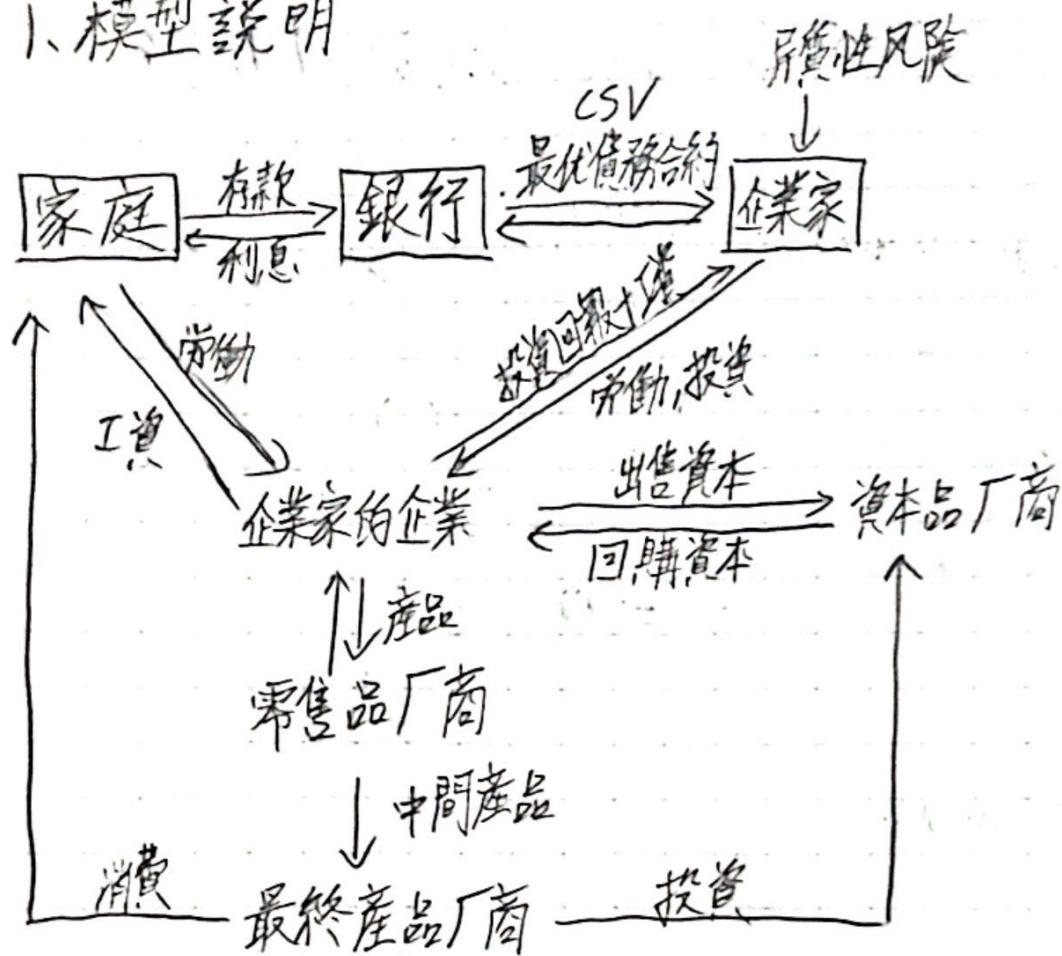


BGG Ver.2

1. 模型說明



2. 家庭(II)

$$\underset{C_t, M_t, B_t, H_t}{\text{Max}} \quad E_t \sum_{t=0}^{\infty} \beta^t \left\{ \ln(C_t + \beta \ln(M_t) + \beta \ln(1-H_t)) \right\}$$

$$\text{s.t. } C_t + M_t + B_t = W_t H_t + \frac{M_{t+1}}{\pi_{t+1}} + R_{t+1} B_{t+1} + J_t - T_t$$

$$\frac{\partial \mathcal{L}}{\partial C_t} = \frac{1}{C_t} - \lambda_t = 0 \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial M_t} = 3 \frac{1}{M_t} - \lambda_t + \beta \lambda_{t+1} \frac{1}{\pi_{t+1}} = 0 \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial B_t} = -\lambda_t + \beta \lambda_t R_t = 0 \quad (3)$$

$$\frac{\partial \mathcal{L}}{\partial H_t} = -\beta \frac{1}{1-H_t} + \lambda_t W_t = 0 \quad (4)$$

C_t : 實質消費, M_t : 實質貨幣需求, B_t : 儲蓄

H_t : 工人勞動, W_t : 實質工資, π_t : 通脹率(gross)

R_t : 實質存款利率(gross)

2. 企業家

- ① 企業家異質，企業家在 t 期購買資本 K_t^k 用于 $t+1$ 期生產。
- ② 視資本價格 Q_t^k 與投資回報率 R_t^k 外生。
- ③ 投資面臨異質性風險 ω_t^k ，回報率為 $w_t^k R_t^k$ 。

ω_t^k : 企業家在 t 期面臨的風險，隨機变量

ω_t^k 獨立同分布，分布函數不隨時問變化，記作 $F(\omega_t^k)$

計算方便起見，令 $E(\omega_t^k) = 1$

- ④ 七期末，企業家的淨資產為 N_{t+1}^k ，向銀行貸款 B_t^k 。

$$Q_t^k K_{t+1}^k = N_{t+1}^k + B_t^k \quad (\text{balance sheet})$$

資產 = 净資產 + 債務

3. 企業家與銀行的債務合約 (CSV)

- ① 若異質性風險 $\omega_t^k \geq$ 風險臨界值 $\bar{\omega}_t^k$ ，則企業家向銀行還本付息。
- ② 若 $\omega_t^k < \bar{\omega}_t^k$ ，則企業家中請破產清算，但需支付清算成本。

