

# Empirical Analysis of the Role of Energy in Economic Growth

Caleb Reese<sup>a</sup>, Lucas Timmer<sup>a</sup>, Matthew Kuperus Heun<sup>a,\*</sup>

<sup>a</sup>*Engineering Department, Calvin College, Grand Rapids, MI 49546, USA*

---

## Abstract

\*\*\*\*\* Add abstract \*\*\*\*\*

*Keywords:* economic growth, energy, cobb-douglas, CES, LINEX

---

Caleb, put your LaTeX code here.

## 1. Cobb-Douglas Without Energy

```
Warning: Convergence failure: false convergence (8)
Nonlinear regression model
  model: iGDP ~ exp(lambda * iYear) * iCapStk^min(a, b) * iLabor^abs(b - a)
  data: dataTable
    lambda      a      b
0.00877 0.54436 0.88424
residual sum-of-squares: 0.262

Algorithm "port", convergence message: false convergence (8)
$isConv
[1] FALSE

$finIter
[1] 3
```

---

\*Corresponding author

*Email address:* mkh2@calvin.edu, tel: +1 (616) 526-6663, fax: +1 (616) 526-6501 (Matthew Kuperus Heun)

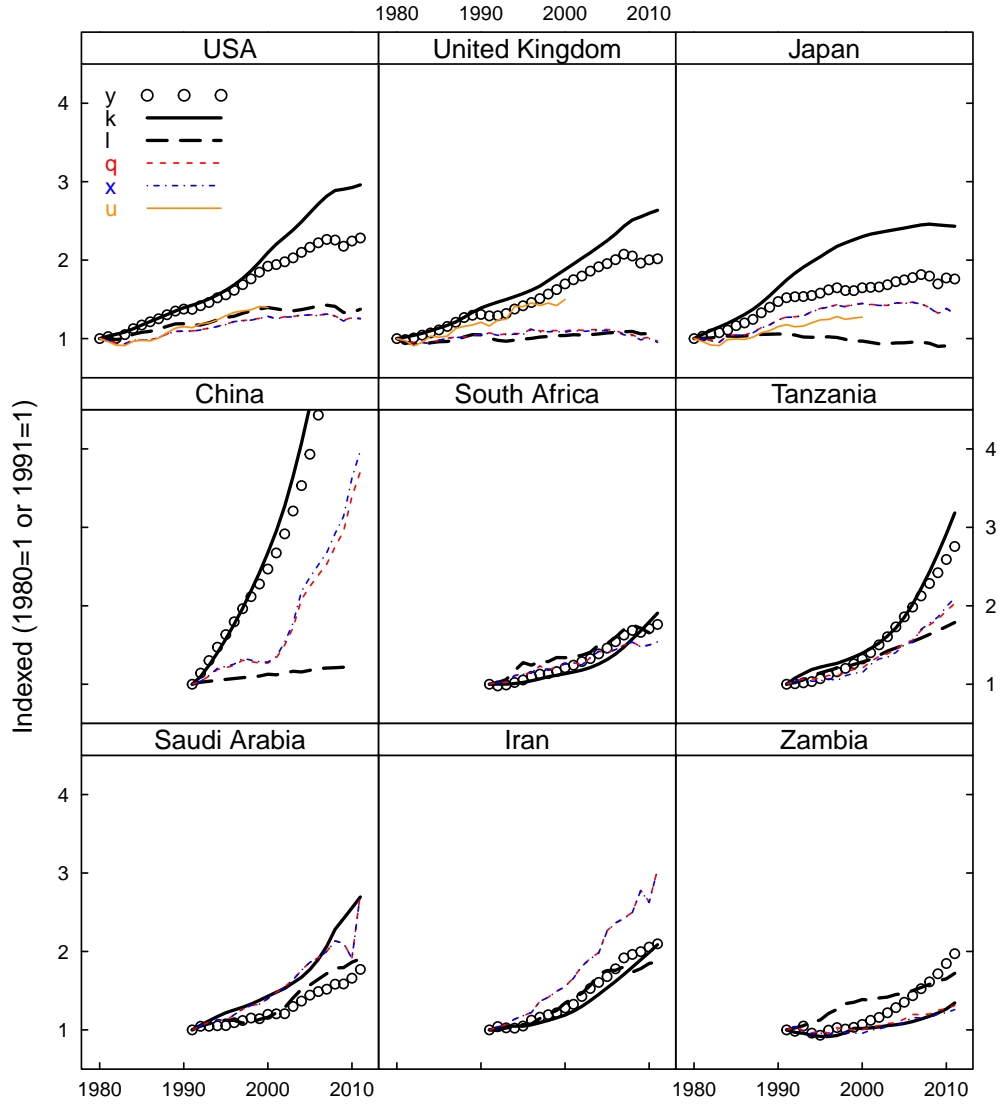


Figure 1: Indexed GDP ( $y$ ), capital stock ( $k$ ), labor ( $l$ ), thermal energy ( $q$ ), exergy ( $x$ ), and useful work ( $u$ ) for all economies. (China's indexed GDP and indexed capital stock rise to  $y = 7.3$  and  $k = 9.2$  in 2011.)

```

$finTol
[1] -1.9

$nEval
