Empirical Analysis of the Role of Energy in Economic Growth

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Abstract

*********** Add abstract ***********

Keywords: economic growth, energy, cobb-douglas, CES, LINEX

Caleb, put your LaTeX code here.

```
createCountryFactorsGraph <- function(countryName){
  dataTable <- loadData(countryName)
  graphType <- "l"
  lineTypes <- c(1, 5, 2, 4, 6) #line types. See http://en.wikibooks.org/wiki/R_Pr
  lineWidths <- c(1, 1, 1, 1, 1) #line widths
  colors <- c("black", "black", "black", "black", "black") #line and point colors
  lineSpec <- list(lty=lineTypes, lwd=lineWidths, col=colors)
  graph <- xyplot(iCapStk+iLabor+iQ+iX+iU ~ Year, data=dataTable,
                  type=graphType,
                  par.settings = list(superpose.line = lineSpec),
                  key=list(text=list(c("k", "l", "q", "x", "u")),
                           type=graphType,
                           lines=lineSpec,
                           columns=1, x=0.0, y=0.98),
                  scales=list(cex=1.0, #controls text size on scales
                              tck=-0.5), #controls tick mark length
                  ylab="Indexed (1980=1)")
```

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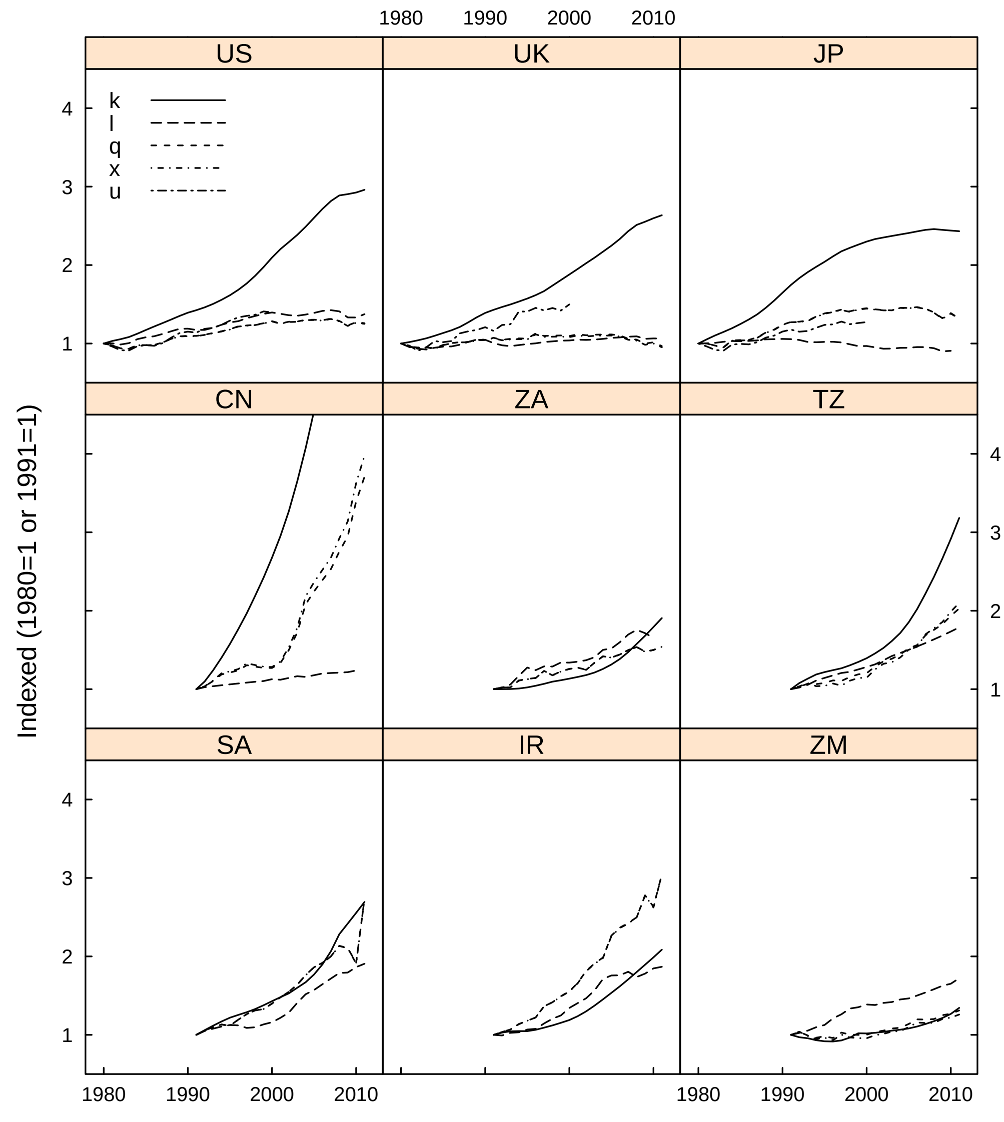

