An Empirical Analysis of the Role of Energy in Economic Growth

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Abstract

*********** Add abstract ***********

Keywords: economic growth, energy, cobb-douglas, CES, LINEX

To-do List

- Reese: Add text from Word verson of paper to the LaTeX version.
- Heun: Deal with missing covariance numbers when columns have unequal length.
- Heun: Add k, l, q, x, and u ONLY fits
- Heun: Add CES production function fits
- Heun: Add Linex production function fits
- Heun: Eliminate blanks in the coefficient tables for the 95% CIs in the Cobb-Douglas with energy rows. Asked Pruim about this via email but have not heard a response.
  - ZA: lower bound on λ and upper bound on α
  - ZM: lower bound on λ

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- Heun: Fix warnings of the form “Warning: step factor 0.000488281 reduced below minFactor of 0.000976562” in the code that generates the Cobb-Douglas with energy fits.
- ~~Heun: Create a table of AIC values for each fit.~~
- ~~Heun: Add u predictions for Cobb-Douglas~~
- ~~Heun: move tables to an appendix at the end.~~
- ~~Heun: Add acknowledgements (Dad, Pruim)~~
- ~~Heun: Add u parameter table and graph~~
- ~~Heun: Add covariance metrics.~~

1. Introduction

*********** Caleb, put your LaTeX code here. *************

2. Historical Data

Figure 1 shows historical data. GDP (y), capital stock (k), labor (l), and the various energy types (q, x, and u) track each other closely. Covariances among these variables are rather high as shown in Tables A.2 through A.10.

