An Empirical Analysis of the Role of Energy in Economic Growth

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Abstract

********** Add abstract **********

Keywords: economic growth, energy, cobb-douglas, CES, LINEX

********** To Do List **********

- Reese: Add text from Word verson of paper to the LaTeX version.
- ~~Heun: Create a table of AIC values for each fit.~~
- ~~Heun: Add u predictions for Cobb-Douglas~~
- Heun: Add u parameter table and graph
- Heun: Add covariance metrics.
- Heun: Eliminate blanks in the coefficient tables for the 95% CIs in the Cobb-Douglas with energy rows. Asked Pruim about this via email but have not heard a response.
  - ZA: lower bound on λ and upper bound on α
  - ZM: lower bound on λ
- Heun: Fix warnings of the form "Warning: step factor 0.000488281 reduced below minFactor of 0.000976562" in the code that generates the Cobb-Douglas with energy fits.

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- Heun: Add CES production function fits
- Heun: Add Linex production function fits
- Heun: Add k, l, q, x, and u ONLY fits

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Caleb, put your LaTeX code here.

1. The Facts

Figure 1 shows the "facts" as they are known.

2. Cobb-Douglas Without Energy

The Cobb-Douglas model without energy is given by

$$y = a^{\lambda(t-t_0)} k^{\alpha} l^{\beta}. \tag{1}$$

Table 1 gives the parameters for the Cobb-Douglas model without energy.

Table 1: Cobb-Douglas (without energy) for 1980-2011 (US, UK, JP), 1991-2010 (CN and ZA), and 1991-2011 (SA, IR, TZ, and ZM). (Parameter estimates beneath symbol. 95% confidence interval bounds to left and right.)

	λ			α			β		
US	0.0087	0.0102	0.0116	0.21	0.27	0.34	0.66	0.73	0.79
UK	-0.0104	0.0097	0.0303	-0.25	0.44	1.12	-0.13	0.56	1.24
JP	0.0021	0.0052	0.0082	0.44	0.52	0.59	0.41	0.48	0.56
CN	-0.0405	0.0188	0.0779	0.11	0.71	1.32	-0.32	0.29	0.89
ZA	-0.0007	0.0008	0.0022	0.46	0.60	0.73	0.26	0.40	0.54
SA	-0.0159	-0.0123	-0.0087	0.21	0.45	0.68	0.32	0.55	0.78
IR	0.0032	0.0039	0.0045	0.49	0.60	0.70	0.30	0.40	0.51
TZ	-0.0039	0.0015	0.0068	0.50	0.73	0.95	0.05	0.27	0.50
ZM	0.0218	0.0249	0.0280	1.25	1.41	1.57	-0.57	-0.41	-0.25

Figure 2 shows values and 95% confidence intervals for the parameters for the Cobb-Douglas model (without energy).

3. Cobb-Douglas With Energy

We can force α, β, and γ to be in $[0, 1]$ by a reparameterization:
$a \in [0, 1], b \in [0, 1], \alpha = \min(a, b), \beta = |b - a|, \gamma = 1 - \max(a, b)$
The Cobb-Douglas model with energy is given by

$$y = a^{\lambda(t-t_0)} k^{\alpha} l^{\beta} e^{\gamma}, \tag{2}$$

where e can be any of thermal energy (q), exergy (x), or useful work (u).

