Empirical Analysis of the Role of Energy in Economic Growth

Caleb Reese[a], Lucas Timmer[a], Matthew Kuperus Heun[a,*]

[a]*Engineering Department, Calvin College, Grand Rapids, MI 49546, USA*

Abstract

********** Add abstract **********

Keywords: economic growth, energy, cobb-douglas, CES, LINEX

Caleb, put your LaTeX code here.

1. Cobb-Douglas Without Energy

```
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
```

2. Cobb-Douglas With Energy

We can force α, β, and γ to be in $[0, 1]$ by a reparameterization:

$$a \in [0,1], b \in [0,1], \alpha = \min(a,b), \beta = |b-a|, \gamma = 1 - \max(a,b)$$

2.1. Cobb-Douglas with Q

*Corresponding author

Email address: mkh2@calvin.edu, tel: +1 (616) 526-6663, fax: +1 (616) 526-6501 (Matthew Kuperus Heun)

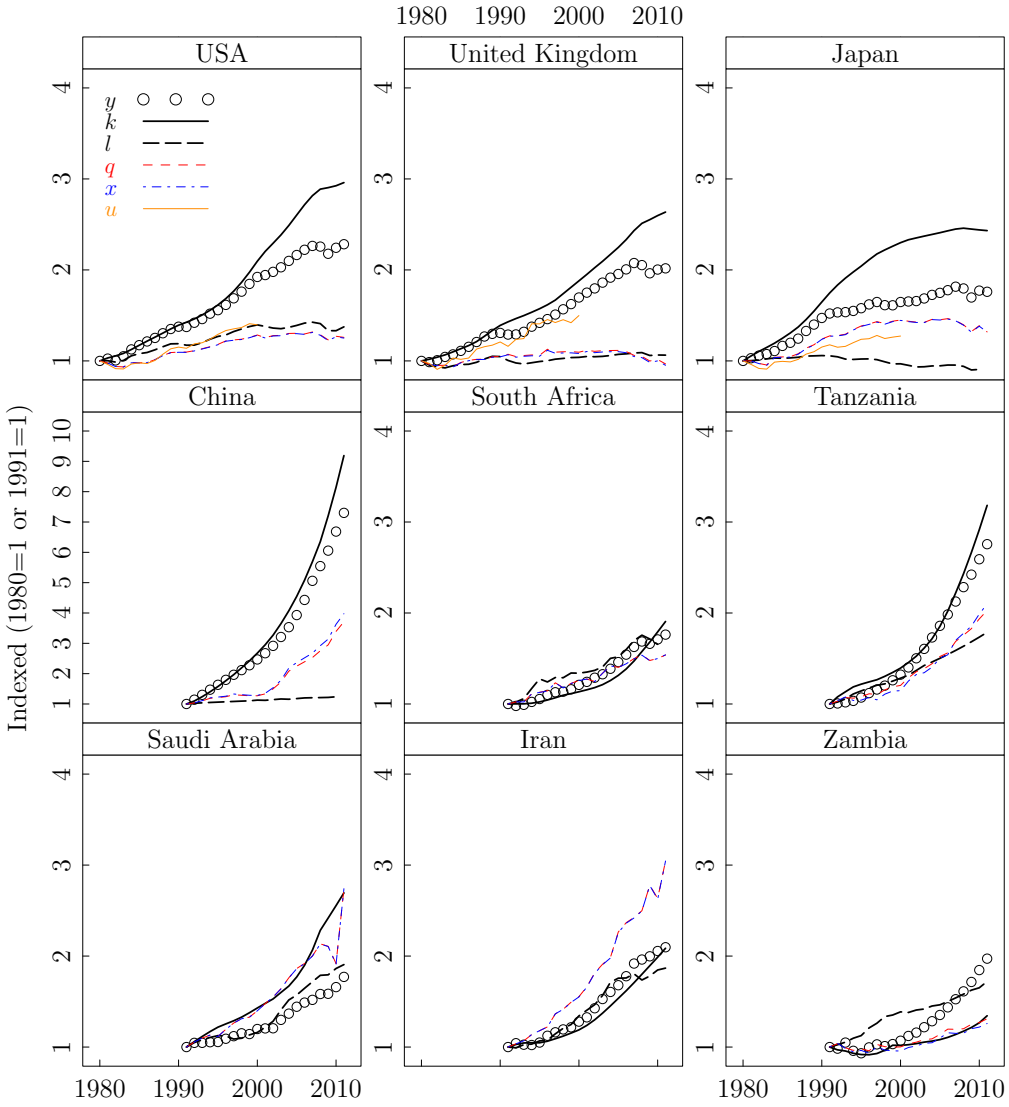

Figure 1: The facts. Indexed GDP (y), capital stock (k), labor (l), thermal energy (q), exergy (x), and useful work (u) for all economies.