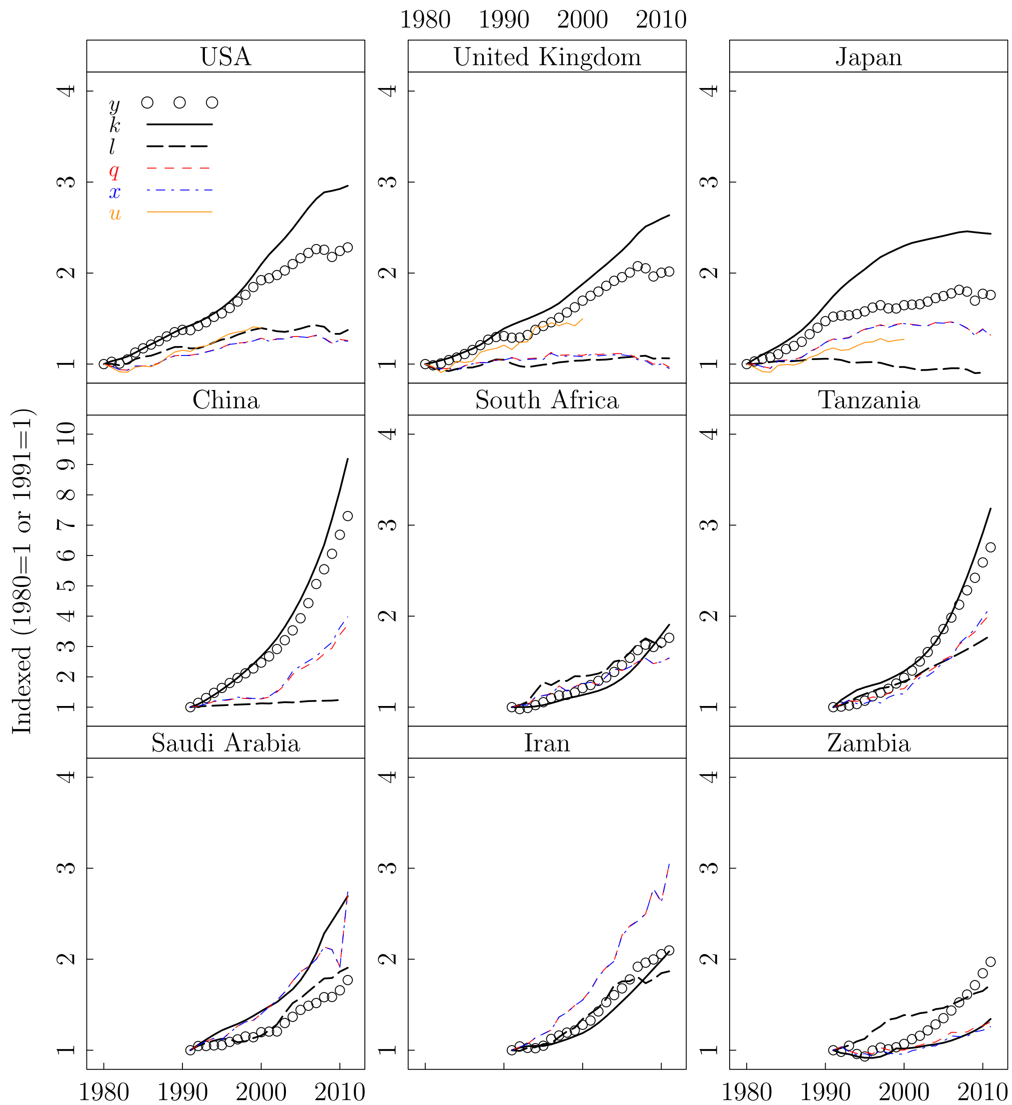

Figure 1: The facts. Indexed GDP (y), capital stock (k), labor (l), thermal energy (q), exergy (x), and useful work (u).

3. Single-factor Models

We begin by fitting single-factor economic models of the form

$$y = a^{\lambda(t-t_0)} f^m \tag{1}$$

to historical data where the factor of production (f) is any of k, l, q, x, or u and the exponent (m) is α for k, β for l, and γ for q, x, or u.

4. Cobb-Douglas Without Energy

The Cobb-Douglas model without energy is given by

$$y = a^{\lambda(t-t_0)} k^\alpha l^\beta. \tag{2}$$

Figure 2 shows values and 95% confidence intervals for the parameters for the Cobb-Douglas model (without energy).

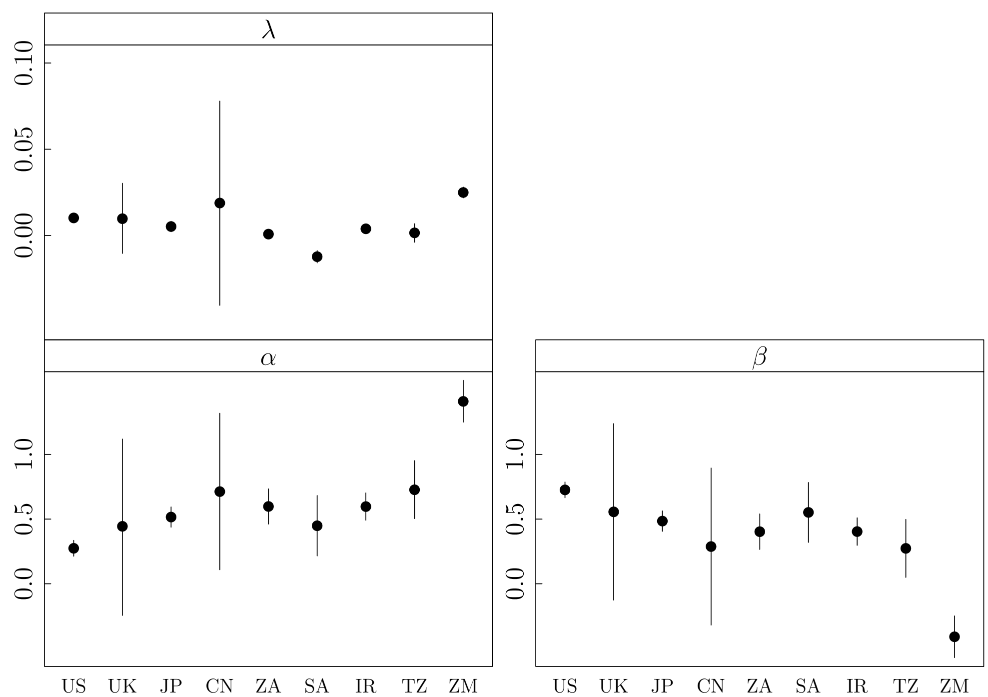

Figure 2: Cobb-Douglas (without energy) model parameters. Vertical bars indicate 95% confidence intervals.

5. Cobb-Douglas With Energy

We can force α, β, and γ to be in $[0, 1]$ by a reparameterization:
$a \in [0, 1], b \in [0, 1], \alpha = \min(a, b), \beta = |b - a|, \gamma = 1 - \max(a, b)$
The Cobb-Douglas model with energy is given by

$$y = a^{\lambda(t-t_0)} k^{\alpha} l^{\beta} e^{\gamma}, \tag{3}$$

where e can be any of thermal energy (q), exergy (x), or useful work (u).

