911	2.5214	NA	CN
113	2007	16	5.0601	1.2063	5.6910	2.5309	2.6764	NA	CN
114	2008	17	5.5458	1.2108	6.3512	2.7533	2.9220	NA	CN
115	2009	18	6.0561	1.2169	7.2033	2.9489	3.1387	NA	CN
116	2010	19	6.6845	1.2381	8.1442	3.3926	3.6274	NA	CN
117	2011	20	7.3013	NA	9.1896	3.7111	3.9776	NA	CN
118	1991	0	1.0000	1.0000	1.0000	1.0000	1.0000	NA	ZA
119	1992	1	0.9786	1.0211	1.0012	1.0229	1.0232	NA	ZA
120	1993	2	0.9907	1.0597	1.0019	1.0228	1.0226	NA	ZA
121	1994	3	1.0227	1.1704	1.0084	1.1113	1.1103	NA	ZA
122	1995	4	1.0546	1.2757	1.0227	1.1294	1.1270	NA	ZA
123	1996	5	1.1000	1.2423	1.0436	1.1437	1.1420	NA	ZA
124	1997	6	1.1291	1.2909	1.0683	1.2329	1.2323	NA	ZA
125	1998	7	1.1350	1.2885	1.0960	1.1784	1.1763	NA	ZA
126	1999	8	1.1617	1.3403	1.1139	1.2262	1.2249	NA	ZA
127	2000	9	1.2100	1.3387	1.1343	1.2584	1.2576	NA	ZA
128	2001	10	1.2431	1.3506	1.1560	1.2739	1.2737	NA	ZA
129	2002	11	1.2887	1.3694	1.1798	1.2477	1.2460	NA	ZA
130	2003	12	1.3267	1.4076	1.2126	1.3375	1.3377	NA	ZA
131	2004	13	1.3871	1.5013	1.2579	1.4192	1.4193	NA	ZA

132	2005	14	1.4603	1.5185	1.3144	1.4029	1.4023	NA	ZA
133	2006	15	1.5422	1.5982	1.3845	1.4428	1.4414	NA	ZA
134	2007	16	1.6268	1.6966	1.4726	1.4988	1.4963	NA	ZA
135	2008	17	1.6866	1.7563	1.5763	1.5387	1.5367	NA	ZA
136	2009	18	1.6613	1.7141	1.6803	1.4780	1.4758	NA	ZA
137	2010	19	1.7095	1.6557	1.7923	1.5004	1.4994	NA	ZA
138	2011	20	1.7625	NA	1.9066	1.5383	1.5402	NA	ZA
139	1991	0	1.0000	1.0000	1.0000	1.0000	1.0000	NA	SA
140	1992	1	1.0463	1.0492	1.0599	1.0553	1.0553	NA	SA
141	1993	2	1.0466	1.0792	1.1165	1.0984	1.0982	NA	SA
142	1994	3	1.0535	1.1059	1.1703	1.1317	1.1313	NA	SA
143	1995	4	1.0556	1.1257	1.2188	1.1168	1.1160	NA	SA
144	1996	5	1.0914	1.1208	1.2543	1.1945	1.1939	NA	SA
145	1997	6	1.1197	1.0884	1.2888	1.2642	1.2638	NA	SA
146	1998	7	1.1514	1.0968	1.3289	1.3114	1.3113	NA	SA
147	1999	8	1.1428	1.1337	1.3774	1.3287	1.3288	NA	SA
148	2000	9	1.1984	1.1607	1.4293	1.3986	1.3985	NA	SA
149	2001	10	1.2049	1.2172	1.4799	1.4779	1.4776	NA	SA
150	2002	11	1.2065	1.2836	1.5290	1.5490	1.5485	NA	SA
151	2003	12	1.2989	1.4080	1.6000	1.6407	1.6397	NA	SA
152	2004	13	1.3673	1.5176	1.6705	1.7614	1.7599	NA	SA
153	2005	14	1.4432	1.5717	1.7699	1.8632	1.8614	NA	SA
154	2006	15	1.4888	1.6458	1.8990	1.9187	1.9166	NA	SA
155	2007	16	1.5188	1.7179	2.0655	2.0012	1.9996	NA	SA
156	2008	17	1.5830	1.7884	2.2830	2.1347	2.1330	NA	SA
157	2009	18	1.5856	1.7941	2.4183	2.1067	2.1051	NA	SA
158	2010	19	1.6590	1.8635	2.5547	1.9178	1.9096	NA	SA
159	2011	20	1.7712	1.9066	2.6944	2.7388	2.7406	NA	SA
160	1991	0	1.0000	1.0000	1.0000	1.0000	1.0000	NA	IR
161	1992	1	1.0425	0.9923	1.0287	1.0355	1.0358	NA	IR
162	1993	2	1.0261	1.0258	1.0452	1.0686	1.0689	NA	IR
163	1994	3	1.0225	1.0326	1.0479	1.1368	1.1377	NA	IR
164	1995	4	1.0496	1.0677	1.0471	1.1794	1.1797	NA	IR
165	1996	5	1.1241	1.0762	1.0644	1.2217	1.2214	NA	IR
166	1997	6	1.1622	1.1451	1.0918	1.3648	1.3651	NA	IR
167	1998	7	1.1940	1.2058	1.1206	1.4154	1.4144	NA	IR
168	1999	8	1.2171	1.2475	1.1536	1.4927	1.4909	NA	IR

