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An Empirical Analysis of the Role of Energy in Economic Growth

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Abstract

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Keywords: economic growth, energy, Cobb-Douglas, CES, LINEX

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1. Introduction

**** Additional references requested by Martin. Incorporate into paper? ****

Lindenberger and Kümmel (2011)

Stresing et al. (2008)

The availability of adequate energy at affordable prices has become an important strategic issue in recent years. Despite a slowdown in economic growth worldwide, longer-term trends in energy use remain strongly upwards, as the dynamics of energy markets are increasingly determined by emerging economies, notably China, India, and the Middle East (International Energy Agency, 2012). A large body of literature points to the fact that economic growth and energy consumption are closely correlated for a diverse set of countries such as the United States (Stern, 2000; Cleveland et al., 1984), India (Paul and Bhattacharya, 2004), China (Wang et al., 2011), South Africa (Menyah and Wolde-Rufael, 2010; Odhiambo, 2009), and sixteen Asian countries (Lee and Chang, 2008) to name a few, although the causal relationship

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from energy consumption to economic growth remains controversial (Chen et al., 2012; Ozturk, 2010; Stern and Enflow, 2013). Ozturk (2010, p. 347) in a survey of the literature on the topic, concluded that there is "...no consensus neither on the existence nor on the direction of causality between these variables in the literature," but does point out that "...causality is running from electricity consumption to economic growth in the most of these studies [surveyed]." Stern and Enflow (2013) found similar results, noting that the direction of causality may change over time.

Many who accept that energy use is closely correlated with economic performance, regardless of the direction of causality, express concern regarding the negative effects of rising energy prices and fossil fuel depletion on current and future economic growth. Benes et al. (2012), incorporating views on both geological physical constraints and technological solutions to oil production, suggest a near doubling of the real price of oil over the next decade. **** Should we re-think this discussion in the light of the shale oil boom and the recent drop in oil prices? –MKH **** **** Yes, we need to explain in a sentence or two what we think is going on. Saudi's playing for market share and increased supply from the US. Potential crippling impacts on green energy upstarts. Possible rebound of oil price soon? –MdW **** Kumhof and Muir (2012) point out that significant impacts on GDP growth could occur for larger oil supply shocks, assuming production functions where oil plays a prominent role and reduced possibilities for substitution away from oil. Nel and van Zyl (2010) argue for the "...vital importance of energy security..." on the basis of an economic growth model with explicit energy-based formulation. Hamilton (2013) argues that "[c]oping with a final peak in world oil production...[will] have much in common with previous historical episodes that resulted from temporary geopolitical conflict, being associated with significant declines in employment and output. If the future decades look like the last 5 years, we are in for a rough time." **** Possibly insert here what we make of the last few months' developments in oil prices. But, as it will be speculative, it needs to be very short and focused. –MdW ****

The current trends in energy production and consumption and the possible significant impacts on the world economy as argued by the above-mentioned scholars raise the question of whether and how to include appropriate energy indicators in macroeconomic growth modeling. Mainstream economic growth theory attributes output (Y) to factors of production: capital stock (K) and labor (L). The Cobb-Douglas production function is the canonical representation of indexed economic output (y) as a function of indexed labor (l), indexed capital stock (k), and an additional exogenous, time-dependent term. This production function can be expressed as

$$y = \theta A k^{\alpha} l^{\beta} \; ; \; A \equiv \mathrm{e}^{\lambda(t-t_0)}, \tag{1}$$

where $y \equiv Y/Y_0$, θ is a scale parameter, e is the base of the natural logarithm, λ is represents the pace of technological progress, t (time) is measured in years, $k \equiv K/K_0$, $l \equiv L/L_0$, Y is represented by GDP, K is expressed in currency units, L is expressed in workers or work-hours/year, and 0 subscripts indicate values at an initial year. The parameters α and β are output elasticities that indicate the relative roles of capital and labor in the economy. Typically, the constraint $\alpha + \beta = 1$ is assumed, indicating constant returns to scale.[1]

As shown in Equation 1 above, mainstream economic theory does not include an explicit term for the role of energy in economic growth. However, Bashmakov (2007), Hamilton (2009), Murphy and Hall (2011), Benes et al. (2012), Kumhof and Muir (2012), Ayres and Warr (2010), Ayres et al. (2013), and others indicate that the importance of energy for economic growth may far outweigh its cost share in the economy. That mainstream economic growth theory excludes energy warrants an inquiry into the proper approach for including energy as a factor of production in economic growth modeling is both relevant and important.

Some energy economists augment economic growth models by including energy as an additional (or the only) factor of production. However, there is little agreement on how this should be done. Neither the mathematical form of the energy-augmented production function nor the quantification of energy[2] is consistent in the literature. Stern (2011, p. 26) provides an overview of approaches to including energy in ecological economic and resource economic models of growth. Stern points out that energy is seen as the *central driver of growth* in ecological economic (including biophysical economic) models and that energy *is included in* resource economic models, but *remains isolated* within the sub-discipline. To break through this theoretical impasse, Stern proposes a synthesized Constant Elasticity of Substitution (CES) production function where the constraints on or abundance of energy determines whether

[1]Dimensionless, indexed quantities are represented by lower-case symbols (y, k, l, and q), and dimensional quantities are represented by upper-case symbols (Y, K, L, and Q). Model parameters are represented by Greek letters (α, β, λ, θ).

[2] Various energy quantifications have been used in economic growth modeling: thermal energy (primary and final), exergy (primary and final), and useful work.

to include energy explicitly as a factor of production. On the basis of this model, Stern (2011, p. 45) points out that energy availability plays "...a key role in enabling growth," despite evidence of decreasing energy consumed per unit of output. He concludes, based on theory and time-series results, that including energy in the production function is warranted, and that further clarification is needed on the role of energy in economic growth.

The inclusion of energy as a factor of production in Cobb-Douglas and CES production functions is generally accepted by most resource and energy economists (Smulders and De Nooij, 2003; van der Zwaan et al., 2002; Kemfert, 1998). Researchers interested in explaining long-term economic growth and possible constraints on growth have begun to include energy as an explicit factor of production, too (Fröling, 2009; Tahvonen and Salo, 2001).

An alternative approach, attempting to be "...more consistent with physical laws ..." (Ayres, 2001) and classified by Stern (2011) as an ecological economic model of growth, is the LINEX model, wherein energy is the only factor of production, and capital (k), labor (l), and energy (e) combine to provide efficiencies that augment energy. There is no Solow-style term ($\mathrm{e}^{\lambda t}$) for technological change. Rather, technological change is manifest as increased efficiency: more useful work done per joule of exergy input to the economy (Ayres and Warr, 2005). Kümmel (1980) developed the LINEX function using energy quantified as primary thermal energy input to the economy. Kümmel represented energy efficiency coefficients in the LINEX function by logistic functions, effectively converting from primary thermal energy to energy at the point of use within the LINEX function itself.

In a series of papers culminating in a book (Ayres and Warr, 2010), Ayres and Warr used the LINEX function with energy quantified as "useful work," which is much closer to the point of use than the primary thermal energy quantification favored by Kümmel. Ayres and Warr do not find the need to use logistic functions representing energy efficiency, presumably because useful work "already incorporate[s] the energy-related efficiency changes in the economic system" (Kümmel, 2013, p. 35). Both Kümmel and Ayres and Warr report satisfactory agreement between the LINEX function and historical economic output.

Unsatisfactorily, both the choice of a production function and the parameters used to populate these functions affect results significantly. This situation leaves far too much latitude for misuse of model specifications and selection of model parameters to support certain theories, rather than empirically testing the theories and parameters *before* they are employed. For example, Lecca et al. (2011) pointed out that the modellers' choice for the structure of a production function reflects his or her assumptions about the elasticities among inputs, and "...the point at which energy enters the [nested CES] production function has a significant impact on CGE [computable general equilibrium] model results." Duffy and Papageorgiou (2000) rejected the Cobb-Douglas production function altogether, finding that capital and labour are more substitutable in richer countries and less substitutable in poorer countries than Cobb-Douglas implies. However, Miller (2008) points to the "...reasonably accurate long-term economic forecasts," achieved with the use of Cobb-Douglas production functions in economic models. Shen and Whalley (2013) provide a recent example wherein empirical analysis on the estimated substitution elasticities is performed before choosing any particular nested CES production function structure. As a result, Shen and Whalley propose an atypical CES nesting structure for the Chinese economy. Saunders (2008) evaluated several production and cost functions (including the Cobb-Douglas, CES, and LINEX functions studied herein, see Section 2.1) to assess whether they are capable of reproducing rebound and backfire effects.[3] Saunders concludes that, of the production functions evaluated in the present study, only the CES function is a candidate for rebound analysis. The Cobb-Douglas function was found to exhibit only backfire. The LINEX function was found to "not satisfy the standard concavity (diminishing marginal productivity) conditions." Saunders (2008, p. 2195)

In addition to model structure, the way in which model parameters are determined is also subject to debate. The *cost share theorem*[4] from mainstream economics states that output elasticities can be calibrated from national accounts data as the cost shares of factors of production: capital, labor, and energy (if augmenting the production function by energy). However, the cost share theorem applies only under conditions of perfect competition in the markets for the factors of production. Energy markets are not "perfect." Many governments have parastatal or full control of electricity generation and fuel markets. Other governments subsidize or incentivize energy markets. And, there are always health and environmental externalities that are not priced into fossil fuel energy markets. If energy markets are further from "perfect" than capital and labor markets, then energy is the factor of production

[3] See Sorrell (2007) for background on rebound and backfire.

