

# Empirical Analysis of the Role of Energy in Economic Growth

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## Abstract

\*\*\*\*\* Add abstract \*\*\*\*\*

*Keywords:* economic growth, energy, cobb-douglas, CES, LINEX

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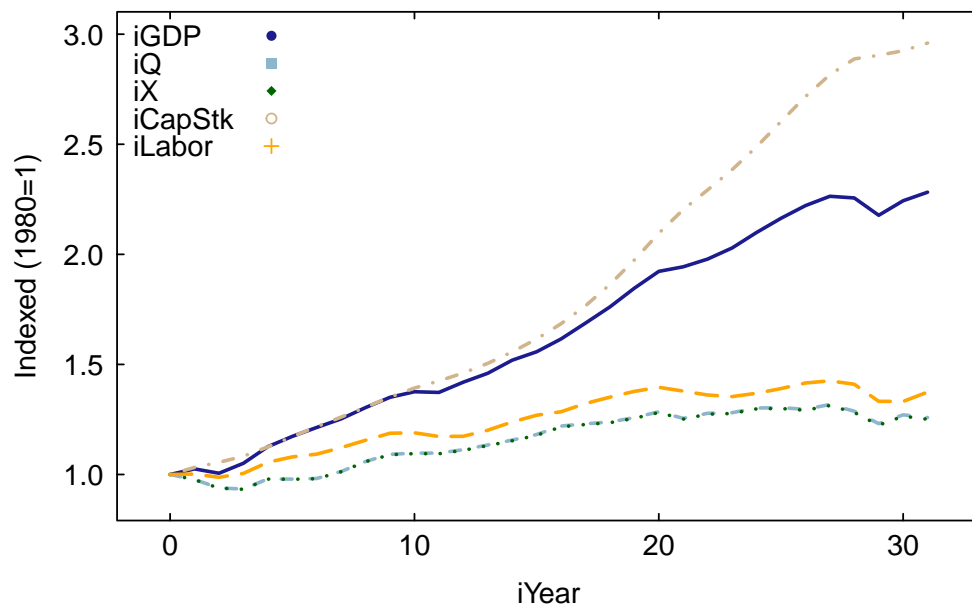
Caleb, put your LaTeX code here.

```
createCountryInputGraph <- function(countryName){  
  dataTable <- loadData(countryName)  
  graph <- xyplot(iGDP+iQ+iX+iCapStk+iLabor ~ iYear, data=dataTable,  
                 type="l",  
                 scales=list(cex=1.0, tck=-0.5),  
                 ylab="Indexed (1980=1)",  
                 auto.key=list(columns=1, x=0.0, y=0.98))  
  return(graph)  
}  
  
createCountryInputGraph("US")
```