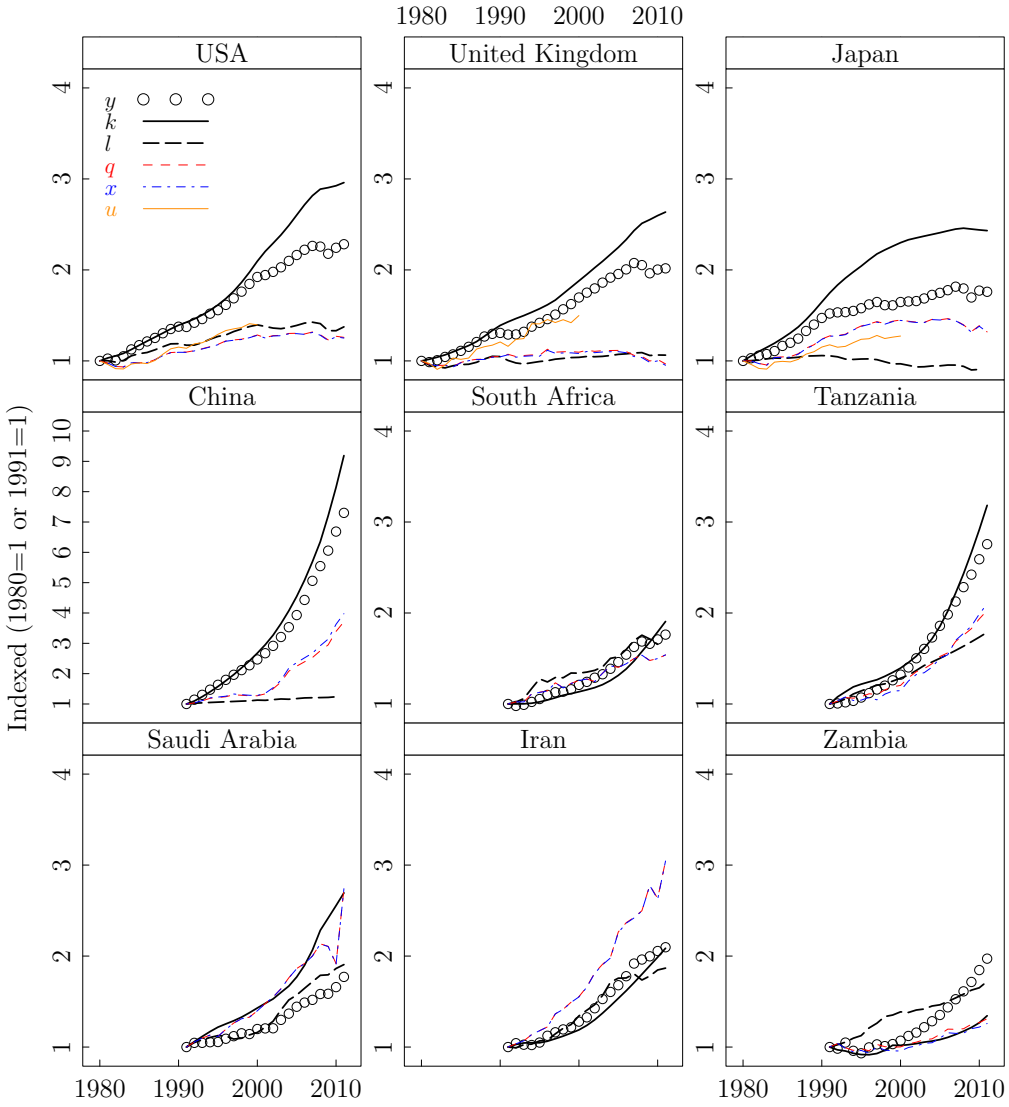

Figure 1: The facts. Indexed GDP (y), capital stock (k), labor (l), thermal energy (q), exergy (x), and useful work (u).

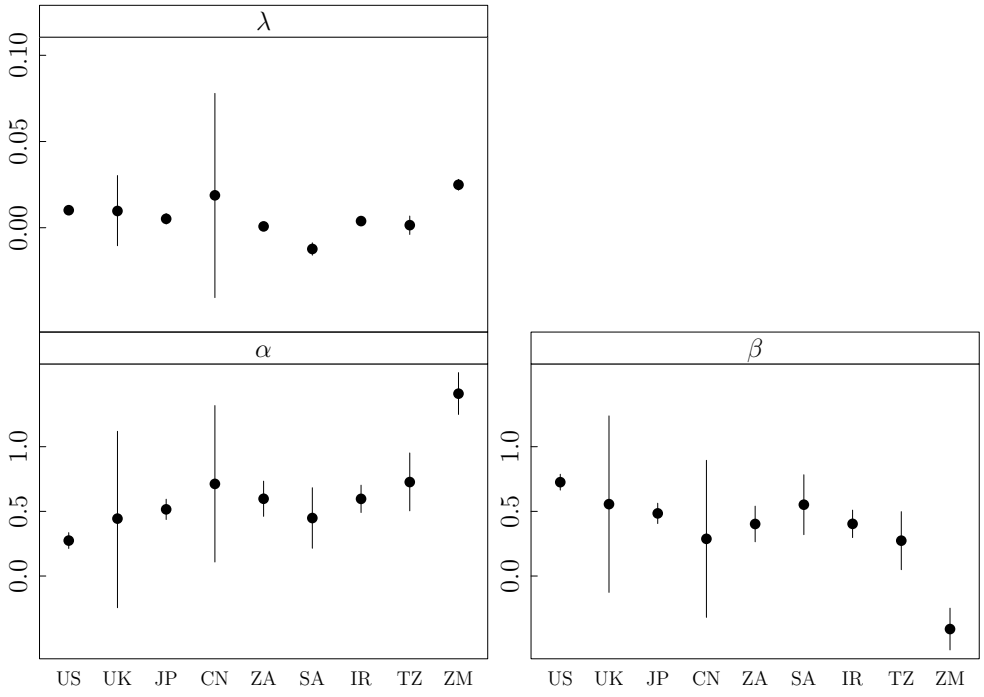

Figure 2: Cobb-Douglas (without energy) model parameters. Vertical bars indicate 95% confidence intervals.

