An Empirical Analysis of the Role of Energy in Economic Growth

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Abstract

*********** Add abstract ***********

Keywords: economic growth, energy, cobb-douglas, CES, LINEX

To-do List

- Reese: Add text from Word verson of paper to the LaTeX version.
- Heun: Add setup variables for figure height and width where not already included.
- Heun: Deal with missing covariance numbers when columns have unequal length.
- Heun: Add single factor fits
  - parameter graphs (λ and m) for each country
  - parameter tables for each country in the appendix
  - ~~GDP comparison graph~~
  - ~~AIC rows in the AIC table~~
- Heun: Add CES production function fits

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- GDP comparison graph
  - parameter graphs for each country
  - parameter tables for each country in the appendix
  - AIC rows in the AIC table

- Heun: Add Linex production function fits
  - GDP comparison graph
  - parameter graphs for each country
  - parameter tables for each country in the appendix
  - AIC rows in the AIC table

- Heun: Eliminate blanks in the coefficient tables for the 95% CIs in the Cobb-Douglas with energy rows. Asked Pruim about this via email on 4 Feb 2013 but have not heard a response.
  - ZA: lower bound on λ and upper bound on α
  - ZM: lower bound on λ

- Heun: Fix warnings of the form "Warning: step factor 0.000488281 reduced below minFactor of 0.000976562" in the code that generates the Cobb-Douglas with energy fits.

- Heun: Ask Martin if he can supply an "Economics" affiliation. Will probably make a stronger paper.

- ~~Heun: Create a table of AIC values for each fit.~~

- ~~Heun: Add u predictions for Cobb-Douglas~~

- ~~Heun: move tables to an appendix at the end.~~

- ~~Heun: Add acknowledgements (Dad, Pruim)~~

- ~~Heun: Add u parameter table and graph~~

- ~~Heun: Add covariance metrics.~~

1. Introduction

*********** Caleb, put your LaTeX code here. *************

2. Historical Data

Figure 1 shows historical data, including GDP (y), capital stock (k), labor (l), and the various energy types (q, x, and u). Factors of production track each other closely, and large covariances among these variables can be seen in Tables A.1 through A.9.