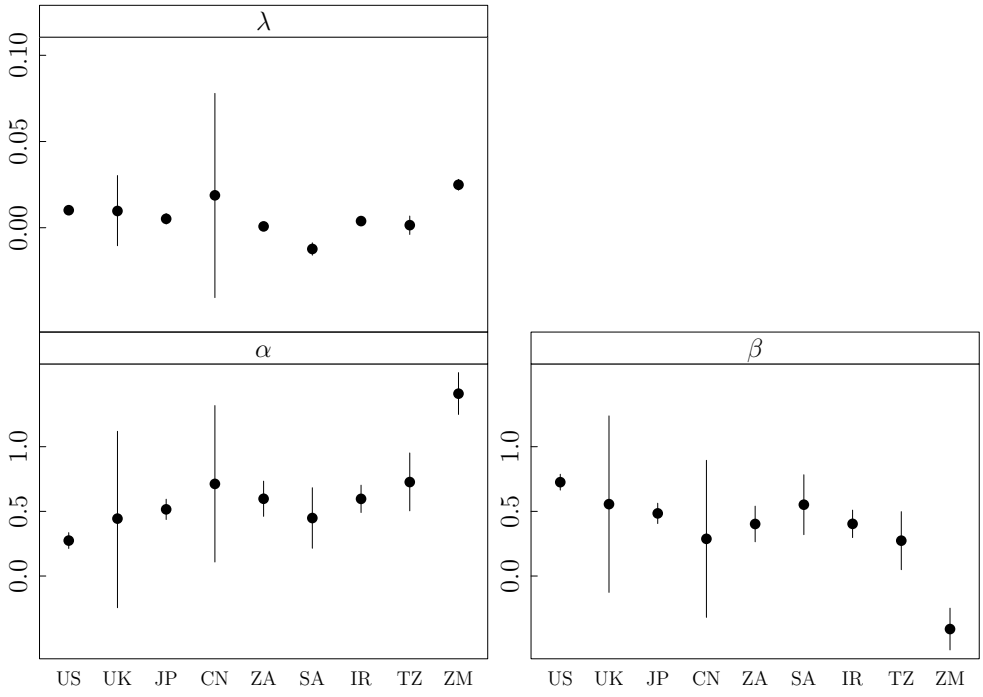

Figure 2: Cobb-Douglas (without energy) model parameters. Vertical bars indicate 95% confidence intervals.

