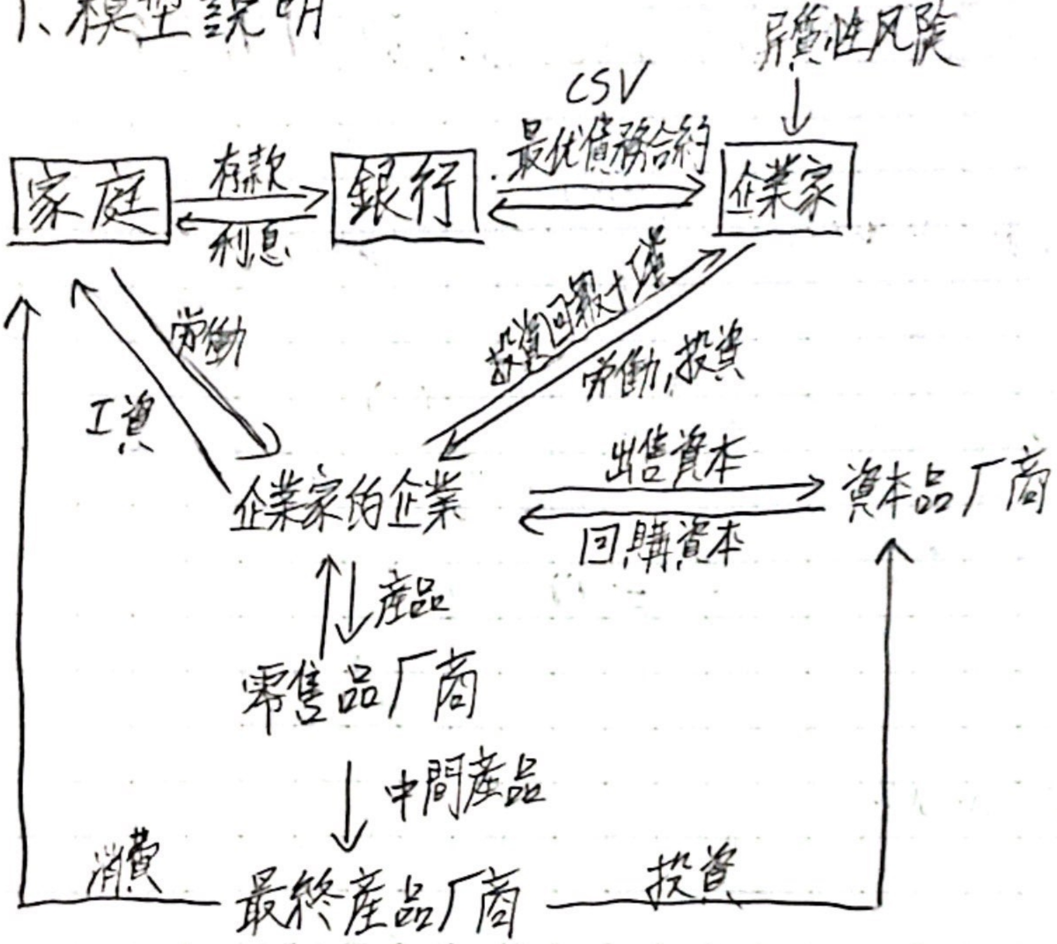
BGG Ver.2

1、模型説明



2. 家庭(工人)

$$\underset{C_t, M_t, B_t, H_t}{Max} \quad E_t \sum_{t=0}^{\infty} \beta^t \{ \ln C_t + \zeta \ln(M_t) + \xi \ln(1-H_t) \}$$

$$s.t.\ C_t + M_t + B_t = W_t H_t + \frac{M_{t-1}}{\pi_t} + R_{t-1} B_{t-1} + J_t - T_t$$

$$\frac{\partial \mathcal{L}}{\partial C_t}: \frac{1}{C_t} - \lambda_t = 0 \qquad (1)$$

$$\frac{\partial \mathcal{L}}{\partial M_t}: \zeta \frac{1}{M_t} - \lambda_t + \beta \lambda_{t+1} \frac{1}{\pi_{t+1}} = 0 \qquad (2)$$

$$\frac{\partial \mathcal{L}}{\partial B_t}: -\lambda_t + \beta \lambda_t R_t = 0 \qquad (3)$$

$$\frac{\partial \mathcal{L}}{\partial H_t}: -\xi \frac{1}{(1-H_t)} + \lambda_t W_t = 0 \qquad (4)$$

C_t：実質消費，M_t：実質貨币需求，B_t：儲蓄

H_t：工人労働，W_t：実質工資，π_t：通脹率(gross)

R_t：実質存款利率(gross)

2、企業家

① 企業家異質，企業家 j 在 t 期購入資本 K_t^j 用于 $t+1$ 期生產

② 視資本価格 Q_t 与投資回報率 R_t^k 外生

③ 投資面临异質性风険 ω_t^j，回報率为 $\omega_t^j R_t^k$.

ω^j：企業家 j 在 t 期面临的风険，随机变量

ω^j 独立同分布，分布函数不随時間変化，記作 $F(\omega^j)$

計算方便起見，令 $E(\omega^j)=1$

④ t 期末，企業家 j 的净資産为 N_{t+1}^j，向銀行貸款 B_t^j.

$$Q_t K_{t+1}^j = N_{t+1}^j + B_t^j \qquad \text{(balance sheet)}$$

資産 = 净資産 + 債務

3、企業家与銀行的債務合約 (CSV)

① 若异質性风険 $\omega_t^j \geq$ 风険临界值 $\bar{\omega}_t^j$，則企業家向銀行还本付息.