• 風險臨界值定義： $\bar{\omega}_t^k R_{t+1}^k Q_t^k K_{t+1}^k = R_t^L B_t^k$

其中， R_t^L 是企業與銀行的合約利率。

- 企業家破產清算時，銀行得到

$$(1-\alpha) \bar{\omega}_t^k R_{t+1}^k Q_t^k K_{t+1}^k$$

即清算成本為企業家資產的比例 α 。

③假設銀行之間完全競爭，利潤為0，該假設下
銀行貸款預期收益 = 銀行吸收存款的成本

$$\begin{aligned}
 \text{预期收益} &= \underbrace{\int_0^{\bar{w}_t} (1-\alpha) w^\alpha R_{t+1}^k Q_t K_{t+1}^\alpha dF(w^\alpha)}_{+ \int_{\bar{w}_t}^{\infty} R_t^L B_t^\alpha dF(w^\alpha)} \\
 &= \underbrace{(1-\alpha) \int_0^{\bar{w}_t} w dF(w^\alpha) R_{t+1}^k Q_t K_{t+1}^\alpha}_{+ \int_{\bar{w}_t}^{\infty} 1 dF(w^\alpha) \bar{w}_t^\alpha R_{t+1}^k Q_t K_{t+1}^\alpha} \\
 &= (1-\alpha) \int_0^{\bar{w}_t} w^\alpha dF(w^\alpha) R_{t+1}^k Q_t K_{t+1}^\alpha \\
 &\quad + (1 - F(\bar{w}_t)) \bar{w}_t^\alpha R_{t+1}^k Q_t K_{t+1}^\alpha \quad \left. \right\} \text{银行}
 \end{aligned}$$

4. 企業家價值最大化問題（只有我知道風險，成不成功我說了算）

$$V_t^{\dot{w}} = \max_{\bar{w}_t^{\dot{w}}, K_{t+1}^{\dot{w}}} E_t \left\{ \int_{\bar{w}_t^{\dot{w}}}^{\infty} w^{\dot{w}} R_{t+1}^k Q_t K_{t+1}^{\dot{w}} dF(w^{\dot{w}}) - (1 - F(\bar{w}_t^{\dot{w}})) \bar{w}_t^{\dot{w}} R_{t+1}^k Q_t K_{t+1}^{\dot{w}} \right\}$$

← 投資成功的收益
 ← 投資成功支付的成本

s.t. 銀行零利潤條件: $R_e B_e =$

$$(1-u) \int_0^{\bar{w}_t} w^{\frac{1}{\alpha}} dF(w^{\frac{1}{\alpha}}) R_{t+1}^k Q_t K_{t+1}^{\frac{1}{\alpha}} + (1-F(\bar{w}_t)) \bar{w}_t^{\frac{1}{\alpha}} R_{t+1}^k Q_t K_{t+1}^{\frac{1}{\alpha}}$$

变量替换：

$$F_t \equiv F(\bar{w}_t^{\hat{a}}) \quad (5), \quad G_t = \int_0^{\bar{w}_t^{\hat{a}}} w^{\hat{a}} dF(w^{\hat{a}}) \quad (6)$$

$$\begin{aligned} P_t &\equiv \int_0^{\bar{w}_t^{\hat{a}}} w^{\hat{a}} dF(w^{\hat{a}}) + \bar{w}_t^{\hat{a}} (1 - F(\bar{w}_t^{\hat{a}})) \\ &= G_t + \bar{w}_t^{\hat{a}} (1 - F(\bar{w}_t^{\hat{a}})) \end{aligned} \quad (7)$$

目标函数：

$$\begin{aligned} &\int_{\bar{w}_t^{\hat{a}}}^{\infty} w^{\hat{a}} dF(w^{\hat{a}}) R_{t+1}^k Q_t K_{t+1}^{\hat{a}} - (1 - F(\bar{w}_t^{\hat{a}})) \bar{w}_t^{\hat{a}} R_{t+1}^k Q_t K_{t+1}^{\hat{a}} \\ &= \left\{ \int_{\bar{w}_t^{\hat{a}}}^{\infty} w^{\hat{a}} dF(w^{\hat{a}}) - (1 - F(\bar{w}_t^{\hat{a}})) \bar{w}_t^{\hat{a}} \right\} R_{t+1}^k Q_t K_{t+1}^{\hat{a}} \\ &= \left\{ \int_0^{\infty} w^{\hat{a}} dF(w^{\hat{a}}) - \int_0^{\bar{w}_t^{\hat{a}}} w^{\hat{a}} dF(w^{\hat{a}}) - (1 - F(\bar{w}_t^{\hat{a}})) \bar{w}_t^{\hat{a}} \right\} \\ &\quad \times R_{t+1}^k Q_t K_{t+1}^{\hat{a}} \\ &= \left\{ E(w^{\hat{a}}) - P_t \right\} R_{t+1}^k Q_t K_{t+1}^{\hat{a}} \\ &= (1 - P_t) R_{t+1}^k Q_t K_{t+1}^{\hat{a}} \end{aligned}$$

銀行零利潤條件左側：

$$\begin{aligned}
 & (1 - F(\bar{w}_t)) \bar{w}_t^{\frac{1}{\alpha}} R_{t+1}^k Q_t K_{t+1}^{\frac{1}{\alpha}} + (1-\mu) \int_0^{\bar{w}_t^{\frac{1}{\alpha}}} w^{\frac{1}{\alpha}} R_{t+1}^k Q_t K_{t+1}^{\frac{1}{\alpha}} dF(w^{\frac{1}{\alpha}}) \\
 &= \{(1 - F(\bar{w}_t)) \bar{w}_t^{\frac{1}{\alpha}} + (1-\mu) \int_0^{\bar{w}_t^{\frac{1}{\alpha}}} w^{\frac{1}{\alpha}} dF(w^{\frac{1}{\alpha}})\} R_{t+1}^k Q_t K_{t+1}^{\frac{1}{\alpha}}. \\
 &= \underbrace{\{(1 - F(\bar{w}_t)) \bar{w}_t^{\frac{1}{\alpha}} + \int_0^{\bar{w}_t^{\frac{1}{\alpha}}} w^{\frac{1}{\alpha}} dF(w^{\frac{1}{\alpha}})\}}_{\mu} R_{t+1}^k Q_t K_{t+1}^{\frac{1}{\alpha}} \\
 &= (P_t - \mu G_t) R_{t+1}^k Q_t K_{t+1}^{\frac{1}{\alpha}}.
 \end{aligned}$$