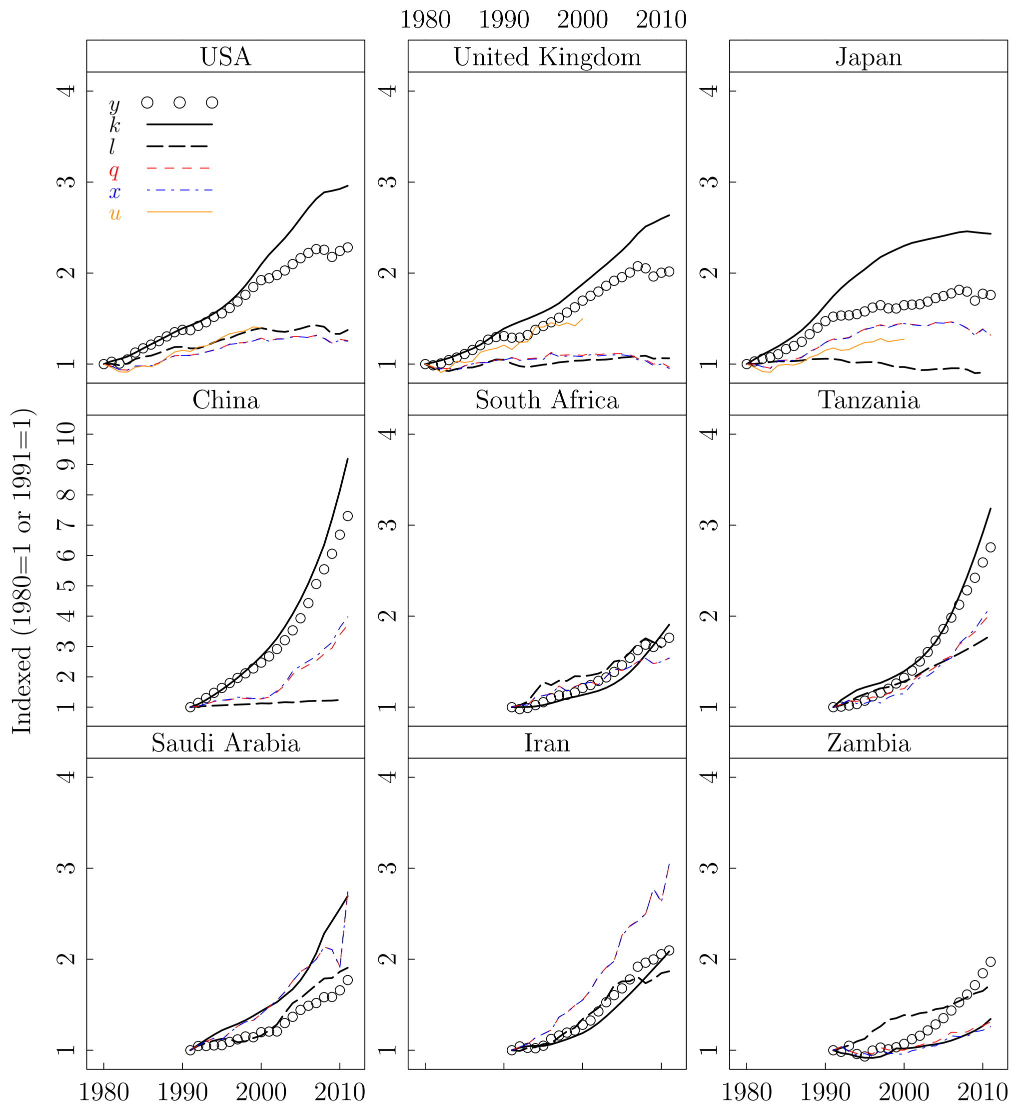

Figure 1: Historical data. Indexed GDP (y), capital stock (k), labor (l), thermal energy (q), exergy (x), and useful work (u).

3. Single-factor Models (SF)

We begin by fitting single-factor economic models of the form

$$y = a^{\lambda(t-t_0)} f^m \tag{1}$$

to historical data where the factor of production (f) is any of k, l, q, x, or u, and the exponent (m) is α for k, β for l, or γ for q, x, or u.

Figure 2 compares single-factor model predictions with historica data.

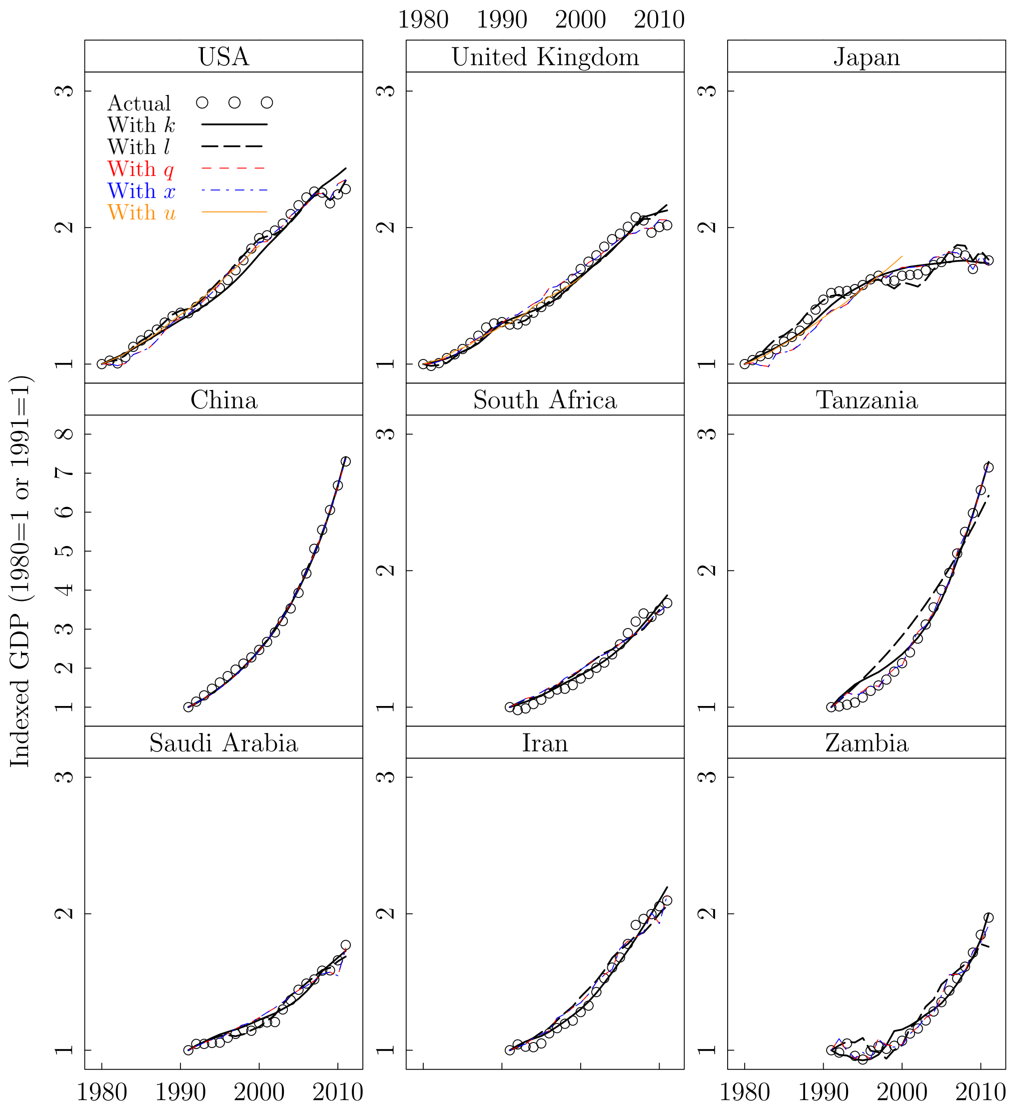

Figure 2: Single-factor results.