② 若 $\omega_t^j < \bar{\omega}_t^j$，則企業家申請破産清算，但需支付清算成本。

• 风険临界值定義：$\bar{\omega}_t^j R_{t+1}^k Q_t K_{t+1}^j = R_t^L B_t^j$.

其中，R_t^L 是企業与銀行的合約利率。

• 企業家破産清算時，銀行得到

$$(1-\mu)\,\omega^j R_{t+1}^k Q_t K_{t+1}^j$$

即清算成本为企業家資産的比例 μ.

③假設銀行之間完全競争，利潤为0，該假設下

銀行貸款予期收益＝銀行吸收存款的成本.

$$予期收益 = \underline{\int_0^{\overline{\omega}_t^e}(1-u)\omega^j R_{t+1}^k Q_t K_{t+1}^j \, dF(\omega^j)} + \int_{\overline{\omega}_t^e}^{\infty} R_t^L B_t^e \, dF(\omega^j)$$

$$= \underline{(1-u)\int_0^{\overline{\omega}_t^e}\omega \, dF(\omega^j) R_{t+1}^k Q_t K_{t+1}^j} + \int_{\overline{\omega}_t^e}^{\infty} 1 \, dF(\omega^j)\, \overline{\omega}_t^e R_{t+1}^k Q_t K_{t+1}^j$$

$$= (1-u)\int_0^{\overline{\omega}_t^e}\omega^j dF(\omega^j) R_{t+1}^k Q_t K_{t+1}^j + (1-F(\overline{\omega}_t^e))\overline{\omega}_t^e R_{t+1}^k Q_t K_{t+1}^j$$

$$成本 = R_t B_t^e.$$

銀行零利潤条件

4、企業家価値最大化問題（只有我知道风険，成不成功我説了算）

$$V_t^e = \max_{\overline{\omega}_t^e,\, K_{t+1}^j} E_t\left\{\int_{\overline{\omega}_t^e}^{\infty}\omega^j R_{t+1}^k Q_t K_{t+1}^j \, dF(\omega^j) - (1-F(\overline{\omega}_t^e))\overline{\omega}_t^e R_{t+1}^k Q_t K_{t+1}\right\}$$

← 投資成功的收益

投資成功支付的成本

s.t. 銀行零利潤条件：$R_t B_t^e =$

$$(1-u)\int_0^{\overline{\omega}_t^e}\omega^j dF(\omega^j) R_{t+1}^k Q_t K_{t+1}^j + (1-F(\overline{\omega}_t^e))\overline{\omega}_t^e R_{t+1}^k Q_t K_{t+1}^j$$

变量替换：

$$\Gamma_t \equiv F(\overline{\omega}_t^j) \quad (5), \quad G_t = \int_0^{\overline{\omega}_t^j} \omega^j \, dF(\omega^j) \quad (6)$$

$$\Gamma_t \equiv \int_0^{\overline{\omega}_t^j} \omega^j dF(\omega^j) + \overline{\omega}_t^j (1 - F(\overline{\omega}_t^j))$$

$$= G_t + \overline{\omega}_t^j (1 - F(\overline{\omega}_t^j)) \quad (7)$$

目标函数：

$$\int_{\overline{\omega}_t^j}^{\infty} \omega^j dF(\omega^j) R_{t+1}^k Q_t K_{t+1}^j - (1 - F(\overline{\omega}_t^j)) \overline{\omega}_t^j R_{t+1}^k Q_t K_{t+1}^j$$

$$= \left\{ \int_{\overline{\omega}_t^j}^{\infty} \omega^j dF(\omega^j) - (1 - F(\overline{\omega}_t^j)) \overline{\omega}_t^j \right\} R_{t+1}^k Q_t K_{t+1}^j$$

$$= \left\{ \int_0^{\infty} \omega^j dF(\omega^j) - \int_0^{\overline{\omega}_t^j} \omega^j dF(\omega^j) - (1 - F(\overline{\omega}_t^j)) \overline{\omega}_t^j \right\}$$

$$\times R_{t+1}^k Q_t K_{t+1}^j$$

$$= \{ E(\omega^j) - \Gamma_t \} R_{t+1}^k Q_t K_{t+1}^j$$

$$= (1 - \Gamma_t) R_{t+1}^k Q_t K_{t+1}^j$$

銀行零利潤条件左側：

$$(1-F(\bar{\omega}_t^{e}))\bar{\omega}_t^{e} R_{t+1}^{k} Q_t K_{t+1}^{ej} + (1-\mu)\int_0^{\bar{\omega}_t^{e}} \omega^{j} R_{t+1}^{k} Q_t K_{t+1}^{ej}\, dF(\omega^{j})$$

$$= \left\{(1-F(\bar{\omega}_t^{e}))\bar{\omega}_t^{e} + (1-\mu)\int_0^{\bar{\omega}_t^{e}} \omega^{j}\, dF(\omega^{j})\right\} R_{t+1}^{k} Q_t K_{t+1}^{ej}.$$

$$= \left\{\underline{(1-F(\bar{\omega}_t^{e}))\bar{\omega}_t^{e} + \int_0^{\bar{\omega}_t^{e}} \omega^{j}\, dF(\omega^{j})} - \underline{\mu\int_0^{\bar{\omega}_t^{e}} \omega^{j}\, dF(\omega^{j})}\right\} R_{t+1}^{k} Q_t K_{t+1}^{ej}$$

$$= (\Gamma_t - \mu G_t) R_{t+1}^{k} Q_t K_{t+1}^{ej}.$$

零利潤条件右側： ↲代入企業家 balance sheet.