5.1. Cobb-Douglas with thermal energy (q)

Figure 3 shows values and 95% confidence intervals for the parameters for the Cobb-Douglas model (with q).

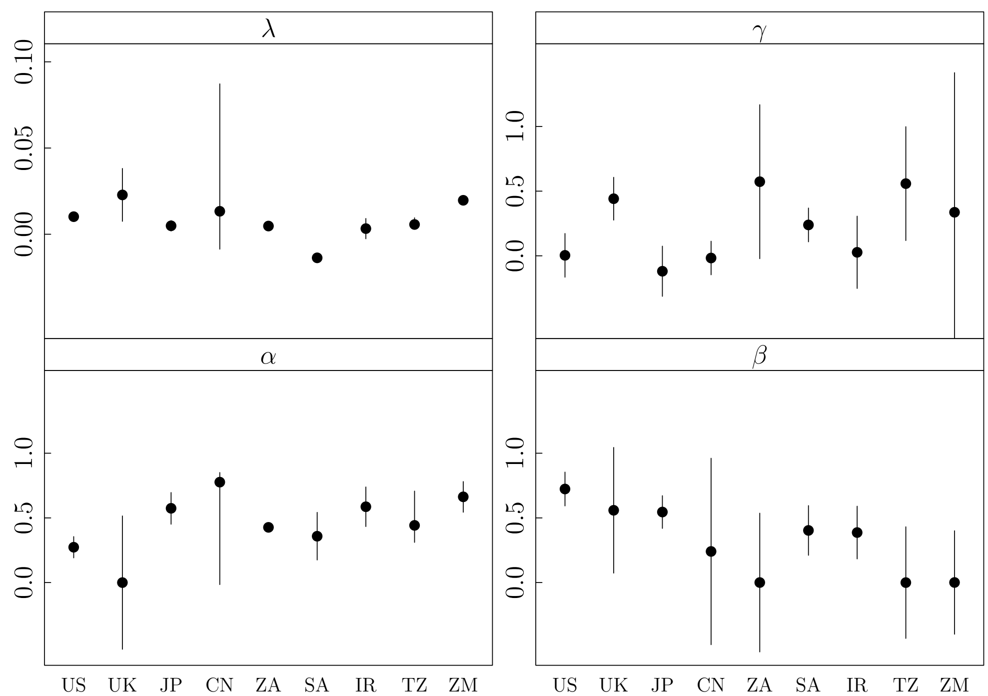

Figure 3: Cobb-Douglas (with q) model parameters. Vertical bars indicate 95% confidence intervals.

5.2. Cobb-Douglas with exergy (x)

The Cobb-Douglas (with exergy) parameters are given in Table A.13.

Figure 4 shows values and 95% confidence intervals for the parameters for the Cobb-Douglas model (with x).