零利潤條件右側：代入企業家 balance sheet.

$$R_t B_t^{\frac{1}{\alpha}} = R_t (Q_t K_{t+1}^{\frac{1}{\alpha}} - N_{t+1}^{\frac{1}{\alpha}})$$

企業家價值最大化問題：

$$\bar{v}_t^{\frac{1}{\alpha}} = \max_{\bar{w}_t^{\frac{1}{\alpha}}, K_{t+1}^{\frac{1}{\alpha}}} (1 - P_t) R_{t+1}^k Q_t K_{t+1}^{\frac{1}{\alpha}}$$

$$\text{s.t. } (P_t - \mu G_t) R_{t+1}^k Q_t K_{t+1}^{\frac{1}{\alpha}} = R_t (Q_t K_{t+1}^{\frac{1}{\alpha}} - N_{t+1}^{\frac{1}{\alpha}})$$

$$f = (1 - P_t) R_{t+1}^k Q_t K_{t+1}^{\frac{1}{\alpha}} + \gamma_t \{(P_t - \mu G_t) R_{t+1}^k Q_t K_{t+1}^{\frac{1}{\alpha}} - R_t (Q_t K_{t+1}^{\frac{1}{\alpha}} - N_{t+1}^{\frac{1}{\alpha}})\}$$

$$\begin{aligned}
 \frac{\partial f}{\partial \bar{w}_t^{\frac{1}{\alpha}}} : & -P_t' R_{t+1}^k Q_t K_{t+1}^{\frac{1}{\alpha}} + \gamma_t (P_t' - \mu G_t') R_{t+1}^k Q_t K_{t+1}^{\frac{1}{\alpha}} = 0 \\
 \Rightarrow & P_t' = \gamma_t (P_t' - \mu G_t') \quad \textcircled{1}
 \end{aligned}$$

$$\frac{\partial f}{\partial K_{t+1}^{\frac{1}{\alpha}}} = (1 - P_t) R_{t+1}^k Q_t + \gamma_t \{(P_t - \mu G_t) R_{t+1}^k Q_t - R_t Q_t\} = 0$$

$$\frac{\partial L}{\partial K_t} : (P_t - uG_t) R_{t+1}^k Q_t K_{t+1}^{\dot{a}} = R_t (Q_t K_t^{\dot{a}} - N_{t+1}^{\dot{a}}) \quad (3)$$

令 $S_{t+1} \equiv R_{t+1}^k / R_t$ (8, 風險利差), $L_{t+1} = \frac{Q_t K_{t+1}^{\dot{a}}}{N_{t+1}^{\dot{a}}}$ (9, 杠杆率)

$$(1) \quad r_t = \frac{P'_t}{P_t - uG_t}$$

$$(2) \quad (1-P_t) \frac{R_{t+1}^k}{R_t} + r_t \{ (P_t - uG_t) \frac{R_{t+1}^k}{R_t} - 1 \} = 0$$

將(1)代入(2)消去 r_t , 再用(8)替換 $\frac{R_{t+1}^k}{R_t}$ 得到.

$$(1-P_t) S_{t+1} + \frac{P'_t}{P_t - uG_t} \{ (P_t - uG_t) S_{t+1} - 1 \} = 0$$

$$\Rightarrow \frac{P'_t - uG_t}{P_t} (1-P_t) + (P_t - uG_t) = \frac{1}{S_{t+1}} \quad (10)$$

將(3)兩邊同除 $R_t N_{t+1}^{\dot{a}}$ 可得

$$(P_t - uG_t) \frac{R_{t+1}^k}{R_t} \frac{Q_t K_{t+1}^{\dot{a}}}{N_{t+1}^{\dot{a}}} = \frac{Q_t K_{t+1}^{\dot{a}}}{N_{t+1}^{\dot{a}}} - 1$$

$$\Rightarrow (P_t - uG_t) S_{t+1} = \frac{L_{t+1} - 1}{L_{t+1}} \quad (11)$$

注意, 由於所有企業家來說, 資本市場環境相同 (即, R_t , R_t^k , P_t , uG_t) 都是一樣的, 所以企業家的最優反應相同, 可以忽略異質性符号 \dot{a} .

5. 補足：最優債務協約為模型帶來了什麼？

• 注意(10)式左側都是 \bar{w}_t^j 的函數，利差 s_{t+1} 與 \bar{w}_t^j 存在一一對應的函數關係，記作 $s_{t+1} = h(\bar{w}_t^j) \Leftrightarrow h^{-1}(s_{t+1}) = \bar{w}_t^j$.

• (11)式可以寫作 $g(\bar{w}_t^j) s_{t+1} = 1 - \frac{1}{L_{t+1}}$
 組合(10)與(11)可知
 $\Rightarrow g(h^{-1}(s_{t+1})) s_{t+1} = 1 - \frac{1}{L_{t+1}}$

那麼利差與企業杠桿率存在一一對應的關係。

$$s_{t+1} = m\left(\frac{1}{L_{t+1}}\right)$$

通過隱函數定理，可以證明 $m'(\cdot) < 0$.

即企業杠桿越高，利差越大。

• $\frac{N_{t+1}}{Q_t K_{t+1}} = m^{-1}\left(\frac{R_{t+1}^k}{R_t}\right)$, 那麼

$$\left(1 - \frac{N_{t+1}}{Q_t K_{t+1}}\right) Q_t K_{t+1} = \left(1 - m^{-1}\left(\frac{R_{t+1}^k}{R_t}\right)\right) Q_t K_{t+1}$$

$$B_t = \underbrace{\left(1 - m^{-1}\left(\frac{R_{t+1}^k}{R_t}\right)\right)}_{\begin{array}{l} \text{企業貸款} \\ (\text{居民存款}) \end{array}} \underbrace{Q_t K_{t+1}}_{\begin{array}{l} \text{可貸資金比例} \\ (\text{依存於利差}) \end{array}}$$

企業資產價值

6. 均衡狀態下(即最優合約下)的企業價值

$$\begin{aligned}
 V_t &= (1 - \rho_t) R_{t+1}^k Q_t K_{t+1} \\
 &= (1 - \rho_t) \frac{R_{t+1}^k}{R_t} R_t Q_t K_{t+1} \\
 &= (1 - \rho_t) S_{t+1} R_t Q_t K_{t+1} \quad (12)
 \end{aligned}$$