[4] See Kümmel et al. (2010, p. 150) for a description of the cost share theorem.

least likely to have its output elasticity be well-represented by cost share as reported in national accounts (Ayres et al., 2013; Kümmel, 2013). Thus, when augmenting production functions by energy, much care should be taken in estimating the relative importance of the factors of production. Kümmel et al. (2010) showed that the cost share theorem applies only when economic growth is unconstrained. Real-world constraints such as the degree of automation and the degree of utilization of capital stock result in shadow prices that move output elasticities away from their cost shares in the economy (Kümmel et al., 2010, Equations 12 and 13). Thus, for this study, we have no *a-priori* expectation that output elasticities estimated from historical output data will match cost shares in national accounts.

Thus, there are stong indications in the literature that both the choice of production function structure and the values of the model parameters drive model results, and empirical and statistical testing of such approaches is prudent, but rarely conducted.

These are not idle academic debates. Macroeconomic growth models are used often to inform economic policy, and the paths of economies worldwide are determined by such policy choices. Therefore, we agree with those who express serious concerns about the reliability of the results from economic growth models (Lecca et al., 2011). We understand, however, that macroeconomic policymaking is needed and is increasingly informed by integrated energy-economy models, in a world where energy constraints are possible (Beckman et al., 2011; Benes et al., 2012; Nel and van Zyl, 2010; Kümmel et al., 2010). Unfortunately, there has been little empirical work assessing the predictive ability of various energy-augmented production functions for different types of economies. And, very few studies of the importance of energy in economic growth include rigorous statistical assessment of energy-related fitting parameters.

This paper contributes to the literature on the role of energy in economic growth by applying statistically rigorous methods along two coordinates of analysis: form of the energy augmented production function and type of economy. We support the call from Shen and Whalley (2013) that both the structure of the production function and the parameters that populate it need empirical testing. Building on the argument of Stern (2011) that energy may or may not be required in production functions depending on its abundance or scarcity, we will test whether different types of countries may need different factors of production to explain their respective growth paths, leaving open the possibility for different explanations of growth in various countries at different stages of their development, a central point of long-wave economic growth modeling (Fröling, 2009; Jones, 2001; Galor and Weil, 2000).

The remainder of this paper is organized as follows. Section 2 discusses the coordinates of analysis for the present study. Section 3 details sources of data and calculations to generate time series for economic output (y) and factors of production (k, l, and q). The procedures for and results from parameter estimation for the various production functions are discussed in Section 4. Section 5 discusses implications of the parameter estimation results, and Section 6 concludes.

2. Coordinates of Analysis

This section describes our two coordinates of analysis.

2.1. Mathematical Forms of the Energy-augmented Production Function

We assess three prominent energy-augmented production functions that appear in the literature: Cobb-Douglas (CD), Constant Elasticity of Substitution (CES), and LINear EXponential (LINEX). The following subsections describe each.

2.1.1. Cobb-Douglas (CD) Production Function

The capital-labor Cobb-Douglas production function shown in Equation 1 can be augmented by energy:

$$y = \theta A k^{\alpha} l^{\beta} e^{\gamma}\ ;\ A \equiv \mathrm{e}^{\lambda(t-t_0)}, \tag{2}$$

where $e \equiv E/E_0$, and E is in units of energy per time, typically TJ/year. The energy-augmented Cobb-Douglas production function is often assumed to have constant returns to scale for the three factors of production: $\alpha + \beta + \gamma = 1$.

2.1.2. Constant Elasticity of Substitution Production Function (CES)

Other energy economists use an energy-augmented Constant Elasticity of Substitution (CES) production function to describe economic growth. The `R` (R Core Team, 2012) package `micEconCES` (Henningsen and Henningsen, 2011) estimates CES production functions of the following forms (among others).

$$y = \gamma\, A\, \left[\delta_1 k^{-\rho_1} + (1-\delta_1) l^{-\rho_1}\right]^{-1/\rho_1} ; A = \mathrm{e}^{\lambda(t-t_0)} \tag{3}$$

$$y = \gamma\, A\, \left\{\delta \left[\delta_1 k^{-\rho_1} + (1-\delta_1) l^{-\rho_1}\right]^{\rho/\rho_1} + (1-\delta) e^{-\rho}\right\}^{-1/\rho} ; A = \mathrm{e}^{\lambda(t-t_0)} \tag{4}$$

Equation 3 is a CES production function with capital stock (k) and labor (l) factors of production. Equation 4 augments Equation 3 with energy using a (kl) + (e) nesting structure, as is typical in the literature. Equation 3 is a degenerate form of Equation 4 where $\delta \to 1$.

γ is a scale factor that adjusts for atypicality of the initial year of the time series. The fitting parameters ρ_1 and ρ indicate elasticities of substitution (σ_1 and σ). The elasticity of substitution between capital (k) and labor (l) is given by $\sigma_1 = \frac{1}{1+\rho_1}$, and the elasticity of substitution between (kl) and (e) is given by $\sigma = \frac{1}{1+\rho}$. As $\rho_1 \to 0$, $\sigma_1 \to 1$, and the embedded CES production function for k and l degenerates to the Cobb-Douglas production function. Similarly, as $\rho \to 0$, $\sigma \to 1$, and the CES production function for (kl) and (e) degenerates to the Cobb-Douglas production function. As $\sigma \to \infty$ ($\rho \to -1$), (kl) and (e) are perfect substitutes. As $\sigma \to 0$ ($\rho \to \infty$), (kl) and (e) are perfect complements: no substitution is possible. Similarly, as $\sigma_1 \to 0$ ($\rho_1 \to \infty$), k and l are perfect complements. δ_1 describes the relative importance of capital (k) and labor (l), and δ describes the importance of (kl) relative to (e).

Constraints on the fitting parameters include $\delta_1 \in [0, 1]$, $\delta \in [0, 1]$, $\rho_1 \in [-1, 0) \cup (0, \infty)$, and $\rho \in [-1, 0) \cup (0, \infty)$.

Two other nestings of the factors of production (k, l, and e) are possible with the CES model.

$$y = \gamma\, A\, \left\{\delta \left[\delta_1 l^{-\rho_1} + (1-\delta_1) e^{-\rho_1}\right]^{\rho/\rho_1} + (1-\delta) k^{-\rho}\right\}^{-1/\rho} ; A = \mathrm{e}^{\lambda(t-t_0)} \quad (5)$$

$$y = \gamma\, A\, \left\{\delta \left[\delta_1 e^{-\rho_1} + (1-\delta_1) k^{-\rho_1}\right]^{\rho/\rho_1} + (1-\delta) l^{-\rho}\right\}^{-1/\rho} ; A = \mathrm{e}^{\lambda(t-t_0)} \quad (6)$$

Note that the parameters ρ (σ) and δ have different meanings depending upon the nesting of the factors of production.

2.1.3. LINEX Production Function

A third production function, the LINear EXponential (LINEX) function

$$y = \theta\, \mathrm{e}^{a_0\left[2\left(1-\frac{1}{\rho_k}\right)+c_t\left(\rho_l-1\right)\right]}\, e\,, \quad (7)$$

was derived from thermodynamic considerations (Kümmel, 1980, 1982; Kümmel et al., 1985; Kümmel, 1989; Kümmel et al., 2002, 2010). In contrast to both the Cobb-Douglas model (Equations 1 and 2) and the CES model (Equations 3–6), the LINEX model (Equation 7) does not include a generic, time-dependent augmentation term (A).

The LINEX model assumes energy (e) is the only factor of production. However, the LINEX model also includes terms that represent efficiencies among capital, labor, and energy, in the form of the ratios ρ_k and ρ_l which are defined by[5]

$$\rho_k \equiv \frac{k}{(1/2)(l+e)} \tag{8}$$

and

$$\rho_l \equiv \frac{l}{e} \tag{9}$$

and represent (8) capital stock increase relative to the average of labor and energy and (9) labor increase relative to energy. The effect of the only factor of production, energy (e), on production is enhanced if either

- capital stock (k) has increased more than the average of labor (l) and energy (e), i.e. if $\rho_k > 1$, or
- labor (l) has increased more than energy (e), i.e. if $\rho_l > 1$.