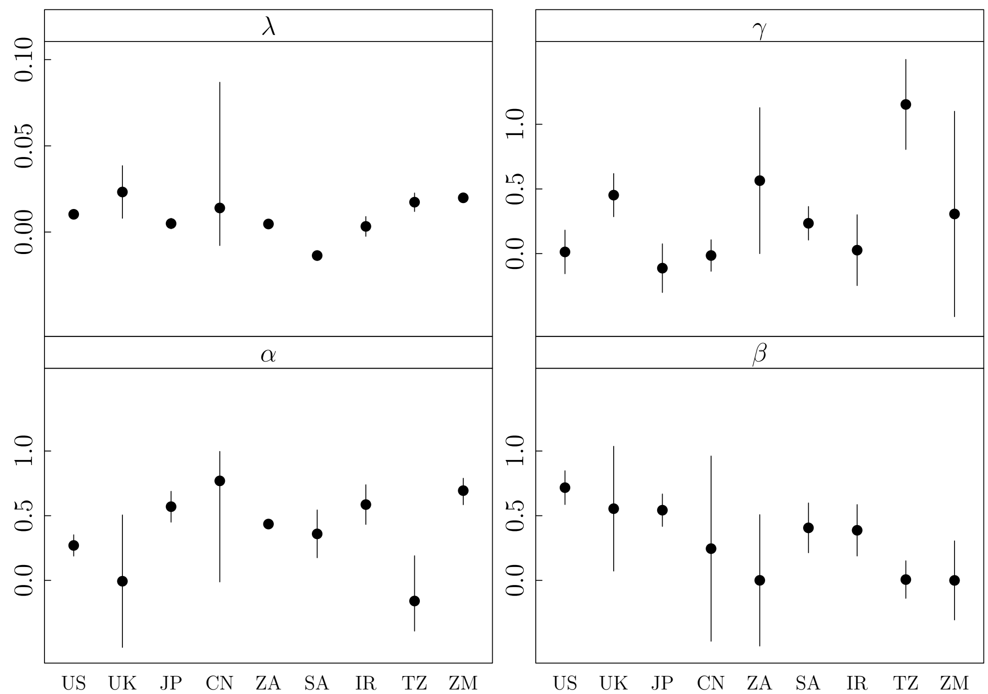

Figure 4: Cobb-Douglas (with x) model parameters. Vertical bars indicate 95% confidence intervals.

5.3. Cobb-Douglas with useful work (u)

Figure 5 shows values and 95% confidence intervals for the parameters for the Cobb-Douglas model (with u).

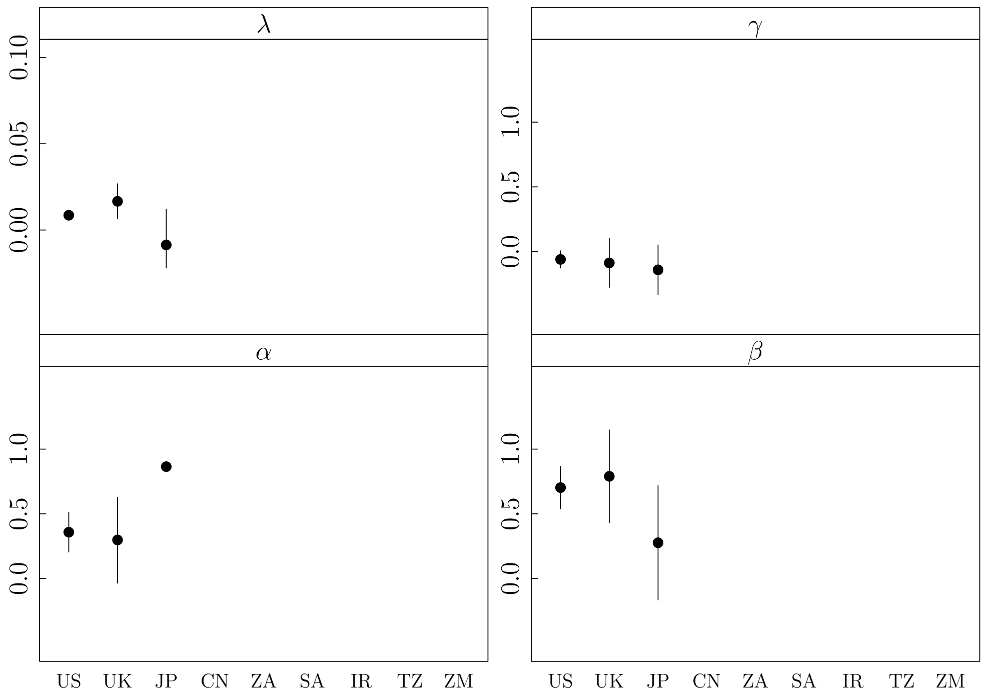

Figure 5: Cobb-Douglas (with u) model parameters. Vertical bars indicate 95% confidence intervals.

5.4. Cobb-Douglas Comparisons

Figure 6 compares predictions from the Cobb-Douglas models (without energy, with Q, and with x) to historical data.