$$R_t B_t^{e} = R_t (Q_t K_{t+1}^{ej} - N_{t+1}^{ej})$$

企業家価値最大化問題：

$$V_t^{e} = \max_{\bar{\omega}_t^{e},\, K_{t+1}^{ej}} (1-\Gamma_t) R_{t+1}^{k} Q_t K_{t+1}^{ej}$$

$$\text{s.t. } (\Gamma_t - \mu G_t) R_{t+1}^{k} Q_t K_{t+1}^{ej} = R_t (Q_t K_{t+1}^{ej} - N_{t+1}^{ej})$$

$$\mathcal{L} = (1-\Gamma_t) R_{t+1}^{k} Q_t K_{t+1}^{ej} + \lambda_t \left\{(\Gamma_t - \mu G_t) R_{t+1}^{k} Q_t K_{t+1}^{ej} - R_t (Q_t K_{t+1}^{ej} - N_{t+1}^{ej})\right\}$$

$$\frac{\partial \mathcal{L}}{\partial \bar{\omega}_t^{e}} : -\Gamma_t' \cancel{R_{t+1}^{k} Q_t K_{t+1}^{ej}} + \lambda_t (\Gamma_t' - \mu G_t') \cancel{R_{t+1}^{k} Q_t K_{t+1}^{ej}} = 0$$

$$\Rightarrow \underline{\Gamma_t' = \lambda_t (\Gamma_t' - \mu G_t') \quad ①}$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}^{ej}} = (1-\Gamma_t) R_{t+1}^{k} Q_t + \lambda_t \left\{(\Gamma_t - \mu G_t) R_{t+1}^{k} Q_t - R_t Q_t\right\} = 0$$

$\frac{\partial \mathcal{L}}{\partial \nu_t}$: $(P_t - uG_t) R^k_{t+1} Q_t K^a_{t+1} = R_t (Q_t K^a_t - N^a_{t+1})$ ③.

· 令 $S_{t+1} \equiv R^k_{t+1} / R_t$ (8, 风险利差), $L_{t+1} = \frac{Q_t K^a_{t+1}}{N^a_{t+1}}$ (9, 杠杆率)

①′ $\nu_t = \frac{P_t'}{P_t' - uG_t'}$

②′ $(1-P_t)\frac{R^k_{t+1}}{R_t} + \nu_t \left\{ (P_t - uG_t)\frac{R^k_{t+1}}{R_t} - 1 \right\} = 0$

将①′代入②′消去 ν_t，再用(8)替换 $\frac{R^k_{t+1}}{R_t}$ 得到.

$$(1-P_t) S_{t+1} + \frac{P_t'}{P_t' - uG_t'} \left\{ (P_t - uG_t) S_{t+1} - 1 \right\} = 0$$

$$\Rightarrow \frac{P_t' - uG_t'}{P_t'}(1-P_t) + (P_t - uG_t) = \frac{1}{S_{t+1}} \quad (10)$$

将③两边同除 $R_t N^a_{t+1}$ 可得

$$(P_t - uG_t)\frac{R^k_{t+1}}{R_t}\frac{Q_t K^a_{t+1}}{N^a_{t+1}} = \frac{Q_t K^a_{t+1}}{N^a_{t+1}} - 1$$

$$\Rightarrow (P_t - uG_t) S_{t+1} = \frac{L_{t+1} - 1}{L_{t+1}} \quad (11)$$

注意，由于对所有企業家来説，資本市場環境相同(即，R_t，R^k_t，…)都是一样的，所以企業家的最优反应相同，可以忽略异質性符号 a.

5. 補足：最优債務協約为模型带来了什么？

- 注意(10)式左側都是 $\bar{\omega}_t^e$ 的函数，利差 S_{t+1} 与 $\bar{\omega}_t^e$ 存在一一对应的函数関係，記作 $S_{t+1} = h(\bar{\omega}_t^e) \Leftrightarrow h^{-1}(S_{t+1}) = \bar{\omega}_t^e$.

- (11)式可以写作 $g(\bar{\omega}_t^e) S_{t+1} = 1 - \frac{1}{L_{t+1}}$ 結合(10)与(11)可知

$$\Rightarrow g(h^{-1}(S_{t+1}))S_{t+1} = 1 - \frac{1}{L_{t+1}}$$

那么利差与企業杠杆率存在一一对应的関係

$$S_{t+1} = m\left(\frac{1}{L_{t+1}}\right)$$

通過隐函数定理，可以証明 $m'(\cdot) < 0$.

即企業杠杆越高，利差越大.

- $\frac{N_{t+1}}{Q_t K_{t+1}} = m^{-1}\left(\frac{R_{t+1}^k}{R_t}\right)$，那么

$$\underline{\left(1 - \frac{N_{t+1}}{Q_t K_{t+1}}\right) Q_t K_{t+1}} = \left(1 - m^{-1}\left(\frac{R_{t+1}^k}{R_t}\right)\right) Q_t K_{t+1}.$$

$$\underbrace{B_t}_{\substack{\text{企業貸款}\\\text{(居民存款)}}} = \underbrace{\left(1 - m^{-1}\left(\frac{R_{t+1}^k}{R_t}\right)\right)}_{\substack{\text{可貸資金比例}\\\text{(依存于利差)}}} \underbrace{Q_t K_{t+1}}_{\text{企業資産価値}}.$$

6、均衡狀態下（即最优合約下）的企業価值

$$V_t = (1-P_t) R^k_{t+1} Q_t K_{t+1}$$
$$= (1-P_t) \frac{R^k_{t+1}}{R_t} R_t Q_t K_{t+1}$$
$$= (1-P_t) S_{t+1} R_t Q_t K_{t+1} \quad (12)$$