function gradient
      44      3

$stopCode
[1] 8

$stopMessage
[1] "false convergence (8)"

```

Table 1: Cobb-Douglas parameters for 1980-2011 (US, UK, JP) or 1991-2011 (others). (Parameter estimates beneath symbol. 95% confidence interval bounds to left and right.)

	$\lambda$			$\alpha$			$\beta$		
US	0.0087	0.0102	0.0116	0.21	0.27	0.34	0.66	0.73	0.79
UK	-0.0066	0.0097	0.0215		0.44		-0.13	0.56	1.24
JP	0.0021	0.0052	0.0082	0.44	0.52	0.59	0.41	0.48	0.56
CN	-0.0021	0.0188	0.0688	0.11	0.71		-0.32	0.29	0.89
ZA	-0.0007	0.0008	0.0022	0.46	0.60	0.73	0.26	0.40	0.54
SA	-0.0159	-0.0123	-0.0087	0.21	0.45	0.68	0.32	0.55	0.78
IR	0.0032	0.0039	0.0045	0.49	0.60	0.70	0.30	0.40	0.51
TZ	-0.0039	0.0015	0.0068	0.50	0.73	0.95	0.05	0.27	0.50
ZM	0.0154	0.0174	0.0191	0.93	1.00		-0.25	0.00	0.25

```

Waiting for profiling to be done...
Waiting for profiling to be done...
Waiting for profiling to be done...
Waiting for profiling to be done...
Waiting for profiling to be done...
Waiting for profiling to be done...
Waiting for profiling to be done...
Waiting for profiling to be done...

```

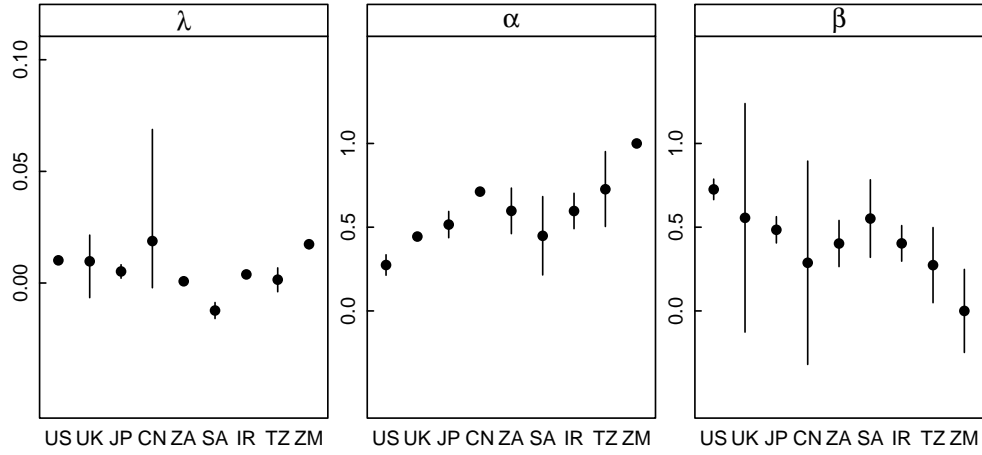


Figure 2: Cobb-Douglas (without energy) model parameters. Vertical bars indicate 95% confidence intervals.