An economy that increases capital stock (k) and labor (l) at the same rate as energy will have capital and labor efficiency ratios (ρ_k and ρ_l, respectively) of unity. In that case, economic output (y) increases at the same rate as energy consumption (e). According to the LINEX model, an economy that increases capital stock (k) without a commensurate increase in labor (l) and energy (e) will experience an increase in output (y) in excess of its increase of energy consumption (e), because

$$\mathrm{e}^{a_0\left[2\left(1-\frac{1}{\rho_k}\right)+c_t\left(\rho_l-1\right)\right]} > 1.$$

Similarly, an economy benefits by increasing labor (l) without a commensurate increase in energy (e). Thus, ρ_k is an efficiency of using additional labor and energy to make additional capital stock available to the economy, and ρ_l is an efficiency of using additional energy to make additional labor available to the economy. Increases in ρ_k and ρ_l provide upward pressure on economic output (y), as the only factor of production (e) is used more efficiently.

The parameter a_0 represents the overall importance of efficiencies ρ_k and ρ_l, and the parameter c_t represents the relative importance of the labor efficiency

[5] The notation for the LINEX model breaks from the conventions used in the Cobb-Douglas and CES models: a_0 and c_t are parameters, and ρ_k and ρ_l are not.

Table 1: Example economies and years of evaluation.

Developed	US	USA	1980–2011
	UK	United Kingdom	
	JP	Japan	
NICs	CN	China	1991–2011
	ZA	South Africa	
OPEC	SA	Saudi Arabia	1991–2011
	IR	Iran	
LDCs	TZ	Tanzania	1991–2011
	ZM	Zambia	

term (ρ_l) compared to the capital stock efficiency term (ρ_k). For this study, we assume both a_0 and c_t are constant with respect to time.

2.2. Economy Types

In the energy-economic growth literature, the focus is almost exclusively on developed economies from North America and Europe. Examples include Kümmel (1980) applying LINEX to describe the West German economy, Serrenho et al. (2010) applying Cobb-Douglas variants to EU-15 economies, van der Werf (2008) applying CES nesting variants to the 12 OECD economies, Lecca et al. (2011) applying CES nesting variants to the Scottish economy, Stern and Kander (2012) applying CES to describe the Swedish economy, and Warr and Ayres (2012) using LINEX and Cobb-Douglas to describe the Austrian, American, British, and Japanese economies.

But, the near-exclusive focus on developed economies closes off the possibility of discovering any role that energy plays in the economic development process. To gain insights from different countries across different stages of development, we present a comparative analysis among representatives of the World Bank's country categories, namely Developed Countries (DCs), Newly Industrialized Countries (NICs), OPEC countries, and Least Developed Countries (LDCs). Table 1 shows our example countries.

3. Sources of Data

To assess energy-augmented models of economic growth along the coordinates of analysis outlined in Section 2, historical data for economic output (Y), capital stock (K), labor (L), and primary thermal energy (Q_p) are required. Subsections below describe sources for and calculations to obtain historical data. A final subsection presents the historical data in graphical form.

3.1. Economic Output

We measure economic output (Y) by gross domestic product (GDP). GDP data in real 2005 US dollars for all countries was obtained from the RGDP^{NA} (Real GDP at constant 2005 national prices) time series of the Penn World Table (Feenstra et al., 2013).

3.2. Capital Stock

Gross capital stock in real 2005 U.S. dollars for all countries was obtained from the RK^{NA} (Capital stock at constant 2005 national prices) time series of the Penn World Table (Feenstra et al., 2013).

3.3. Labor and Employment

Total employment for all countries was obtained from the EMP (Number of persons engaged) time series of the Penn World Table (Feenstra et al., 2013). The EMP time series was used as the labor factor of production for CN, ZA, SA, IR, TZ, and ZM. For the developed nations (US, UK, and JP), average hours worked per worker per year was obtained from the AVH (Average annual hours worked by persons engaged) time series of the Penn World Table (Feenstra et al., 2013). The product of EMP and AVH was used as the labor factor of production for the developed nations.

3.4. Energy

To account for all energy input to an economy, we summed consumption data for many energy sources, including coal, oil and petroleum products, dry natural gas and natural gas plant liquids, nuclear power generation, and renewable energy sources (solar, wind, geothermal, biomass/fuel wood, and hydro). We also added animal muscle work. Similar to the economic data discussed in Sections 3.1–3.3, we obtained energy data for developed countries from 1980–2011. For all other countries, we obtained energy data for 1991–2011.

Thermal energy (Q) for fuels (in units of TJ/year) was calculated by

$$Q_{fuel} = C_{fuel} h_{fuel} \tag{10}$$

where C_{fuel} is the consumption rate of the fuel (in units of short tons/year for coal, barrels/year for oil and petroleum, ft^3/year for dry natural gas and natural gas plant liquids, and m^3/year for biomass and fuel wood) and h_{fuel} is the heat content of the fuel (in kJ/short ton for coal, kJ/barrel for oil, kJ/ft^3 for dry natural gas and natural gas plant liquids, and kJ/kg for biomass and fuel wood). Fuel wood density of 300 kg/m^3 was taken from Ragland and Aerts (1991) at the low end of the range of density of dry wood and bark.

Yearly consumption rates (C_{fuel}) of coal, oil and petroleum, dry natural gas and natural gas plant liquids, and biofuels (ethanol and biodiesel) were obtained from the EIA (2013) for all economies. Yearly average heat content data for coal, oil, and natural gas and natural gas plant liquids was obtained from the EIA (2013) for all economies. We used lower (net) heating values of 26.8 MJ/kg (Çengel and Boles, 2011, Table A-27) and 37.8 MJ/kg (Gupta and Demirbas, 2010, Table 7.3) for the heat content of ethanol and corn-based biodiesel, respectively. Yearly consumption and heat content data for biomass/fuel wood was obtained from the "Fuelwood" time series of the FAO Statistics Database (FAO, 2013).

For renewable and nuclear energy sources, yearly electricity generation (in BTU/year) was available directly from the EIA (2013) for all economies, so Equation 10 was unnecessary.

Animal muscle work was estimated using an approach similar to Ayres and Warr (2010), using data from Warr's REXS database (Warr, 2012) and results from Wirsenius (2000).[6] For animal work, our analysis was restricted to draught animals (horses and mules). Muscle work from animals ($Q_{animals}$) was calculated using

$$Q_{animals} = Q_{m,horse} N_{horses} + Q_{m,mule} N_{mules} \tag{11}$$

where Q_m is metabolizable energy (in kcal/day-animal) and N is the number of animals. Metabolizable energy values ($Q_{m,horses}$ = 19,110 kcal/day-horse and $Q_{m,mules}$ = 7,215 kcal/day-mule) were taken from Warr (2012). Q_m represents the amount of energy in the feed consumed by animals for a day of

[6] Following Ayres and Warr (2010), human muscle work is excluded from the energy time series, on the grounds that labor terms are already present in all production functions.

work, and they are assumed constant for all countries and years. The number of horses and mules for each economy was taken from the "Live Animals" section of the FAO Statistics online database (FAO, 2013).

3.5. Historical Data

Figure **??** presents historical data, including indexed values for output (y), capital stock (k), labor (l), and energy (q) for 1980–2011 (developed economies) and 1991–2011 (other economies). **** Remove word "Calvin" from top. –MKH ****

4. Parameter Estimation

4.1. Point Estimates

Each of the economic growth models discussed in Section 2.1 has parameters which must be estimated (fitted) using data for each economy (see Sections 2.2 and 3). We obtained parameter estimates by applying the method of least-squares to log-transformed data using the `R` (R Core Team, 2012) functions `lm` (for Cobb-Douglas and LINEX models) and `cesEst` (Henningsen and Henningsen, 2011) (for CES models).[7]

Although the details vary from model to model, each of our three models has the general form

$$y = f(t, k, l, e; \theta) + \text{error} \ , \tag{12}$$

where θ is a vector of parameters for the function f. **** if we use θ this way, we need a new scaling parameter. –rjp **** Parameter estimates are chosen to minimize **** Randy edit the following equation to reflect the fact that we're fitting in log space. –MKH ****

$$sse = \sum_i (y_i - f(t, k, l, e; \theta))^2 \tag{13}$$

within constraints. For the optimal parameter values $\hat{\theta}$, we define the fitted response by

$$\hat{y}_i = f(t_i, k_i, l_i, e_i; \hat{\theta}) \tag{14}$$

[7] As is typical in the literature, we employ the natural logarithm transform to allow linear fitting of exponential coefficients.

and the residuals (r_i) as the difference between the observed response and the fitted response **** Randy edit the following equation to reflect the fact that we're fitting in log space. –MKH ****

$$r_i = y_i - \hat{y}_i = y_i - f(t, k, l, e; \hat{\theta}) \tag{15}$$

Details for each of the models are discussed in Sections 4.3–4.5 below.