169	2000	9	1.2797	1.3428	1.1887	1.5540	1.5511	NA	IR
170	2001	10	1.3267	1.4047	1.2379	1.6620	1.6583	NA	IR
171	2002	11	1.4264	1.4662	1.2995	1.8149	1.8108	NA	IR
172	2003	12	1.5279	1.5670	1.3728	1.9104	1.9054	NA	IR
173	2004	13	1.6056	1.7123	1.4524	1.9856	1.9790	NA	IR
174	2005	14	1.6798	1.7569	1.5353	2.2649	2.2580	NA	IR
175	2006	15	1.7788	1.7594	1.6185	2.3667	2.3611	NA	IR
176	2007	16	1.9180	1.8045	1.7074	2.4251	2.4189	NA	IR
177	2008	17	1.9621	1.7369	1.8002	2.4982	2.4960	NA	IR
178	2009	18	1.9974	1.7804	1.8939	2.7790	2.7763	NA	IR
179	2010	19	2.0555	1.8470	1.9876	2.6340	2.6238	NA	IR
180	2011	20	2.0971	1.8676	2.0861	3.0449	3.0448	NA	IR
181	1991	0	1.0000	1.0000	1.0000	1.0000	1.0000	NA	TZ
182	1992	1	1.0059	1.0211	1.0783	1.0310	1.0364	NA	TZ
183	1993	2	1.0180	1.0556	1.1342	1.0704	1.0771	NA	TZ
184	1994	3	1.0340	1.1100	1.1878	1.0643	1.0385	NA	TZ
185	1995	4	1.0708	1.1436	1.2178	1.0797	1.0424	NA	TZ
186	1996	5	1.1196	1.1756	1.2428	1.1121	1.0720	NA	TZ
187	1997	6	1.1590	1.2065	1.2664	1.1053	1.0455	NA	TZ
188	1998	7	1.2020	1.2211	1.3040	1.1548	1.1080	NA	TZ
189	1999	8	1.2603	1.2516	1.3468	1.1892	1.1422	NA	TZ
190	2000	9	1.3224	1.2833	1.3944	1.2041	1.1463	NA	TZ
191	2001	10	1.4017	1.3164	1.4558	1.2881	1.2521	NA	TZ
192	2002	11	1.5021	1.3685	1.5255	1.3549	1.3238	NA	TZ
193	2003	12	1.6055	1.4230	1.6143	1.3880	1.3437	NA	TZ
194	2004	13	1.7312	1.4616	1.7172	1.4427	1.4050	NA	TZ
195	2005	14	1.8588	1.5020	1.8532	1.5215	1.5025	NA	TZ
196	2006	15	1.9841	1.5443	2.0209	1.5621	1.5409	NA	TZ
197	2007	16	2.1259	1.5885	2.2198	1.6875	1.6998	NA	TZ
198	2008	17	2.2840	1.6349	2.4314	1.7563	1.7766	NA	TZ
199	2009	18	2.4215	1.6836	2.6653	1.8280	1.8560	NA	TZ
200	2010	19	2.5907	1.7345	2.9149	1.9348	1.9882	NA	TZ
201	2011	20	2.7564	1.7878	3.1830	2.0288	2.1001	NA	TZ
202	1991	0	1.0000	1.0000	1.0000	1.0000	1.0000	NA	ZM
203	1992	1	0.9827	1.0254	0.9692	1.0367	1.0401	NA	ZM
204	1993	2	1.0494	1.0539	0.9566	0.9919	0.9852	NA	ZM
205	1994	3	0.9589	1.0974	0.9314	0.9638	0.9473	NA	ZM

```
206 1995     4 0.9318 1.1260  0.9180 0.9815 0.9636      NA      ZM
207 1996     5 0.9966 1.2100  0.9155 0.9591 0.9322      NA      ZM
208 1997     6 1.0294 1.2621  0.9291 1.0287 1.0025      NA      ZM
209 1998     7 1.0104 1.3351  0.9650 1.0042 0.9673      NA      ZM
210 1999     8 1.0328 1.3521  1.0201 0.9994 0.9585      NA      ZM
211 2000     9 1.0697 1.3869  1.0181 1.0018 0.9569      NA      ZM
212 2001    10 1.1221 1.3801  1.0260 1.0356 0.9952      NA      ZM
213 2002    11 1.1592 1.4070  1.0379 1.0539 1.0124      NA      ZM
214 2003    12 1.2183 1.4183  1.0541 1.0812 1.0412      NA      ZM
215 2004    13 1.2839 1.4513  1.0659 1.0903 1.0475      NA      ZM
216 2005    14 1.3525 1.4644  1.0819 1.1387 1.0993      NA      ZM
217 2006    15 1.4365 1.5012  1.1066 1.1973 1.1573      NA      ZM
218 2007    16 1.5256 1.5402  1.1382 1.1953 1.1505      NA      ZM
219 2008    17 1.6123 1.5816  1.1745 1.2013 1.1531      NA      ZM
220 2009    18 1.7155 1.6256  1.2152 1.2507 1.2019      NA      ZM
221 2010    19 1.8461 1.6515  1.2668 1.2690 1.2196      NA      ZM
222 2011    20 1.9716 1.7216  1.3440 1.3110 1.2591      NA      ZM
```