假設每期企業家消費掉一部分：

$$C_t^e = (1 - \nu^e) V_t \quad (13)$$

並且企業家投入自己的工資，使之與殘存的企業價值形成新的淨資產：

$$N_{t+1} = \nu^e V_t + H_t^e W_t^e \quad (14)$$

7. 企業家持有企業的生產經營活動

① 企業家向資本品工商購買生產性資本，向家庭雇佣勞動生產：

$$Y_t^e = A_t K_t^\alpha [H_t^{e2} (H_t^e)^{1-\alpha}]^{1-\alpha} \quad (15)$$

其中， H_t^e 勞動來自家庭， H_t^e 來自企業家（假設供給無彈性 $H_t^e = 1$ ）

② 企業家在七期末將折旧后的資本以 Q_{t+1} 價格賣給資本品工商，向資本品工商購入 K_t 的新資本，將產出以 $P_{m,t}$ 的價格賣給零售商。

$$\underset{H_t, H_t^e}{\text{Max}} \quad J_t = P_{m,t} Y_t^e + Q_t(1-\delta)K_t - R_t^k Q_{t-1} K_t - w_t H_t - w_t^e H_t^e,$$

$$\text{s.t. } Y_t^e = A_t K_t^\alpha [H_t^{\Omega^2} (H_t^e)^{1-\Omega}]^{1-\alpha}$$

$$\begin{aligned} \frac{\partial \varphi}{\partial H_t} : \quad & w_t = P_{m,t} A_t K_t^\alpha (1-\alpha) [H_t^{\Omega^2} (H_t^e)^{1-\Omega}]^{-\alpha} \Omega H_t^{\Omega-1} (H_t^e)^{1-\Omega} \\ \Rightarrow w_t H_t = & (1-\alpha) \Omega P_{m,t} \underline{A_t K_t^\alpha [H_t^{\Omega^2} (H_t^e)^{1-\Omega}]^{1-\alpha}} \\ = & (1-\alpha) \Omega P_{m,t} \underline{Y_t^e}. \end{aligned} \quad (16)$$

$$\frac{\partial \varphi}{\partial H_t^e} : \quad \underline{w_t^e H_t^e} = (1-\alpha)(1-\Omega) P_{m,t} \underline{Y_t^e} \quad (17)$$

假設企業家完全競爭，零利潤條件下

$$0 = P_{m,t} Y_t^e + Q_t(1-\delta)K_t - R_t^k Q_{t-1} K_t - w_t H_t - w_t^e H_t^e.$$

$$0 = \alpha P_{m,t} Y_t^e + Q_t(1-\delta)K_t - R_t^k Q_{t-1} K_t.$$

$$\Rightarrow \underline{R_t^k} = \frac{\alpha P_{m,t} Y_t^e}{Q_{t-1} K_t} + \frac{Q_t(1-\delta)}{Q_{t-1}} \quad (18)$$

8. 最終生產品工商

以產值價格從零售商處購買異質性中間商品 Y_t^i ，打包後以價格 P_t^i 出售給家庭與資本品工商：

$$\begin{aligned} \text{Max}_{Y_t^i} \quad & P_t Y_t - \int_0^1 P_t^i Y_t^i \, di \\ \text{s.t. } & Y_t = \left[\int_0^1 (Y_t^i)^{\frac{\varepsilon-1}{\varepsilon}} \, di \right]^{\frac{\varepsilon}{\varepsilon-1}} \end{aligned}$$

$$\text{FOC: } P_t \frac{\frac{1}{\varepsilon-1} \left[\int_0^1 (Y_t^i)^{\frac{\varepsilon-1}{\varepsilon}} \, di \right]^{\frac{1}{\varepsilon-1}} \frac{1}{\varepsilon} (Y_t^i)^{-\frac{1}{\varepsilon}}}{= Y_t^{\frac{1}{\varepsilon}}} = P_t^i.$$

$$\Rightarrow Y_t^i = \left(\frac{P_t^i}{P_t} \right)^{-\varepsilon} Y_t.$$

將FOC代入目標函數，根據0利潤條件可得

$$P_t Y_t - \int_0^1 P_t^i \left(\frac{P_t^i}{P_t} \right)^{-\varepsilon} Y_t \, di = 0.$$

$$\Rightarrow P_t = \left(\int_0^1 P_t^i^{1-\varepsilon} \, di \right)^{\frac{1}{1-\varepsilon}}$$

9. 需售商

从 P_{mt} 从企业家处买入产品，转化为中间产品，以 P_t^* 市场价格卖给最终品厂商，Calvo 定价。

可最优调整价格的概率为 $1-\theta$ ，

$$\max_{P_t^*} E_t \sum_{k=0}^{\infty} (\theta\beta)^k \frac{\lambda_{t+k}}{\lambda_t} \left(\frac{P_t^*}{P_{t+k}} - P_{m,t+k} \right) Y_{t+k}^{i*}$$

$$\text{s.t. } Y_{t+k}^{i*} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k}$$

$$\text{FOC: } \sum_{k=0}^{\infty} (\theta\beta)^k \frac{\lambda_{t+k}}{\lambda_t} \left[(1-\varepsilon) \frac{\frac{P_t^*}{P_{t+k}}^{-\varepsilon}}{P_{t+k}^{1-\varepsilon}} - (-\varepsilon) P_{m,t+k} \left(\frac{P_t^*}{P_{t+k}}^{-\varepsilon} \right) \right] Y_{t+k}$$

$$\Rightarrow P_t^* = \frac{\varepsilon}{1-\varepsilon} \frac{\sum_k (\theta\beta)^k \lambda_{t+k} P_{m,t+k} P_{t+k}^{\varepsilon} Y_{t+k}}{\sum_k (\theta\beta)^k \lambda_{t+k} P_{t+k}^{1-\varepsilon} Y_{t+k}}$$

