Empirical Analysis of the Role of Energy in Economic Growth

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Abstract

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Keywords: economic growth, energy, cobb-douglas, CES, LINEX

Caleb, put your LaTeX code here.

Figure 1 shows the "facts" as they are known.

1. Cobb-Douglas Without Energy

The Cobb-Douglas model without energy is given by

$$y = a^{\lambda(t-t_0)} k^\alpha l^\beta. \tag{1}$$

Table 1 gives the parameters for the Cobb-Douglas model without energy.

Table 1: Cobb-Douglas (without energy) for 1980-2011 (US, UK, JP), 1991-2010 (CN and ZA), and 1991-2011 (SA, IR, TZ, and ZM). (Parameter estimates beneath symbol. 95% confidence interval bounds to left and right.)

	λ			α			β		
US	0.0087	0.0102	0.0116	0.21	0.27	0.34	0.66	0.73	0.79
UK	-0.0104	0.0097	0.0303	-0.25	0.44	1.12	-0.13	0.56	1.24
JP	0.0021	0.0052	0.0082	0.44	0.52	0.59	0.41	0.48	0.56
CN	-0.0405	0.0188	0.0779	0.11	0.71	1.32	-0.32	0.29	0.89
ZA	-0.0007	0.0008	0.0022	0.46	0.60	0.73	0.26	0.40	0.54
SA	-0.0159	-0.0123	-0.0087	0.21	0.45	0.68	0.32	0.55	0.78
IR	0.0032	0.0039	0.0045	0.49	0.60	0.70	0.30	0.40	0.51
TZ	-0.0039	0.0015	0.0068	0.50	0.73	0.95	0.05	0.27	0.50
ZM	0.0218	0.0249	0.0280	1.25	1.41	1.57	-0.57	-0.41	-0.25