---

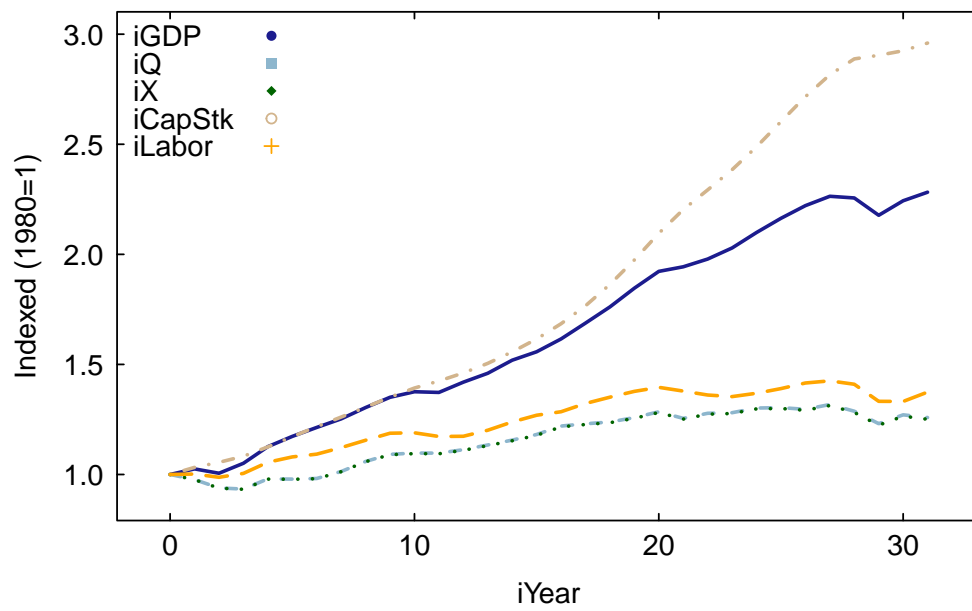
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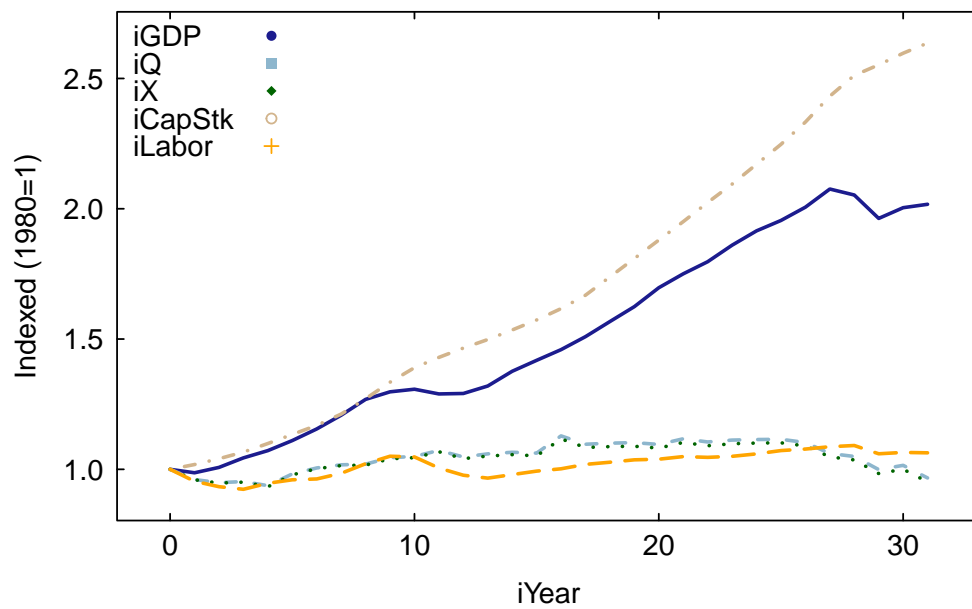


```
lapply(countries, createCountryInputGraph)
```