4. Cobb-Douglas Models Without Energy (CD)

The Cobb-Douglas model without energy is given by

$$y = a^{\lambda(t-t_0)} k^{\alpha} l^{\beta}. \tag{2}$$

Figure 3 shows values and 95% confidence intervals for the parameters for the Cobb-Douglas model (without energy).

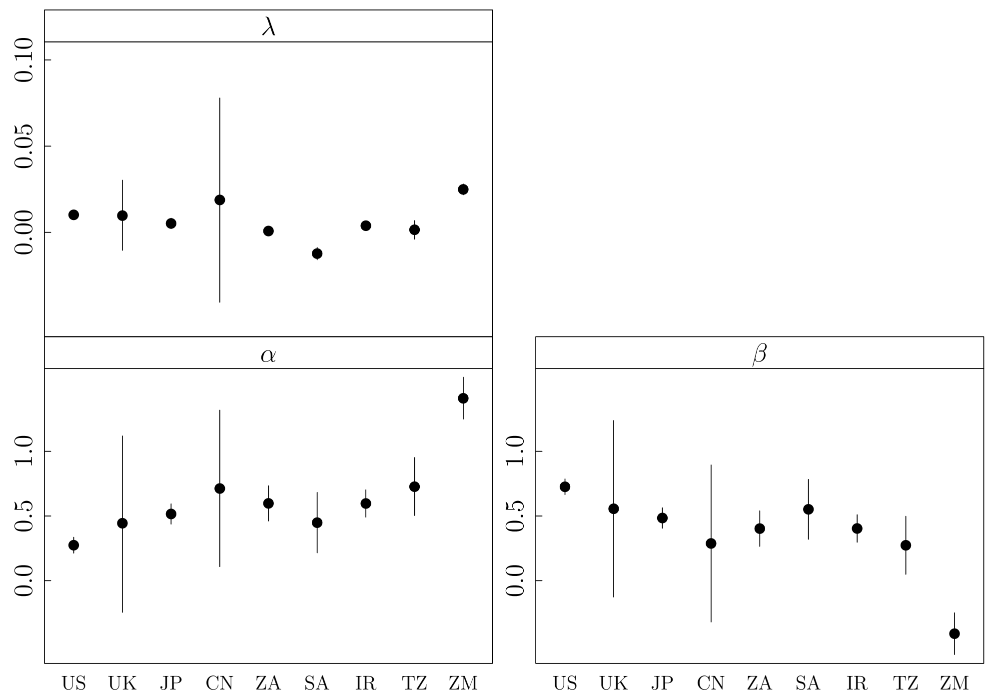

Figure 3: Cobb-Douglas (without energy) model parameters. Vertical bars indicate 95% confidence intervals.

5. Cobb-Douglas Models With Energy (CDe)

We can force α, β, and γ to be in $[0, 1]$ by a reparameterization:
$a \in [0, 1], b \in [0, 1], \alpha = \min(a, b), \beta = |b - a|, \gamma = 1 - \max(a, b)$
The Cobb-Douglas model with energy is given by

$$y = a^{\lambda(t-t_0)} k^{\alpha} l^{\beta} e^{\gamma}, \tag{3}$$

where e can be any of thermal energy (q), exergy (x), or useful work (u).

5.1. Cobb-Douglas with thermal energy (q)

Figure 4 shows values and 95% confidence intervals for the parameters for the Cobb-Douglas model (with q).