*Waiting for profiling to be done...*

## 2. Cobb-Douglas With Energy

We can force  $\alpha$ ,  $\beta$ , and  $\gamma$  to be in  $[0, 1]$  by a reparameterization:

$$a \in [0, 1], b \in [0, 1], \alpha = \min(a, b), \beta = |b - a|, \gamma = 1 - \max(a, b)$$

2.1. Cobb-Douglas with  $Q$

2.2. Cobb-Douglas With  $X$

2.3. Cobb-Douglas With  $U$

## 3. CES

3.1. CES with  $Q$

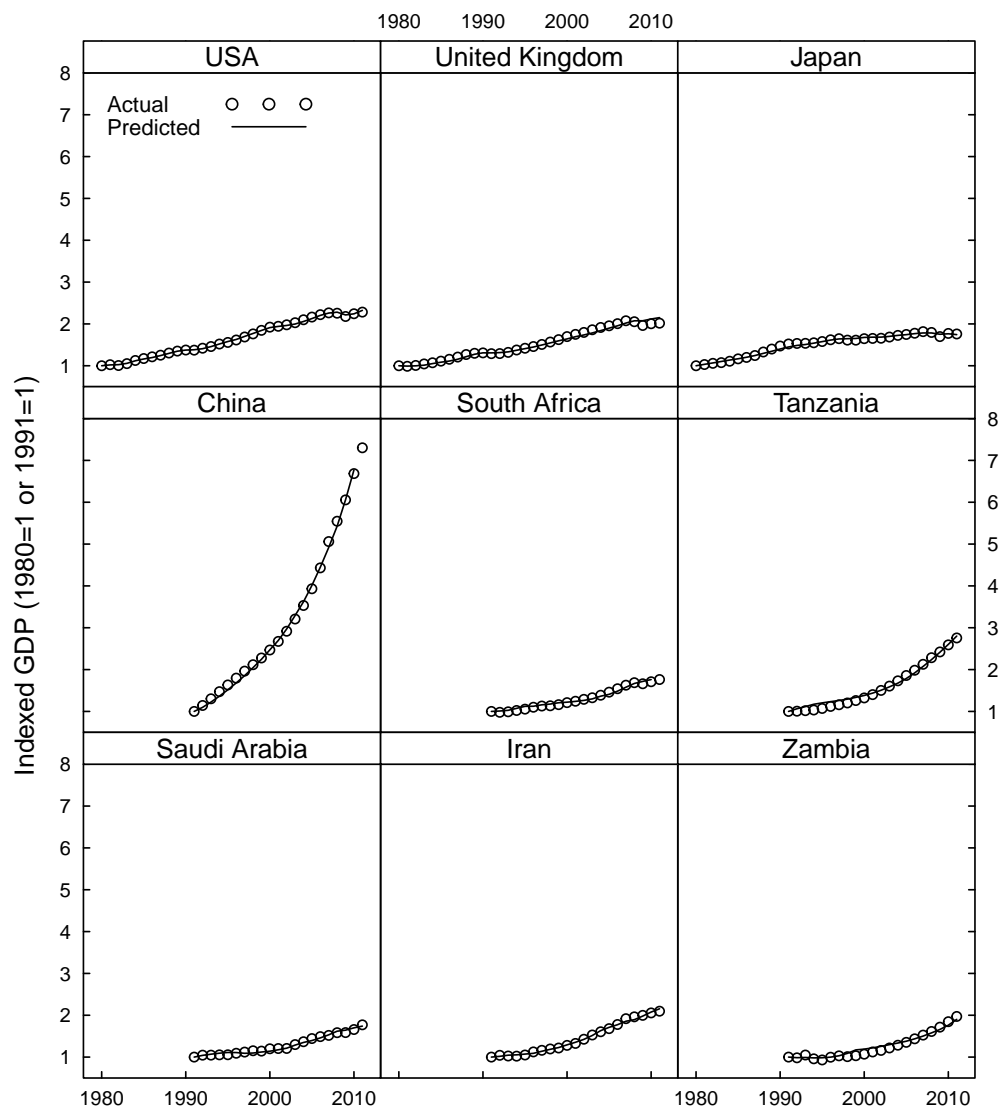


Figure 3: Cobb-Douglas (without energy) results.