4.2. Resampling Methods

Bootstrapping is a statistical resampling technique for assigning measures of accuracy and precision to sample estimates by measuring properties of an estimator when sampling from a resampling distribution. Resampling distributions can be formed in a number of ways in accordance with the type of data, experimental design, and modeling assumptions involved. In each case, many new resamples are created, each of which is a randomized version of the original sample data to which the desired analysis method can be applied. By investigating, for example, the variability of a parameter estimate from one resample to another, one can learn about the precision of the estimation method. **** Add some references here. –rjp ****

In the context of linear models (regression), resamples are generally created by residual resampling. In our case, we formed resamples by adding to the fitted response ($\hat{y}$) the product of a residual from the original model fit and random sign (-1 or 1, each with probability 0.5). For the case of Cobb-Douglas and LINEX models, this residual resampling occurs on the log-transformed data.

Intuitively, this method assumes that the residuals are indicative of the variability (from many potential sources) inherent in the data such that it would be unsurprising if the residual from any particular year had been observed in a different year. Thus, a resampled response $\tilde{Y}'$ can be computed as **** Randy edit the following equations to reflect the fact that we're fitting in log space? –MKH ****

$$\tilde{Y}_i' = \hat{y}_i \pm r_j \tag{16}$$

where

$$r_i = y_i - \hat{y}_i \tag{17}$$

both the sign ($\pm$) and the index of the residual (j, typically different from i) are chosen at random (with replacement). We repeated the resampling process 1000 times for each combination of growth model and country, both with and without energy as appropriate.

The coefficients from the fit to a resampled time series (the "resample coefficients") will be different from the coefficients obtained from the fit to historical data (the "base coefficients"). When these resample coefficients are highly variable, it is an indication that the data do not determine the parameter estimates very precisely. Lack of precision can stem from a number of factors, most obviously a poor model fit, low model sensitivity to one or more parameters, or correlations between parameter estimates.

It is important to note that large residuals can arise from either (a) poor quality historical time series data, or (b) a mathematical model that does not describe the underlying phenomena well. It is also important to note that (c) even when the residuals are small and the model produces fitted values that track the observed data closely, it may yet be difficult to estimate some or all of the model parameters precisely. The resampling method employed herein reflects all three of these potential sources of uncertainty in parameter estimates.

We choose the resampling approach instead of the more-common technique of estimating confidence intervals from standard errors for two reasons. First, model parameters are highly constrained, and confidence intervals often violate the constraints. For example a result of $\alpha = 0.25 \pm 0.3$ is nonsensical, because $\alpha \in [0, 1]$. Second, true confidence intervals for constrained parameters are often asymmetric (e.g., $\alpha = 0.2 + 0.1, -0.05$), but the standard error approach yields symmetric confidence intervals. The resampling approach that we employed both respects constraints and allows for asymmetric confidence intervals.[8]

4.3. Cobb-Douglas Models

The Cobb-Douglas model without energy is given by Equation 1. Equation 1 was reparameterized as

$$y = \theta \mathrm{e}^{\lambda(t-t_0)} k^{\alpha} l^{1-\alpha} \tag{18}$$

to ensure $\alpha + \beta = 1$ for constant returns to scale. θ, λ, and α were estimated by the `R` (R Core Team, 2012) function `lm` in log-transform space. If the estimated value for α was found outside the interval $[0, 1]$, we set α to its boundary value and re-estimated λ. The value of β was found with $\beta = 1 - \alpha$.

[8] In addition, we choose to present parameter uncertainty results visually (see Figures **??**, **??**, **??**–**??**, and **??**), both because the resampling technique lends itself to visual presentation and because we believe visual comparison is more effective than tables of numbers.

To estimate the parameters θ, λ, α, β, and γ in the energy-augmented Cobb-Douglas model, Equation 2 was reparameterized as

$$y = \theta \mathrm{e}^{\lambda(t-t_0)} k^{\alpha} l^{\beta} e^{1-\alpha-\beta} \tag{19}$$

ensuring that $\alpha + \beta + \gamma = 1$, thereby providing constant returns to scale. The `R` (R Core Team, 2012) function `lm` was used to estimate values of λ, θ, α, and β in log-transform space. If the fitted value for α or β fell outside the interval $[0, 1]$, we fit along all boundaries ($\alpha = 1$, $\beta = 1$, $\gamma = 1$, $\alpha = 0$, $\beta = 0$, and $\gamma = 0$) and chose the boundary fit with minimum sse (Equation 13) as the winner. γ was recovered with $\gamma = 1 - \alpha - \beta$.

Figure **??** shows λ and θ values for Cobb-Douglas resample models with and without energy. Each reample is shown as a gray dot on the graph. The fit to historical data is shown as a gray crosshairs.

Figure **??** shows ternary plots for α, β, and γ parameter values for Cobb-Douglas (without and with energy) resample models.

Figure **??** compares predictions from the Cobb-Douglas models (without and with energy) to historical data. Historical data are shown as a black line. The fit to historical data is shown as a white line. The gray band encompasses ??% of the fits to resampled data.

4.4. CES Models

The CES model without and with energy is given by Equations 3–6. The `R` (R Core Team, 2012) package `micEconCES` (Henningsen and Henningsen, 2011) was used to estimate parameters λ, γ, δ, δ_1, ρ, and ρ_1. The `cesEst` function of `micEconCES` provides several algorithm options for parameter estimation. The default algorithm (Levenberg-Marquardt) does not respect parameter constraints (see Section 2.1.2) and, in our testing, nearly always violated them, often returning negative values for elasticity of substitution parameters σ and σ_1. Thus, we used the two fitting algorithms available in `cesEst` that respect coefficient constraints: `PORT` and `L-BFGS-B`.

Our CES parameter estimation algorithm starts with an eleven-value grid search in ρ and ρ_1 (9, 2, 1, 0.43, 0.25, 0.1, -0.1, -0.5, -0.75, -0.9, -0.99), which corresponds to σ and σ_1 values of 0.1, 0.33, 0.5, 0.7, 0.8, 0.9, 1.11, 2, 4, 10, and 100, respectively. During the grid search, values of ρ and ρ_1 are fixed, and values of λ, γ, δ, and δ_1 are estimated by gradient search with the `PORT` and `L-BFGS-B` algorithms. In all, 121 gradient searches in λ, γ, δ, and δ_1 at grid points representing all combinations of ρ and ρ_1 are attempted. During the grid search portion of our algorithm, starting values for the free parameters are $\lambda = 0.015/\text{year}$, $\delta = 0.5$, $\delta_1 = 0.5$, and γ is set to a value such that the mean of the residuals is zero by the `cesEst` function.

Next, a gradient search (using both `PORT` and `L-BFGS-B`) is attempted wherein all fitting parameters (λ, γ, δ, δ_1, ρ, and ρ_1) are allowed to float. The start values for fitting parameters are taken from the grid search point that provided the lowest *sse* (Equation 13).

If resampled data are being fitted, a prior fit to historical data is available. In the final step of our algorithm, a gradient search (using both `PORT` and `L-BFGS-B`) uses coefficients from the best fit to historical data as its starting point. In this final gradient search, all model parameters (λ, γ, δ, δ_1, ρ, and ρ_1) are considered free parameters.

The fit with lowest *sse* of all above trials is deemed the winner, and its fitting parameters are used as the model.

Henningsen and Henningsen (2011), in their detailed analysis of Kemfert (1998), found that

> ... the Levenberg-Marquardt and the `PORT` algorithms are—at least in this study—most likely to find the coefficients that give the best fit to the model, where the `PORT` algorithm can be used to restrict the estimates to the economically meaningful region.

Table 2: Equations for α, β, and γ for the various CES nestings.

Nesting	Equation	α	β	γ
$(kl) + ()$	3	δ_1	$1 - \delta_1$	0
$(kl) + (e)$	4	$\delta\delta_1$	$\delta(1 - \delta_1)$	$1 - \delta$
$(le) + (k)$	5	$1 - \delta$	$\delta\delta_1$	$\delta(1 - \delta_1)$
$(ek) + (l)$	6	$\delta(1 - \delta_1)$	$1 - \delta$	$\delta\delta_1$

In our testing, we found that to be mostly true. `PORT` nearly always provided lower *sse* than `L-BFGS-B`, despite the fact that `L-BFGS-B` often reports convergence and `PORT` does not for the same data (i.e., for the same y, k, l, and e time series).

Coefficients for each resampled CES model for all countries and all nests are shown in Figures **??**–**??**. The white crosshair on each graph shows the location of the fit to historical data. In Figures **??** and **??**, the bottom of the vertical axes represents $\sigma = 0$ (perfect complements). The top of the vertical axes represents $\sigma = \infty$ (perfect substitutes).

Note that as δ or δ_1 approaches 0 (1) in Equations 3–6, the corresponding value of σ or σ_1 becomes irrelevant, and resample points along the left (right) edge of Figures **??** and **??** are equivalent. That is, all points along the left (right) edge of Figures **??** and **??** have the same *sse*. As a consequence, the `cesEst` fitting algorithm has no way to discriminate among points along the left (right) edge of Figures **??** and **??**. Thus, resample point estimates where δ or δ_1 approaches 0 (1) tend to cluster at the locations of the initial grid search in σ or σ_1 (0.1, 0.33, 0.5, 0.7, 0.8, 0.9, 1.11, 2, 4, 10, and 100).