Figure 1: Factors of Production for All Countries. (China's indexed capital stock rises to $k = 9.2$ in 2011.)

```r
  return(graph)
}

createGDPComparisonGraph <- function(countryName){
  dataTable <- loadData(countryName)
  graphType <- "l"
  lineTypes <- c(1) #line types. See http://en.wikibooks.org/wiki/R_Programming/Gr
  lineWidths <- c(1) #line widths
  colors <- c("black") #line and point colors
  lineSpec <- list(lty=lineTypes, lwd=lineWidths, col=colors)
  graph <- xyplot(iGDP ~ Year, data=dataTable,
                  key=list(text=list(c("GDP")),
                           type=graphType,
                           lines=lineSpec,
                           columns=1, x=0.0, y=0.98),
                  type=graphType,
                  par.settings = list(superpose.line = lineSpec),
                  scales=list(cex=1.0, #controls text size on scales
                              tck=-0.5), #controls tick mark length
                  ylab="Indexed (1980=1)")
  return(graph)
}


# createGDPComparisonGraph <- function(countryName){
#   dataTable <- loadData(countryName)
#   graphType <- "l"
#   colors <- c("black", "black", "black", "black") #line and point colors
#   lineWidths <- c(1, 1, 1, 1) #line widths
#   lineTypes <- c(1, 5, 2, 4) #line types. See http://en.wikibooks.org/wiki/R_Pro
#   lineSpec <- list(lty=lineTypes, lwd=lineWidths, col=colors)
#   pointSpec <- list(col=colors)
#   graph <- xyplot(iGDP ~ iYear, data=dataTable,
#                   key=list(text=list(c("GDP")),
#                            type=graphType,
#                            lines=lineSpec,
#                            columns=1, x=0.0, y=0.98),
```

```
#                   type=graphType,
#                   par.settings = list(superpose.line = lineSpec),
# #                                     superpose.symbol = pointSpec),
#                   scales=list(cex=1.0, #controls text size on scales
#                               tck=-0.5), #controls tick mark length
#                   ylab="Indexed (1980=1)")
#   return(graph)
#
#
# }



#createGDPComparisonGraph("US")

#createCountryFactorsGraph("US")
#lapply(countries, createCountryFactorsGraph)
```