3.1. Cobb-Douglas with q

The Cobb-Douglas (with thermal energy, q) parameters are given in Table 2.

Table 2: Cobb-Douglas (with q) for 1980-2011 (US, UK, JP), 1991-2010 (CN and ZA), and 1991-2011 (SA, IR, TZ, and ZM). (Parameter estimates beneath symbol. 95% confidence interval bounds to left and right.)

		λ			α			β			γ	
US	0.0078	0.0102	0.0126	0.19	0.27	0.36	0.59	0.72	0.85	-0.17	0.00	0.17
UK	0.0075	0.0228	0.0382	-0.52	-0.00	0.52	0.07	0.56	1.04	0.28	0.44	0.61
JP	0.0019	0.0049	0.0079	0.45	0.57	0.70	0.42	0.55	0.67	-0.31	-0.12	0.07
CN	-0.0087	0.0133	0.0872	-0.02	0.78	0.85	-0.48	0.24	0.96	-0.15	-0.02	0.11
ZA		0.0048	0.0054	0.35	0.43		-0.54	0.00	0.54	-0.02	0.57	1.17
SA	-0.0165	-0.0137	-0.0109	0.17	0.36	0.54	0.21	0.40	0.60	0.11	0.24	0.37
IR	-0.0026	0.0033	0.0092	0.43	0.59	0.74	0.18	0.39	0.59	-0.25	0.03	0.31
TZ	0.0044	0.0057	0.0095	0.31	0.44	0.71	-0.43	-0.00	0.43	0.12	0.56	1.00
ZM		0.0197	0.0208	0.54	0.66	0.78	-0.40	0.00	0.40	-0.74	0.34	1.42

Figure 3 shows values and 95% confidence intervals for the parameters for the Cobb-Douglas model (with q).

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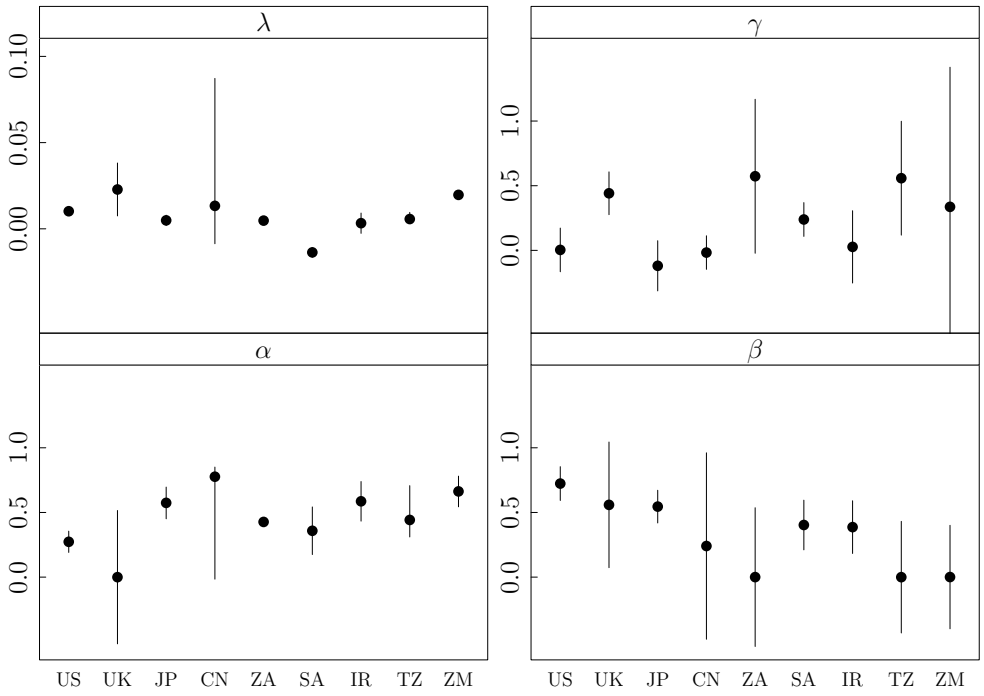

Figure 3: Cobb-Douglas (with q) model parameters. Vertical bars indicate 95% confidence intervals.