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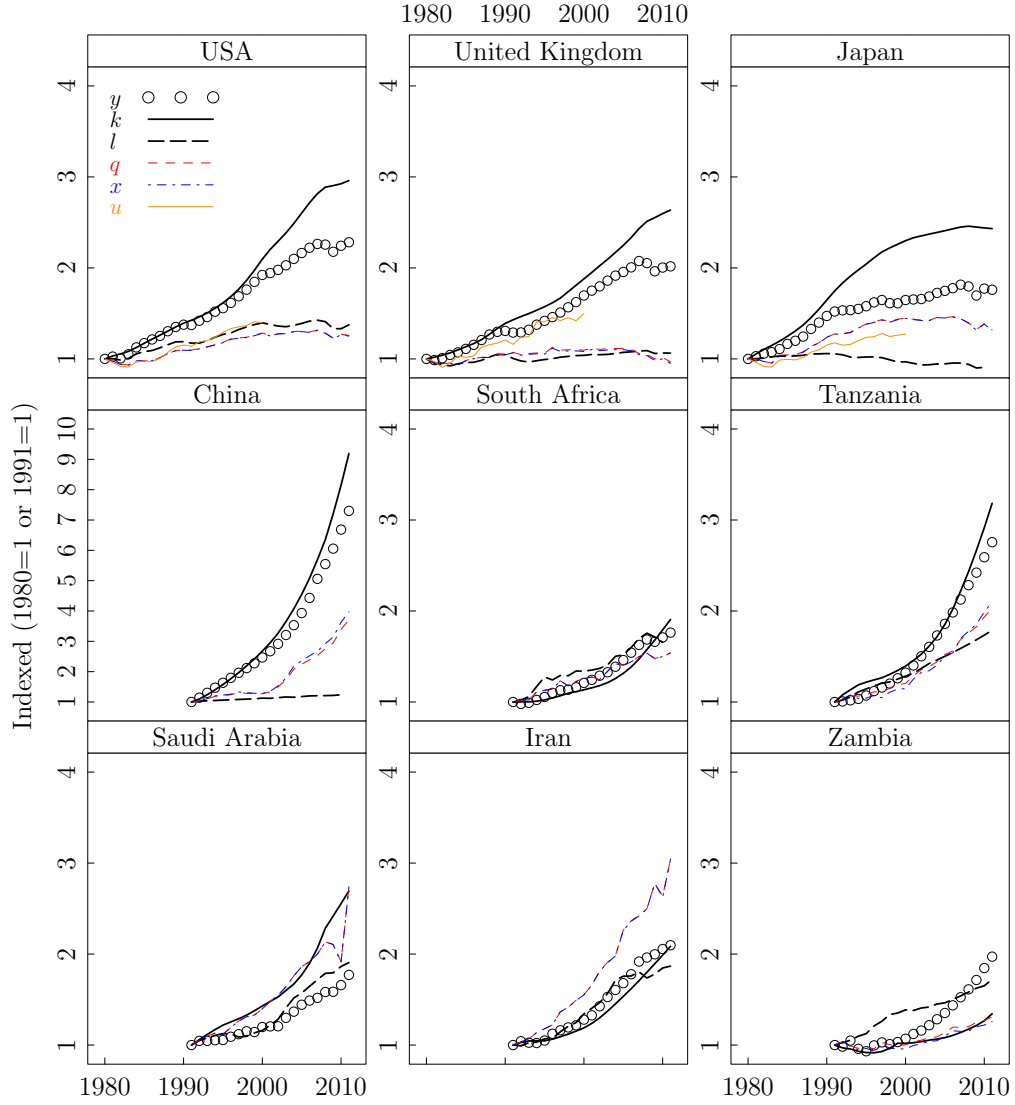
Figure 1: The facts. Indexed GDP (y), capital stock (k), labor (l), thermal energy (q), exergy (x), and useful work (u).
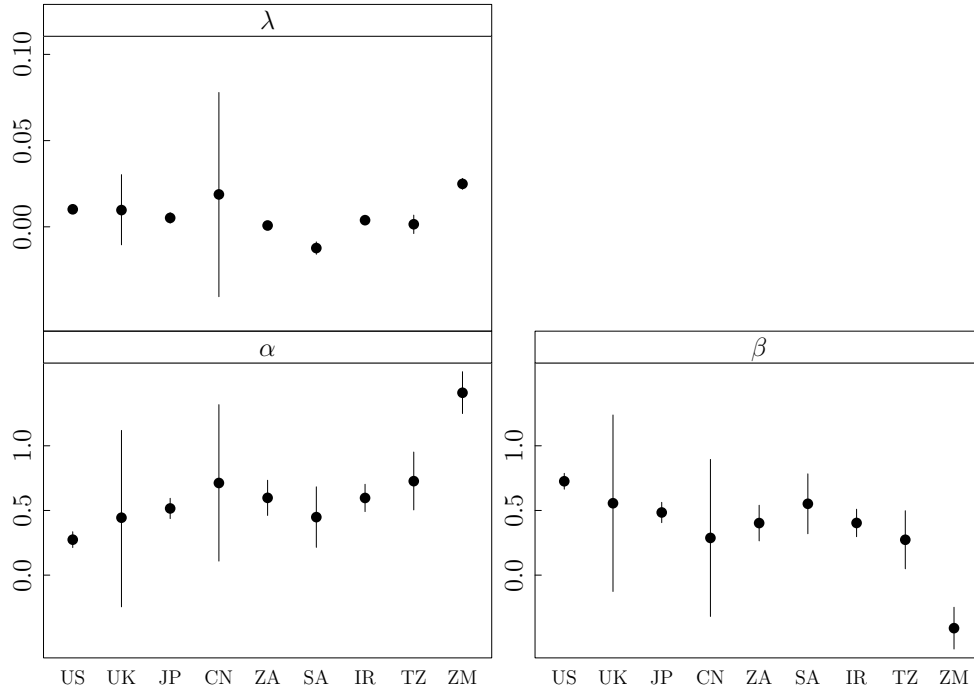
Figure 2: Cobb-Douglas (without energy) model parameters. Vertical bars indicate 95% confidence intervals.

Figure 2 shows values and 95% confidence intervals for the parameters for the Cobb-Douglas model (without energy).