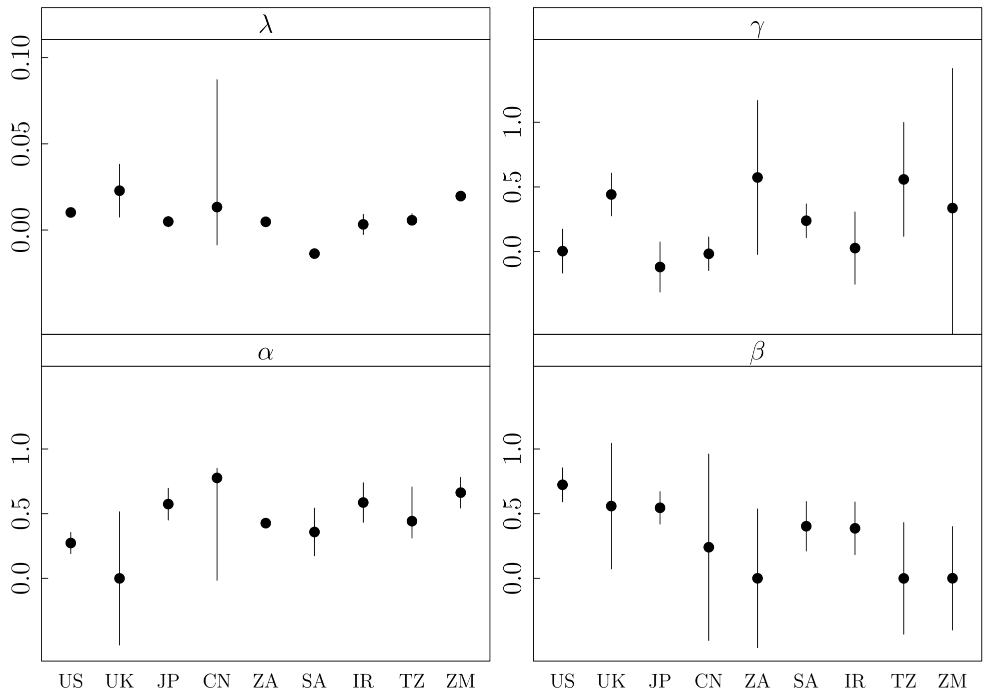

Figure 4: Cobb-Douglas (with q) model parameters. Vertical bars indicate 95% confidence intervals.

5.2. Cobb-Douglas with exergy (x)

The Cobb-Douglas (with exergy) parameters are given in Table B.6.

Figure 5 shows values and 95% confidence intervals for the parameters for the Cobb-Douglas model (with x).

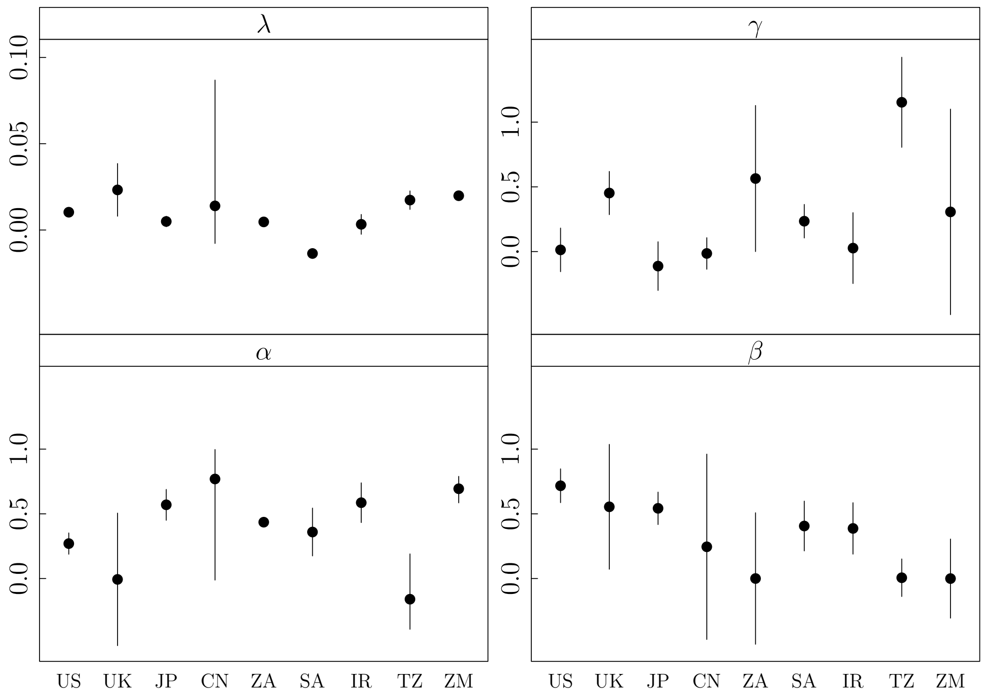

Figure 5: Cobb-Douglas (with x) model parameters. Vertical bars indicate 95% confidence intervals.

5.3. Cobb-Douglas with useful work (u)

Figure 6 shows values and 95% confidence intervals for the parameters for the Cobb-Douglas model (with u).