$$\Rightarrow \frac{P_t^*}{P_t} \equiv \pi_t^* = \frac{\varepsilon}{1-\varepsilon} \frac{\sum_k (\theta\beta)^k \left(\frac{P_{t+k}}{P_t} \right)^{\varepsilon} P_{m,t+k} Y_{t+k}}{\sum_k (\theta\beta)^k \left(\frac{P_{t+k}}{P_t} \right)^{1-\varepsilon} Y_{t+k}}$$

$$\text{令 } X_t' = \sum_{k=0}^{\infty} (\theta\beta)^k \lambda_{t+k} P_{m,t+k} \left(\frac{P_{t+k}}{P_t} \right)^{\varepsilon} Y_{t+k}$$

$$= \lambda_t P_{mt} Y_t + \theta\beta \sum_{k=1}^{\infty} (\theta\beta)^{k-1} \lambda_{t+k} P_{m,t+k} \left(\frac{P_{t+k}}{P_t} \right)^{\varepsilon} Y_{t+k}$$

$$(令 k = t+1)$$

$$= \lambda_t P_{mt} Y_t + \theta\beta \sum_{r=0}^{\infty} (\theta\beta)^r \lambda_{t+r+1} P_{m,t+r+1} \left(\frac{P_{t+r+1}}{P_{t+1}} \frac{P_{t+1}}{P_t} \right)^{\varepsilon} Y_{t+r+1}$$

$$= \lambda_t P_{mt} Y_t + \theta\beta \pi_{t+1}^{\varepsilon} X_{t+1}'$$

同理，

$$\begin{aligned} X_t^2 &= \sum_{k=0}^{\infty} (\theta P)^k \lambda_{t+k} \left(\frac{P_{t+k}}{P_t}\right)^{1-\varepsilon} Y_{t+k} \\ &= \lambda_t Y_t + \theta P \pi_{t+1}^{\varepsilon-1} X_{t+1}^2 \end{aligned}$$

綜上可得

$$\underline{\pi_t^*} = \frac{\varepsilon}{\varepsilon-1} \frac{X_t^1}{X_t^2} \quad (19)$$

$$\underline{X_t^1} = \lambda_t P_{t+1} Y_t + \theta P \pi_{t+1}^{\varepsilon-1} X_{t+1}^1 \quad (20)$$

$$\underline{X_t^2} = \lambda_t Y_t + \theta P \pi_{t+1}^{\varepsilon-1} X_{t+1}^2 \quad (21)$$

根據最終產品廠商的零利潤條件：

$$\begin{aligned} P_t^{1-\varepsilon} &= \int_0^1 P_t^i{}^{1-\varepsilon} di \\ &= \theta \int_0^1 P_{t+1}^i{}^{1-\varepsilon} di + (1-\theta) \int_0^1 P_t^i{}^*{}^{1-\varepsilon} di \\ &= \theta P_{t+1}^{1-\varepsilon} + (1-\theta) (P_t^i{}^*)^{1-\varepsilon} \end{aligned}$$

兩邊同除 $P_t^{1-\varepsilon}$

$$\left(\frac{P_t}{P_t}\right)^{1-\varepsilon} = \theta \left(\frac{P_{t+1}}{P_t}\right)^{1-\varepsilon} + (1-\theta) \left(\frac{P_t^i{}^*}{P_t}\right)^{1-\varepsilon}$$

$$\Rightarrow \underline{1} = \theta \pi_t^{\varepsilon-1} + (1-\theta) \pi_t^i{}^*{}^{1-\varepsilon} \quad (22)$$

10. 資本累積與資本品企業

$$\text{資本累積方程 } \underline{k_{t+1} = (1-\delta)k_t + I_t - \frac{\chi}{2} \left(\frac{I_t}{k_t} - \delta \right)^2 k_t} \quad (23)$$

其中, $I_t - \frac{\chi}{2} \left(\frac{I_t}{k_t} - \delta \right)^2$ 為新增資本量, 由資本品廠商提供。

資本品企業 (假設資本生產的費用為 1 單位)

$$\max_{I_t} Q_t \left[I_t - \frac{\chi}{2} \left(\frac{I_t}{k_t} - \delta \right)^2 \right] - I_t$$

$$\Rightarrow \underline{Q_t = \left[1 - \chi \left(\frac{I_t}{k_t} - \delta \right) \right]^{-1}} \quad (24)$$

11. 市場出清條件與加總

$$Y_t^e = \int_0^1 Y_t^i di = \int_0^1 \left(\frac{P_t^i}{P_t} \right)^{-\varepsilon} di Y_t = P_t Y_t \quad (25)$$

$$\begin{aligned} D_t &= \int_0^1 \left(\frac{P_t^i}{P_t} \right)^{-\varepsilon} di \\ &= \theta \int_0^1 \left(\frac{P_{t-1}^i}{P_t} \right)^{-\varepsilon} di + (1-\theta) \int_0^1 \left(\frac{P_t^{i*}}{P_t} \right)^{-\varepsilon} di \\ &= \theta \int_0^1 \left(\frac{P_{t-1}^i}{P_{t-1}} \frac{P_{t-1}}{P_t} \right)^{-\varepsilon} di + (1-\theta) \left(\frac{P_t^{i*}}{P_t} \right)^{-\varepsilon} \\ &= \underline{\theta \pi_t^\varepsilon D_{t-1} + (1-\theta) \pi_t^{i* - \varepsilon}} \end{aligned} \quad (26)$$

$$\underline{Y_t = C_t + C_t^e + I_t + \frac{\chi}{2} \left(\frac{I_t}{k_t} - \delta \right)^2 k_t + u G_t R_t^k Q_{t-1} k_t} \quad (27)$$

12. 宏观，外生冲击与一些补充方程

$$R_t = \frac{i_t}{\pi_t + 1} \quad (28)$$

$$\frac{i_t}{\pi_t} = \left(\frac{i_{t-1}}{\pi_t} \right)^p \left[\left(\frac{\pi_t}{\pi_{t-1}} \right)^{\frac{1-p}{p}} \right]^{1-p} \exp(\varepsilon_t^r) \quad (29)$$

$$\ln(A_t) = \rho \ln(A_{t-1}) + \varepsilon_t^a \quad (30)$$

$$H_t^e = 1 \quad (31)$$

$$P_t^e = P(\bar{w}_t^e) \quad (32)$$

$$G_t^e = G(\bar{w}_t^e) \quad (33)$$

13. 方程代换

方才(5), (6), (7), (32), (33)没写成显性方程, 因此需要
替换.