2. Cobb-Douglas With Energy

We can force α, β, and γ to be in $[0, 1]$ by a reparameterization:

$$a \in [0, 1], b \in [0, 1], \alpha = \min(a, b), \beta = |b - a|, \gamma = 1 - \max(a, b)$$

The Cobb-Douglas model with energy is given by

$$y = a^{\lambda(t-t_0)} k^\alpha l^\beta e^\gamma, \tag{2}$$

where e can be any of thermal energy (q), exergy (x), or useful work (u).

2.1. Cobb-Douglas with q

The Cobb-Douglas (with thermal energy, q) parameters are given in Table 2.

Table 2: Cobb-Douglas (with q) for 1980-2011 (US, UK, JP), 1991-2010 (CN and ZA), and 1991-2011 (SA, IR, TZ, and ZM). (Parameter estimates beneath symbol. 95% confidence interval bounds to left and right.)

	λ			α			β			γ		
US	0.0078	0.0102	0.0126	0.19	0.27	0.36	0.59	0.72	0.85	-0.17	0.00	0.17
UK	0.0075	0.0228	0.0382	-0.52	-0.00	0.52	0.07	0.56	1.04	0.28	0.44	0.61
JP	0.0019	0.0049	0.0079	0.45	0.57	0.70	0.42	0.55	0.67	-0.31	-0.12	0.07
CN	-0.0087	0.0133	0.0872	-0.02	0.78	0.85	-0.48	0.24	0.96	-0.15	-0.02	0.11
ZA		0.0048	0.0054	0.35	0.43		-0.54	0.00	0.54	-0.02	0.57	1.17
SA	-0.0165	-0.0137	-0.0109	0.17	0.36	0.54	0.21	0.40	0.60	0.11	0.24	0.37
IR	-0.0026	0.0033	0.0092	0.43	0.59	0.74	0.18	0.39	0.59	-0.25	0.03	0.31
TZ	0.0044	0.0057	0.0095	0.31	0.44	0.71	-0.43	-0.00	0.43	0.12	0.56	1.00
ZM		0.0197	0.0208	0.54	0.66	0.78	-0.40	0.00	0.40	-0.74	0.34	1.42

Figure 3 shows values and 95% confidence intervals for the parameters for the Cobb-Douglas model (with q).

The Cobb-Douglas (with exergy) parameters are given in Table 3.

Figure 4 shows values and 95% confidence intervals for the parameters for the Cobb-Douglas model (with x).

Table 3: Cobb-Douglas (with x) for 1980-2011 (US, UK, JP), 1991-2010 (CN and ZA), and 1991-2011 (SA, IR, TZ, and ZM). (Parameter estimates beneath symbol. 95% confidence interval bounds to left and right.)

	λ			α			β			γ		
US	0.0079	0.0103	0.0127	0.19	0.27	0.35	0.59	0.72	0.85	-0.16	0.01	0.18
UK	0.0080	0.0232	0.0385	-0.52	-0.01	0.51	0.07	0.55	1.04	0.29	0.45	0.62
JP	0.0019	0.0049	0.0080	0.45	0.57	0.69	0.42	0.54	0.67	-0.30	-0.11	0.08
CN	-0.0078	0.0140	0.0869	-0.01	0.77	1.00	-0.47	0.25	0.96	-0.14	-0.01	0.11
ZA		0.0047	0.0054	0.36	0.44		-0.51	0.00	0.51	0.00	0.56	1.13
SA	-0.0164	-0.0136	-0.0108	0.17	0.36	0.54	0.21	0.41	0.60	0.11	0.23	0.36
IR	-0.0025	0.0033	0.0090	0.43	0.59	0.74	0.19	0.39	0.59	-0.25	0.03	0.30
TZ	0.0119	0.0173	0.0227	-0.39	-0.16	0.19	-0.14	0.01	0.15	0.81	1.15	1.50
ZM		0.0199	0.0209	0.58	0.69	0.79	-0.31	-0.00	0.31	-0.49	0.31	1.10