假設每期企業家消費掉一部分：

$$C^e_t = (1-\gamma^e) V_t \quad (13)$$

并且企業家投入自己的工資，使之与残存的企業価值形成新的净資產：

$$N_{t+1} = \gamma^e V_t + H^e_t W^e_t \quad (14)$$

7. 企業家持有企業的生產經営活動

①企業家向資本品厂商購入生產性資本，向家庭雇佣劳働生產：

$$Y^e_t = A_t K_t^{\alpha} [H_t^{\Omega} (H^e_t)^{1-\Omega}]^{1-\alpha} \quad (15)$$

其中，H_t 劳働来自家庭，H^e_t 来自企業家（假設供給无弹性 $H^e_t = 1$）

② 企業家在 t 期末将折旧后的資本以 Q_{t-1} 価格卖給資本品厂商，向資本品厂商購入 K_t 的新資本，将產出以 $P_{m,t}$ 的価格卖給零售商.

$$\max_{H_t, H_t^e} J_t = P_{m,t} Y_t^e + Q_t(1-\delta)K_t - R_t^k Q_{t-1} K_t - w_t H_t - W_t^e H_t^e.$$

$$s.t.\ Y_t^e = A_t K_t^{\alpha} [H_t^{\Omega} (H_t^e)^{1-\Omega}]^{1-\alpha}$$

$$\frac{\partial \mathcal{L}}{\partial H_t} : W_t = P_{m,t} A_t K_t^{\alpha} (1-\alpha) [H_t^{\Omega} (H_t^e)^{1-\Omega}]^{-\alpha} \Omega H_t^{\Omega-1} (H_t^e)^{1-\Omega}$$

$$\Rightarrow w_t H_t = (1-\alpha)\Omega P_{m,t} \underline{A_t K_t^{\alpha} [H_t^{\Omega} (H_t^e)^{1-\Omega}]^{1-\alpha}} \quad (16)$$

$$= (1-\alpha)\Omega P_{m,t} \underline{Y_t^e}.$$

$$\frac{\partial \mathcal{L}}{\partial H_t^e} : \underline{W_t^e H_t^e = (1-\alpha)(1-\Omega) P_{m,t} Y_t^e} \quad (17)$$

假設企業家完全競爭,零利潤条件下

$$0 = P_{m,t} Y_t^e + Q_t(1-\delta)K_t - R_t^k Q_{t-1} K_t - W_t H_t - W_t^e H_t^e.$$

$$0 = \alpha P_{m,t} Y_t^e + Q_t(1-\delta)K_t - R_t^k Q_{t-1} K_t.$$

$$\Rightarrow \underline{R_t^k = \frac{\alpha P_{m,t} Y_t^e}{Q_{t-1} K_t} + \frac{Q_t(1-\delta)}{Q_{t-1}}} \quad (18)$$

8、最終生產品厂商

以P_t^i価格从零售商処購买異質性中間商品Y_t^i，打包後以価格P_t出售給家庭与資本品厂商.

$$\max_{Y_t^i} \quad P_t Y_t - \int_0^1 P_t^i Y_t^i \, di$$

$$\text{s.t.} \quad Y_t = \left[\int_0^1 (Y_t^i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$\text{FOC}: \quad P_t \cancel{\frac{\varepsilon}{\varepsilon-1}} \underbrace{\left[\int_0^1 (Y_t^i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{1}{\varepsilon-1}}}_{=Y_t^{\frac{1}{\varepsilon}}} \cancel{\frac{\varepsilon-1}{\varepsilon}} (Y_t^i)^{-\frac{1}{\varepsilon}} = P_t^i$$

$$\Rightarrow \quad Y_t^i = \left(\frac{P_t^i}{P_t}\right)^{-\varepsilon} Y_t$$

将FOC代入目标函数，根据0利潤条件可得

$$P_t Y_t - \int_0^1 P_t^i \left(\frac{P_t^i}{P_t}\right)^{-\varepsilon} Y_t \, di = 0$$

$$\Rightarrow \quad P_t = \left(\int_0^1 {P_t^i}^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$$

9. 零售商

以 $P_{m,t}$ 从企業家处买入產品，轉化为中間產品，以 P_t^i 価格卖給最終品厂商，Calvo 定価.

可最优調整価格的概率为 $1-\theta$,

$$\max_{P_t^{i*}} E_t \sum_{k=0}^{\infty} (\theta\beta)^k \frac{\lambda_{t+k}}{\lambda_t} \left(\frac{P_t^{i*}}{P_{t+k}} - P_{m,t+k}\right) Y_{t+k}^{i*}$$

$$s.t.\quad Y_{t+k}^{*} = \left(\frac{P_t^{i*}}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k}$$

$$FOC: \sum_{k=0}^{\infty} (\theta\beta)^k \cancel{\frac{\lambda_{t+k}}{\lambda_t}} \left[(1-\varepsilon)\frac{\cancel{P_t^{i*-\varepsilon}}}{P_{t+k}^{1-\varepsilon}} - (-\varepsilon) P_{m,t+k}\left(\frac{P_t^{i*-1-\varepsilon}}{P_{t+k}^{-\varepsilon}}\right)\right] Y_{t+k}$$

$$\Rightarrow P_t^{i*} = \frac{\varepsilon}{1-\varepsilon} \frac{\sum_k (\theta\beta)^k \lambda_{t+k} P_{m,t+k} P_{t+k}^{\varepsilon} Y_{t+k}}{\sum_k (\theta\beta)^k \lambda_{t+k} P_{t+k}^{1-\varepsilon} Y_{t+k}}$$

$$\Rightarrow \frac{P_t^{i*}}{P_t} \equiv \pi_t^* = \frac{\varepsilon}{1-\varepsilon} \frac{\sum_k (\theta\beta)^k \left(\frac{P_{t+k}}{P_t}\right)^{\varepsilon} P_{m,t+k} Y_{t+k}}{\sum_k (\theta\beta)^k \left(\frac{P_{t+k}}{P_t}\right)^{1-\varepsilon} Y_{t+k}}$$