We can develop pseudo α, β, and γ parameters for the CES model, based on values of δ and δ_1. Table 2 shows the permuted equations for each CES nesting.

Figure **??** shows α, β, and γ parameters for the CES model for all nests and countries. Figure **??** is comparable to the ternary graph for the Cobb-Douglas models, Figure **??**.

Figure **??** compares CES model predictions to historical data for all nestings. As with the Cobb-Douglas results, historical data are shown as a black line, the fit to historical data is shown as a white line, and the gray band encompasses ??% of the fits to resampled data.

4.5. LINEX Models

The LINEX model is given in Equation 7. If the fitting algorithm drives $a_0 \to 0$, any changes to c_t have no effect on sse. Therefore, we transformed the LINEX model into

$$y = \theta A e; A = \mathrm{e}^{\left[2a_0\left(1-\frac{1}{\rho_k}\right)+a_1\left(\rho_l-1\right)\right]}, \tag{20}$$

where $a_1 \equiv a_0 c_t$. We used the `lm` function in `R` (R Core Team, 2012) to estimate least-squares values for a_0 and a_1 in Equation 20 in log-transformed space. We recovered the value of c_t with $c_t = a_1/a_0$. Figure **??** shows the coefficients estimated from resampled data. We fit a_0 and c_t as unconstrained constants.

Figure **??** compares LINEX model predictions to historical data. Again, historical data are shown as a black line, the fit to historical data is shown as a white line and the gray band encompasses ??% of the fits to resampled data.

The LINEX model coefficients a_0 and c_t imply time-varying values of α, β, and γ as shown by Warr and Ayres (2012, Equations 7). Thus, each fitted model implies a trajectory of α, β, and γ values as time proceeds. We found unconstrained fitting produced α, β, γ trajectories that violated parameter constraints ($[0, 1]$) for all countries, with the exception of Iran. If the LINEX model were forced to respect the constraints, it would fit worse than it already does for those countries where the constraints are violated (that is, for every country except Iran). Warr and Ayres (2012) selected values of a_0 and c_t, while still holding them constant for the entire time series, such that the constraints on α, β, and γ were respected throughout their time series. But, Warr's approach necessarily leads to larger *sse* values than unconstrained fitting. Because in our results unconstrained LINEX models already fit worse than constrained Cobb-Douglas and CES models, we chose to report unconstrained LINEX model fits in this paper.

5. Discussion

**** Other notes from our Word document that didn't fit anywhere above. ****

Lastly, what do aggregate models say?

What direction can take from this work?

What is the status of ME growth modeling after this?

**** End other notes ****

We find it beneficial to discuss the results through a series of questions, answers to which are illuminated by the techniques and data presented in Section 4 above.

**** Martin: Do you think it would be helpful to tie the conclusions of each section to the literature? If so, give it a shot. –MKH ****

5.1. Is it important to include energy in the production function?

For the Cobb-Douglas and CES production functions, augmentation by energy improves goodness of fit when $\gamma > 0$.[9] First, we evaluate the results of the Cobb-Douglas functions in Figures **??** and **??**. Visually, $\gamma > 0$ is shown in the right column of Figure **??** as data points that are above the horizontal

[9] The LINEX function is unhelpful in answering the question "Is it important to include energy in production functions?" because LINEX presupposes that energy is the only factor of production.

base of the ternary plots. We note that the list of countries for which $\gamma \gg 0$ includes Tanzania, South Africa, Saudi Arabia, and the United Kingdom. Especially for Tanzania, but also for Saudi Arabia and the United Kingdom, it appears certain that including energy improves goodness of fit: both the fit to historical data (crosshairs) and all resample data points (black dots) are above the base of the triangle in the energy columns of Figure **??**. However, for South Africa, significant uncertainty in γ, as shown by the large vertical spread of resample data points, leads us to exercise caution in concluding that augmenting the Cobb-Douglas production function by energy is helpful for improving the goodness of fit to historical data. A robust result (γ equal or close to zero and very little vertical spread of the resample points) indicates that including energy does not help to explain economic growth in Zambia, while for United States, Japan, China, and especially Iran, it is less certain that augmenting with energy is unhelpful (larger vertical spread of resample points).

The results for the CES analysis (Figure **??**) indicate that including energy is clearly helpful in explaining economic growth in Saudi Arabia, because all resample data points are consistently far above the base of the ternary plots ($\gamma \gg 0$) for all possible ways of nesting energy with capital and labor. Two out of the three possible nestings suggest the importance of including energy for Japan and Tanzania. For Iran, the crosshairs indicate that energy may explain some growth, but this is highly uncertain as the resample data points have a very large vertical spread. Explaining economic growth in the United States, China, and Zambia will benefit relatively less by including energy, but the vertical spreads indicate some uncertainty in the cases of the United States and Zambia. The results for both the United Kingdom and South Africa exhibit significant uncertainty and are contradictory for the various nests.

Based on this analysis of the empirical data, we find no grounds to argue for augmenting the production function with energy for all economies. Our results support Stern's (2011) conclusion that energy may or may not be required in production functions, depending on its abundance or scarcity. One cannot know *a-priori* the economies for which augmenting by energy will improve goodness of fit. Statistical testing is clearly warranted.

5.2. *Are the World Bank country categories helpful for understanding the role of energy in economic growth?*

Some developed economies (United States) benefit little from including energy as a factor of production, while some others (United Kingdom) benefit in a minor way. For other developed economies (Japan), the results of Cobb-Douglas and CES models point in opposite directions. For NICs, the results are mixed: China does not convincingly benefit from energy-augmented production functions, and results are uncertain for South Africa due to lack of precision in determining γ. In one OPEC country (Saudi Arabia) adding energy clearly helps to explain economic growth, while in another (Iran), the benefit of energy augmentation is small and uncertain. For LDCs there is a either a strong indication that energy-augmented production functions are beneficial as an explanation for growth (Tanzania) or strong indication otherwise (Zambia).

In summary, we observe that World Bank country categories (Table 1) do not predict whether energy augmentation will be beneficial. In fact, there is as much variation of the role of energy in economic growth within World Bank country categories as between World Bank country categories. Thus, we conclude that the World Banks country categories are not helpful in explaining the role of energy in economic growth. Rather, a nuanced approach is necessary wherein the energy situation in each economy is assessed individually.

5.3. *Does augmentation by energy reduce the Solow residual?*

The so-called Solow residual (λ in Equations 1–6) accounts for slowdowns, speed-ups, improvements in the education of the labor force, and all sorts of things [that] appear as “technical change.” (Solow, 1957) The time-dependent term is often associated with “technological progress,” but that is a very narrow reading of Solow’s original intent:

> It will be seen that I am using the phrase “technical change” as a short-hand expression for any kind of shift in the production function. Thus slowdowns, speed-ups, improvements in the education of the labor force, and all sorts of things will appear as “technical change.” (Solow, 1957)

It has been shown elsewhere (Ayres and Warr, 2010) that consideration of energy (specifically, useful work) in the production function eliminates the need for the Solow residual (λ) over the very long run (e.g., 1900–2000).

The purpose of this paper, however, is to assess common production functions *as used in practice*, so we include λ terms in all fitted production functions. The present approach allows us to answer the question of whether augmentation by primary energy in the production function reduces the *value of* the Solow residual, as opposed to eliminating the *need for* the Solow residual altogether (Ayres and Warr's question). After augmenting by energy, the Solow-residual (λ) in the Cobb-Douglas (Figure **??**) and CES models (Figure **??**) still explains a part of economic growth for several economies, especially China, but also for the United Kingdom, Japan, and Tanzania, and to a lesser extent for the United States and Iran. In South Africa and Zambia, the Solow-residual is zero, while for Saudi Arabia it is negative.[10] For the Chinese economy, energy augmentation significantly reduces the uncertainty of the value of the Solow residual (Figure **??**). Furthermore, no economies show reduced magnitude of the Solow residual from left to right in Figures **??** and **??**.

Thus, we conclude that energy augmentation of the production function by primary thermal energy does not decrease the value of the Solow residual (λ) over the years studied (1980–2011 and 1991–2011). Longer-term analyses involving other energy quantifications (exergy or useful work) are warranted.

5.4. What are the implications of parameter uncertainty?

In addition to the usual parameter estimation process by linear coefficient determination (Cobb-Douglas and LINEX) and non-linear optimization algorithms (CES), the resampling analysis performed herein provides insight into the questions of (a) how accurately parameters are known and (b) the implications of parameter imprecision.