2.2. Cobb-Douglas Comparisons

Figure 5 compares predictions from the Cobb-Douglas models (without energy, with Q, and with x) to historical data.
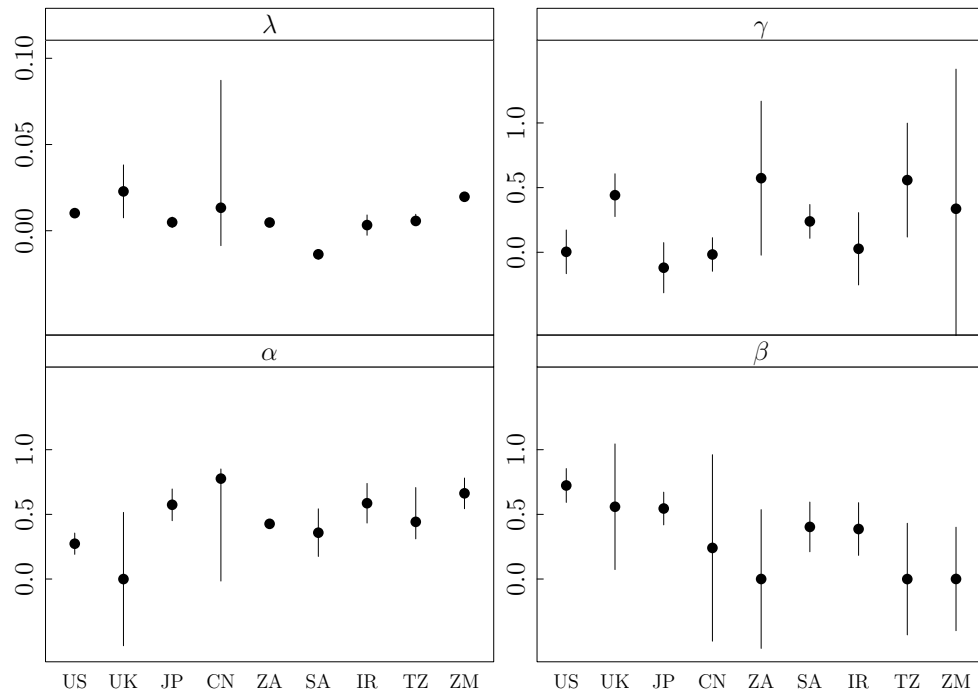
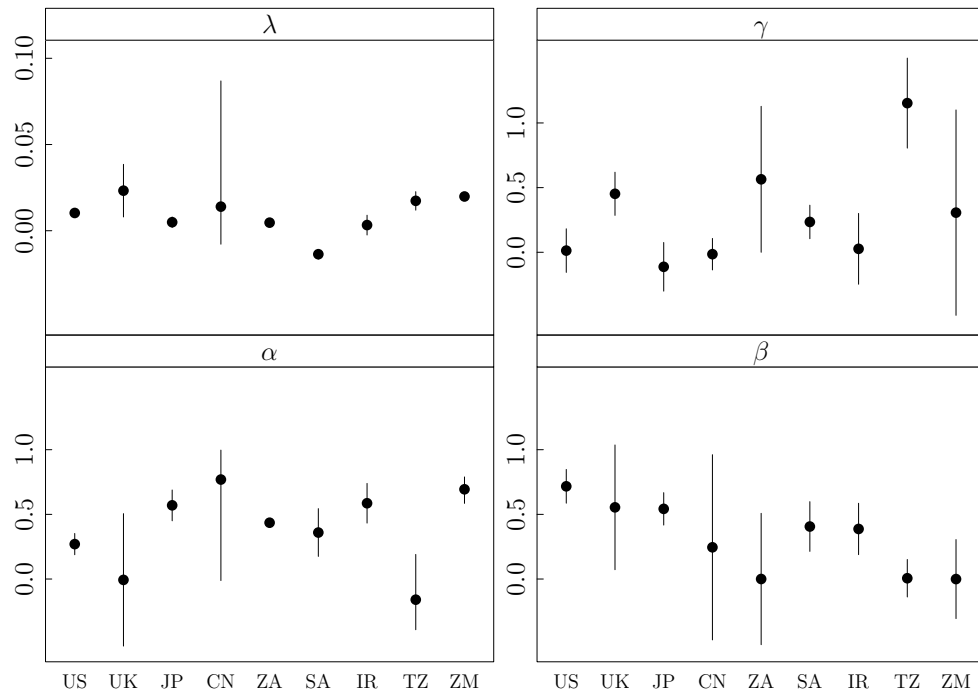Figure 3: Cobb-Douglas (with q) model parameters. Vertical bars indicate 95% confidence intervals.