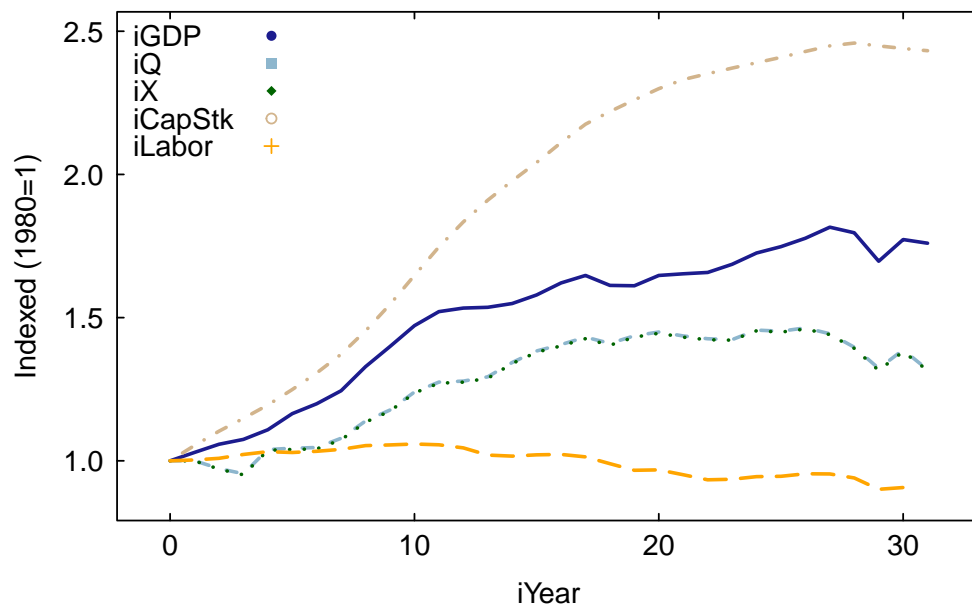
\$US



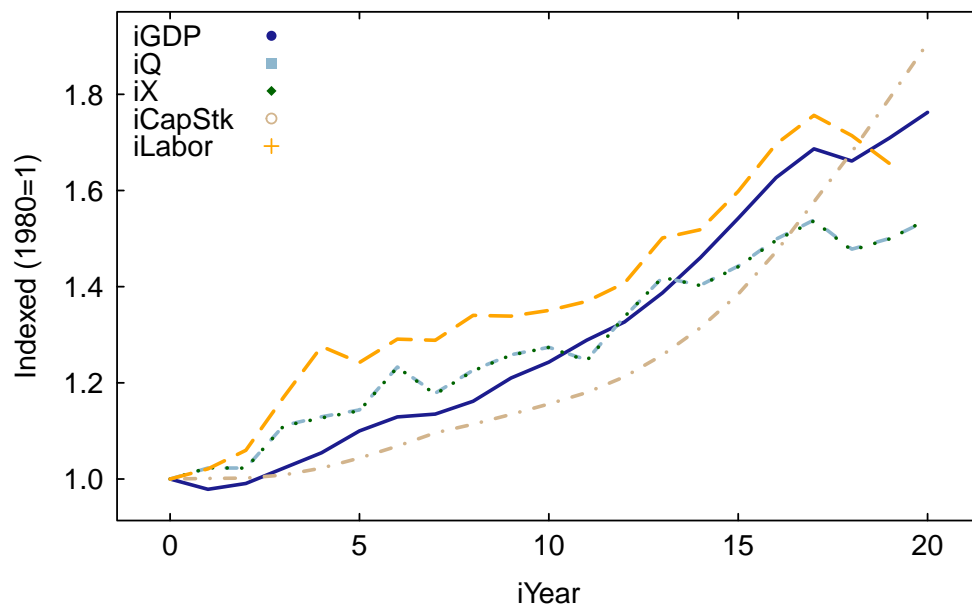
\$UK



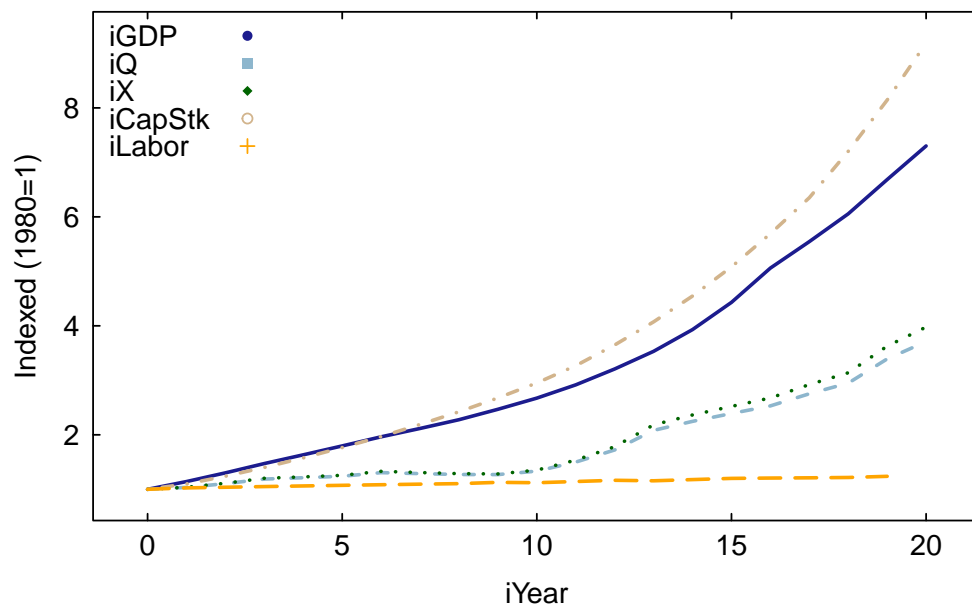
\$JP



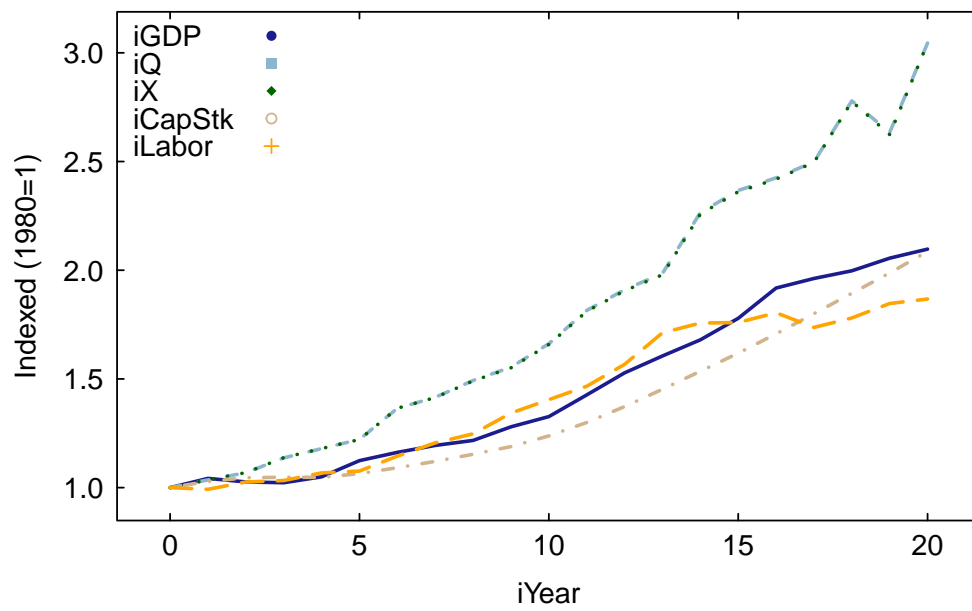
\$ZA



\$CN

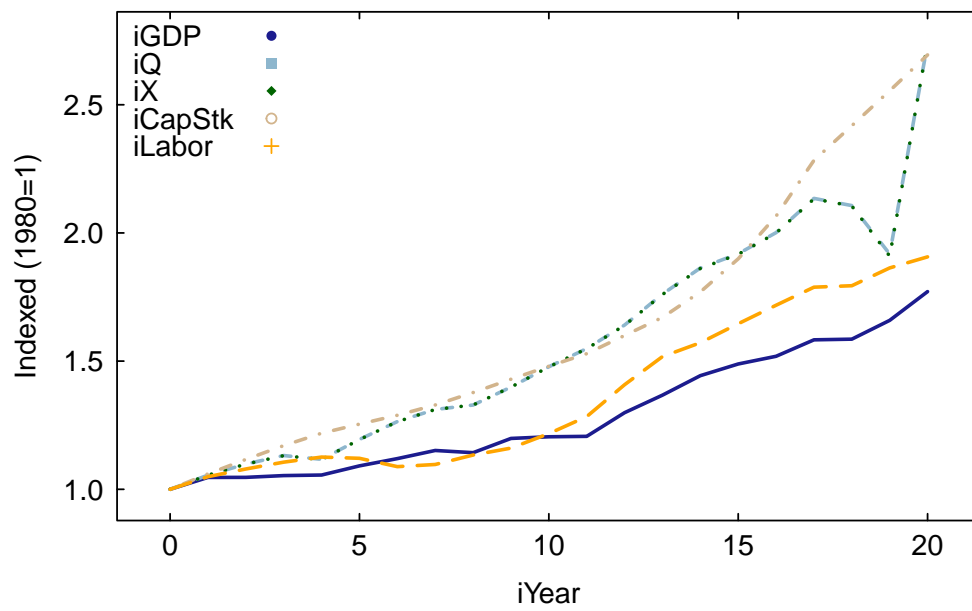


\$IR

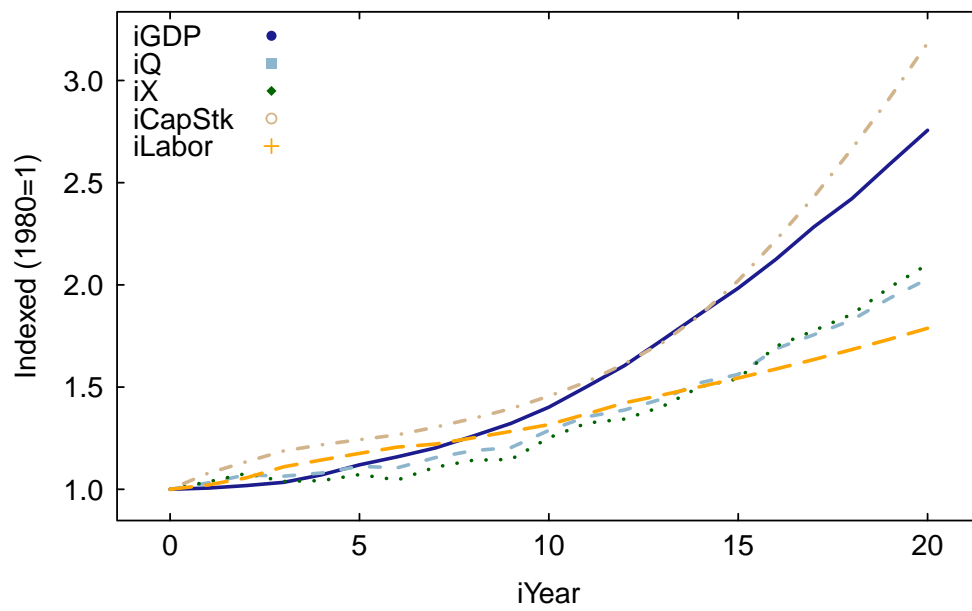


\$SA

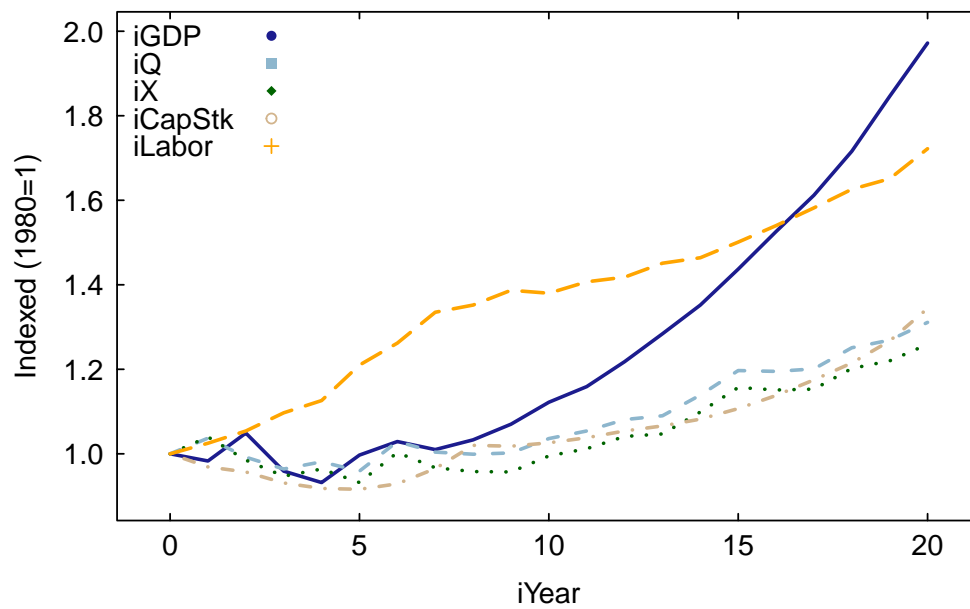




\$TZ



\$ZM



## 1. Cobb-Douglas Without Energy

```
#####
# Calculates parameter estimates and confidence intervals
# for the Cobb-Douglas production function given a country.
#
# countryName is a string containing the 2-letter abbreviation for the country, e.
#
# returns a vector of data for the Cobb-Douglas model.
# First item is the +95% CI on all parameters
# Second item contains the parameter estimates
# Third item is the -95% CI on all parameters
# Each row has names: lambda, alpha, beta, gamma, corresponding to the parameters
##
```

