Empirical Analysis of the Role of Energy in Economic Growth

Caleb Reese[a], Lucas Timmer[a], Matthew Kuperus Heun[a,*]

[a]*Engineering Department, Calvin College, Grand Rapids, MI 49546, USA*

Abstract

*********** Add abstract ***********

Keywords: economic growth, energy, cobb-douglas, CES, LINEX

1. Cobb-Douglas Without Energy

```
##################################################
# Calculates parameter estimates and confidence intervals
# for the Cobb-Douglas production function given a country.
#
# countryName is a string containing the 2-letter abbreviation for the country, e.
#
# returns a vector of data for the Cobb-Douglas model.
# First item is the +95% CI on all parameters
# Second item contains the parameter estimates
# Third item is the -95% CI on all parameters
# Each row has names: lambda, alpha, beta, gamma, corresponding to the parameters
##
cobbDouglasData <- function(countryName){

  # Load the data that we need.
  dataTable <- loadData(countryName)
```

*Corresponding author

Email address: mkh2@calvin.edu, tel: +1 (616) 526-6663, fax: +1 (616) 526-6501 (Matthew Kuperus Heun)

```
# Establish guess values for alpha and lambda.
lambdaGuess <- 0.0 # guessing lambda = 0 means there is no technological progres
alphaGuess <- 0.3 # a typical value for alpha, the coefficient on capital stock

# Runs a non-linear least squares fit to the data. We've replaced beta with 1-al
modelCD <- nls(iGDP ~ exp(lambda*iYear) * iCapStk^alpha * iLabor^(1 - alpha),
               start=(list(lambda=lambdaGuess,alpha=alphaGuess)),
               data=dataTable)

# Checks validity of the model. AIC stands for Akaike's Information Criterion.
aicCD  <- AIC(modelCD, k=2)
#print(aicCD)

summaryCD <- summary(modelCD) # Gives the nls summary table.
#print(summaryCD)
ciCD <- confint(modelCD, level = ciLevel); ciCD # Displays confidence intervals

# Calculate beta and its confidence interval and report it.
alpha <- as.numeric(coef(modelCD)["alpha"])
beta <- 1.0 - alpha
beta.est <- deltaMethod(modelCD, "1 - alpha"); beta.est # Estimates beta and its
# Now calculate a confidence interval on beta
dofCD <- summaryCD$df[2]; dofCD # Gives the degrees of freedom for the model.
tvalCD <- qt(ciHalfLevel, df = dofCD); tvalCD
betaCICD <- with(beta.est, Estimate + c(-1.0, 1.0) * tvalCD * SE); betaCICD # Gi

#print(coef(modelCD))

# Combine all estimates and their confidence intervals into data frames with int
estCD <- data.frame(lambda = coef(modelCD)["lambda"], alpha = coef(modelCD)["alp
#print(estCD)
row.names(estCD) <- "CD"
#row.names(estCD) <- "Cobb-Douglas: $y = e^{\\lambda t}k^{\\alpha}l^{\\beta}$"
# The [1] subscripts pick off the lower confidence interval
lowerCD <- data.frame(lambda = ciCD["lambda","2.5%"], alpha = ciCD["alpha", "2.5
row.names(lowerCD) <- "- 95% CI"
# The [2] subscripts pick off the lower confidence interval
```

```
  upperCD <- data.frame(lambda = ciCD["lambda","97.5%"], alpha = ciCD["alpha", "97
  row.names(upperCD) <- "+ 95% CI"

  # Now create the data for a table.
  dataCD <- rbind(upperCD, estCD, lowerCD)
  #print(dataCD)
  return(dataCD)
}

####################################
# Creates a LaTeX printable table from the Cobb Douglas data. This function first
#
# countryName is a string containint the 2-letter abbreviation for the country, e.
#
# returns a printable LaTeX table from xtable.
##
cobbDouglasTable <- function(countryName){
  dataCD <- cobbDouglasData(countryName)
  colnames(dataCD) <- c("$\\lambda$", "$\\alpha$", "$\\beta$", "$\\gamma$")
  tableCD <- xtable(dataCD, caption=paste(countryName, " Cobb-Douglas, 1980-2011",
  #print(tableCD)
  return(tableCD)
}
```

```
tablesCD <- lapply(countries, cobbDouglasTable)

Waiting for profiling to be done...
Waiting for profiling to be done...
```

```
print(tablesCD[["US"]], caption.placement="top")
```

```
print(tablesCD[["ZA"]], caption.placement="top")
```