假设 w^e 服从对数正态分布, 即

$$\ln(w^e) \sim N(\mu, \sigma^2)$$

$$\text{那么 } f(w^e) = \frac{1}{\sqrt{2\pi}\sigma w^e} \exp\left[-\frac{(\ln(w^e)-\mu)^2}{2\sigma^2}\right]$$

$$\text{易证得 } E(w^e) = \exp(\mu + \frac{1}{2}\sigma^2)$$

$$\text{Var}(w^e) = \exp(\sigma^2 + 2\mu)(\exp(\sigma^2) - 1)$$

由于我们假设 $E(w^e) = 1$, 那么 $\mu = -\frac{1}{2}\sigma^2$.

$$\text{于是 } \text{Var}(w^e) = \exp(\sigma^2) - 1 \approx \sigma^2$$

$$(5)' F_t = F(\bar{w}^{\frac{1}{2}})$$

$$= \int_0^{\bar{w}^{\frac{1}{2}}} \frac{1}{\sqrt{2\pi}\delta w^{\frac{1}{2}}} \exp\left[-\frac{(\ln(w^{\frac{1}{2}}) + \frac{\delta^2}{2})^2}{2\delta^2}\right] dw^{\frac{1}{2}}$$

$$\hat{z} = \ln(w^{\frac{1}{2}})$$

$$= \int_{-\infty}^{\ln(\bar{w}^{\frac{1}{2}})} \frac{1}{\sqrt{2\pi}\delta} \exp\left[-\frac{(z + \frac{\delta^2}{2})^2}{2\delta^2}\right] \underbrace{\frac{dw^{\frac{1}{2}}}{w^{\frac{1}{2}}}}_{= d\ln(w^{\frac{1}{2}}) = dz}$$

$$\hat{m} = \frac{z + \frac{\delta^2}{2}}{\delta}$$

$$= \int_{-\infty}^{\frac{\ln(\bar{w}^{\frac{1}{2}}) + \frac{\delta^2}{2}}{\delta}} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{m^2}{2}\right] \underbrace{\frac{dz}{\delta}}_{= dm}$$

$$\underline{F_t = \text{Re}\left(\frac{\ln(\bar{w}^{\frac{1}{2}}) + \frac{\delta^2}{2}}{\delta}\right)}$$

$$(6)' G_t = \int_0^{\bar{w}^{\frac{1}{2}}} \omega dF(\omega)$$

$$= \int_0^{\bar{w}^{\frac{1}{2}}} \frac{1}{\sqrt{2\pi}\delta} \exp\left[-\frac{(\ln(\omega) + \frac{\delta^2}{2})^2}{2\delta^2}\right] d\omega$$

$$\hat{z} = \ln(\omega)$$

$$= \int_{-\infty}^{\ln(\bar{w}^{\frac{1}{2}})} \frac{1}{\sqrt{2\pi}\delta} \exp\left[-\frac{(z + \frac{\delta^2}{2})^2}{2\delta^2}\right] \underbrace{\omega \frac{d\omega}{\omega}}_{= dz} = dz$$

$$\text{注意到 } \omega = \exp(\ln(\omega)) = \exp(z)$$

$$= \int_{-\infty}^{\ln(\bar{w}^{\frac{1}{2}})} \frac{1}{\sqrt{2\pi}\delta} \exp\left[-\frac{2\delta^2 z - z^2 - \frac{\delta^4}{4} - \delta^2 z}{2\delta^2}\right] dz$$

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$$= \int_{-\infty}^{\ln(\bar{w}_t)} \frac{1}{\sqrt{2\pi}\delta} \exp\left[-\frac{(z-\frac{\delta^2}{2})^2}{2\delta^2}\right] dz$$

$$\text{令 } m = \frac{z-\frac{\delta^2}{2}}{\delta}$$

$$= \int_{-\infty}^{\frac{\ln(\bar{w}_t)-\frac{\delta^2}{2}}{\delta}} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{m^2}{2}\right] \frac{dm}{\delta} = dm$$

$$\underline{G_t = \Phi\left(\frac{\ln(\bar{w}_t)}{\delta} - \frac{\delta}{2}\right)} \quad (6)'$$

$$\underline{P_t = G_t + \bar{w}_t(1 - F_t)} \quad (7)'$$

$$F'_t = \phi\left(\frac{\ln(\bar{w}_t)}{\delta} + \frac{\delta}{2}\right) \times \frac{1}{\delta \bar{w}_t}$$

$$G'_t = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{\ln(\bar{w}_t)}{\delta} - \frac{\delta}{2}\right)^2\right\} \frac{1}{\delta \bar{w}_t}$$

$$= \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left[\left(\frac{\ln(\bar{w}_t)}{\delta}\right)^2 + \left(\frac{\delta}{2}\right)^2 - \ln(\bar{w}_t)\right]\right\} \frac{1}{\delta \bar{w}_t}$$

$$= \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left[\underbrace{\left(\frac{\ln(\bar{w}_t)}{\delta}\right)^2 + \left(\frac{\delta}{2}\right)^2}_{\ln(\bar{w}_t) - 2\ln(\bar{w}_t)}\right]\right\} \frac{1}{\delta \bar{w}_t}$$

$$= \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{\ln(\bar{w}_t)}{\delta} + \frac{\delta}{2}\right)^2\right\} \bar{w}_t \times \frac{1}{\delta \bar{w}_t}$$

$$\underline{G'_t = \phi\left(\frac{\ln(\bar{w}_t)}{\delta} + \frac{\delta}{2}\right) \frac{1}{\delta}} \quad (33)'$$

$$\underline{P'_t = G'_t + (1 - F_t) - \frac{\bar{w}_t F'_t}{G'_t} = 1 - F_t} \quad (32)'$$