1. Cobb-Douglas Without Energy

```
#################################################
# Calculates parameter estimates and confidence intervals
# for the Cobb-Douglas production function given a country.
#
# countryName is a string containing the 2-letter abbreviation for the country, e.
#
# returns a vector of data for the Cobb-Douglas model.
# First item is the +95% CI on all parameters
# Second item contains the parameter estimates
# Third item is the -95% CI on all parameters
# Each row has names: lambda, alpha, beta, gamma, corresponding to the parameters
##
```

```
cobbDouglasData <- function(countryName){

  # Load the data that we need.
  dataTable <- loadData(countryName)

  # Establish guess values for alpha and lambda.
  lambdaGuess <- 0.0 # guessing lambda = 0 means there is no technological progres
  alphaGuess <- 0.3 # a typical value for alpha, the coefficient on capital stock

  # Runs a non-linear least squares fit to the data. We've replaced beta with 1-al
  modelCD <- nls(iGDP ~ exp(lambda*iYear) * iCapStk^alpha * iLabor^(1 - alpha),
#                  algorithm = "port",
                 start = list(lambda=lambdaGuess, alpha=alphaGuess),
#                  lower = list(lambda=-Inf, alpha=0),
#                  upper = list(lambda=Inf, alpha=1),
                 data=dataTable)

  # Checks validity of the model. AIC stands for Akaike's Information Criterion.
  aicCD  <- AIC(modelCD, k=2)
  #print(aicCD)

  summaryCD <- summary(modelCD) # Gives the nls summary table.
  #print(summaryCD)
  ciCD <- confint(modelCD, level = ciLevel); ciCD # Displays confidence intervals

  # Calculate beta and its confidence interval and report it.
  alpha <- as.numeric(coef(modelCD)["alpha"])
  beta <- 1.0 - alpha
  beta.est <- deltaMethod(modelCD, "1 - alpha"); beta.est # Estimates beta and its
  # Now calculate a confidence interval on beta
  dofCD <- summaryCD$df[2]; dofCD # Gives the degrees of freedom for the model.
  tvalCD <- qt(ciHalfLevel, df = dofCD); tvalCD
  betaCICD <- with(beta.est, Estimate + c(-1.0, 1.0) * tvalCD * SE); betaCICD # Gi

  #print(coef(modelCD))

  # Combine all estimates and their confidence intervals into data frames with int
```

```
  estCD <- data.frame(lambda = coef(modelCD)["lambda"], alpha = coef(modelCD)["alp
  #print(estCD)
  row.names(estCD) <- "CD"
  #row.names(estCD) <- "Cobb-Douglas: $y = e^{\\lambda t}k^{\\alpha}l^{\\beta}$"
  # The [1] subscripts pick off the lower confidence interval
  lowerCD <- data.frame(lambda = ciCD["lambda","2.5%"], alpha = ciCD["alpha", "2.5
  row.names(lowerCD) <- "- 95% CI"
  # The [2] subscripts pick off the lower confidence interval
  upperCD <- data.frame(lambda = ciCD["lambda","97.5%"], alpha = ciCD["alpha", "97
  row.names(upperCD) <- "+ 95% CI"

  # Now create the data for a table.
  dataCD <- rbind(upperCD, estCD, lowerCD)
  #print(dataCD)
  return(dataCD)
}

############
# Creates a row for the Cobb Douglas parameters table
##
cobbDouglasCountryRow <- function(countryName){
  dataCD <- cobbDouglasData(countryName)
  # Create CI ranges
  out <- cbind(countryName,
               dataCD["- 95% CI", "lambda"], dataCD["CD", "lambda"], dataCD["+ 95%
               dataCD["- 95% CI", "alpha"],  dataCD["CD", "alpha"],  dataCD["+ 95%
               dataCD["- 95% CI", "beta"],   dataCD["CD", "beta"],   dataCD["+ 95%

  return(out)
}

cobbDouglasParamsTable <- function(){
  dataCD <- rbind(cobbDouglasCountryRow("US"),
                cobbDouglasCountryRow("UK"),
                cobbDouglasCountryRow("JP"),
                cobbDouglasCountryRow("CN"),
                cobbDouglasCountryRow("ZA"),
```

```
              cobbDouglasCountryRow("SA"),
              cobbDouglasCountryRow("IR"),
              cobbDouglasCountryRow("TZ"),
              cobbDouglasCountryRow("ZM"))
  colnames(dataCD) <- c("", "-95% CI", "$\\lambda$", "+95% CI",
                        "-95% CI", "$\\alpha$",  "+95% CI",
                        "-95% CI", "$\\beta$",   "+95% CI")
  rownames(dataCD) <- countries
  print("************** Trying to print dataCD *********************")
#   print(class(dataCD))
#   print(dataCD)
  tableCD <- xtable(dataCD, caption="Cobb-Douglas, 1980-2011 or 1991-2011", digit
  #print(tableCD)
  return(tableCD)
}
cobbDouglasParamsTable()

Waiting for profiling to be done...
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[1] "************** Trying to print dataCD *********************"
% latex table generated in R 2.15.2 by xtable 1.7-0 package
% Sun Jan 27 16:43:01 2013
\begin{table}[ht]
\begin{center}
\begin{tabular}{rllllllllll}
  \hline
 & V1 & -95\% CI & \$$\backslash$lambda\$ & +95\% CI & -95\% CI & \$$\backslash$al
  \hline
US & US & 0.00867713811541703 & 0.0101554649771947 & 0.0116267632506195 & 0.213128
  UK & UK & -0.0104339533284891 & 0.0097166097229806 & 0.0302750369133842 & -0.245
```

```
  JP & JP & 0.00144591559700194 & 0.00479965989492084 & 0.00814370765620374 & 0.43
  CN & CN & -0.0405221051255718 & 0.0187921739305594 & 0.0779058376028465 & 0.1085
  ZA & ZA & -0.000717427211566538 & 0.000771177746585204 & 0.00222325751849382 & 0
  SA & SA & -0.0159263027425623 & -0.0123103576408377 & -0.00873591015395777 & 0.2
  IR & IR & 0.0031544365568454 & 0.00385069982960034 & 0.00453844391610371 & 0.491
  TZ & TZ & -0.00391419988823917 & 0.00149948729754192 & 0.00678367569691732 & 0.5
  ZM & ZM & 0.0217845209774593 & 0.0249136301557912 & 0.0280398287326144 & 1.24947
   \hline
\end{tabular}
\caption{Cobb-Douglas, 1980-2011 or 1991-2011}
\end{center}
\end{table}
```

```
####################################
# Creates a LaTeX printable table from the Cobb Douglas data. This function first
#
# countryName is a string containint the 2-letter abbreviation for the country, e.
#
# returns a printable LaTeX table from xtable.
##
cobbDouglasCountryTable <- function(countryName){
  dataCD <- cobbDouglasData(countryName)
  colnames(dataCD) <- c("$\\lambda$", "$\\alpha$", "$\\beta$", "$\\gamma$")
  tableCD <- xtable(dataCD, caption=paste(countryName, " Cobb-Douglas, 1980-2011",
  #print(tableCD)
  return(tableCD)
}
```

```
tablesCD <- lapply(countries, cobbDouglasCountryTable)

Waiting for profiling to be done...
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Waiting for profiling to be done...
Waiting for profiling to be done...
Waiting for profiling to be done...
```

```
Waiting for profiling to be done...
Waiting for profiling to be done...
Waiting for profiling to be done...
Waiting for profiling to be done...
```