Table 1: US Cobb-Douglas, 1980-2011

	λ	α	β	γ
+ 95% CI	0.0116	0.33	0.79	
CD	0.0103	0.27	0.73	0.0
- 95% CI	0.0089	0.21	0.67	

Table 2: ZA Cobb-Douglas, 1980-2011

	λ	α	β	γ
+ 95% CI	0.0023	0.74	0.54	
CD	0.0008	0.60	0.40	0.0
- 95% CI	-0.0007	0.46	0.26	

```
# According to http://cran.r-project.org/web/packages/xtable/vignettes/xtableGalle
# be able to use the "sanitize.text.function" parameter to allow markup in column
# line is not working at the present time. --MKH, 18 Jan 2012.
# print(tableCD, caption.placement="top", sanitize.text.function = function(x){x})
```

2. Cobb-Douglas With Energy

We can force α, β, and γ to be in $[0, 1]$ by a reparameterization:

$$a \in [0,1], b \in [0,1], \alpha = \min(a,b), \beta = |b-a|, \gamma = 1 - \max(a,b)$$

```
###############################
# This function fits the Cobb Douglas production function including energy to hist
# The functional form is
#
# iGDP = exp(lambda * itime) * iCapStk^alpha * iLabor^beta * iEnergy^gamma
#
# iYear: time indexed to 0.0 at the first year [years since beginning of data set]
# iGDP = time series GDP data indexed to 1.0 at the first year [-]
# iCapStk = time series capital stock data indexed to 1.0 at the first year [-]
# iLabor = time series labor data indexed to 1.0 at the first year [-]
```

```
# iEnergy = time series energy data indexed to 1.0 at the first year [-].
#      This may be any of heat (iQ), exergy (iX), or useful work (iU).
#
# returns a data object containing three rows and 5 columns.
#   col 1: lambda
#   col 2: alpha
#   col 3: beta
#   col 4: gamma
#   row 1: -95% CI for each parameter
#   row 2: estimate for each parameter
#   row 3: +95% CI for each parameter
#
# countryName is a string containint the 2-letter abbreviation for the country, e.
# energyType is a string to be used in table captions reprsenting the type of ener
##
cobbDouglasEnergyData <- function(countryName, energyType){

  print(countryName)
  energyColumnName  <- paste("i", energyType, sep="")
  #print(energyColumnName)

  # Load the data that we need.
  dataTable <- loadData(countryName)

  # Reparameterize to ensure that we meet the constraints:
  # * alpha + beta + gamma = 1.0.
  # * alpha, beta, and gamma are all between 0.0 and 1.0.
  # To do this, we reparameterize as
  # * 0 < a < 1
  # * 0 < b < 1
  # * alpha = min(a, b)
  # * beta = b - a
  # * gamma = 1 - max(a, b)
  iEnergy <- dataTable[energyColumnName] #grabs the desired energy column
  lambdaGuess <- 0.0 # guessing lambda = 0 means there is no technological progres
  alphaGuess <- 0.3 # a typical value for alpha
  betaGuess <- 0.7 # a typical value for beta
```

```
modelCDe <- nls(iGDP ~ exp(lambda*iYear) *
                  iCapStk^min(a,b) * iLabor^abs(b-a) *
                  iEnergy^(1.0 - max(a,b)),
                algorithm = "port",
                start = list(lambda=lambdaGuess, a=alphaGuess, b=alphaGuess+beta
                lower = list(lambda=-Inf, a=0, b=0),
                upper = list(lambda=Inf, a=1, b=1),
                data = dataTable)
aicCDe <- AIC(modelCDe, k=2) # Checks validity of the model. AIC stands for Akai
#print(aicCDe)