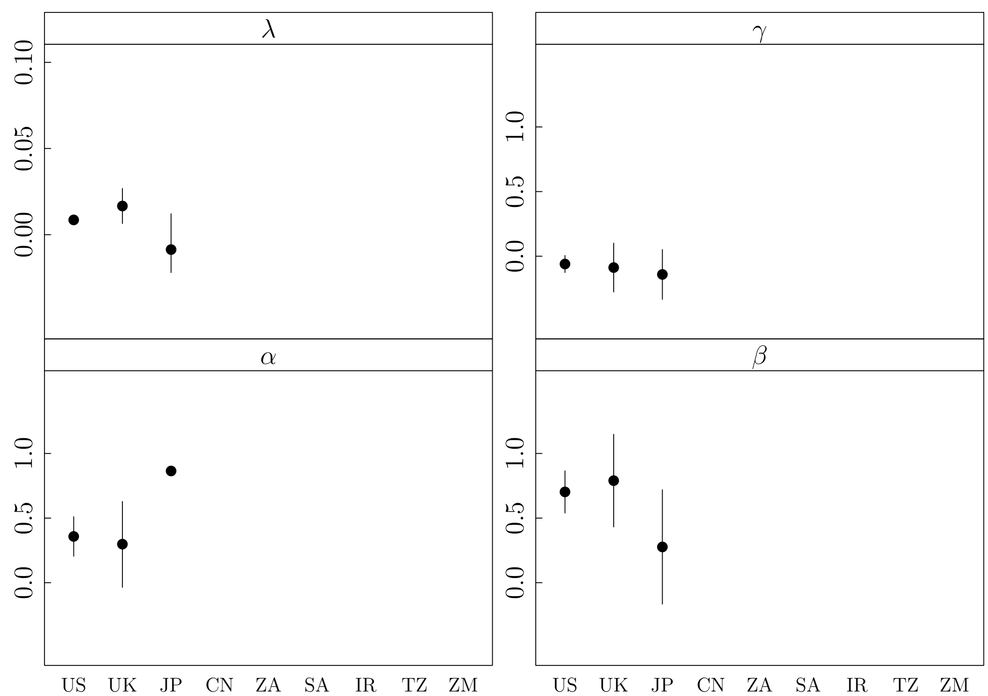

Figure 6: Cobb-Douglas (with u) model parameters. Vertical bars indicate 95% confidence intervals.

5.4. Cobb-Douglas Comparisons

Figure 7 compares predictions from the Cobb-Douglas models (without energy, with Q, and with x) to historical data.

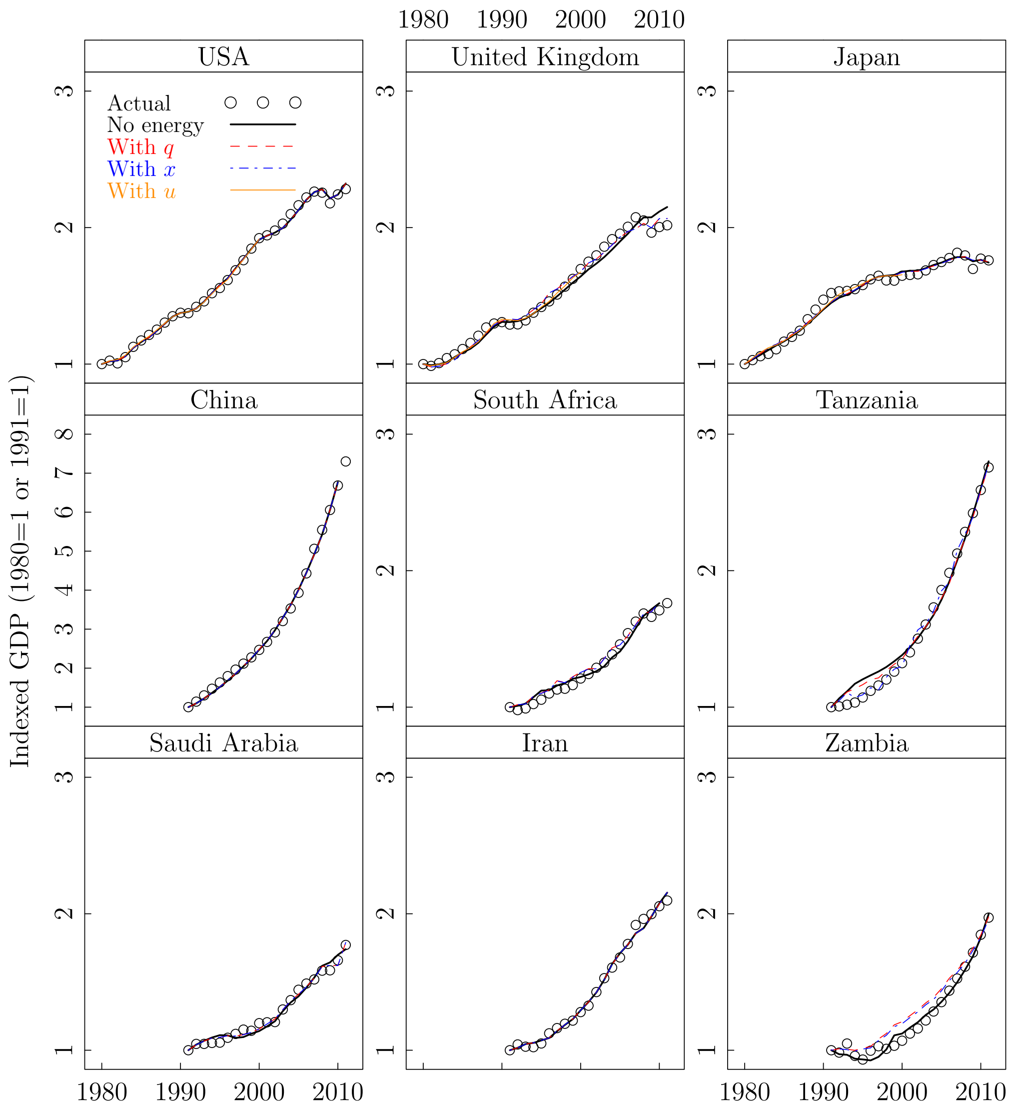

Figure 7: Cobb-Douglas results.