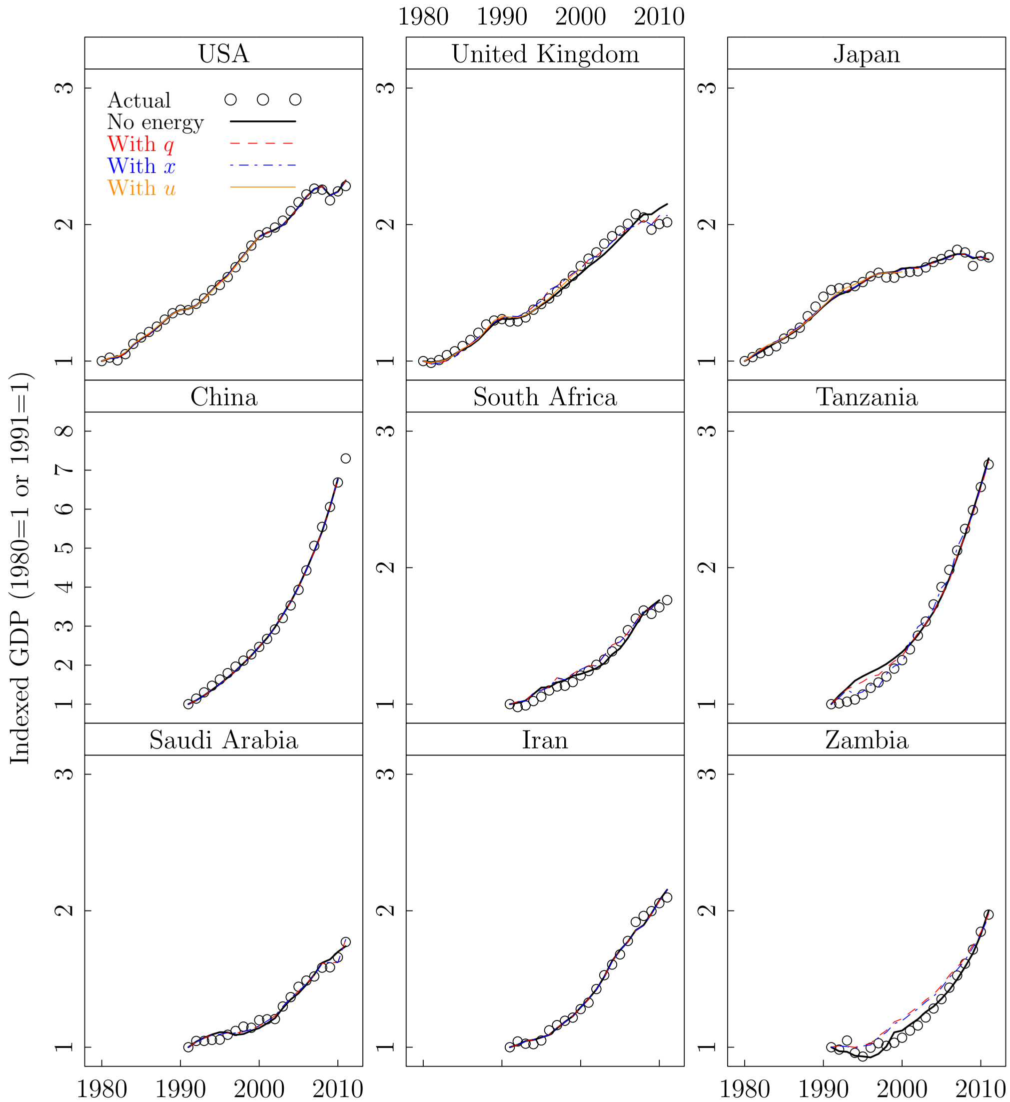

Figure 6: Cobb-Douglas results.

6. CES

6.1. *CES with Q*

6.2. *CES with X*

6.3. *CES with U*

7. LINEX

7.1. *LINEX with Q*

7.2. *LINEX with X*

7.3. *LINEX with U*

8. Goodness of Fit

We assess goodness of fit via the Akaike Information Criterion (AIC). AIC values for all models and all countries are shown in Table 1. Increasing goodness of fit is indicated by smaller (i.e., more negative) AIC values. AIC values can be compared per data set (i.e., per country) but not across data sets (i.e., not across countries).

Table 1: AIC values for all models.

	US	UK	JP	CN	ZA	SA	IR	TZ	ZM
CD	-159.4	-92.4	-126.8	-38.0	-66.5	-75.8	-81.1	-44.4	-67.9
CDq	-157.4	-113.2	-126.5	-36.0	-63.7	-86.4	-79.1	-52.5	-35.1
CDx	-157.5	-113.7	-126.4	-36.0	-65.0	-86.3	-79.1	-70.9	-40.4
CDu	-128.2	-92.9	-79.6						

The AIC results show *****************.

9. Conclusion

****************** Add conclusion here. *********************

Acknowledgements

The authors thank Randall Pruim (Calvin College) and Loren L. Heun (Western Michigan University) for their insightful comments on and assistance with the statistical analyses presented herein.