A significant result to emerge from this work is that large parameter uncertainty does not necessarily lead to poor model fit to historical economic output. To choose one example of many, the value of γ for the Cobb-Douglas model for Tanzania (0.59) shows a resampling range of 0.2–0.9. (See Figure **??**.) Yet, Cobb-Douglas models for Tanzania, with and without energy, both (a) fit historical data very well (white line compared to black line in Tanzania row of Figure **??**) and (b) exhibit a very narrow range of resampled fits (thin gray band in Tanzania row of Figure **??**). This means that a good

[10] The negative Solow residual (λ) for Saudi Arabia is a consequence of the fact that all indexed factors of production (k, l, and e) are greater than economic output (y) for most of the period under study (1991–2011). See Figure **??**.

fit to historical data does not indicate that model parameters have small uncertainty. In fact, similar to Tanzania, many economies exhibit both large uncertainty in model parameters and very good fit to historical data. (See Figures **??** and **??** compared to Figure **??** for the Cobb-Douglas models and Figure **??**–**??** compared to Figure **??** for CES models.) This result is consistent across almost all economies and models. Thus, we conclude that great care must be taken when interpreting estimated parameters: it is very easy to claim more certainty than is warranted. Where large uncertainty is observed, additional work is needed to contextualize the results, for example, sector-based analysis or ****. **** Martin, add more here? ****

**** Not sure whether contextualization will work. Could be read that macroeconomic growth modeling in such cases is not really useful at all? –MdW ****

The implications of parameter uncertainties should be obvious, but we will state them here. Policy based upon parameters with high uncertainty can lead to bad, or completely wrong, policy. **** Martin: can you find an example in our results that drives this point home? –MKH May be barking up wrong production factor, may assume subs when complementarity: ****

5.5. Is R^2 a good criterion for choosing a model?

We caution against using R^2 as the only criterion for choosing a model, because, in our experience, both Cobb-Douglas and CES models yield high R^2 values for all countries (see Figures **??** and **??**), yet very different uncertainties for fitted parameters. In some cases, LINEX models fare poorly regarding R^2 and might be excluded on that basis. (See South Africa and Tanzania in Figure **??**.) In our opinion, it is equally, if not more, important to know the uncertainty of estimated parameters than to assess R^2. With rigorous uncertainty testing as we have done here, the quality of parameter estimates is exposed: at a glance, the tightness of resampled results indicates whether or not confidence in the parameters or model is warranted. We conclude that it is insufficient to argue in favor of a particular production function on the grounds of R^2 alone.

5.6. Does the CES model structure (nesting) matter?

The subsections below assess whether the choice of CES nesting structure influences model results for both relative importance of factors of production (δ and δ_1 values) and substitutability among factors of production (σ and σ_1 values).

5.6.1. Relative importance of factors of production (δ and δ_1 values)

For most economies (United Kingdom, Japan, China, South Africa, Saudi Arabia, Iran, and Tanzania), Figure **??** shows that the relative importance of production factors capital (k), labor (l), and energy (e) is different for the three different CES nestings (Equations 4–6). In other words, you cannot view two of the ternary plots in the rightmost three columns of Figure **??** and predict the third. Nesting structure matters little for the economies of the United States and Zambia.

Certain combinations of economy, parameter values, and CES nesting structure come with very large resampling spreads, indicating large fitted parameter uncertainties. For example, for Iran's $(k{+}l){+}(e)$ nesting in Figures **??**–**??**, there is virtually no useful information in δ, δ_1, α, β, and γ.

Thus, we find that nesting has significant influence on model results.

5.6.2. Elasticity of substitution among factors of production (σ and σ_1 values)

The interpretation of elasticities of substitution among factors of production (k, l, and e) is challenging, because a version of each elasticity of substitution appears three or four times among the various nests. For example, an elasticity of substitution between capital (k) and labor (l) appears in the following nests: $(\underline{k}{+}\underline{l})$, $(\underline{k}{+}\underline{l}){+}(e)$, $(\underline{l}{+}e){+}\underline{k}$, and $(e{+}\underline{k}){+}\underline{l}$. Tables 3–11 provide interpretation of Figures **??** and **??**. In the tables, C (c) indicates strong (weak) complementarity, S (s) indicates strong (weak) substitutability, and $\sim\sim\sim$ ($\sim$) indicates large (small) uncertainty.

**** If we stick with these tables, do we want to use $\sim\sim\sim$ to indicate uncertainty? –MKH ****

**** Need to think this presentation through, want to see how balances with the rest of the paper; do we have the space seems to be the limiting factor? –MdW ****

**** I'm fine if we want to change to another form of these tables. But, I found the presentation below to be a good way to think about the results. My idea was to say "hang on! We have several ways to get at some of these elasticities. Let's compare all the different ways to look at the elasticity between any two factors of production. Do they agree? Are they in conflict?" Another point to make is that these comparisons need to be done honestly. We should not claim anything when the data are uncertain. –MKH ****

**** Regarding Martin's question about space, I've seen other papers in *Energy Economics* that are 51 printed journal pages. (Saunders, 2008) I expect we are fine on length. –MKH ****

Table 3: Elasticities of substitution (US).

	k & l		k & e		l & e	
σ_1	$(\underline{k}+\underline{l})+()$	C				
	$(\underline{k}+\underline{l})+(e)$	C	$(\underline{e}+\underline{k})+(l)$	$\sim\sim\sim$	$(\underline{l}+\underline{e})+(k)$	$\sim\sim\sim$
			$(\underline{k}+l)+(\underline{e})$	$\sim\sim\sim$	$(k+\underline{l})+(\underline{e})$	$\sim\sim\sim$
σ	$(\underline{l}+e)+(\underline{k})$	C	$(l+\underline{e})+(\underline{k})$	C		
	$(e+\underline{k})+(\underline{l})$	C			$(\underline{e}+k)+(\underline{l})$	C
	C		C		C	

Table 4: Elasticities of substitution (UK).

	k & l		k & e		l & e	
σ_1	$(\underline{k}+\underline{l})+()$	C				
	$(\underline{k}+\underline{l})+(e)$	$\sim\sim\sim$	$(\underline{e}+\underline{k})+(l)$	C	$(\underline{l}+\underline{e})+(k)$	C
			$(\underline{k}+l)+(\underline{e})$	C	$(k+\underline{l})+(\underline{e})$	C
σ	$(\underline{l}+e)+(\underline{k})$	C	$(l+\underline{e})+(\underline{k})$	C		
	$(e+\underline{k})+(\underline{l})$	$\sim\sim\sim$			$(\underline{e}+k)+(\underline{l})$	$\sim\sim\sim$
	C		C		C	

Table 5: Elasticities of substitution (JP).

	k & l		k & e		l & e	
σ_1	$(\underline{k}+\underline{l})+()$	c				
	$(\underline{k}+\underline{l})+(e)$	C	$(\underline{e}+\underline{k})+(l)$	∼∼∼	$(\underline{l}+\underline{e})+(k)$	C
σ			$(\underline{k}+l)+(\underline{e})$	C	$(k+\underline{l})+(\underline{e})$	C
	$(\underline{l}+e)+(\underline{k})$	C	$(l+\underline{e})+(\underline{k})$	C		
	$(e+\underline{k})+(\underline{l})$	C			$(\underline{e}+k)+(\underline{l})$	C
	C		C		C	

Table 6: Elasticities of substitution (CN).

	k & l		k & e		l & e	
σ_1	$(\underline{k}+\underline{l})+()$	∼∼∼				
	$(\underline{k}+\underline{l})+(e)$	C	$(\underline{e}+\underline{k})+(l)$	C	$(\underline{l}+\underline{e})+(k)$	S
σ			$(\underline{k}+l)+(\underline{e})$	S[11]	$(k+\underline{l})+(\underline{e})$	S
	$(\underline{l}+e)+(\underline{k})$	C	$(l+\underline{e})+(\underline{k})$	C		
	$(e+\underline{k})+(\underline{l})$	c			$(\underline{e}+k)+(\underline{l})$	c[12]
	C		C		S	

Table 7: Elasticities of substitution (ZA).

	k & l		k & e		l & e	
σ_1	$(\underline{k}+\underline{l})+()$	c$\sim$				
	$(\underline{k}+\underline{l})+(e)$	$\sim\sim\sim$	$(\underline{e}+\underline{k})+(l)$	C	$(\underline{l}+\underline{e})+(k)$	$\sim\sim\sim$
σ			$(\underline{k}+l)+(\underline{e})$	C	$(k+\underline{l})+(\underline{e})$	C
	$(\underline{l}+e)+(\underline{k})$	C	$(l+\underline{e})+(\underline{k})$	C		
	$(e+\underline{k})+(\underline{l})$	$\sim\sim\sim$			$(\underline{e}+k)+(\underline{l})$	C[13]
	c		C		C	

Table 8: Elasticities of substitution (SA).

	k & l		k & e		l & e	
σ_1	$(\underline{k}+\underline{l})+()$	$\sim\sim\sim$				
	$(\underline{k}+\underline{l})+(e)$	$\sim\sim\sim$	$(\underline{e}+\underline{k})+(l)$	C	$(\underline{l}+\underline{e})+(k)$	C
σ			$(\underline{k}+l)+(\underline{e})$	C	$(k+\underline{l})+(\underline{e})$	C
	$(\underline{l}+e)+(\underline{k})$	$\sim\sim\sim$	$(l+\underline{e})+(\underline{k})$	$\sim\sim\sim$		
	$(e+\underline{k})+(\underline{l})$	$\sim\sim\sim$			$(\underline{e}+k)+(\underline{l})$	$\sim\sim\sim$
	$\sim\sim\sim$		C		C	

For many estimates of elasticity of substitution (σ and σ_1), the resampling technique indicates high uncertainty ($\sim\sim\sim$). Where results are not uncertain, we find complementarity (C and c) among factors of production for most economies with the exception of China, where an indication of substitutability between labor and energy is observed.