summaryCDe <- summary(modelCDe) # Gives the nls summary table
print(summaryCDe)

# Provides confidence intervals on lambda, a, and b. But, we need CIs on alpha a
ciCDe <- confint(modelCDe, level = ciLevel)
#print(ciCDe)

a <- as.numeric(coef(modelCDe)["a"])
b <- as.numeric(coef(modelCDe)["b"])
lambda <- as.numeric(coef(modelCDe)["lambda"])
alpha <- a
beta <- b - a
gamma <- 1.0 - alpha - beta

# Report results with SE
beta.est <- deltaMethod(modelCDe, "b-a") # Reports results for beta, because bet
gamma.est <- deltaMethod(modelCDe, "1-b") # Reports results for gamma, because g

# Now calculate confidence intervals.
dofCDe <- summaryCDe$df[2] # Gives the degrees of freedom for the model.
#print(dofCDe)
tvalCDe <- qt(ciHalfLevel, df = dofCDe)
#print(tvalCDe)
betaCICDe <- with(beta.est, Estimate + c(-1.0, 1.0) * tvalCDe * SE) # Gives the
#print(betaCICDe)
gammaCICDe <- with(gamma.est, Estimate + c(-1.0, 1.0) * tvalCDe * SE) # Gives th
```

```
  #print(gammaCICDe)

  # Combine all estimates and their confidence intervals into data frames with int
  estCDe <- data.frame(lambda = lambda, alpha = alpha, beta = beta, gamma = gamma)
  #print(estCDe);
  #row.names(estCDe) <- "Cobb-Douglas with e: $y = e^{\\lambda t}k^{\\alpha}l^{\\b
  row.names(estCDe) <- paste("C-D with ", energyType, sep="")
  # The [1] subscripts pick off the lower confidence interval
  lowerCDe <- data.frame(lambda = ciCDe["lambda","2.5%"], alpha = ciCDe["a", "2.5%
  row.names(lowerCDe) <- "- 95% CI"
  # The [2] subscripts pick off the lower confidence interval
  upperCDe <- data.frame(lambda = ciCDe["lambda","97.5%"], alpha = ciCDe["a", "97.
  row.names(upperCDe) <- "+ 95% CI"

  # Now create the data for a table.
  dataCDe <- rbind(upperCDe, estCDe, lowerCDe)
  #print(dataCDe)
  return(dataCDe)
}

####################################
# Creates a LaTeX printable table from the CES data. This function first calls cob
#
# countryName is a string containint the 2-letter abbreviation for the country, e.
# energyType is a string to be used in table captions reprsenting the type of ener
#
# returns a printable LaTeX table from xtable.
##
cobbDouglasEnergyTable <- function(countryName, energyType){
  dataCDe <- cobbDouglasEnergyData(countryName, energyType)
  tableCDe <- xtable(dataCDe, caption=paste(countryName, ", 1980-2011.", sep=""),
  return(tableCDe)
}
```

2.1. Cobb-Douglas with Q

```
print(CDqTables[["US"]], caption.placement="top")
```

Table 3: US, 1980-2011.

	lambda	alpha	beta	gamma
+ 95% CI	0.0127	0.33	0.84	0.177
C-D with Q	0.0105	0.26	0.72	0.021
- 95% CI	0.0089	0.19	0.59	-0.135

```
print(CDqTables[["ZA"]], caption.placement="top")
```

Table 4: ZA, 1980-2011.

	lambda	alpha	beta	gamma
+ 95% CI	0.0031	0.73	0.55	0.261
C-D with Q	0.0013	0.59	0.34	0.073
- 95% CI	-0.0006	0.45	0.12	-0.116

```
# According to http://cran.r-project.org/web/packages/xtable/vignettes/xtableGalle
# be able to use the "sanitize.text.function" parameter to allow markup in column
# line is not working at the present time. --MKH, 18 Jan 2012.
# print(tableCDe, sanitize.text.function = function(x){x})

#print(tableAll, caption.placement="top")
```