6. CES Models

6.1. CES with Q

6.2. CES with X

6.3. CES with U

7. LINEX Models

7.1. LINEX with Q

7.2. LINEX with X

7.3. LINEX with U

8. Conclusion

****************** Add conclusion here. *********************

Acknowledgements

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Appendix A. Covariance Tables

The following tables show covariances among GDP (y) and factors of production (k, l, q, x, and u).

Table A.1: Covariance table for US.

	y	k	l	q	x	u
y	1.00	0.99	0.95	0.95	0.95	
k	0.99	1.00	0.88	0.89	0.88	
l	0.95	0.88	1.00	0.98	0.98	
q	0.95	0.89	0.98	1.00	1.00	
x	0.95	0.88	0.98	1.00	1.00	
u						1.00

Table A.2: Covariance table for UK.

	y	k	l	q	x	u
y	1.00	0.99	0.89	0.53	0.47	
k	0.99	1.00	0.85	0.43	0.36	
l	0.89	0.85	1.00	0.56	0.52	
q	0.53	0.43	0.56	1.00	1.00	
x	0.47	0.36	0.52	1.00	1.00	
u						1.00

Table A.3: Covariance table for JP.

	y	k	l	q	x	u
y	1.00	0.98	-0.62	0.96	0.96	
k	0.98	1.00	-0.72	0.96	0.96	
l	-0.62	-0.72	1.00	-0.55	-0.55	
q	0.96	0.96	-0.55	1.00	1.00	
x	0.96	0.96	-0.55	1.00	1.00	
u						1.00

Table A.4: Covariance table for CN.

	y	k	l	q	x
y	1.00	1.00		0.99	0.99
k	1.00	1.00		0.99	0.99
l			1.00		
q	0.99	0.99		1.00	1.00
x	0.99	0.99		1.00	1.00

Table A.5: Covariance table for ZA.

	y	k	l	q	x
y	1.00	0.96		0.97	0.97
k	0.96	1.00		0.89	0.89
l			1.00		
q	0.97	0.89		1.00	1.00
x	0.97	0.89		1.00	1.00

Table A.6: Covariance table for SA.

	y	k	l	q	x
y	1.00	0.99	0.99	0.97	0.97
k	0.99	1.00	0.97	0.95	0.95
l	0.99	0.97	1.00	0.95	0.95
q	0.97	0.95	0.95	1.00	1.00
x	0.97	0.95	0.95	1.00	1.00

Table A.7: Covariance table for IR.

	y	k	l	q	x
y	1.00	0.99	0.97	0.99	0.99
k	0.99	1.00	0.93	0.98	0.98
l	0.97	0.93	1.00	0.97	0.97
q	0.99	0.98	0.97	1.00	1.00
x	0.99	0.98	0.97	1.00	1.00

Table A.8: Covariance table for TZ.

	y	k	l	q	x
y	1.00	0.99	0.98	1.00	1.00
k	0.99	1.00	0.96	0.99	0.99
l	0.98	0.96	1.00	0.98	0.97
q	1.00	0.99	0.98	1.00	1.00
x	1.00	0.99	0.97	1.00	1.00

Table A.9: Covariance table for ZM.

	y	k	l	q	x
y	1.00	0.98	0.88	0.98	0.96
k	0.98	1.00	0.87	0.96	0.94
l	0.88	0.87	1.00	0.86	0.79
q	0.98	0.96	0.86	1.00	0.99
x	0.96	0.94	0.79	0.99	1.00

Appendix B. Model Parameters

Appendix B.1. Single-factor Models

Tables **??** below provide tabular results for single-factor models.

Table B.1: Single-factor model (with k) parameters for 1980-2011 (US, UK, JP), 1991-2010 (CN and ZA), and 1991-2011 (SA, IR, TZ, and ZM). (Parameter estimates beneath symbol. 95% confidence interval bounds to left and right.)