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The Cobb-Douglas (with exergy) parameters are given in Table 3.

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Figure 4 shows values and 95% confidence intervals for the parameters for the Cobb-Douglas model (with x).

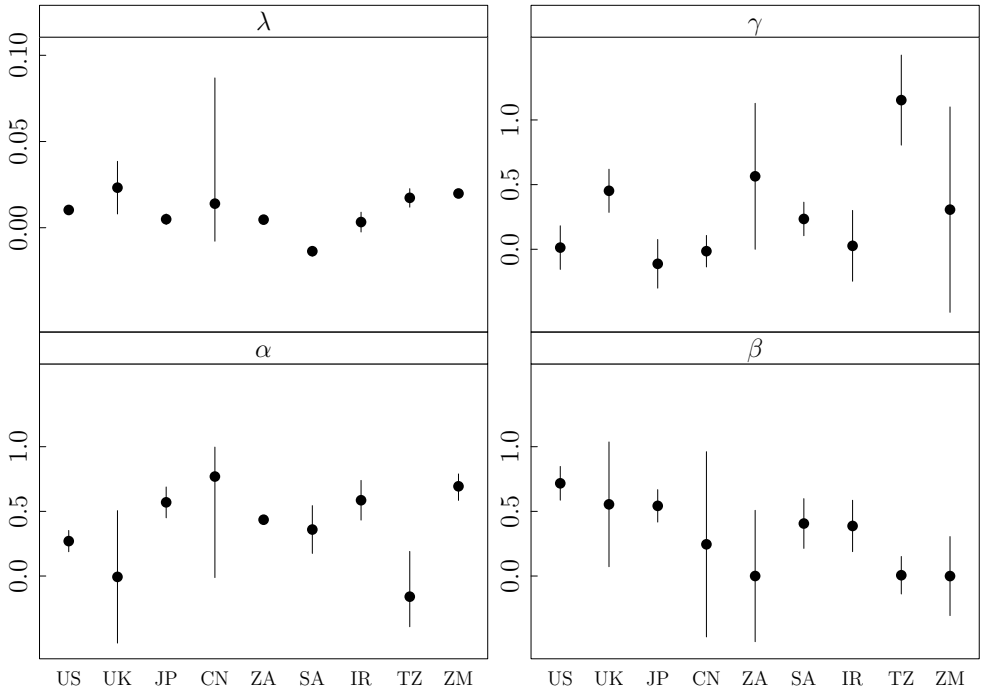

Figure 4: Cobb-Douglas (with x) model parameters. Vertical bars indicate 95% confidence intervals.

Table 3: Cobb-Douglas (with x) for 1980-2011 (US, UK, JP), 1991-2010 (CN and ZA), and 1991-2011 (SA, IR, TZ, and ZM). (Parameter estimates beneath symbol. 95% confidence interval bounds to left and right.)

	λ			α			β			γ		
US	0.0079	0.0103	0.0127	0.19	0.27	0.35	0.59	0.72	0.85	-0.16	0.01	0.18
UK	0.0080	0.0232	0.0385	-0.52	-0.01	0.51	0.07	0.55	1.04	0.29	0.45	0.62
JP	0.0019	0.0049	0.0080	0.45	0.57	0.69	0.42	0.54	0.67	-0.30	-0.11	0.08
CN	-0.0078	0.0140	0.0869	-0.01	0.77	1.00	-0.47	0.25	0.96	-0.14	-0.01	0.11
ZA		0.0047	0.0054	0.36	0.44		-0.51	0.00	0.51	0.00	0.56	1.13
SA	-0.0164	-0.0136	-0.0108	0.17	0.36	0.54	0.21	0.41	0.60	0.11	0.23	0.36
IR	-0.0025	0.0033	0.0090	0.43	0.59	0.74	0.19	0.39	0.59	-0.25	0.03	0.30
TZ	0.0119	0.0173	0.0227	-0.39	-0.16	0.19	-0.14	0.01	0.15	0.81	1.15	1.50
ZM		0.0199	0.0209	0.58	0.69	0.79	-0.31	-0.00	0.31	-0.49	0.31	1.10

3.2. Cobb-Douglas Comparisons

Figure 5 compares predictions from the Cobb-Douglas models (without energy, with Q, and with x) to historical data.

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4. CES

4.1. CES with Q
4.2. CES with X
4.3. CES with U

5. LINEX

5.1. LINEX with Q
5.2. LINEX with X
5.3. LINEX with U

6. Goodness of Fit

We assess goodness of fit via the Akaike Information Criterion (AIC). AIC values for all models and all countries are shown in Table 4. Increasing goodness of fit is indicated by smaller (i.e., more negative) AIC values. AIC values can be compared per data set (i.e., per country) but not across data sets (i.e., not across countries).