2.2. Cobb-Douglas With X

```r
print(CDxTables[["US"]], caption.placement="top")
```

Table 5: US, 1980-2011.

	lambda	alpha	beta	gamma
+ 95% CI	0.0129	0.33	0.82	0.196
C-D with X	0.0107	0.26	0.70	0.043
- 95% CI	0.0089	0.18	0.58	-0.110

```r
print(CDxTables[["ZA"]], caption.placement="top")
```

Table 6: ZA, 1980-2011.

	lambda	alpha	beta	gamma
+ 95% CI	0.0031	0.73	0.55	0.263
C-D with X	0.0013	0.59	0.34	0.075
- 95% CI	-0.0006	0.45	0.12	-0.114

2.3. Cobb-Douglas With U

```r
CDuTables <- lapply(countries, cobbDouglasEnergyTable, energyType="U")

[1] "US"

Error: Missing value or an infinity produced when evaluating the
model
```

```r
print(CDuTables[["US"]], caption.placement="top")

Error: error in evaluating the argument 'x' in selecting a method
for function 'print': Error: object 'CDuTables' not found

print(CDuTables[["ZA"]], caption.placement="top")

Error: error in evaluating the argument 'x' in selecting a method
for function 'print': Error: object 'CDuTables' not found
```

3. CES

```
cesData <- function(countryName, energyType){
  energyColumnName <- paste("i", energyType, sep="")
  # Load the data that we need.
  dataTable <- loadData(countryName)

  # Establish guess values for phi beta, zeta, lambda_L and lambda_E.
  phiGuess <- -20
  betaGuess <- 0.5 # a typical value for beta (exponent on labor)
  zetaGuess <- 0.0004 # a small value
  lambda_LGuess <- 0.007 #assuming no technical progress on the labor-capital port
  lambda_EGuess <- 0.008 #assuming no technical progress on the energy portion of

  # Runs a non-linear least squares fit to the data with constraints
  modelCES <- nls(iGDP ~ ((1-zeta) * (exp(lambda_L*iYear) * iCapStk^(1-beta) * iLa
                          + zeta*(exp(lambda_E*iYear) * iQ)^phi)^(1/phi),
                  algorithm = "port",
                  control = nls.control(maxiter = 500, tol = 1e-06, minFactor = 1
                                        printEval = FALSE, warnOnly = FALSE),
                  start = list(phi=phiGuess, beta=betaGuess, zeta=zetaGuess, lamb
                               lambda_E=lambda_EGuess),
                  lower = list(phi=-Inf, beta=0, zeta=0, lambda_L=-Inf, lambda_E=
                  upper = list(phi=0, beta=1, zeta=1, lambda_L=Inf, lambda_E=Inf)
                  data=dataTable)

  aicCES <- AIC(modelCES, k=2) # Checks validity of the model. AIC stands for Akai
  print(aicCES)

  # Gives the nls summary table
  summaryCES <- summary(modelCES) # Gives the nls summary table
  print(summaryCES)

  # Provides confidence intervals on phi, beta, zeta, lambda_L, and lambda_E. But,
  ciCES <- confint(modelCES, level = ciLevel)
  print(ciCES)
```

```
# Get the estimate for alpha
beta <- as.numeric(coef(modelCES)["beta"])
alpha <- 1.0 - beta
alpha.est <- deltaMethod(modelCES, "1 - beta") # Estimates alpha and its standar
print(alpha.est)

# Now calculate a confidence interval on alpha
dofCES <- summaryCES$df[2]
print(dofCES) # Gives the degrees of freedom for the model.
tvalCES <- qt(ciHalfLevel, df = dofCES); tvalCES
# Get confidence intervals for each parameter in the model
alphaCICES <- with(alpha.est, Estimate + c(-1.0, 1.0) * tvalCES * SE) # CI on al
print(alphaCICES)

# Assemble the data into data frames for the table.
estCES <- data.frame(phi = coef(modelCES)["phi"], alpha = alpha,
                     beta = coef(modelCES)["beta"], zeta = coef(modelCES)["zeta'
                     lambda_L = coef(modelCES)["lambda_L"], lambda_E = coef(mode
row.names(estCES) <- paste("CES with ", energyType, sep="")
#print(estCES)
# The [1] subscripts pick off the lower confidence interval
lowerCES <- data.frame(phi = ciCES["phi","2.5%"], alpha = alphaCICES[1],
                       beta = ciCES["beta", "2.5%"], zeta = ciCES["zeta", "2.5%'
                       lambda_L = ciCES["lambda_L", "2.5%"], lambda_E = ciCES["l
row.names(lowerCES) <- "- 95% CI"
# The [2] subscripts pick off the lower confidence interval
upperCES <- data.frame(phi = ciCES["phi","97.5%"], alpha = alphaCICES[2],
                       beta = ciCES["beta", "97.5%"], zeta = ciCES["zeta", "97.5
                       lambda_L = ciCES["lambda_L", "97.5%"], lambda_E = ciCES['
row.names(upperCES) <- "+ 95% CI"

# Now create the data for a table.
dataCES <- rbind(upperCES, estCES, lowerCES)
print(dataCES)
return(dataCES)

#xyplot( resid(modelCESQ) ~ fitted(modelCESQ) )
```

```
  #histogram( ~resid(modelCESQ) )
  #qqmath( ~resid(modelCESQ) )
}

####################################
# Creates a LaTeX printable table from the CES data. This function first calls ces
#
# countryName is a string containint the 2-letter abbreviation for the country, e.
# energyType is a string to be used in table captions reprsenting the type of ener
#
# returns a printable LaTeX table from xtable.
##
cesTable <- function(countryName, energyType){
  dataCESe <- cesData(countryName, energyType)
  tableCESq <- xtable(dataCESe, caption=paste(countryName, ", 1980-2011.", sep="")
}
```