2. Cobb-Douglas With Energy

We can force α, β, and γ to be in $[0, 1]$ by a reparameterization:

$$a \in [0,1], b \in [0,1], \alpha = \min(a,b), \beta = |b-a|, \gamma = 1 - \max(a,b)$$

2.1. *Cobb-Douglas with Q*

```
# Note that the anlaysis of ZA is taking a long time here. Need to figure out why.
CDqTables <- lapply(countries, cobbDouglasEnergyTable, energyType="Q")
```

```
print(CDqTables[["US"]], caption.placement="top")
print(CDqTables[["ZA"]], caption.placement="top")
# According to http://cran.r-project.org/web/packages/xtable/vignettes/xtableGalle
# be able to use the "sanitize.text.function" parameter to allow markup in column
# line is not working at the present time. --MKH, 18 Jan 2012.
# print(tableCDe, sanitize.text.function = function(x){x})

#print(tableAll, caption.placement="top")
```

2.2. Cobb-Douglas With X

```
# Note that the anlaysis of ZA is taking a long time here. Need to figure out why.
CDxTables <- lapply(countries, cobbDouglasEnergyTable, energyType="X")
```

```
print(CDxTables[["US"]], caption.placement="top")
print(CDxTables[["ZA"]], caption.placement="top")
```

2.3. Cobb-Douglas With U

```
CDuTables <- lapply(countries, cobbDouglasEnergyTable, energyType="U")
```

```
print(CDuTables[["US"]], caption.placement="top")
print(CDuTables[["ZA"]], caption.placement="top")
```