[1] "************* Trying to print dataCD ********************"

Table 1: Cobb-Douglas, 1980-2011 or 1991-2011

	V1	-95% CI	λ	+95% CI	-95% CI
US	US	0.00867713811541703	0.0101554649771947	0.0116267632506195	0.21312859
UK	UK	-0.0104339533284891	0.0097166097229806	0.0302750369133842	-0.2450552
JP	JP	0.00144591559700194	0.00479965989492084	0.00814370765620374	0.43868300
CN	CN	-0.0405221051255718	0.0187921739305594	0.0779058376028465	0.10850595
ZA	ZA	-0.000717427211566538	0.000771177746585204	0.00222325751849382	0.46144149
SA	SA	-0.0159263027425623	-0.0123103576408377	-0.00873591015395777	0.21482043
IR	IR	0.0031544365568454	0.00385069982960034	0.00453844391610371	0.49113172
TZ	TZ	-0.00391419988823917	0.00149948729754192	0.00678367569691732	0.50416691
ZM	ZM	0.0217845209774593	0.0249136301557912	0.0280398287326144	1.24947924

Table 2: US Cobb-Douglas, 1980-2011

	λ	α	β	γ
+ 95% CI	0.0116	0.34	0.79	
CD	0.0102	0.27	0.73	0.0
- 95% CI	0.0087	0.21	0.66	

2. Cobb-Douglas With Energy

We can force α, β, and γ to be in $[0, 1]$ by a reparameterization:

$$a \in [0, 1], b \in [0, 1], \alpha = \min(a, b), \beta = |b - a|, \gamma = 1 - \max(a, b)$$

Table 3: UK Cobb-Douglas, 1980-2011

	λ	α	β	γ
+ 95% CI	0.0303	1.12	1.24	
CD	0.0097	0.44	0.56	0.0
- 95% CI	-0.0104	-0.25	-0.13	

Table 4: JP Cobb-Douglas, 1980-2011

	λ	α	β	γ
+ 95% CI	0.0081	0.61	0.56	
CD	0.0048	0.52	0.48	0.0
- 95% CI	0.0014	0.44	0.39	

2.1. Cobb-Douglas with Q

```
# Note that the anlaysis of ZA is taking a long time here. Need to figure out why.
CDqTables <- lapply(countries, cobbDouglasEnergyTable, energyType="Q")
```

```
print(CDqTables[["US"]], caption.placement="top")
print(CDqTables[["ZA"]], caption.placement="top")
# According to http://cran.r-project.org/web/packages/xtable/vignettes/xtableGalle
# be able to use the "sanitize.text.function" parameter to allow markup in column
# line is not working at the present time. --MKH, 18 Jan 2012.
# print(tableCDe, sanitize.text.function = function(x){x})

#print(tableAll, caption.placement="top")
```