令 $X_t^1 = \sum_{k=0}^{\infty} (\theta\beta)^k \lambda_{t+k} P_{m,t+k} \left(\frac{P_{t+k}}{P_t}\right)^{\varepsilon} Y_{t+k}$

$$= \lambda_t P_{m,t} Y_t + \theta\beta \sum_{k=1}^{\infty} (\theta\beta)^{k-1} \lambda_{t+k} P_{m,t+k} \left(\frac{P_{t+k}}{P_t}\right)^{\varepsilon} Y_{t+k}$$

令 $k=r+1$

$$= \lambda_t P_{m,t} Y_t + \theta\beta \sum_{r=0}^{\infty} (\theta\beta)^r \lambda_{t+r+1} P_{m,t+k+1} \left(\frac{P_{t+r+1}}{P_{t+1}} \frac{P_{t+1}}{P_t}\right)^{\varepsilon} Y_{t+r+1}$$

$$= \lambda_t P_{m,t} Y_t + \theta\beta\, \pi_{t+1}^{\varepsilon} X_{t+1}^1 .$$

同理，

$$X_t^2 = \sum_{k=0}^{\infty} (\theta\beta)^k \lambda_{t+k} \left(\frac{P_{t+k}}{P_t}\right)^{1-\varepsilon} Y_{t+k}$$

$$= \lambda_t Y_t + \theta\beta \Pi_{t+1}^{\varepsilon-1} X_{t+1}^2$$

綜上可得

$$\underline{\Pi_t^* = \frac{\varepsilon}{\varepsilon-1}\frac{X_t^1}{X_t^2}} \tag{19}$$

$$\underline{X_t^1 = \lambda_t P_{mt} Y_t + \theta\beta \Pi_{t+1}^{\varepsilon} X_{t+1}^1} \tag{20}$$

$$\underline{X_t^2 = \lambda_t Y_t + \theta\beta \Pi_{t+1}^{\varepsilon-1} X_{t+1}^2} \tag{21}$$

根据最終產品厂商的零利潤条件：

$$P_t^{1-\varepsilon} = \int_0^1 P_t^{i\,1-\varepsilon}\, di$$

$$= \theta \underline{\int_0^1 P_{t-1}^{i\,1-\varepsilon}\, di} + (1-\theta)\int_0^1 P_t^{i*\,1-\varepsilon}\, di$$

$$= \theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^{i*})^{1-\varepsilon}$$

两边同除 $P_t^{1-\varepsilon}$

$$\left(\frac{P_t}{P_t}\right)^{1-\varepsilon} = \theta\left(\frac{P_{t-1}}{P_t}\right)^{1-\varepsilon} + (1-\theta)\left(\frac{P_t^{i*}}{P_t}\right)^{1-\varepsilon}$$

$$\Rightarrow \underline{1 = \theta \Pi_t^{\varepsilon-1} + (1-\theta)\Pi_t^{*\,1-\varepsilon}} \tag{22}$$

10. 資本累积与資本品企業

資本累积方程 $K_{t+1}=(1-\delta)K_t+I_t-\frac{\chi}{2}\left(\frac{I_t}{K_t}-\delta\right)^2K_t$ (23)

其中，$I_t-\frac{\chi}{2}\left(\frac{I_t}{K_t}-\delta\right)^2$ 为新增資本量，由資本品厂商提供.

資本品企業（假設資本生産的費用为1单位）

$$\max_{I_t}\ Q_t\left[I_t-\frac{\chi}{2}\left(\frac{I_t}{K_t}-\delta\right)^2\right]-I_t$$

$$\Rightarrow\ Q_t=\left[1-\chi\left(\frac{I_t}{K_t}-\delta\right)\right]^{-1}\quad(24)$$

11. 市場出清条件与加总.

$$Y_t^e=\int_0^1Y_t^i\,di=\int_0^1\left(\frac{P_t^i}{P_t}\right)^{-\varepsilon}di\,Y_t\equiv D_tY_t\quad(25)$$

$$\begin{aligned}D_t&=\int_0^1\left(\frac{P_t^i}{P_t}\right)^{-\varepsilon}di\\&=\theta\int_0^1\left(\frac{P_{t-1}^i}{P_t}\right)^{-\varepsilon}di+(1-\theta)\int_0^1\left(\frac{P_t^{i*}}{P_t}\right)^{-\varepsilon}di\\&=\theta\int_0^1\left(\frac{P_{t-1}^i}{P_{t-1}}\frac{P_{t-1}}{P_t}\right)^{-\varepsilon}di+(1-\theta)\left(\frac{P_t^{i*}}{P_t}\right)^{-\varepsilon}\\&=\theta\Pi_t^{\varepsilon}D_{t-1}+(1-\theta)\Pi_t^{*-\varepsilon}\quad(26)\end{aligned}$$

$$Y_t=C_t+C_t^e+I_t+\frac{\chi}{2}\left(\frac{I_t}{K_t}-\delta\right)^2K_t+uG_tR_t^kQ_{t-1}K_t\quad(27)$$

12. 央行，外生冲击与一些补充方程

$$R_t = \frac{i_t}{\pi_{t+1}} \quad (28)$$

$$\frac{i_t}{i} = \left(\frac{i_{t-1}}{i}\right)^{\rho} \left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi}\right]^{1-\rho} \exp(\varepsilon_t^r) \quad (29)$$

$$\ln(A_t) = \rho \ln(A_{t-1}) + \varepsilon_t^a \quad (30)$$

$$H_t^e = 1 \quad (31)$$

$$\Gamma_t = \Gamma(\overline{\omega}_t^j) \quad (32)$$

$$G_t = G(\overline{\omega}_t^j) \quad (33)$$

13. 方程代换

方才(5),(6),(7),(32),(33)没写成显性方程，因此需要替换.