		λ			α	
US	0.0006	0.0154	0.0300	-0.02	0.38	0.78
UK	-0.0326	-0.0099	0.0126	0.39	1.09	1.80
JP	-0.0046	-0.0014	0.0017	0.59	0.67	0.76
CN	-0.0074	0.0644	0.1360	-0.33	0.32	0.98
ZA	0.0143	0.0187	0.0231	0.18	0.35	0.51
SA	-0.0102	0.0019	0.0138	0.26	0.51	0.77
IR	0.0115	0.0182	0.0248	0.38	0.57	0.77
TZ	-0.0017	0.0098	0.0212	0.50	0.72	0.94
ZM	0.0099	0.0128	0.0156	1.23	1.49	1.75

Table B.2: Single-factor model (with l) parameters for 1980-2011 (US, UK, JP), 1991-2010 (CN and ZA), and 1991-2011 (SA, IR, TZ, and ZM). (Parameter estimates beneath symbol. 95% confidence interval bounds to left and right.)

		λ			β	
US	0.0177	0.0194	0.0211	0.65	0.79	0.92
UK	0.0223	0.0234	0.0245	0.33	0.78	1.24
JP	0.0260	0.0268	0.0276	1.75	2.03	2.31
CN	0.0762	0.0947	0.1135	-1.19	0.46	2.08
ZA	0.0204	0.0350	0.0496	-0.73	-0.25	0.22
SA	0.0028	0.0087	0.0144	0.36	0.54	0.73
IR	0.0108	0.0250	0.0391	-0.06	0.36	0.77
TZ	-0.1038	0.0450	0.1977	-5.21	0.06	5.20
ZM	0.0735	0.1000	0.1281	-3.65	-2.64	-1.69

Table B.3: Single-factor model (with q) parameters for 1980-2011 (US, UK, JP), 1991-2010 (CN and ZA), and 1991-2011 (SA, IR, TZ, and ZM). (Parameter estimates beneath symbol. 95% confidence interval bounds to left and right.)