That so many estimates of elasticity of substitution exhibit large uncertainty, often spanning the range from perfect complements (σ, $\sigma_1 = 0$) to

[11] Driven by le?
[12] Driven by kl?
[13] Points on right are all same.

Table 9: Elasticities of substitution (IR).

	k & l		k & e		l & e	
σ_1	$(\underline{k}+\underline{l})+()$	$\sim\sim\sim$				
	$(\underline{k}+\underline{l})+(e)$	$\sim\sim\sim$	$(\underline{e}+\underline{k})+(l)$	$\sim\sim\sim$	$(\underline{l}+\underline{e})+(k)$	$\sim\sim\sim$
σ			$(\underline{k}+l)+(\underline{e})$	$\sim\sim\sim$	$(k+\underline{l})+(\underline{e})$	$\sim\sim\sim$
	$(\underline{l}+e)+(\underline{k})$	$\sim\sim\sim$	$(l+\underline{e})+(\underline{k})$	$\sim\sim\sim$		
	$(e+\underline{k})+(\underline{l})$	C			$(\underline{e}+k)+(\underline{l})$	C
	$\sim\sim\sim$		$\sim\sim\sim$		$\sim\sim\sim$	

Table 10: Elasticities of substitution (TZ).

	k & l		k & e		l & e	
σ_1	$(\underline{k}+\underline{l})+()$	C				
	$(\underline{k}+\underline{l})+(e)$	C	$(\underline{e}+\underline{k})+(l)$	$\sim\sim\sim$	$(\underline{l}+\underline{e})+(k)$	C
σ			$(\underline{k}+l)+(\underline{e})$	$\sim\sim\sim$	$(k+\underline{l})+(\underline{e})$	$\sim\sim\sim$
	$(\underline{l}+e)+(\underline{k})$	C	$(l+\underline{e})+(\underline{k})$	C		
	$(e+\underline{k})+(\underline{l})$	C			$(\underline{e}+k)+(\underline{l})$	C
	C		$\sim\sim\sim$		C	

Table 11: Elasticities of substitution (ZM).

	k & l		k & e		l & e	
σ_1	$(\underline{k}+\underline{l})+()$	$\sim\sim\sim$				
	$(\underline{k}+\underline{l})+(e)$	$\sim\sim\sim$	$(\underline{e}+\underline{k})+(l)$	$\sim\sim\sim$	$(\underline{l}+\underline{e})+(k)$	$\sim\sim\sim$
σ			$(\underline{k}+l)+(\underline{e})$	$\sim\sim\sim$	$(k+\underline{l})+(\underline{e})$	$\sim\sim\sim$
	$(\underline{l}+e)+(\underline{k})$	$\sim\sim\sim$	$(l+\underline{e})+(\underline{k})$	$\sim\sim\sim$		
	$(e+\underline{k})+(\underline{l})$	$\sim\sim\sim$			$(\underline{e}+k)+(\underline{l})$	$\sim\sim\sim$
	$\sim\sim\sim$		$\sim\sim\sim$		$\sim\sim\sim$	

perfect substitutes (σ, $\sigma_1 = \infty$), is a troubling characteristic of the estimated CES coefficients that is rarely investigated in the literature. Indeed, for some economies (Iran and Zambia), estimating any elasticity of substitution at all is problematic and fraught with peril. Earlier, we noted that even when residuals for the fit to historical data are small and the model produces fitted values that track the historical data closely, it may yet be difficult to estimate some or all of the model parameters precisely. That appears to be the case for values of ρ and ρ_1 and associated elasticities of substitution (σ and σ_1) in the CES production function. Note that this result is unlikely to be caused by poor-quality data alone. Elasticities of substitution for the economies with presumably the best data in the world (United States, United Kingdom, and Japan) exhibit large uncertainty.

Some of the uncertainty in parameter estimates is explained by the mathematical structure of the CES equations. Table 12 summarizes these observations. Numbers in parentheses below correspond to row numbers in the table. (1, 2) Equations 3–6 indicate that if $\delta_1 = 0$ or $\delta_1 = 1$, ρ_1 cancels. Consequently, *sse* (Equation 13) is insensitive to ρ_1 (and, therefore, σ_1). As $\delta_1 \to 0$ or $\delta_1 \to 1$, the precision with which σ_1 can be estimated declines. (3) If $\delta = 0$, δ_1 and ρ_1 are eliminated. Furthermore, with $\delta = 0$, ρ cancels and *sse* becomes insensitive to ρ (σ). Thus, as $\delta \to 0$, the precision with which δ_1, σ_1, and σ can be estimated declines. (4) If $\delta = 1$, ρ cancels and *sse* becomes insensitive to values of σ. Given that both (a) the `cesEst` fitting algorithm still uses a meaningless value for ρ when $\delta \to 1$ and (b) the ratio ρ/ρ_1 appears in the CES equation, the value of ρ_1 will also become imprecise as $\delta \to 1$ due to the large uncertainty of ρ. Essentially, ρ_1 compensates for imprecise values of ρ on each successive resample, thereby taking on ρ's uncertainty. Thus, as $\delta \to 1$, the precision with which both σ and σ_1 can be estimated declines. (5) If $\rho_1 = \infty$, $\sigma_1 = 0$ and the CES function (Equation 3) simplifies to the Leontief function (Csontos and Ray, 1992):

$$y = \gamma A \min\left[k,\, l\right] \,, \tag{21}$$

and δ_1 is eliminated. Thus, as $\sigma_1 \to 0$, the precision with which we can estimate δ_1 declines. (6) If $\rho = \infty$, $\sigma = 0$. Using Equation 4 as an example, we find that

$$y = \gamma A \min\left[\delta_1 k^{-\rho_1} + (1-\delta_1) l^{-\rho_1},\, e\right] \,, \tag{22}$$

and δ is eliminated (Csontos and Ray, 1992). Thus, as $\sigma \to 0$, the precision with which δ can be estimated declines.

**** From the above discussion, it is clear that when $\min sse$ occurs at any boundary (with the exception of σ, $\sigma_1 = 0$, ρ, $\rho_1 = -1$), it will be difficult to estimate many CES parameters with precision. It is concerning that so many of our CES results end up on boundaries. I don't think we know why we hit the boundaries so frequently. I think it's real, i.e., lowest *sse* occurs at the boundary. But, we can't know for sure, because the `cesEst` function is a black box to us. One thing in our favor is that we do a thorough grid search in ρ and ρ_1 first. Presumably, the grid search finds any potential *sse* valleys before starting the full gradient search. This is probably an issue to be discussed with Randy. –MKH ****

**** We do a thorough job of fitting along the boundaries for the Cobb-Douglas function. Perhaps we should do something similar for the CES model? I.e., we could fit the CES model unconstrained. We could set up a list of models to try along each boundary. Similar to Cobb-Douglas, if an unconstrained CES point estimate lands beyond the boundary, we could simply choose the lowest *sse* anywhere on the boundary as the winner. –MKH ****

**** This raises another question, namely whether we should be reporting coefficients that are, by definition unknowable. In the worst case, when $\delta = 0$ (as it is for the fit to historical data for US; (le)k nesting, Equation 5; in Figure **??**), CES simplifies to $y = \gamma A l$. All of σ, δ, σ_1, and δ_1 are eliminated from the model and are, therefore, unknowable. At present, we report the values of uknowable coefficients on the resample graphs. One approach would be to decide *not* to report unknowable values. But, there are a few resample

Table 12: Implications of CES model structure on parameter precision.

	Mathematical effect		Affect on precision	
1	$\delta_1 = 0$	ρ_1 cancels	$\delta_1 \to 0$	σ_1 imprecise
2	$\delta_1 = 1$	ρ_1 cancels	$\delta_1 \to 1$	σ_1 imprecise
3	$\delta = 0$	δ_1 eliminated ρ_1 eliminated ρ cancels	$\delta \to 0$	δ_1 imprecise σ_1 imprecise σ imprecise
4	$\delta = 1$	ρ cancels	$\delta \to 1$	σ imprecise σ_1 imprecise
5	$\rho_1 = \infty$	$\sigma_1 = 0$ δ_1 eliminated	$\sigma_1 \to 0$	δ_1 imprecise
6	$\rho = \infty$	$\sigma = 0$ δ eliminated	$\sigma \to 0$	δ imprecise

models that are rather close to but not exactly at $\delta = 1$ in Figure **??**. Should we report parameters for the resample models where $\delta \neq 1$? –MKH ****

Some of the uncertainty in parameter estimates may be explained by structural change in the economies. For example, **** Martin: can you select one of the highly uncertain economies and discuss it? Perhaps Iran or Zambia? ****. To account for the large uncertainty, it may be useful to perform piecewise analyses of smaller time periods. For the present study, doing piecewise analysis of, say, Iran **** use the economy that Martin discusses above **** would mean that the CES model is estimated on two 10-year periods. Considering that there are six fitted parameters in the energy-augmented CES model (γ, λ, δ, δ_1, ρ, and ρ_1 in Equations 4–6), ten years of historical data leaves precious few (only 4) degrees of freedom for the fitting process.