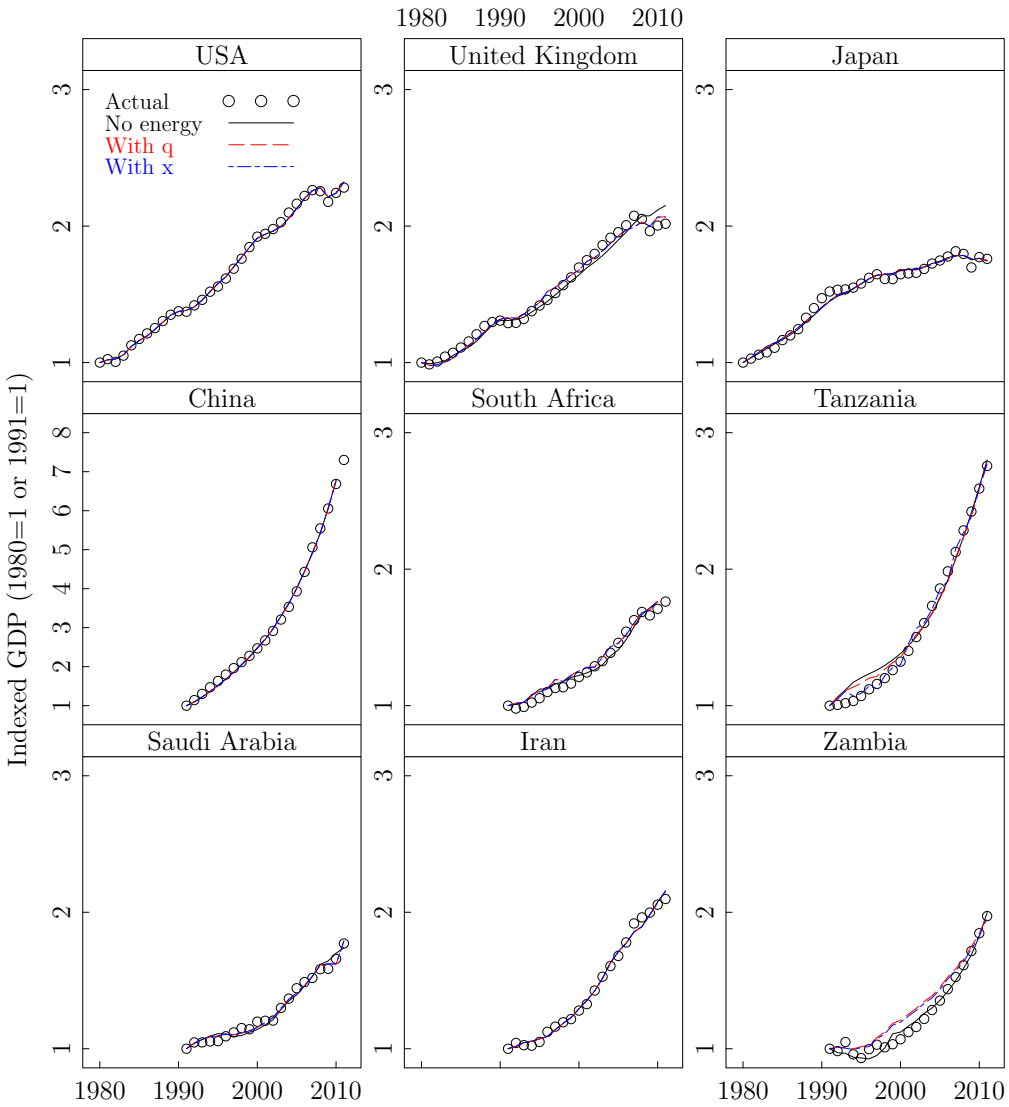

Figure 3: Cobb-Douglas results.

Table 1: Cobb-Douglas (with q) for 1980-2011 (US, UK, JP) or 1991-2011 (others). (Parameter estimates beneath symbol. 95% confidence interval bounds to left and right.)

		λ			α			β			γ	
US	0.0078	0.0102	0.0126	0.19	0.27	0.36	0.59	0.72	0.85	-0.17	0.00	0.17
UK	0.0075	0.0228	0.0382	-0.52	-0.00	0.52	0.07	0.56	1.04	0.28	0.44	0.61
JP	0.0019	0.0049	0.0079	0.45	0.57	0.70	0.42	0.55	0.67	-0.31	-0.12	0.07
CN	-0.0087	0.0133	0.0872	-0.02	0.78	0.85	-0.48	0.24	0.96	-0.15	-0.02	0.11
ZA		0.0048	0.0054	0.35	0.43		-0.54	0.00	0.54	-0.02	0.57	1.17
SA	-0.0165	-0.0137	-0.0109	0.17	0.36	0.54	0.21	0.40	0.60	0.11	0.24	0.37
IR	-0.0026	0.0033	0.0092	0.43	0.59	0.74	0.18	0.39	0.59	-0.25	0.03	0.31
TZ	0.0044	0.0057	0.0095	0.31	0.44	0.71	-0.43	-0.00	0.43	0.12	0.56	1.00
ZM		0.0197	0.0208	0.54	0.66	0.78	-0.40	0.00	0.40	-0.74	0.34	1.42

```
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  number of iterations exceeded maximum of 200
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
```