14 Summary

$$\bar{C}_t - \lambda_t = 0 \quad (1)$$

$$3\frac{1}{M_t} - \lambda_t + \beta \lambda_{t+1} \frac{1}{\pi_{t+1}} = 0 \quad (2)$$

$$-\lambda_t + \beta \lambda_{t+1} R_t = 0 \quad (3)$$

$$-\frac{3}{T-H_t} + \lambda_t w_t = 0 \quad (4)$$

$$F_t = \bar{\Psi}\left(\frac{\ln(w_t)}{6} + \frac{\epsilon}{2}\right) \quad (5)$$

$$G_t = \bar{\Psi}\left(\frac{\ln(w_t)}{6} - \frac{\epsilon}{2}\right) \quad (6)$$

$$\bar{P}_t = G_t + \bar{w}_t(1-F_t) \quad (7)$$

$$S_{t+1} = \frac{R_{t+1}^k}{R_t} \quad (8)$$

$$L_{t+1} = Q_t K_{t+1} / N_{t+1} \quad (9)$$

$$\frac{\bar{P}'_t - u G'_t}{\bar{P}'_t} (1 - \bar{P}_t) + (\bar{P}'_t - u G_t) = \frac{1}{S_{t+1}} \quad (10)$$

$$(\bar{P}'_t - u G_t) S_{t+1} = 1 - \frac{1}{I_{t+1}} \quad (11)$$

$$V_t = (1 - \bar{P}_t) S_{t+1} R_t Q_t K_{t+1} \quad (12)$$

$$C_t^e = (1 - \gamma^e) V_t \quad (13)$$

$$N_{t+1} = r^e V_t + H_t^e W_t^e \quad (14)$$

$$Y_t^e = A_t K_t^\alpha (H_t^{e^2} H_t^{e^2})^{1-\alpha} \quad (15)$$

$$W_t H_t = (1-\alpha) \Omega P_{mt} Y_t^e \quad (16)$$

$$W_t^e H_t^e = (1-\alpha)(1-\Omega) P_{mt} Y_t^e \quad (17)$$

$$R_t^k = \frac{\alpha P_{mt} Y_t^e}{Q_{t-1} K_t} + \frac{Q_t(1-\delta)}{Q_{t-1}} \quad (18)$$

$$\pi_t^* = \frac{\varepsilon}{\varepsilon-1} \frac{x_t^1}{x_t^2} \quad (19)$$

$$x_t^1 = \lambda_t P_{mt} Y_t + \theta \beta \pi_{t+1}^{\varepsilon} x_{t+1}^1 \quad (20)$$

$$x_t^2 = \lambda_t Y_t + \theta \beta \pi_{t+1}^{\varepsilon-1} x_{t+1}^2 \quad (21)$$

$$1 = \theta \pi_t^{\varepsilon-1} + (1-\theta) \pi_t^{*\varepsilon-1} \quad (22)$$

$$K_{t+1} = (1-\delta) K_t + I_t - \frac{\chi}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t \quad (23)$$

$$Q_t = [1 - \chi \left(\frac{I_t}{K_t} - \delta \right)]^{-1} \quad (24)$$

$$Y_t^e = D_t Y_t \quad (25)$$

$$D_t = \theta \pi_t^{\varepsilon} D_{t-1} + (1-\theta) \pi_t^{*\varepsilon} \quad (26)$$

$$Y_t = C_t + G_t^e + I_t + \frac{\chi}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t + \mu G_t R_t^k Q_{t-1} K_t \quad (27)$$

$$R_t = \frac{\pi_t}{\pi_{t+1}} \quad (28)$$

$$\frac{\pi_t}{\pi_{t+1}} = \left(\frac{\pi_{t+1}}{\pi_t} \right)^{\rho} \left[\left(\frac{\pi_t}{\pi_{t+1}} \right)^{\phi_{\pi}} \right]^{1-\rho} \exp(\epsilon_t^r) \quad (29)$$

$$\ln(A_t) = \rho_a \ln(A_{t-1}) + \epsilon_t^a \quad (30)$$

$$H_t^e = 1 \quad (31)$$

$$\bar{P}_t' = 1 - F_t \quad (32)$$

$$G_t' = \phi \left(\frac{\ln(\bar{W}_t)}{\delta} + \frac{\sigma}{2} \right) \frac{1}{\delta} \quad (33)$$

变量 (33)

$G_t, \lambda_t, M_t, \bar{P}_t, R_t,$
 $H_t, W_t, \bar{F}_t, G_t, P_t,$
 $\bar{W}_t, S_t, R_t^k, L_t, Q_t,$
 $K_t, N_t, P_t', G_t', V_t,$
 $C_t^e, H_t^e, W_t^e, Y_t^e, A_t,$
 $I_t, \bar{P}_t^*, X_t^1, X_t^2, D_t,$
 Y_t, i_t, P_m, t

Dynare 程序要注意 L_t, K_t, N_t, S_t 的時間點選取。

BGG ver.2. 穩態求解第二部分

$$(30) \underline{A=1} \text{ (外生給定)} \quad ⑨$$

$$\underline{\pi=1} \text{ (外生給定)} \quad ⑩$$

$$(22) 1 = \theta \pi^{\varepsilon-1} + (1-\theta) \pi^{*\varepsilon-1} \Rightarrow \underline{\pi^*=1} \quad ⑪$$

$$(20) X^1 = \frac{\lambda P_m Y}{1-\theta P}$$

$$(21) X^2 = \frac{\lambda Y}{1-\theta P}$$

$$(19) \pi^* = \frac{\varepsilon}{\varepsilon-1} \frac{X^1}{X^2}$$

粘性價格
部分

$$P_m = \frac{\varepsilon-1}{\varepsilon} \quad ⑫$$

$$(26) D = \theta D \pi^\varepsilon + (1-\theta) \pi^{*\varepsilon-1} \Rightarrow \underline{D=1} \quad ⑬$$

$$(3) \beta \lambda R - \lambda = 0 \Rightarrow \underline{R=\frac{1}{\beta}} \quad ⑭$$

$$(28) R = \frac{i}{\pi} \Rightarrow \underline{i=R} \quad ⑮$$

$$(8) \underbrace{S}_{\text{外生}} = \frac{R^k}{R} \Rightarrow \underline{R^k = SR} \quad ⑯$$

$$(23) K = (1-\delta)K + I - \frac{\chi}{2} \left(\frac{I}{K} - \delta \right) K \Rightarrow \underline{\frac{I}{K} = \delta}$$