The AIC results show **********.

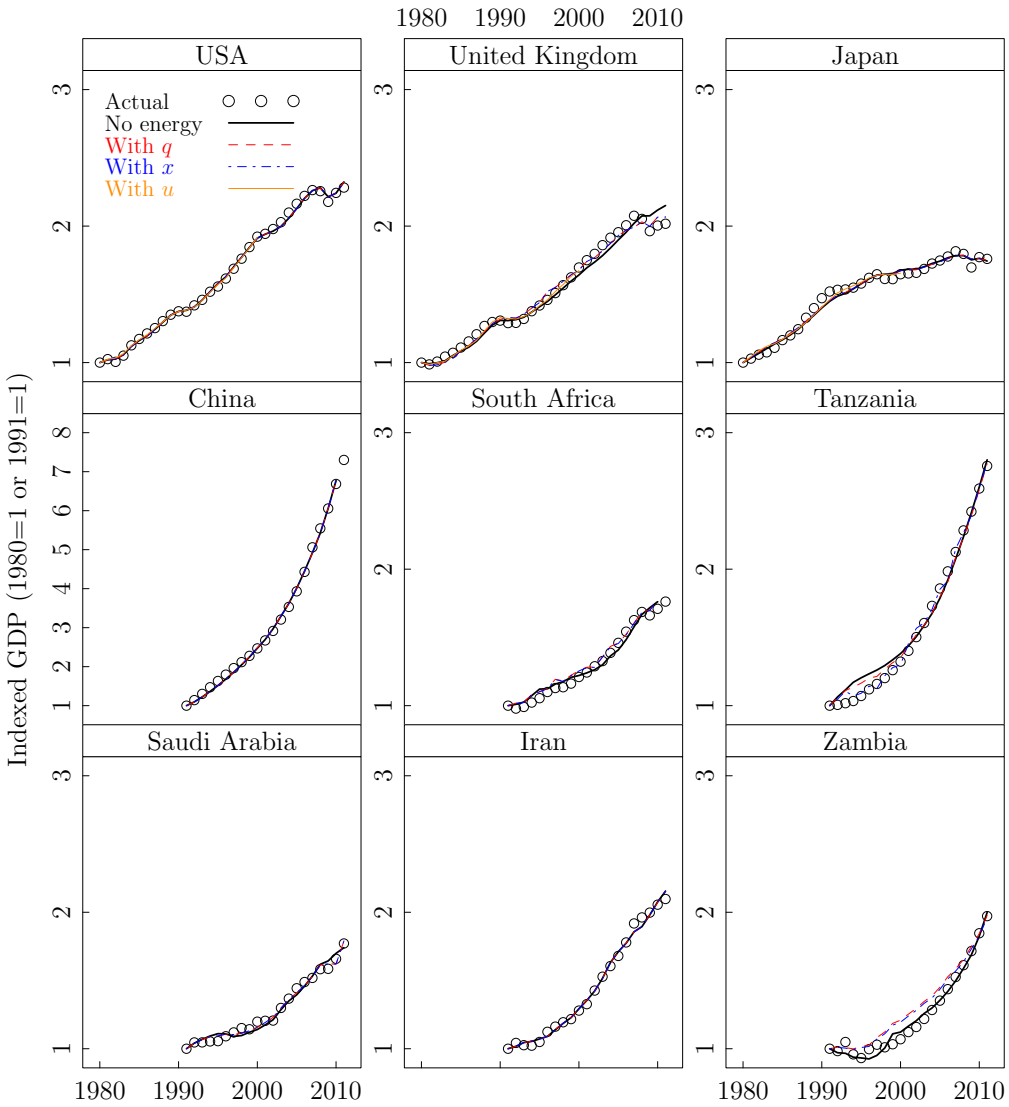

Figure 5: Cobb-Douglas results.

Table 4: AIC values for all models.

	US	UK	JP	CN	ZA	SA	IR	TZ	ZM
CD	-159.4	-92.4	-126.8	-38.0	-66.5	-75.8	-81.1	-44.4	-67.9
CDq	-157.4	-113.2	-126.5	-36.0	-63.7	-86.4	-79.1	-52.5	-35.1
CDx	-157.5	-113.7	-126.4	-36.0	-65.0	-86.3	-79.1	-70.9	-40.4
CDu	-128.2	-92.9	-79.6						