```

cobbDouglasData <- function(countryName){

  # Load the data that we need.
  dataTable <- loadData(countryName)

  # Establish guess values for alpha and lambda.
  lambdaGuess <- 0.0 # guessing lambda = 0 means there is no technological progress
  alphaGuess <- 0.3 # a typical value for alpha, the coefficient on capital stock

  # Runs a non-linear least squares fit to the data. We've replaced beta with 1-alpha
  modelCD <- nls(iGDP ~ exp(lambda*iYear) * iCapStk^alpha * iLabor^(1 - alpha),
    #           algorithm = "port",
    #           start = list(lambda=lambdaGuess, alpha=alphaGuess),
    #           lower = list(lambda=-Inf, alpha=0),
    #           upper = list(lambda=Inf, alpha=1),
    #           data=dataTable)

  # Checks validity of the model. AIC stands for Akaike's Information Criterion.
  aicCD <- AIC(modelCD, k=2)
  #print(aicCD)

  summaryCD <- summary(modelCD) # Gives the nls summary table.
  #print(summaryCD)
  ciCD <- confint(modelCD, level = ciLevel); ciCD # Displays confidence intervals

  # Calculate beta and its confidence interval and report it.
  alpha <- as.numeric(coef(modelCD)["alpha"])
  beta <- 1.0 - alpha
  beta.est <- deltaMethod(modelCD, "1 - alpha"); beta.est # Estimates beta and its
  # Now calculate a confidence interval on beta
  dofCD <- summaryCD$df[2]; dofCD # Gives the degrees of freedom for the model.
  tvalCD <- qt(ciHalfLevel, df = dofCD); tvalCD
  betaCICD <- with(beta.est, Estimate + c(-1.0, 1.0) * tvalCD * SE); betaCICD # Gives
  #print(coef(modelCD))

  # Combine all estimates and their confidence intervals into data frames with int