2.2. Cobb-Douglas With X

```
# Note that the anlaysis of ZA is taking a long time here. Need to figure out why.
CDxTables <- lapply(countries, cobbDouglasEnergyTable, energyType="X")
```

Table 5: CN Cobb-Douglas, 1980-2011

	λ	α	β	γ
+ 95% CI	0.0779	1.32	0.89	
CD	0.0188	0.71	0.29	0.0
- 95% CI	-0.0405	0.11	-0.32	

Table 6: ZA Cobb-Douglas, 1980-2011

	λ	α	β	γ
+ 95% CI	0.0022	0.73	0.54	
CD	0.0008	0.60	0.40	0.0
- 95% CI	-0.0007	0.46	0.26	

```
print(CDxTables[["US"]], caption.placement="top")
print(CDxTables[["ZA"]], caption.placement="top")
```

2.3. Cobb-Douglas With U

```
CDuTables <- lapply(countries, cobbDouglasEnergyTable, energyType="U")
```

```
print(CDuTables[["US"]], caption.placement="top")
print(CDuTables[["ZA"]], caption.placement="top")
```

3. CES

```
cesData <- function(countryName, energyType){
  energyColumnName <- paste("i", energyType, sep="")
  # Load the data that we need.
  dataTable <- loadData(countryName)

  # Establish guess values for phi beta, zeta, lambda_L and lambda_E.
  phiGuess <- -20
  betaGuess <- 0.5 # a typical value for beta (exponent on labor)
```

Table 7: SA Cobb-Douglas, 1980-2011

	λ	α	β	γ
+ 95% CI	-0.0087	0.68	0.78	
CD	-0.0123	0.45	0.55	0.0
- 95% CI	-0.0159	0.21	0.32	

Table 8: IR Cobb-Douglas, 1980-2011

	λ	α	β	γ
+ 95% CI	0.0045	0.70	0.51	
CD	0.0039	0.60	0.40	0.0
- 95% CI	0.0032	0.49	0.30	

```
zetaGuess <- 0.0004 # a small value
lambda_LGuess <- 0.007 #assuming no technical progress on the labor-capital port
lambda_EGuess <- 0.008 #assuming no technical progress on the energy portion of

# Runs a non-linear least squares fit to the data with constraints
modelCES <- nls(iGDP ~ ((1-zeta) * (exp(lambda_L*iYear) * iCapStk^(1-beta) * iLa
                         + zeta*(exp(lambda_E*iYear) * iQ)^phi)^(1/phi),
                 algorithm = "port",
                 control = nls.control(maxiter = 500, tol = 1e-06, minFactor = 1
                                       printEval = FALSE, warnOnly = FALSE),
                 start = list(phi=phiGuess, beta=betaGuess, zeta=zetaGuess, lamb
                              lambda_E=lambda_EGuess),
                 lower = list(phi=-Inf, beta=0, zeta=0, lambda_L=-Inf, lambda_E=
                 upper = list(phi=0, beta=1, zeta=1, lambda_L=Inf, lambda_E=Inf)
                 data=dataTable)

aicCES <- AIC(modelCES, k=2) # Checks validity of the model. AIC stands for Akai
print(aicCES)

# Gives the nls summary table
summaryCES <- summary(modelCES) # Gives the nls summary table
print(summaryCES)
```

Table 9: TZ Cobb-Douglas, 1980-2011

	λ	α	β	γ
+ 95% CI	0.0068	0.95	0.50	
CD	0.0015	0.73	0.27	0.0
- 95% CI	-0.0039	0.50	0.05	