		λ			γ	
US	0.0190	0.0212	0.0234	0.63	0.86	1.08
UK	0.0230	0.0238	0.0246	0.29	0.51	0.73
JP	0.0056	0.0090	0.0122	0.74	0.96	1.18
CN	0.0932	0.0997	0.1061	-0.10	0.00	0.11
ZA	0.0172	0.0329	0.0486	-0.90	-0.23	0.43
SA	0.0009	0.0122	0.0233	0.05	0.31	0.57
IR	-0.0312	-0.0016	0.0276	0.18	0.71	1.25
TZ	-0.0085	-0.0012	0.0060	1.27	1.49	1.71
ZM	-0.0001	0.0042	0.0084	1.73	2.10	2.48

```
Error:  error in evaluating the argument 'x' in selecting a method
for function 'print':  Error in prof$getProfile() :
step factor 0.000488281 reduced below 'minFactor' of 0.000976562
Calls:  singleFactorParamsTable ...  confint.nls -> profile -> profile.nls
-> <Anonymous>
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Appendix B.2. Cobb-Douglas Models

Table B.4 gives the parameters for the Cobb-Douglas model without energy.

Table B.4: Cobb-Douglas model (without energy) parameters for 1980-2011 (US, UK, JP), 1991-2010 (CN and ZA), and 1991-2011 (SA, IR, TZ, and ZM). (Parameter estimates beneath symbol. 95% confidence interval bounds to left and right.)

		λ			α			β	
US	0.0087	0.0102	0.0116	0.21	0.27	0.34	0.66	0.73	0.79
UK	-0.0104	0.0097	0.0303	-0.25	0.44	1.12	-0.13	0.56	1.24
JP	0.0021	0.0052	0.0082	0.44	0.52	0.59	0.41	0.48	0.56
CN	-0.0405	0.0188	0.0779	0.11	0.71	1.32	-0.32	0.29	0.89
ZA	-0.0007	0.0008	0.0022	0.46	0.60	0.73	0.26	0.40	0.54
SA	-0.0159	-0.0123	-0.0087	0.21	0.45	0.68	0.32	0.55	0.78
IR	0.0032	0.0039	0.0045	0.49	0.60	0.70	0.30	0.40	0.51
TZ	-0.0039	0.0015	0.0068	0.50	0.73	0.95	0.05	0.27	0.50
ZM	0.0218	0.0249	0.0280	1.25	1.41	1.57	-0.57	-0.41	-0.25

The Cobb-Douglas (with thermal energy, q) parameters are given in Table B.5.

Table B.5: Cobb-Douglas model (with q) parameters for 1980-2011 (US, UK, JP), 1991-2010 (CN and ZA), and 1991-2011 (SA, IR, TZ, and ZM). (Parameter estimates beneath symbol. 95% confidence interval bounds to left and right.)

		λ			α			β			γ	
US	0.0078	0.0102	0.0126	0.19	0.27	0.36	0.59	0.72	0.85	-0.17	0.00	0.17
UK	0.0075	0.0228	0.0382	-0.52	-0.00	0.52	0.07	0.56	1.04	0.28	0.44	0.61
JP	0.0019	0.0049	0.0079	0.45	0.57	0.70	0.42	0.55	0.67	-0.31	-0.12	0.07
CN	-0.0087	0.0133	0.0872	-0.02	0.78	0.85	-0.48	0.24	0.96	-0.15	-0.02	0.11
ZA		0.0048	0.0054	0.35	0.43		-0.54	0.00	0.54	-0.02	0.57	1.17
SA	-0.0165	-0.0137	-0.0109	0.17	0.36	0.54	0.21	0.40	0.60	0.11	0.24	0.37
IR	-0.0026	0.0033	0.0092	0.43	0.59	0.74	0.18	0.39	0.59	-0.25	0.03	0.31
TZ	0.0044	0.0057	0.0095	0.31	0.44	0.71	-0.43	-0.00	0.43	0.12	0.56	1.00
ZM		0.0197	0.0208	0.54	0.66	0.78	-0.40	0.00	0.40	-0.74	0.34	1.42

Table B.6 gives the parameters for the Cobb-Douglas model with exergy (x).

Table B.6: Cobb-Douglas model (with x) parameters for 1980-2011 (US, UK, JP), 1991-2010 (CN and ZA), and 1991-2011 (SA, IR, TZ, and ZM). (Parameter estimates beneath symbol. 95% confidence interval bounds to left and right.)

		λ			α			β			γ	
US	0.0079	0.0103	0.0127	0.19	0.27	0.35	0.59	0.72	0.85	-0.16	0.01	0.18
UK	0.0080	0.0232	0.0385	-0.52	-0.01	0.51	0.07	0.55	1.04	0.29	0.45	0.62
JP	0.0019	0.0049	0.0080	0.45	0.57	0.69	0.42	0.54	0.67	-0.30	-0.11	0.08
CN	-0.0078	0.0140	0.0869	-0.01	0.77	1.00	-0.47	0.25	0.96	-0.14	-0.01	0.11
ZA		0.0047	0.0054	0.36	0.44		-0.51	0.00	0.51	0.00	0.56	1.13
SA	-0.0164	-0.0136	-0.0108	0.17	0.36	0.54	0.21	0.41	0.60	0.11	0.23	0.36
IR	-0.0025	0.0033	0.0090	0.43	0.59	0.74	0.19	0.39	0.59	-0.25	0.03	0.30
TZ	0.0119	0.0173	0.0227	-0.39	-0.16	0.19	-0.14	0.01	0.15	0.81	1.15	1.50
ZM		0.0199	0.0209	0.58	0.69	0.79	-0.31	-0.00	0.31	-0.49	0.31	1.10

The Cobb-Douglas (with useful work, u) parameters are given in Table B.7.

Table B.7: Cobb-Douglas model (with u) parameters for 1980-2011 (US, UK, JP), 1991-2010 (CN and ZA), and 1991-2011 (SA, IR, TZ, and ZM). (Parameter estimates beneath symbol. 95% confidence interval bounds to left and right.)

		λ			α			β			γ	
US	0.0058	0.0086	0.0113	0.20	0.36	0.51	0.54	0.70	0.87	-0.13	-0.06	0.01
UK	0.0065	0.0166	0.0268	-0.04	0.30	0.63	0.43	0.79	1.15	-0.28	-0.09	0.10
JP	-0.0220	-0.0087	0.0121	0.39	0.86		-0.17	0.28	0.72	-0.33	-0.14	0.05
CN												
ZA												
SA												
IR												
TZ												
ZM												

Appendix C. Goodness of fit

We assess goodness of fit via the Akaike Information Criterion (AIC). AIC values for all models and all countries are shown in Table C.1. Increasing goodness of fit is indicated by smaller (i.e., more negative) AIC values. AIC values can be compared per data set (i.e., per country) but not across data sets (i.e., not across countries).

Table C.1: AIC values for all models.

	US	UK	JP	CN	ZA	SA	IR	TZ	ZM
SFk	-73.1	-89.6	-103.6	-35.8	-69.0	-71.0	-65.3	-45.0	-60.9
SFl	-126.2	-91.6	-98.2	-32.6	-52.1	-80.9	-45.5	-18.2	-38.0
SFq	-104.5	-98.0	-73.9	-34.6	-54.9	-63.3	-49.4	-70.5	-61.5
SFx	-105.1	-98.8	-72.5	-34.6	-54.9	-63.1	-49.3	-71.4	-61.7
SFu	-83.6	-76.5	-43.1						
CD	-159.4	-92.4	-126.8	-38.0	-66.5	-75.8	-81.1	-44.4	-67.9
CDq	-157.4	-113.2	-126.5	-36.0	-63.7	-86.4	-79.1	-52.5	-35.1
CDx	-157.5	-113.7	-126.4	-36.0	-65.0	-86.3	-79.1	-70.9	-40.4
CDu	-128.2	-92.9	-79.6						