```

```

estCD <- data.frame(lambda = coef(modelCD)["lambda"], alpha = coef(modelCD)["alpha"])
#print(estCD)
row.names(estCD) <- "CD"
#row.names(estCD) <- "Cobb-Douglas: $y = e^{\lambda t}k^{\alpha}l^{\beta}$"
# The [1] subscripts pick off the lower confidence interval
lowerCD <- data.frame(lambda = ciCD["lambda", "2.5%"], alpha = ciCD["alpha", "2.5%"])
row.names(lowerCD) <- "- 95% CI"
# The [2] subscripts pick off the lower confidence interval
upperCD <- data.frame(lambda = ciCD["lambda", "97.5%"], alpha = ciCD["alpha", "97.5%"])
row.names(upperCD) <- "+ 95% CI"

# Now create the data for a table.
dataCD <- rbind(upperCD, estCD, lowerCD)
#print(dataCD)
return(dataCD)
}

#####
# Creates a LaTeX printable table from the Cobb Douglas data. This function first
#
# countryName is a string containint the 2-letter abbreviation for the country, e.
#
# returns a printable LaTeX table from xtable.
##
cobbDouglasTable <- function(countryName){
  dataCD <- cobbDouglasData(countryName)
  colnames(dataCD) <- c("$\\lambda$", "$\\alpha$", "$\\beta$", "$\\gamma$")
  tableCD <- xtable(dataCD, caption=paste(countryName, " Cobb-Douglas, 1980-2011"),
  #print(tableCD)
  return(tableCD)
}

```

```

tablesCD <- lapply(countries, cobbDouglasTable)

```

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```
print(tablesCD[["US"]], caption.placement="top")
```

	Table 1: US Cobb-Douglas, 1980-2011			
	$\lambda$	$\alpha$	$\beta$	$\gamma$
+ 95% CI	0.0116	0.34	0.79	
CD	0.0102	0.27	0.73	0.0
- 95% CI	0.0087	0.21	0.66	

```
print(tablesCD[["UK"]], caption.placement="top")
```

	Table 2: UK Cobb-Douglas, 1980-2011			
	$\lambda$	$\alpha$	$\beta$	$\gamma$
+ 95% CI	0.0303	1.12	1.24	
CD	0.0097	0.44	0.56	0.0
- 95% CI	-0.0104	-0.25	-0.13	

```
print(tablesCD[["JP"]], caption.placement="top")
```

```
print(tablesCD[["ZA"]], caption.placement="top")
```

Table 3: JP Cobb-Douglas, 1980-2011				
	$\lambda$	$\alpha$	$\beta$	$\gamma$
+ 95% CI	0.0081	0.61	0.56	
CD	0.0048	0.52	0.48	0.0
- 95% CI	0.0014	0.44	0.39	

Table 4: ZA Cobb-Douglas, 1980-2011				
	$\lambda$	$\alpha$	$\beta$	$\gamma$
+ 95% CI	0.0022	0.73	0.54	
CD	0.0008	0.60	0.40	0.0
- 95% CI	-0.0007	0.46	0.26	

```
print(tablesCD[["CN"]], caption.placement="top")
```

Table 5: CN Cobb-Douglas, 1980-2011				
	$\lambda$	$\alpha$	$\beta$	$\gamma$
+ 95% CI	0.0779	1.32	0.89	
CD	0.0188	0.71	0.29	0.0
- 95% CI	-0.0405	0.11	-0.32	

```
print(tablesCD[["SA"]], caption.placement="top")
```

```
print(tablesCD[["IR"]], caption.placement="top")
```

```
print(tablesCD[["TZ"]], caption.placement="top")
```

Table 6: SA Cobb-Douglas, 1980-2011				
	$\lambda$	$\alpha$	$\beta$	$\gamma$
+ 95% CI	-0.0087	0.68	0.78	
CD	-0.0123	0.45	0.55	0.0
- 95% CI	-0.0159	0.21	0.32	

Table 7: IR Cobb-Douglas, 1980-2011				
	$\lambda$	$\alpha$	$\beta$	$\gamma$
+ 95% CI	0.0045	0.70	0.51	
CD	0.0039	0.60	0.40	0.0
- 95% CI	0.0032	0.49	0.30	