假設 ω^j 服从对数正态分布，即

$$\ln(\omega^j) \sim N(u, \sigma^2)$$

那么 $f(\omega^j) = \frac{1}{\sqrt{2\pi}\,\sigma\,\omega^j} \exp\left[-\frac{(\ln(\omega^j) - u)^2}{2\sigma^2}\right]$

易証得 $E(\omega^j) = \exp(u + \frac{1}{2}\sigma^2)$

$$Var(\omega^j) = \exp(\sigma^2 + 2u)(\exp(\sigma^2) - 1)$$

由于我们假設 $E(\omega^j) = 1$，那么 $u = -\frac{1}{2}\sigma^2$

于是 $Var(\omega^j) = \exp(\sigma^2) - 1 \approx \sigma^2$

$$(5)' \quad F_t = F(\bar{\omega}_t^j)$$

$$= \int_0^{\bar{\omega}_t^j} \frac{1}{\sqrt{2\pi}\,\sigma\,\omega^j} \exp\left[-\frac{(\ln(\omega^j)+\frac{\sigma^2}{2})^2}{2\sigma^2}\right] d\omega^j$$

令 $z=\ln\omega^j$

$$= \int_{-\infty}^{\ln(\bar{\omega}_t^j)} \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left[-\frac{(z+\frac{\sigma^2}{2})^2}{2\sigma^2}\right] \underbrace{\frac{d\omega^j}{\omega^j}}_{=d\ln(\omega^j)=dz}$$

令 $m=\frac{z+\frac{\sigma^2}{2}}{\sigma}$

$$= \int_{-\infty}^{\frac{\ln(\bar{\omega}_t^j)+\frac{\sigma^2}{2}}{\sigma}} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{m^2}{2}\right] \underbrace{\frac{dz}{\sigma}}_{=dm}$$

$$\underline{F_t = \Phi\left(\frac{\ln(\bar{\omega}_t^j)+\frac{\sigma^2}{2}}{\sigma}\right)}$$

$$(6)' \quad G_t = \int_0^{\bar{\omega}_t} \omega \, dF(\omega)$$

$$= \int_0^{\bar{\omega}_t} \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left[\frac{-(\ln(\omega)+\frac{\sigma^2}{2})^2}{2\sigma^2}\right] d\omega$$

令 $z=\ln(\omega)$

$$= \int_{-\infty}^{\ln(\bar{\omega}_t)} \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left[\frac{-(z+\frac{\sigma^2}{2})^2}{2\sigma^2}\right] \omega \underbrace{\frac{d\omega}{\omega}}_{=dz}$$

注意到 $\omega = \exp(\ln(\omega)) = \exp(z)$

$$= \int_{-\infty}^{\ln(\bar{\omega}_t)} \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left[\frac{2\sigma^2 z - z^2 - \frac{\sigma^4}{4} - \sigma^2 z}{2\sigma^2}\right] dz$$

$$= \int_{-\infty}^{\ln(\bar{\omega}_t)} \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left[-\frac{\left(z-\frac{\sigma^2}{2}\right)^2}{2\sigma^2}\right] dz$$

令 $m = \frac{z-\frac{\sigma^2}{2}}{\sigma}$

$$= \int_{-\infty}^{\frac{\ln(\bar{\omega}_t)-\frac{\sigma^2}{2}}{\sigma}} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{m^2}{2}\right] \underbrace{\frac{dz}{\sigma}}_{=dm}$$

$$\underline{G_t = \Phi\left(\frac{\ln(\bar{\omega}_t)}{\sigma} - \frac{\sigma}{2}\right)} \qquad (6)'$$

$$\Gamma_t = G_t + \bar{\omega}_t(1-F_t) \qquad (7)'$$

$$F_t' = \phi\left(\frac{\ln(\bar{\omega}_t)}{\sigma} + \frac{\sigma}{2}\right) \times \frac{1}{\sigma\bar{\omega}_t}$$

$$G_t' = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{\ln(\bar{\omega}_t)}{\sigma} - \frac{\sigma}{2}\right)^2\right\} \frac{1}{\sigma\bar{\omega}_t}$$

$$= \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left[\left(\frac{\ln(\bar{\omega}_t)}{\sigma}\right)^2 + \left(\frac{\sigma}{2}\right)^2 - \ln(\bar{\omega}_t)\right]\right\} \frac{1}{\sigma\bar{\omega}_t}$$

$$= \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left[\underline{\left(\frac{\ln(\bar{\omega}_t)}{\sigma}\right)^2 + \left(\frac{\sigma}{2}\right)^2 + \ln(\bar{\omega}_t)}\, \underline{-2\ln(\bar{\omega}_t)}\right]\right\} \frac{1}{\sigma\bar{\omega}_t}$$

$$= \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{\ln(\bar{\omega}_t)}{\sigma} + \frac{\sigma}{2}\right)^2\right\} \bar{\omega}_t \times \frac{1}{\sigma\bar{\omega}_t}$$

$$\underline{G_t' = \phi\left(\frac{\ln(\bar{\omega}_t)}{\sigma} + \frac{\sigma}{2}\right)\frac{1}{\sigma}} \qquad (33)'$$

$$\underline{\Gamma_t' = G_t' + (1-F_t) - \underbrace{\bar{\omega}_t F_t'}_{=G_t'} = 1-F_t} \qquad (32)'$$