3. CES

```
cesData <- function(countryName, energyType){
  energyColumnName <- paste("i", energyType, sep="")
  # Load the data that we need.
  dataTable <- loadData(countryName)

  # Establish guess values for phi beta, zeta, lambda_L and lambda_E.
  phiGuess <- -20
  betaGuess <- 0.5 # a typical value for beta (exponent on labor)
  zetaGuess <- 0.0004 # a small value
  lambda_LGuess <- 0.007 #assuming no technical progress on the labor-capital port
  lambda_EGuess <- 0.008 #assuming no technical progress on the energy portion of

  # Runs a non-linear least squares fit to the data with constraints
  modelCES <- nls(iGDP ~ ((1-zeta) * (exp(lambda_L*iYear) * iCapStk^(1-beta) * iLa
                       + zeta*(exp(lambda_E*iYear) * iQ)^phi)^(1/phi),
                  algorithm = "port",
                  control = nls.control(maxiter = 500, tol = 1e-06, minFactor = 1
```

```
                                      printEval = FALSE, warnOnly = FALSE),
                 start = list(phi=phiGuess, beta=betaGuess, zeta=zetaGuess, lamb
                              lambda_E=lambda_EGuess),
                 lower = list(phi=-Inf, beta=0, zeta=0, lambda_L=-Inf, lambda_E=
                 upper = list(phi=0, beta=1, zeta=1, lambda_L=Inf, lambda_E=Inf)
                 data=dataTable)

aicCES <- AIC(modelCES, k=2) # Checks validity of the model. AIC stands for Akai
print(aicCES)

# Gives the nls summary table
summaryCES <- summary(modelCES) # Gives the nls summary table
print(summaryCES)

# Provides confidence intervals on phi, beta, zeta, lambda_L, and lambda_E. But,
ciCES <- confint(modelCES, level = ciLevel)
print(ciCES)

# Get the estimate for alpha
beta <- as.numeric(coef(modelCES)["beta"])
alpha <- 1.0 - beta
alpha.est <- deltaMethod(modelCES, "1 - beta") # Estimates alpha and its standar
print(alpha.est)

# Now calculate a confidence interval on alpha
dofCES <- summaryCES$df[2]
print(dofCES) # Gives the degrees of freedom for the model.
tvalCES <- qt(ciHalfLevel, df = dofCES); tvalCES
# Get confidence intervals for each parameter in the model
alphaCICES <- with(alpha.est, Estimate + c(-1.0, 1.0) * tvalCES * SE) # CI on al
print(alphaCICES)

# Assemble the data into data frames for the table.
estCES <- data.frame(phi = coef(modelCES)["phi"], alpha = alpha,
                     beta = coef(modelCES)["beta"], zeta = coef(modelCES)["zeta"
                     lambda_L = coef(modelCES)["lambda_L"], lambda_E = coef(mode
row.names(estCES) <- paste("CES with ", energyType, sep="")
```

```
  #print(estCES)
  # The [1] subscripts pick off the lower confidence interval
  lowerCES <- data.frame(phi = ciCES["phi","2.5%"], alpha = alphaCICES[1],
                         beta = ciCES["beta", "2.5%"], zeta = ciCES["zeta", "2.5%'
                         lambda_L = ciCES["lambda_L", "2.5%"], lambda_E = ciCES["l
  row.names(lowerCES) <- "- 95% CI"
  # The [2] subscripts pick off the lower confidence interval
  upperCES <- data.frame(phi = ciCES["phi","97.5%"], alpha = alphaCICES[2],
                         beta = ciCES["beta", "97.5%"], zeta = ciCES["zeta", "97.5
                         lambda_L = ciCES["lambda_L", "97.5%"], lambda_E = ciCES['
  row.names(upperCES) <- "+ 95% CI"

  # Now create the data for a table.
  dataCES <- rbind(upperCES, estCES, lowerCES)
  print(dataCES)
  return(dataCES)

  #xyplot( resid(modelCESQ) ~ fitted(modelCESQ) )
  #histogram( ~resid(modelCESQ) )
  #qqmath( ~resid(modelCESQ) )
}

####################################
# Creates a LaTeX printable table from the CES data. This function first calls ces
#
# countryName is a string containint the 2-letter abbreviation for the country, e.
# energyType is a string to be used in table captions reprsenting the type of ener
#
# returns a printable LaTeX table from xtable.
##
cesTable <- function(countryName, energyType){
  dataCESe <- cesData(countryName, energyType)
  tableCESq <- xtable(dataCESe, caption=paste(countryName, ", 1980-2011.", sep="")
}
```

3.1. CES with Q

```
countryName <- "US"
energyType <- "Q"
tableCESq <- cesTable(countryName, energyType)
```

```
[1] -194

Formula: iGDP ~ ((1 - zeta) * (exp(lambda_L * iYear) * iCapStk^(1 - beta) *
    iLabor^beta)^phi + zeta * (exp(lambda_E * iYear) * iQ)^phi)^(1/phi)

Parameters:
          Estimate Std. Error t value Pr(>|t|)
phi      -3.96e+01   2.43e+01   -1.63   0.1144
beta      6.09e-01   3.45e-02   17.64  2.4e-16
zeta      2.09e-06   1.32e-05    0.16   0.8758
lambda_L  7.98e-03   6.68e-04   11.95  2.8e-12
lambda_E  8.57e-03   2.48e-03    3.45   0.0018

Residual standard error: 0.0105 on 27 degrees of freedom

Algorithm "port", convergence message: relative convergence (4)
```

Waiting for profiling to be done...

```
              2.5%      97.5%
phi             NA -10.290831
beta      0.514667   0.665371
zeta            NA         NA
lambda_L 0.006428   0.009152
lambda_E 0.000715   0.012468
         Estimate      SE
1 - beta   0.3911 0.03453
[1] 27
[1] 0.3202 0.4619
               phi  alpha   beta      zeta lambda_L lambda_E
+ 95% CI    -10.29 0.4619 0.6654        NA 0.009152 0.012468
CES with Q -39.64 0.3911 0.6089 2.085e-06 0.007979 0.008570
- 95% CI        NA 0.3202 0.5147        NA 0.006428 0.000715
```

```
#CESqTables <- lapply(countries, cesTable, energyType="Q")
```

```
print(tableCESq, caption.placement="top")
```

Table 2: US, 1980-2011.

	phi	alpha	beta	zeta	lambda_L	lambda_E
+ 95% CI	-10.3	0.46	0.67		0.00915	0.01247
CES with Q	-39.6	0.39	0.61	0.000002	0.00798	0.00857
- 95% CI		0.32	0.51		0.00643	0.00071

```
#print(CESqTables[["US"]], caption.placement="top")
#print(CESqTables[["ZA"]], caption.placement="top")
```