3.1. CES with Q

```
countryName <- "US"
energyType <- "Q"
tableCESq <- cesTable(countryName, energyType)

[1] -197.8

Formula: iGDP ~ ((1 - zeta) * (exp(lambda_L * iYear) * iCapStk^(1 - beta) *
    iLabor^beta)^phi + zeta * (exp(lambda_E * iYear) * iQ)^phi)^(1/phi)

Parameters:
          Estimate Std. Error t value Pr(>|t|)
phi      -2.22e+01   1.50e+01   -1.48   0.1512
beta      5.82e-01   5.20e-02   11.19  1.2e-11
zeta      3.51e-04   1.38e-03    0.25   0.8014
lambda_L  7.62e-03   8.30e-04    9.18  8.5e-10
lambda_E  8.05e-03   2.84e-03    2.84   0.0085
```

```
Residual standard error: 0.00993 on 27 degrees of freedom

Algorithm "port", convergence message: relative convergence (4)

Waiting for profiling to be done...

              2.5%     97.5%
phi             NA -4.621870
beta      0.462998  0.656248
zeta            NA        NA
lambda_L  0.006024  0.008997
lambda_E -0.002867  0.012398
         Estimate      SE
1 - beta   0.4179 0.05204
[1] 27
[1] 0.3112 0.5247
                 phi  alpha   beta      zeta lambda_L  lambda_E
+ 95% CI    -4.622 0.5247 0.6562        NA 0.008997  0.012398
CES with Q -22.219 0.4179 0.5821 0.0003506 0.007620  0.008053
- 95% CI        NA 0.3112 0.4630        NA 0.006024 -0.002867


#CESqTables <- lapply(countries, cesTable, energyType="Q")
```

```
print(tableCESq, caption.placement="top")
```

Table 7: US, 1980-2011.

	phi	alpha	beta	zeta	lambda_L	lambda_E
+ 95% CI	-4.6	0.52	0.66		0.00900	0.01240
CES with Q	-22.2	0.42	0.58	0.000351	0.00762	0.00805
- 95% CI		0.31	0.46		0.00602	-0.00287

```
#print(CESqTables[["US"]], caption.placement="top")
#print(CESqTables[["ZA"]], caption.placement="top")
```