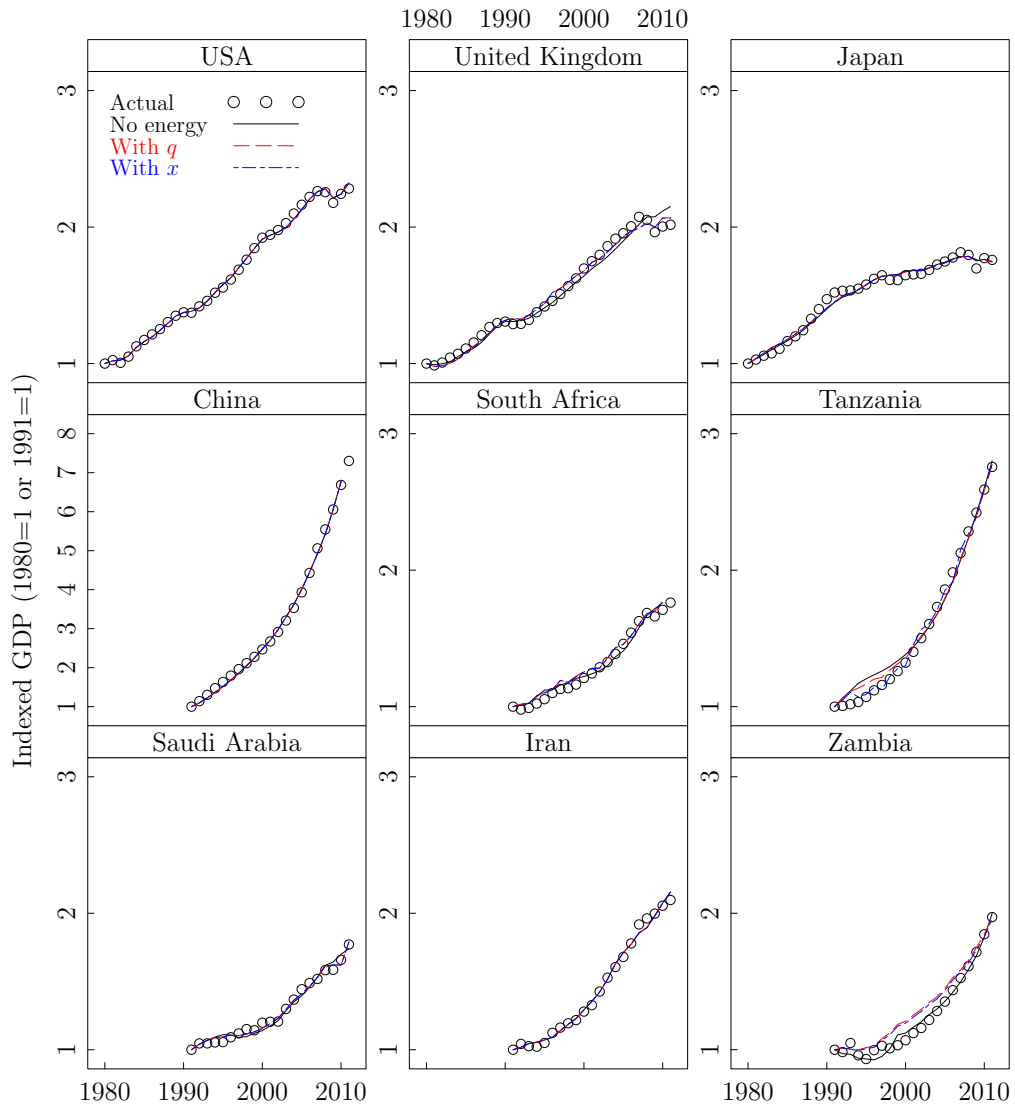Figure 4: Cobb-Douglas (with x) model parameters. Vertical bars indicate 95% confidence intervals.

Figure 5: Cobb-Douglas results.

3. CES

3.1. CES with Q

3.2. CES with X

3.3. CES with U

4. LINEX

4.1. LINEX with Q

4.2. LINEX with X

4.3. LINEX with U