```
print(tablesCD[["ZM"]], caption.placement="top")
```

```
# According to http://cran.r-project.org/web/packages/xtable/vignettes/xtableGalle
# be able to use the "sanitize.text.function" parameter to allow markup in column
# line is not working at the present time. --MKH, 18 Jan 2012.
# print(tableCD, caption.placement="top", sanitize.text.function = function(x){x})
```

## 2. Cobb-Douglas With Energy

We can force  $\alpha$ ,  $\beta$ , and  $\gamma$  to be in  $[0, 1]$  by a reparameterization:

$$a \in [0, 1], b \in [0, 1], \alpha = \min(a, b), \beta = |b - a|, \gamma = 1 - \max(a, b)$$

### 2.1. Cobb-Douglas with $Q$

```
# Note that the analysis of ZA is taking a long time here. Need to figure out why.
CDqTables <- lapply(countries, cobbDouglasEnergyTable, energyType="Q")
```



Table 8: TZ Cobb-Douglas, 1980-2011				
	$\lambda$	$\alpha$	$\beta$	$\gamma$
+ 95% CI	0.0068	0.95	0.50	
CD	0.0015	0.73	0.27	0.0
- 95% CI	-0.0039	0.50	0.05	

Table 9: ZM Cobb-Douglas, 1980-2011				
	$\lambda$	$\alpha$	$\beta$	$\gamma$
+ 95% CI	0.0280	1.57	-0.25	
CD	0.0249	1.41	-0.41	0.0
- 95% CI	0.0218	1.25	-0.57	

```
print(CDqTables[["US"]], caption.placement="top")
print(CDqTables[["ZA"]], caption.placement="top")
# According to http://cran.r-project.org/web/packages/xtable/vignettes/xtableGalle
# be able to use the "sanitize.text.function" parameter to allow markup in column
# line is not working at the present time. --MKH, 18 Jan 2012.
# print(tableCDe, sanitize.text.function = function(x){x})

#print(tableAll, caption.placement="top")
```

## 2.2. Cobb-Douglas With $X$

```
# Note that the analysis of ZA is taking a long time here. Need to figure out why.
CDxTables <- lapply(countries, cobbDouglasEnergyTable, energyType="X")
```

```
print(CDxTables[["US"]], caption.placement="top")
print(CDxTables[["ZA"]], caption.placement="top")
```

## 2.3. Cobb-Douglas With $U$

```
CDuTables <- lapply(countries, cobbDouglasEnergyTable, energyType="U")
```

```
print(CDuTables[["US"]], caption.placement="top")
print(CDuTables[["ZA"]], caption.placement="top")
```

### 3. CES

```
cesData <- function(countryName, energyType){
  energyColumnName <- paste("i", energyType, sep="")
  # Load the data that we need.
  dataTable <- loadData(countryName)

  # Establish guess values for phi beta, zeta, lambda_L and lambda_E.
  phiGuess <- -20
  betaGuess <- 0.5 # a typical value for beta (exponent on labor)
  zetaGuess <- 0.0004 # a small value
  lambda_LGuess <- 0.007 #assuming no technical progress on the labor-capital port
  lambda_EGuess <- 0.008 #assuming no technical progress on the energy portion of

  # Runs a non-linear least squares fit to the data with constraints
  modelCES <- nls(iGDP ~ ((1-zeta) * (exp(lambda_L*iYear) * iCapStk^(1-beta) * iLa
    + zeta*(exp(lambda_E*iYear) * iQ)^phi)^(1/phi),
    algorithm = "port",
    control = nls.control(maxiter = 500, tol = 1e-06, minFactor = 1
      printEval = FALSE, warnOnly = FALSE),
    start = list(phi=phiGuess, beta=betaGuess, zeta=zetaGuess, lamb
      lambda_E=lambda_EGuess),
    lower = list(phi=-Inf, beta=0, zeta=0, lambda_L=-Inf, lambda_E=
    upper = list(phi=0, beta=1, zeta=1, lambda_L=Inf, lambda_E=Inf)
    data=dataTable)

  aicCES <- AIC(modelCES, k=2) # Checks validity of the model. AIC stands for Akai
  print(aicCES)

  # Gives the nls summary table
```

```

summaryCES <- summary(modelCES) # Gives the nls summary table
print(summaryCES)

# Provides confidence intervals on phi, beta, zeta, lambda_L, and lambda_E. But,
ciCES <- confint(modelCES, level = ciLevel)
print(ciCES)

# Get the estimate for alpha
beta <- as.numeric(coef(modelCES)["beta"])
alpha <- 1.0 - beta
alpha.est <- deltaMethod(modelCES, "1 - beta") # Estimates alpha and its standard error
print(alpha.est)

# Now calculate a confidence interval on alpha
dofCES <- summaryCES$df[2]
print(dofCES) # Gives the degrees of freedom for the model.
tvalCES <- qt(ciHalfLevel, df = dofCES); tvalCES
# Get confidence intervals for each parameter in the model
alphaCICES <- with(alpha.est, Estimate + c(-1.0, 1.0) * tvalCES * SE) # CI on alpha
print(alphaCICES)