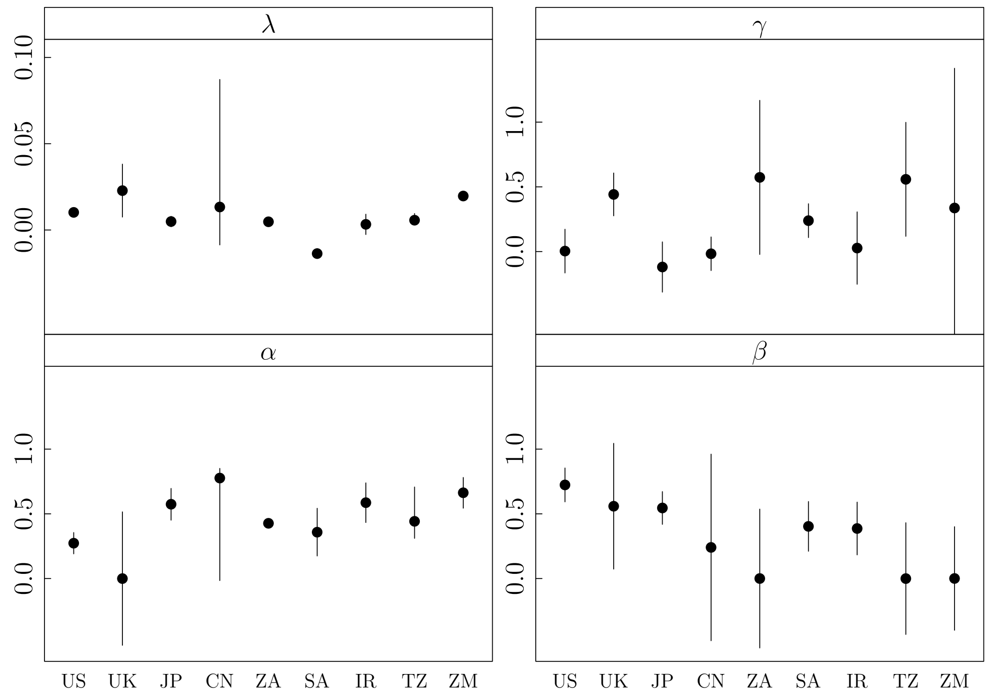

Figure 4: Cobb-Douglas (with q) model parameters. Vertical bars indicate 95% confidence intervals.

```
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
```

```
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
```

```
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Warning:  step factor 0.000488281 reduced below 'minFactor' of 0.000976562
```

Table 2: Cobb-Douglas (with x) for 1980-2011 (US, UK, JP) or 1991-2011 (others). (Parameter estimates beneath symbol. 95% confidence interval bounds to left and right.)

		λ			α			β			γ	
US	0.0079	0.0103	0.0127	0.19	0.27	0.35	0.59	0.72	0.85	-0.16	0.01	0.18
UK	0.0080	0.0232	0.0385	-0.52	-0.01	0.51	0.07	0.55	1.04	0.29	0.45	0.62
JP	0.0019	0.0049	0.0080	0.45	0.57	0.69	0.42	0.54	0.67	-0.30	-0.11	0.08
CN	-0.0078	0.0140	0.0869	-0.01	0.77	1.00	-0.47	0.25	0.96	-0.14	-0.01	0.11
ZA		0.0047	0.0054	0.36	0.44		-0.51	0.00	0.51	0.00	0.56	1.13
SA	-0.0164	-0.0136	-0.0108	0.17	0.36	0.54	0.21	0.41	0.60	0.11	0.23	0.36
IR	-0.0025	0.0033	0.0090	0.43	0.59	0.74	0.19	0.39	0.59	-0.25	0.03	0.30
TZ	0.0119	0.0173	0.0227	-0.39	-0.16	0.19	-0.14	0.01	0.15	0.81	1.15	1.50
ZM		0.0199	0.0209	0.58	0.69	0.79	-0.31	-0.00	0.31	-0.49	0.31	1.10

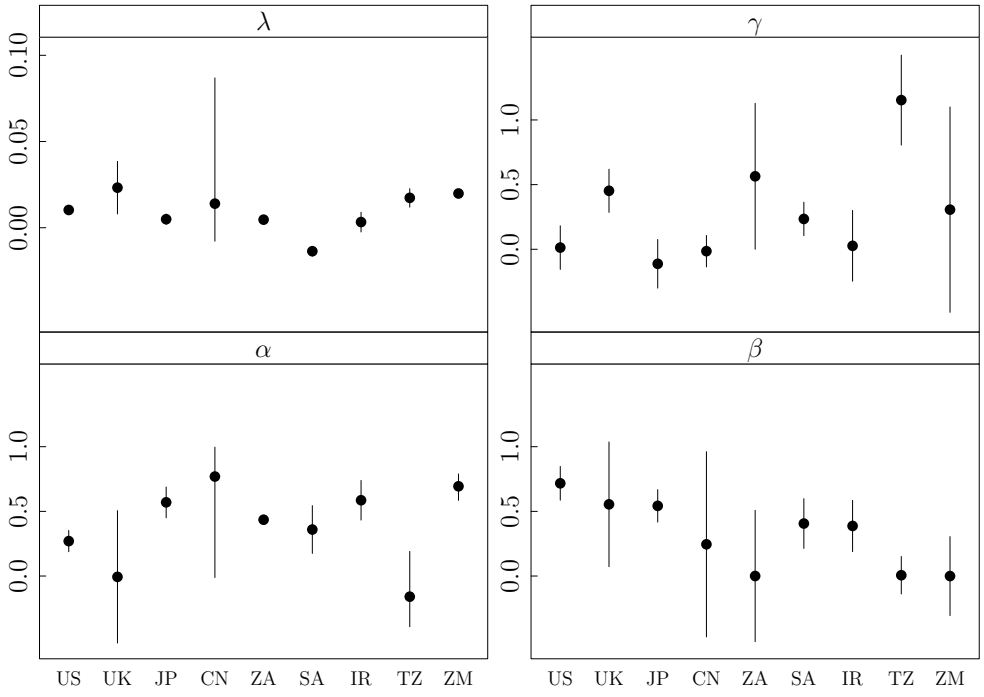

Figure 5: Cobb-Douglas (with x) model parameters. Vertical bars indicate 95% confidence intervals.

3. CES

3.1. CES with Q