Table 10: ZM Cobb-Douglas, 1980-2011

	λ	α	β	γ
+ 95% CI	0.0280	1.57	-0.25	
CD	0.0249	1.41	-0.41	0.0
- 95% CI	0.0218	1.25	-0.57	

```
# Provides confidence intervals on phi, beta, zeta, lambda_L, and lambda_E. But,
ciCES <- confint(modelCES, level = ciLevel)
print(ciCES)

# Get the estimate for alpha
beta <- as.numeric(coef(modelCES)["beta"])
alpha <- 1.0 - beta
alpha.est <- deltaMethod(modelCES, "1 - beta") # Estimates alpha and its standar
print(alpha.est)

# Now calculate a confidence interval on alpha
dofCES <- summaryCES$df[2]
print(dofCES) # Gives the degrees of freedom for the model.
tvalCES <- qt(ciHalfLevel, df = dofCES); tvalCES
# Get confidence intervals for each parameter in the model
alphaCICES <- with(alpha.est, Estimate + c(-1.0, 1.0) * tvalCES * SE) # CI on al
print(alphaCICES)

# Assemble the data into data frames for the table.
estCES <- data.frame(phi = coef(modelCES)["phi"], alpha = alpha,
                     beta = coef(modelCES)["beta"], zeta = coef(modelCES)["zeta"
                     lambda_L = coef(modelCES)["lambda_L"], lambda_E = coef(mode
```

```
  row.names(estCES) <- paste("CES with ", energyType, sep="")
  #print(estCES)
  # The [1] subscripts pick off the lower confidence interval
  lowerCES <- data.frame(phi = ciCES["phi","2.5%"], alpha = alphaCICES[1],
                         beta = ciCES["beta", "2.5%"], zeta = ciCES["zeta", "2.5%'
                         lambda_L = ciCES["lambda_L", "2.5%"], lambda_E = ciCES["l
  row.names(lowerCES) <- "- 95% CI"
  # The [2] subscripts pick off the lower confidence interval
  upperCES <- data.frame(phi = ciCES["phi","97.5%"], alpha = alphaCICES[2],
                         beta = ciCES["beta", "97.5%"], zeta = ciCES["zeta", "97.5
                         lambda_L = ciCES["lambda_L", "97.5%"], lambda_E = ciCES['
  row.names(upperCES) <- "+ 95% CI"

  # Now create the data for a table.
  dataCES <- rbind(upperCES, estCES, lowerCES)
  print(dataCES)
  return(dataCES)

  #xyplot( resid(modelCESQ) ~ fitted(modelCESQ) )
  #histogram( ~resid(modelCESQ) )
  #qqmath( ~resid(modelCESQ) )
}

####################################
# Creates a LaTeX printable table from the CES data. This function first calls ces
#
# countryName is a string containint the 2-letter abbreviation for the country, e.
# energyType is a string to be used in table captions reprsenting the type of ener
#
# returns a printable LaTeX table from xtable.
##
cesTable <- function(countryName, energyType){
  dataCESe <- cesData(countryName, energyType)
  tableCESq <- xtable(dataCESe, caption=paste(countryName, ", 1980-2011.", sep=""))
}
```

3.1. CES with Q

```
countryName <- "US"
energyType <- "Q"
tableCESq <- cesTable(countryName, energyType)
```

```
[1] -194

Formula: iGDP ~ ((1 - zeta) * (exp(lambda_L * iYear) * iCapStk^(1 - beta) *
    iLabor^beta)^phi + zeta * (exp(lambda_E * iYear) * iQ)^phi)^(1/phi)

Parameters:
          Estimate Std. Error t value Pr(>|t|)
phi      -3.96e+01   2.43e+01   -1.63   0.1144
beta      6.09e-01   3.45e-02   17.64  2.4e-16
zeta      2.09e-06   1.32e-05    0.16   0.8758
lambda_L  7.98e-03   6.68e-04   11.95  2.8e-12
lambda_E  8.57e-03   2.48e-03    3.45   0.0018

Residual standard error: 0.0105 on 27 degrees of freedom

Algorithm "port", convergence message: relative convergence (4)
Waiting for profiling to be done...
             2.5%      97.5%
phi            NA -10.290831
beta     0.514667   0.665371
zeta           NA         NA
lambda_L 0.006428   0.009152
lambda_E 0.000715   0.012468
         Estimate      SE
1 - beta   0.3911 0.03453
[1] 27
[1] 0.3202 0.4619
              phi  alpha   beta      zeta lambda_L lambda_E
+ 95% CI   -10.29 0.4619 0.6654        NA 0.009152 0.012468
CES with Q -39.64 0.3911 0.6089 2.085e-06 0.007979 0.008570
- 95% CI       NA 0.3202 0.5147        NA 0.006428 0.000715
```

```
#CESqTables <- lapply(countries, cesTable, energyType="Q")
```

```r
print(tableCESq, caption.placement="top")
```

Table 11: US, 1980-2011.

	phi	alpha	beta	zeta	lambda_L	lambda_E
+ 95% CI	-10.3	0.46	0.67		0.00915	0.01247
CES with Q	-39.6	0.39	0.61	0.000002	0.00798	0.00857
- 95% CI		0.32	0.51		0.00643	0.00071

```r
#print(CESqTables[["US"]], caption.placement="top")
#print(CESqTables[["ZA"]], caption.placement="top")
```