Based on the results and discussions above regarding (a) the relative importance of factors of production and (b) the elasticity of substitution among factors of production, we conclude that use of the CES model for energy analysis (Equations 4–6) should be undertaken with utmost caution. Our results support Lecca et al.'s (2011) conclusion for the need to econometrically specify production function relationships and to perform sensitivity analysis on nesting choices in economies to enhance the reliability of results. **** Would econometrically specifying production function relationships and performing sensitivity analysis really "enhance the reliability" of results? Or, would doing so improve transparency? –MKH **** Indeed, rigorous statistical testing on the relative importance of production factors and elasticities of substitution is strongly recommended.

5.7. Which production function is best for energy augmentation?

Several criteria could be applied to the question of which production function is best for describing the role of energy in economic growth. We consider four criteria here: best match to historical data, minimum ambiguity in model structure, minimum uncertainty in model parameters, and robust parameter estimation process. In the sections below, we evaluate the Cobb-Douglas, CES, and LINEX models on these criteria.

5.7.1. Best match to historical data

Figures **??**, **??**, and **??** indicate that the CES production function is consistently better than Cobb-Douglas or LINEX in terms of matching historical data. Cobb-Douglas is a close second, but LINEX struggles, especially with the Saudi Arabian and Tanzanian economies.[14,15]

Thus, CES is preferred on this criterion.

5.7.2. Minimum ambiguity in model structure and interpretation

For each production function, values of fitted parameters give rise to interpretations of macroeconomic dynamics that, in turn, have significant implications for macroeconomic policy and human well-being. Thus, it is desirable that such interpretations have little ambiguity.

The energy-augmented CES production function presents significant ambiguity to the energy analyst in the form of the choice of factor nesting. We

[14] The order of these results is unsurprising: energy-augmented CES contains 6 fitted parameters; Cobb-Douglas, 4; and LINEX, 3. We expect models with more parameters to provide a better fit to historical data.

[15] We also note that in contrast to our approach of fitting constant a_0 and c_t values, which is shared by Warr and Ayres (2012), Kümmel (2011) used logistic functions to generate time-varying a_0 and c_t values. Kümmel's approach will necessarily produce a better match to historical data, because the logistic curves involve several additional fitting parameters.

showed above that choice of nesting can significantly affect the interpretation of macroecnomic dynamics, especially the relative importance of factors of production (see Section 5.6.1 and Figure **??**) and the understanding of substitutibility among factors of production (see Section 5.6.2). Thus, the CES production function has the undesireable characteristic that model structure (i.e., nesting) affects both parameter values and macroeconomic interpretation. Neither the Cobb-Douglas nor the LINEX production function presents similar structural ambiguity.

Thus, Cobb-Douglas and LINEX are preferred on this criterion.

5.7.3. Minimum uncertainty in model parameters

Uncertainties in fitted parameters give rise to uncertainty in macroeconomic dynamics inferred from fitted parameters. Thus, small uncertainty in fitted parameters is desirable. Figures **??**, **??**, **??**–**??**, and **??** show that Cobb-Douglas and LINEX have, by and large, smaller uncertainties in fitted parameters than CES. Thus, Cobb-Douglas and LINEX are preferred on this criterion.

5.7.4. Robust parameter estimation process

It is desirable that the combination of production function and parameter estimation process has the following characteristics: affords easy mathematical and numerical implementation, provides an easy way to respect constraints on fitted parameters, and allows asymmetric estimates of fitted parameter uncertainty.

Easy mathematical and numerical implementation. For the Cobb-Douglas and LINEX models, logarithmic transformation of the production functions allows simple linear estimation of model parameters. In contrast, it is impossible to fit the CES model linearly, so parameters must be estimated by a more-complicated and time-consuming optimization process. (See Section 4.4.) On the `dahl` supercomputer at Calvin College (Adams, 2008), complete analysis of all data shown in this paper (including 1000 resamples for every model, every economy, and every CES nesting) takes about 15 hours. Approximately 98% of the processing effort is spent on the CES models.[16] Thus, Cobb-Douglas

[16] All Cobb-Douglas and LINEX models run on a single node (using 4 processors per node) in about 1.5 hours (6 processor-hours). All CES models run on 9 nodes (using 3 processors per node for a total of 27 processors) in about 15 hours (405 processor-hours).

and LINEX are preferred on this criterion.

Easy to respect parameter constraints. It is relatively easy to implement constraints on fitting parameter for the Cobb-Douglas production function. (See Section 4.3.) For the CES prodcution function, the capability to respect parameter constraints is a feature of the `cesEst` function (Henningsen and Henningsen, 2011) and is, therefore, also relatively easy to implement. In contrast, because output elasticities α, β, and γ are not directly available as parameters in the LINEX model (see Equations 7 and 20), it is very difficult to enforce constraints during the fitting process.[17] Indeed, we found (Section 4.5) that the LINEX model regularly violates parameter constraints.

Thus, Cobb-Douglas and CES are preferred on this criterion.

Easy asymmetric uncertainties. As discussed in Section 5.4, constraints on fitting parameters make it desirable that the combination of production function and the parameter estimation process allows estimation of asymmetric parameter uncertainties. In the present work, we found that the resampling techniques described in Section 4.2 afford estimates of asymmetric parameter uncertainties. Thus, none of the prodcution functions are preferred over any other on this criterion.

5.8. Summary of discussion

The evidence above indicates that there is no quick or easy answer to the question "which production function is best for augmentation by energy?" We find that the answer depends upon the objective of the analysis. In our opinion, the Cobb-Douglas models provides a good compromise among the competing criteria of ease of parameter estimation, good fit to historical data, and small uncertainties in fitted parameters.

If precise fitting to historical data is paramount and uncertainty in fitted coefficients, especially elasticities of substitution, is not an issue, the CES model is the best.

If one needs to represent rebound effects (not just backfire), CES *must* be employed, because the CES function was found by Saunders (2008) to be the only production function studied here that is capable of representing rebound in addition to backfire. However, the ability of CES to represent

[17] See Kümmel (2011) for an approach to respecting parameter constraints for the LINEX function.

rebound effects is tied to the presence of elasticities of substitution (σ and σ_1) in the CES model. Unfortunately, the resampling analyses show that estimated values of elasticity of substitution are often highly uncertain. Thus, whether the CES function is helpful for estimating rebound in addition to backfire effects remains an open question.

The LINEX model is intriguing because of its thermodynamic roots, but our implementation suffers from poor reproduction of historical data and difficulty in respecting parameter constraints. If the LINEX function is to be used for energy analysis, we recommend that either Kümmel's (2011) fitting approach (energy quantified by primary thermal energy and logistic curves for a_0 and c_t) or Ayres and Warr's (2010) fitting approach (energy quantified by useful work and constant values for a_0 and c_t) be employed and that the results be thoroughly tested and compared against other production functions. Doing so is left as future work.

6. Conclusion

**** Conclusion goes here. ****

Review the issues.

Review our methods.

Review our significant results.

**** Maybe put the discussion of "Which production function is best" here? ****

We conclude that the best policy will be made with a macroeconomic modeling approach that is highly interpretative, uses several production functions, and estimates asymmetric uncertainties of fitted parameters.

7. Future Work

**** Revisit this section when we finish writing the paper. ****

In the future, we intend add an additional coordinate of analysis (Section 2): energy quantification. Specifically, we want to analyze exergy (X) and useful work (U) energy quantifications within the framework established in this paper. Doing so will provide an opportunity to engage the literature around useful work as a driver of economic growth.(Ayres et al., 2003; Ayres and Warr, 2005, 2010; Warr and Ayres, 2006; Warr et al., 2010; Warr and Ayres, 2010, 2012) And, we will be able to assess the interaction between economic models and energy quantification: are there combinations of economic models and energy quantifications that provide significantly better descriptions of growth? We intend to evaluate additional countries as data become available. And, we wish to perform analyses over longer time scales. In particular, we wish to investigate the effects of significant economic processes (such as industrialization) and political events (world wars) using the resampling techniques presented herein.

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Reproducible Research

In the spirit of Reproducible Research (Gandrud, 2013), all data, spreadsheets, `R` code (R Core Team, 2012), and other materials associated with this paper can be found at
`https://github.com/MatthewHeun/Econ-Growth-R-Analysis`.

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[18] Any opinion, findings, and conclusions or recommendations expressed herein are those of the authors, and, therefore, NRF does not accept any liability in regard thereto.

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