14 Summary

$$\frac{1}{C_t} - \lambda_t = 0 \qquad (1)$$

$$\xi \frac{1}{M_t} - \lambda_t + \beta \lambda_{t+1} \frac{1}{\pi_{t+1}} = 0 \qquad (2)$$

$$-\lambda_t + \beta \lambda_{t+1} R_t = 0 \qquad (3)$$

$$-\xi \frac{1}{1-H_t} + \lambda_t W_t = 0 \qquad (4)$$

$$F_t = \Phi\left(\frac{\ln(\bar{\omega}_t)}{\sigma} + \frac{\sigma}{2} \right) \qquad (5)$$

$$G_t = \Phi\left(\frac{\ln(\bar{\omega}_t)}{\sigma} - \frac{\sigma}{2} \right) \qquad (6)$$

$$\Gamma_t = G_t + \bar{\omega}_t (1 - F_t) \qquad (7)$$

$$s_{t+1} = \frac{R^k_{t+1}}{R_t} \qquad (8)$$

$$L_{t+1} = Q_t K_{t+1} / N_{t+1} \qquad (9)$$

$$\frac{\Gamma'_t - \mu G'_t}{\Gamma'_t} (1 - \Gamma_t) + (\Gamma_t - \mu G_t) = \frac{1}{s_{t+1}} \qquad (10)$$

$$(\Gamma_t - \mu G_t)\, s_{t+1} = 1 - \frac{1}{L_{t+1}} \qquad (11)$$

$$V_t = (1 - \Gamma_t)\, s_{t+1} R_t Q_t K_{t+1} \qquad (12)$$

$$C^e_t = (1 - \gamma^e)\, V_t \qquad (13)$$

$$N_{t+1} = \gamma^e V_t + H_t^e W_t^e \qquad (14)$$

$$Y_t^e = A_t K_t^{\alpha} \left(H_t^{\Omega} H_t^{e\,1-\Omega}\right)^{1-\alpha} \qquad (15)$$

$$W_t H_t = (1-\alpha)\Omega P_{m,t} Y_t^e \qquad (16)$$

$$W_t^e H_t^e = (1-\alpha)(1-\Omega) P_{m,t} Y_t^e \qquad (17)$$

$$R_t^k = \frac{\alpha P_{m,t} Y_t^e}{Q_{t-1} K_t} + \frac{Q_t(1-\delta)}{Q_{t-1}} \qquad (18)$$

$$\Pi_t^* = \frac{\varepsilon}{\varepsilon-1} \frac{X_t^1}{X_t^2} \qquad (19)$$

$$X_t^1 = \lambda_t P_{m,t} Y_t + \theta\beta \Pi_{t+1}^{\varepsilon} X_{t+1}^1 \qquad (20)$$

$$X_t^2 = \lambda_t Y_t + \theta\beta \Pi_{t+1}^{\varepsilon-1} X_{t+1}^2 \qquad (21)$$

$$1 = \theta \Pi_t^{\varepsilon-1} + (1-\theta) \Pi_t^{*\,1-\varepsilon} \qquad (22)$$

$$K_{t+1} = (1-\delta) K_t + I_t - \frac{\chi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t \qquad (23)$$

$$Q_t = \left[1 - \chi\left(\frac{I_t}{K_t} - \delta\right)\right]^{-1} \qquad (24)$$

$$Y_t^e = D_t Y_t \qquad (25)$$

$$D_t = \theta \Pi_t^{\varepsilon} D_{t-1} + (1-\theta) \Pi_t^{*\,-\varepsilon} \qquad (26)$$

$$Y_t = C_t + C_t^e + I_t + \frac{\chi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t + \mu G_t R_t^k Q_{t-1} K_t \qquad (27)$$

$$R_t = \frac{i_t}{\Pi_{t+1}} \qquad (28)$$

$$\frac{i_t}{i} = \left(\frac{i_{t-1}}{i}\right)^{\rho} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi}\right]^{1-\rho} \exp(\varepsilon_t^r) \qquad (29)$$

$$\ln(A_t) = \rho_a \ln(A_{t-1}) + \varepsilon_t^a \qquad (30)$$

$H_t^e = 1$ (131)

$P_t' = 1 - F_t$ (132)

$G_t' = \phi\left(\frac{\ln(\varpi_t)}{\sigma} + \frac{\sigma}{2}\right)\frac{1}{\sigma}$ (133)

变量 (33)

C_t, λ_t, M_t, Π_t, R_t,
H_t, W_t, F_t, G_t, P_t,
ϖ_t, S_t, R_t^k, L_t, Q_t,
K_t, N_t, P_t', G_t', V_t,
C_t^e, H_t^e, W_t^e, Y_t^e, A_t,
I_t, Π_t^*, X_t^1, X_t^2, D_t,
Y_t, i_t, $P_{m,t}$

Dynare 程序要注意 L_t, K_t, N_t, S_t 的時間点選取.

BGG ver.2、稳態求解第二部分.

(30) $A=1$ (外生給定) ⑨

$\Pi=1$ (外生給定) ⑩

(122) $1=\theta\Pi^{\varepsilon-1}+(1-\theta)\Pi^{*1-\varepsilon} \Rightarrow \Pi^*=1$ ⑪

粘性価格部分.