Appendix A. Tabular Results

Appendix A.1. Covariances

The following tables show covariances among GDP (y) and factors of production (k, l, q, x, and u).

Table A.2: Covariance table for US economy.

	y	k	l	q	x	u
y	1.00	0.99	0.95	0.95	0.95	
k	0.99	1.00	0.88	0.89	0.88	
l	0.95	0.88	1.00	0.98	0.98	
q	0.95	0.89	0.98	1.00	1.00	
x	0.95	0.88	0.98	1.00	1.00	
u						1.00

Table A.3: Covariance table for UK economy.

	y	k	l	q	x	u
y	1.00	0.99	0.89	0.53	0.47	
k	0.99	1.00	0.85	0.43	0.36	
l	0.89	0.85	1.00	0.56	0.52	
q	0.53	0.43	0.56	1.00	1.00	
x	0.47	0.36	0.52	1.00	1.00	
u						1.00

Table A.4: Covariance table for JP economy.

	y	k	l	q	x	u
y	1.00	0.98	-0.62	0.96	0.96	
k	0.98	1.00	-0.72	0.96	0.96	
l	-0.62	-0.72	1.00	-0.55	-0.55	
q	0.96	0.96	-0.55	1.00	1.00	
x	0.96	0.96	-0.55	1.00	1.00	
u						1.00

Table A.5: Covariance table for CN economy.

	y	k	l	q	x
y	1.00	1.00		0.99	0.99
k	1.00	1.00		0.99	0.99
l			1.00		
q	0.99	0.99		1.00	1.00
x	0.99	0.99		1.00	1.00

Table A.6: Covariance table for ZA economy.

	y	k	l	q	x
y	1.00	0.96		0.97	0.97
k	0.96	1.00		0.89	0.89
l			1.00		
q	0.97	0.89		1.00	1.00
x	0.97	0.89		1.00	1.00

Table A.7: Covariance table for SA economy.

	y	k	l	q	x
y	1.00	0.99	0.99	0.97	0.97
k	0.99	1.00	0.97	0.95	0.95
l	0.99	0.97	1.00	0.95	0.95
q	0.97	0.95	0.95	1.00	1.00
x	0.97	0.95	0.95	1.00	1.00

Table A.8: Covariance table for IR economy.

	y	k	l	q	x
y	1.00	0.99	0.97	0.99	0.99
k	0.99	1.00	0.93	0.98	0.98
l	0.97	0.93	1.00	0.97	0.97
q	0.99	0.98	0.97	1.00	1.00
x	0.99	0.98	0.97	1.00	1.00

Table A.9: Covariance table for TZ economy.

	y	k	l	q	x
y	1.00	0.99	0.98	1.00	1.00
k	0.99	1.00	0.96	0.99	0.99
l	0.98	0.96	1.00	0.98	0.97
q	1.00	0.99	0.98	1.00	1.00
x	1.00	0.99	0.97	1.00	1.00

Table A.10: Covariance table for ZM economy.

	y	k	l	q	x
y	1.00	0.98	0.88	0.98	0.96
k	0.98	1.00	0.87	0.96	0.94
l	0.88	0.87	1.00	0.86	0.79
q	0.98	0.96	0.86	1.00	0.99
x	0.96	0.94	0.79	0.99	1.00

Appendix A.2. Model Parameters

Table A.11 gives the parameters for the Cobb-Douglas model without energy.

Table A.11: Cobb-Douglas (without energy) for 1980-2011 (US, UK, JP), 1991-2010 (CN and ZA), and 1991-2011 (SA, IR, TZ, and ZM). (Parameter estimates beneath symbol. 95% confidence interval bounds to left and right.)

		λ			α			β	
US	0.0087	0.0102	0.0116	0.21	0.27	0.34	0.66	0.73	0.79
UK	-0.0104	0.0097	0.0303	-0.25	0.44	1.12	-0.13	0.56	1.24
JP	0.0021	0.0052	0.0082	0.44	0.52	0.59	0.41	0.48	0.56
CN	-0.0405	0.0188	0.0779	0.11	0.71	1.32	-0.32	0.29	0.89
ZA	-0.0007	0.0008	0.0022	0.46	0.60	0.73	0.26	0.40	0.54
SA	-0.0159	-0.0123	-0.0087	0.21	0.45	0.68	0.32	0.55	0.78
IR	0.0032	0.0039	0.0045	0.49	0.60	0.70	0.30	0.40	0.51
TZ	-0.0039	0.0015	0.0068	0.50	0.73	0.95	0.05	0.27	0.50
ZM	0.0218	0.0249	0.0280	1.25	1.41	1.57	-0.57	-0.41	-0.25

The Cobb-Douglas (with thermal energy, q) parameters are given in Table A.12.