# Assemble the data into data frames for the table.
estCES <- data.frame(phi = coef(modelCES)["phi"], alpha = alpha,
                     beta = coef(modelCES)["beta"], zeta = coef(modelCES)["zeta"],
                     lambda_L = coef(modelCES)["lambda_L"], lambda_E = coef(modelCES)["lambda_E"])
row.names(estCES) <- paste("CES with ", energyType, sep="")
#print(estCES)
# The [1] subscripts pick off the lower confidence interval
lowerCES <- data.frame(phi = ciCES["phi", "2.5%"], alpha = alphaCICES[1],
                      beta = ciCES["beta", "2.5%"], zeta = ciCES["zeta", "2.5%"],
                      lambda_L = ciCES["lambda_L", "2.5%"], lambda_E = ciCES["lambda_E", "2.5%"])
row.names(lowerCES) <- "- 95% CI"
# The [2] subscripts pick off the upper confidence interval
upperCES <- data.frame(phi = ciCES["phi", "97.5%"], alpha = alphaCICES[2],
                      beta = ciCES["beta", "97.5%"], zeta = ciCES["zeta", "97.5%"],
                      lambda_L = ciCES["lambda_L", "97.5%"], lambda_E = ciCES["lambda_E", "97.5%"])
row.names(upperCES) <- "+ 95% CI"

```

```

# Now create the data for a table.
dataCES <- rbind(upperCES, estCES, lowerCES)
print(dataCES)
return(dataCES)

#xyplot( resid(modelCESQ) ~ fitted(modelCESQ) )
#histogram( ~resid(modelCESQ) )
#qqmath( ~resid(modelCESQ) )
}

#####
# Creates a LaTeX printable table from the CES data. This function first calls ces
#
# countryName is a string containint the 2-letter abbreviation for the country, e.
# energyType is a string to be used in table captions representing the type of ener
#
# returns a printable LaTeX table from xtable.
##
cesTable <- function(countryName, energyType){
  dataCESe <- cesData(countryName, energyType)
  tableCESq <- xtable(dataCESe, caption=paste(countryName, ", 1980-2011.", sep="")
}

```

### 3.1. CES with Q

```

countryName <- "US"
energyType <- "Q"
tableCESq <- cesTable(countryName, energyType)

[1] -194

Formula: iGDP ~ ((1 - zeta) * (exp(lambda_L * iYear) * iCapStk^(1 - beta) *
  iLabor^beta)^phi + zeta * (exp(lambda_E * iYear) * iQ)^phi)^(1/phi)

Parameters:
      Estimate Std. Error t value Pr(>|t|)

```

```

phi      -3.96e+01  2.43e+01  -1.63  0.1144
beta      6.09e-01  3.45e-02  17.64  2.4e-16
zeta      2.09e-06  1.32e-05   0.16  0.8758
lambda_L  7.98e-03  6.68e-04  11.95  2.8e-12
lambda_E  8.57e-03  2.48e-03   3.45  0.0018

```

Residual standard error: 0.0105 on 27 degrees of freedom

Algorithm "port", convergence message: relative convergence (4)

*Waiting for profiling to be done...*

```

           2.5%      97.5%
phi           NA -10.290831
beta      0.514667  0.665371
zeta           NA           NA
lambda_L 0.006428  0.009152
lambda_E 0.000715  0.012468
      Estimate      SE
1 - beta  0.3911 0.03453
[1] 27
[1] 0.3202 0.4619
           phi  alpha  beta      zeta  lambda_L  lambda_E
+ 95% CI  -10.29 0.4619 0.6654      NA  0.009152 0.012468
CES with Q -39.64 0.3911 0.6089 2.085e-06 0.007979 0.008570
- 95% CI      NA 0.3202 0.5147      NA  0.006428 0.000715

```

```
#CESqTables <- lapply(countries, cesTable, energyType="Q")
```

```
print(tableCESq, caption.placement="top")
```

```

#print(CESqTables[["US"]], caption.placement="top")
#print(CESqTables[["ZA"]], caption.placement="top")

```

Table 10: US, 1980-2011.

	phi	alpha	beta	zeta	lambda_L	lambda_E
+ 95% CI	-10.3	0.46	0.67		0.00915	0.01247
CES with Q	-39.6	0.39	0.61	0.000002	0.00798	0.00857
- 95% CI		0.32	0.51		0.00643	0.00071