(20) $X^1=\frac{\lambda P_m Y}{1-\theta\beta}$

(21) $X^2=\frac{\lambda Y}{1-\theta\beta}$

(19) $\Pi^*=\frac{\varepsilon}{\varepsilon-1}\frac{X^1}{X^2}$

$\Rightarrow P_m=\frac{\varepsilon-1}{\varepsilon}$ ⑫

(26) $D=\theta D\Pi^{\varepsilon}+(1-\theta)\Pi^{*-\varepsilon} \Rightarrow D=1$ ⑬

(3) $\beta\lambda R-\lambda=0 \Rightarrow R=\frac{1}{\beta}$ ⑭

(28) $R=\frac{i}{\Pi} \Rightarrow i=R$ ⑮

(8) $\underbrace{S}_{\text{已知}}=\frac{R^k}{R} \Rightarrow R^k=SR$ ⑯

各種利率.

(23) $K=(1-\delta)K+I-\frac{\chi}{2}\left(\frac{I}{K}-\delta\right)K \Rightarrow \frac{I}{K}=\delta$

(24) $Q=\left[1-\chi\left(\frac{I}{K}-\delta\right)\right]^{-1} \Rightarrow Q=1$ ⑰

(18) $R^k=\frac{\alpha P_m Y^e}{QK}+\frac{Q(1-\delta)}{Q} \Rightarrow \frac{Y^e}{K}=\frac{1}{\alpha P_m}(R^k-1+\delta)$

(25) $Y^e=DY \Rightarrow \frac{Y}{K}=\frac{Y^e}{K}$.

(12) $V=(1-P)SRQK \Rightarrow \underline{\frac{V}{K}=(1-P)SRQ}$

(14) $N=r^eV+W^e$

(9) $L=\frac{K}{N}$

$\Rightarrow \frac{1}{L}=\frac{N}{K}=r^e\underbrace{\frac{V}{K}}_{已知}+\frac{W^e}{K}$

(17) $W^eH^e=(1-\alpha)(1-\Omega)P_mY$，又由(31) $\underline{H^e=1}$ ⑱

$\Rightarrow \underline{\frac{W^e}{K}=(1-\alpha)(1-\Omega)P_m\underbrace{\frac{Y}{K}}_{已知}}$

結合以上两个推导:

$\underline{r^e=\left(\frac{1}{L}-\frac{W^e}{K}\right)\left(\frac{V}{K}\right)^{-1}}$ ← r^e被視为内生由$L=\frac{K}{N}$決定

又有 $\underline{\frac{N}{K}=\frac{1}{L}}$

(13) $C^e=(1-r^e)V \Rightarrow \underline{\frac{C^e}{K}=(1-r^e)\frac{V}{K}}$

(27) $\underbrace{\frac{Y}{K}}_{已知}=\frac{C}{K}+\underbrace{\frac{C^e}{K}}_{已知}+\underbrace{\frac{I}{K}}_{=0}+\underbrace{uGR^k}_{已知}$

$\Rightarrow \underline{\frac{C}{K}=\frac{Y}{K}-\frac{C^e}{K}-\frac{I}{K}-uGR^k}$

$$(1)\quad \lambda = \frac{1}{c}$$

$$(4)\quad \lambda W = \zeta \frac{1}{1-H} \quad \Big\} \quad W = \zeta \frac{1}{1-H} C.$$

$$(16)\quad WH = (1-\alpha)\Omega P_m Y^e$$

$$\Rightarrow \frac{W}{K} H = (1-\alpha)\Omega P_m \underbrace{\frac{Y^e}{K}}_{\text{已知}}$$

$$\Rightarrow \underbrace{\frac{C}{K}}_{\text{已知}} \zeta \frac{H}{1-H} = (1-\alpha) P_m \underbrace{\frac{Y^e}{K}}_{\text{已知}}$$

$$\Rightarrow \underline{H = \left\{\zeta \frac{C}{K}\left[(1-\alpha)\Omega P_m \frac{Y^e}{K}\right]^{-1} + 1\right\}^{-1}} \quad (19)$$

$$(15)\quad Y^e = \underbrace{A}_{=1} K^{\alpha}\left[H^{\Omega}(\underbrace{H^e}_{=1})^{1-\Omega}\right]^{1-\alpha}$$

$$\Rightarrow \frac{Y^e}{K} = \left(\frac{H^{\Omega}}{K}\right)^{1-\alpha}$$

$$\Rightarrow \underline{K = H^{\Omega}\left(\frac{Y^e}{K}\right)^{-\frac{1}{1-\alpha}}} \quad (20)$$

$\underline{I = \frac{I}{K} \times K}$ (21) $\underline{Y^e = \frac{Y^e}{K} \times K}$ (22) $\underline{Y = Y^e}$ (23)

$\underline{V = \frac{V}{K} \times K}$ (24) $\underline{W^e = \frac{W^e}{K} \times K}$ (25) $\underline{N = \frac{N}{K} \times K}$ (26)

$\underline{C^e = \frac{C^e}{K} \times K}$ (27) $\underline{C = \frac{C}{K} \times K}$ (28) $\underline{W = \zeta \frac{1}{1-H} C}$ (29).

$\underline{\lambda = \frac{1}{C}}$ (30) $\underline{X^1 = \frac{\lambda P_m Y}{1-\theta\beta}}$ (31) $\underline{X^2 = \frac{\lambda Y}{1-\theta\beta}}$ (32)

No.

Date

$$(2)\quad \xi \frac{1}{M} - \lambda + \beta \frac{\lambda}{\pi} = 0$$

$$\Rightarrow \underline{M = (\lambda - \beta\lambda)^{-1} \xi} \quad (33).$$