Table A.12: Cobb-Douglas (with q) for 1980-2011 (US, UK, JP), 1991-2010 (CN and ZA), and 1991-2011 (SA, IR, TZ, and ZM). (Parameter estimates beneath symbol. 95% confidence interval bounds to left and right.)

		λ			α			β			γ	
US	0.0078	0.0102	0.0126	0.19	0.27	0.36	0.59	0.72	0.85	-0.17	0.00	0.17
UK	0.0075	0.0228	0.0382	-0.52	-0.00	0.52	0.07	0.56	1.04	0.28	0.44	0.61
JP	0.0019	0.0049	0.0079	0.45	0.57	0.70	0.42	0.55	0.67	-0.31	-0.12	0.07
CN	-0.0087	0.0133	0.0872	-0.02	0.78	0.85	-0.48	0.24	0.96	-0.15	-0.02	0.11
ZA		0.0048	0.0054	0.35	0.43		-0.54	0.00	0.54	-0.02	0.57	1.17
SA	-0.0165	-0.0137	-0.0109	0.17	0.36	0.54	0.21	0.40	0.60	0.11	0.24	0.37
IR	-0.0026	0.0033	0.0092	0.43	0.59	0.74	0.18	0.39	0.59	-0.25	0.03	0.31
TZ	0.0044	0.0057	0.0095	0.31	0.44	0.71	-0.43	-0.00	0.43	0.12	0.56	1.00
ZM		0.0197	0.0208	0.54	0.66	0.78	-0.40	0.00	0.40	-0.74	0.34	1.42

Table A.13 gives the parameters for the Cobb-Douglas model with exergy (x).

Table A.13: Cobb-Douglas (with x) for 1980-2011 (US, UK, JP), 1991-2010 (CN and ZA), and 1991-2011 (SA, IR, TZ, and ZM). (Parameter estimates beneath symbol. 95% confidence interval bounds to left and right.)

		λ			α			β			γ	
US	0.0079	0.0103	0.0127	0.19	0.27	0.35	0.59	0.72	0.85	-0.16	0.01	0.18
UK	0.0080	0.0232	0.0385	-0.52	-0.01	0.51	0.07	0.55	1.04	0.29	0.45	0.62
JP	0.0019	0.0049	0.0080	0.45	0.57	0.69	0.42	0.54	0.67	-0.30	-0.11	0.08
CN	-0.0078	0.0140	0.0869	-0.01	0.77	1.00	-0.47	0.25	0.96	-0.14	-0.01	0.11
ZA		0.0047	0.0054	0.36	0.44		-0.51	0.00	0.51	0.00	0.56	1.13
SA	-0.0164	-0.0136	-0.0108	0.17	0.36	0.54	0.21	0.41	0.60	0.11	0.23	0.36
IR	-0.0025	0.0033	0.0090	0.43	0.59	0.74	0.19	0.39	0.59	-0.25	0.03	0.30
TZ	0.0119	0.0173	0.0227	-0.39	-0.16	0.19	-0.14	0.01	0.15	0.81	1.15	1.50
ZM		0.0199	0.0209	0.58	0.69	0.79	-0.31	-0.00	0.31	-0.49	0.31	1.10

The Cobb-Douglas (with useful work, u) parameters are given in Table A.14.

Table A.14: Cobb-Douglas (with u) for 1980-2011 (US, UK, JP), 1991-2010 (CN and ZA), and 1991-2011 (SA, IR, TZ, and ZM). (Parameter estimates beneath symbol. 95% confidence interval bounds to left and right.)

		λ			α			β			γ	
US	0.0058	0.0086	0.0113	0.20	0.36	0.51	0.54	0.70	0.87	-0.13	-0.06	0.01
UK	0.0065	0.0166	0.0268	-0.04	0.30	0.63	0.43	0.79	1.15	-0.28	-0.09	0.10
JP	-0.0220	-0.0087	0.0121	0.39	0.86		-0.17	0.28	0.72	-0.33	-0.14	0.05
CN												
ZA												
SA												
IR												
TZ												
ZM												