$$(24) Q = \left[1 - \chi \left(\frac{I}{K} - \delta \right) \right]^{-1} \Rightarrow \underline{Q=1} \quad ⑰$$

$$(18) R^k = \frac{\alpha P_m Y^e}{QK} + \frac{Q(1-\delta)}{Q} \Rightarrow \underline{\frac{Y^e}{K} = \frac{1}{\alpha P_m} (R^k - 1 + \delta)}$$

$$(25) Y^e = DY \Rightarrow \underline{\frac{Y^e}{K} = \frac{D}{K}}$$

各種利率

$$(12) V = (1-p)SRQK \Rightarrow \frac{V}{K} = (1-p)SRQ$$

$$(14) N = r^e V + w^e$$

$$(9) L = \frac{K}{N}$$

$$\Rightarrow \frac{L}{K} = \frac{N}{K} = r^e \underbrace{\frac{V}{K}}_{已知} + \frac{w^e}{K}$$

$$(17) w^e H^e = (1-\lambda)(1-\mu) P_m Y, \text{ 又由 (31) } H^e = 1 \quad (18)$$

$$\Rightarrow \frac{w^e}{K} = (1-\lambda)(1-\mu) P_m \underbrace{Y}_{已知}$$

结合以上两个推导：

$$\underline{r^e = (1 - \frac{w^e}{K})(\frac{V}{K})^{-1}} \leftarrow r^e \text{ 被视为内生由 } L = \frac{3}{2} \text{ 决定}$$

$$\text{又有 } \frac{N}{K} = \frac{L}{K}$$

$$(13) C^e = (1 - r^e)V \Rightarrow \underline{\frac{C^e}{K} = (1 - r^e)\frac{V}{K}}$$

$$(27) \underline{\frac{Y}{K} = \frac{C^e}{K} + \frac{I}{K} + \frac{UGR^k}{K}}$$

已知 已知 = S 已知

$$\Rightarrow \underline{\frac{C^e}{K} = \frac{Y}{K} - \frac{I}{K} - \frac{UGR^k}{K}}$$

$$(1) \lambda = \frac{1}{C} \quad (4) \lambda W = \frac{1}{1-H} \quad \left. \begin{array}{l} \\ \end{array} \right\} W = \frac{1}{1-H} C$$

$$(16) WH = (1-\lambda) \Omega P_m Y^e$$

$$\Rightarrow \frac{W}{K} H = (1-\lambda) \Omega P_m \frac{Y^e}{K}$$

$$\Rightarrow \frac{C}{K} \frac{W}{K} \frac{H}{1-H} = (1-\lambda) P_m \frac{Y^e}{K}$$

$$\Rightarrow H = \frac{\frac{C}{K} [(1-\lambda) \Omega P_m \frac{Y^e}{K}]^{-1} + 1}{1} \quad (17)$$

$$(15) Y^e = \underbrace{A}_{=1} K^\alpha \underbrace{[H^\alpha (H^e)^{1-\alpha}]^{1-\alpha}}_{=1}$$

$$\Rightarrow \frac{Y^e}{K} = \left(\frac{H^\alpha}{K} \right)^{1-\alpha}$$

$$\Rightarrow K = H^\alpha \left(\frac{Y^e}{K} \right)^{-\frac{1}{1-\alpha}} \quad (18)$$

$$I = \frac{1}{K} \times K \quad (19) \quad Y^e = \frac{Y^e}{K} \times K \quad (20) \quad Y = Y^e \quad (21)$$

$$V = \frac{N}{K} \times K \quad (22) \quad W^e = \frac{W^e}{K} \times K \quad (23) \quad N = \frac{N}{K} \times K \quad (24)$$

$$C^e = \frac{C^e}{K} \times K \quad (25) \quad C = \frac{C}{K} \times K \quad (26) \quad W = \frac{1}{1-H} C \quad (27)$$

$$\lambda = \frac{1}{C} \quad (28) \quad X^1 = \frac{\lambda P_m Y}{1-\theta P} \quad (29) \quad X^2 = \frac{\lambda Y}{1-\theta P} \quad (30)$$

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$$(2) \beta \frac{1}{\lambda} - \lambda + \beta \frac{\lambda}{\pi} = 0$$

$$\Rightarrow M = (\lambda - \beta \lambda)^{-1} \beta \quad (3)$$

15. 穩態求解.

首先求解正的穩態，調整參數 r^e 標準杠杆率為 $\frac{3}{2}$.

$$L = \frac{3}{2} \quad (1)$$

$$(10) \quad \frac{P' - uG'}{P'} (1 - F) + (P - uG) = \frac{1}{S}$$

$$(11) \quad (P - uG)SL = L - 1 \Rightarrow \frac{1}{S} = (P - uG) \frac{L}{L - 1}$$

將(11)代入(10)得到

$$\frac{P' - uG'}{P'} (1 - F) + (P - uG) = (P - uG) \frac{L}{L - 1}$$

$$\Rightarrow \frac{P' - uG'}{P'} (1 - F) - \frac{1}{L - 1} (P - uG) = 0$$

這是 F 的函數與 L 的方程，由於 L 已標準， \bar{w} 可求得，視為已知。

$$\bar{w} \quad (2) \quad F = \bar{w} \left(\frac{\ln(\bar{w})}{8} + \frac{1}{2} \delta \right) \quad (3)$$

$$G = \bar{w} \left(\frac{\ln(\bar{w})}{8} - \frac{1}{2} \delta \right) \quad (4) \quad F' = \phi \left(\frac{\ln(\bar{w})}{8} + \frac{1}{2} \delta \right) \frac{1}{8\bar{w}}$$

$$G' = \phi \left(\frac{\ln(\bar{w})}{8} + \frac{1}{2} \delta \right) \frac{1}{8} \quad (5) \quad P = G + \bar{w}(1 - F) \quad (6)$$

$$P' = 1 - F \quad (7) \quad \text{通過(11)可知: } S = \frac{L-1}{L} (P - uG)^{-1} \quad (8)$$

至此模型中最優債務合約的主要變量求解完成。

(後接 DSGE 2.5)