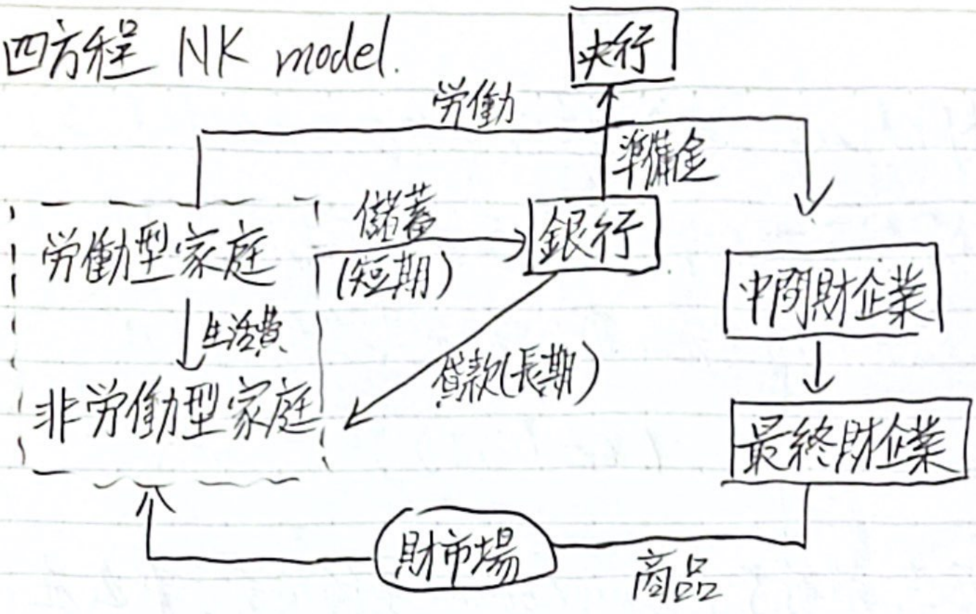
四方程 NK model.

Sims, Wu (2019)
NBER, 26067



1. 勞働型家庭

$$\max_{C_t, L_t, S_t} V_t = E_0 \sum_{t=0}^{\infty} \left[\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \psi \frac{L_t^{1+\chi}}{1+\chi} \right]$$

$$\text{s.t.} \quad P_t C_t + S_t = W_t L_t + R^S_{t-1} S_{t-1} + P_t D_t + P_t D_t^{FI} + P_t T_t - P_t X_t^b - P_t X_t^{FI}$$

$$\Rightarrow C_t + S_t = w_t L_t + \frac{R^S_{t-1}}{\Pi_t} S_{t-1} + D_t + D_t^{FI} - T_t - X_t^b - X_t^{FI}$$

其中、S_t儲蓄，D_t企業分紅，D_t^{FI}銀行分紅，X_t^b生活費，X_t^{FI}金融中介維持費，T_t来自央行的転移支付.

$\frac{\partial \mathcal{L}}{\partial C_t}$: $C_t^{-\sigma} = \lambda_t$ ① marginal utility of income.

$\frac{\partial \mathcal{L}}{\partial L_t}$: $\psi L_t^{\chi} = \lambda_t w_t$ ② 勞働供給

$\frac{\partial \mathcal{L}}{\partial S_t}$: $-\lambda_t + \beta \lambda_{t+1} \frac{R_t^S}{\Pi_{t+1}} = 0$ ③ 欧拉

2、長期債 (woordford, 2001, 永続年金, decaying perpetuities)

假設存在一種永続債，其 coupon 每期衰減，衰減率为 K.

也就是說，如果在 t 期発行 1元，那么接下来需支付 coupon

$$\begin{array}{cccc} t+1 & t+2 & t+3 & \cdots \\ K & K^2 & K^3 & \end{array} \qquad (K \in (0,1))$$

那么，假設过去无限期発行了 $\{CB_{t-1}, CB_{t-2}, \cdots\}$ 的債券，那么在 t-1 期，債权人收到的 coupon 总額 (or 債务人的 coupon liability)

$$B_{t-1} = CB_{t-1} + K CB_{t-2} + K^2 CB_{t-3} + \cdots$$

这種設定下，t 期発行的債券可用 Recursive 的方式表現：

$$\begin{cases} B_t = CB_t + K CB_{t-1} + K^2 CB_{t-2} + \cdots \\ K B_{t-1} = K CB_{t-1} + K^2 CB_{t-2} + \cdots \end{cases}$$

$$\Rightarrow CB_t = B_t - K B_{t-1} \Rightarrow \underline{cb_t = b_t - K \frac{1}{\pi_t} b_{t-1}} \quad ④$$

- 如果新発行的債券価格为 Q_t，那么这種 coupon 結构下，過去 s 期発行的債券価格为 $K^s Q_t$.

3、非劳働型家庭

$$\max_{b_t,\, c_{b,t}} \sum_{t=0}^{\infty} \beta_b^t \left(\frac{c_{b,t}^{1-\gamma}-1}{1-\gamma} \right)$$

s.t. $P_t C_{b,t} + B_{t-1} = Q_t CB_t + \underline{P_t X_t^b}$ 劳働型家型的转移支付. X_t^{H}

$$\Rightarrow P_t C_{b,t} + B_{t-1} = Q_t (B_t - \kappa B_{t-1}) + P_t X_t^b$$

$$\Rightarrow c_{b,t} + \frac{1}{\pi_t} b_{t-1} = Q_t \left(b_t - \kappa \frac{1}{\pi_t} b_{t-1} \right) + X_t^b$$

$\frac{\partial \mathcal{L}^b}{\partial c_{b,t}}$: $\underline{\lambda_t^b = c_{b,t}^{-\gamma}}$ ⑤

$\frac{\partial \mathcal{L}^b}{\partial b_t}$: $\lambda_t^b Q_t - \beta_b \lambda_{t+1}^b \left(Q_{t+1} \kappa \frac{1}{\pi_{t+1}} + \frac{1}{\pi_{t+1}} \right) = 0$.

Let $\underline{R_t^b = \frac{1+\kappa Q_t}{Q_{t-1}}}$ ⑥

$$\Rightarrow \underline{\lambda_t^b} = \beta_b \lambda_{t+1}^b \frac{1}{\pi_{t+1}} \left(\frac{1+\kappa Q_{t+1}}{Q_t} \right)$$

$$= \beta_b \lambda_{t+1}^b \pi_{t+1}^{-1} R_{t+1}^b \quad ⑦$$

注：参数設定時 $\beta_b < \beta$，即非劳働型家庭 impatient。这使得他们有働力从銀行貸款，满足当下消費。

4、銀行

銀行只運作兩期，利潤上交勞動型家庭(D_t^{FI})。在每次運作結束後，从家庭得到新運営資金X_t^{FI}，継続運営。

- Balance sheet

$$Q_t CB_t^{FI} + RE_t^{FI} = S_t^{FI} + P_t X_t^{FI}$$

其中，S_t^{FI}为銀行負債(勞動家庭儲蓄)，X_t^{FI}为銀行淨資産(運営資金)，CB_t^{FI}为銀行資産(非勞動家庭債券)，RE_t^{FI}为準備金(在央行的存儲)

- 銀行利潤交給家庭

$$D_{t+1}^{FI} = (R_{t+1}^{b} - R_t^{s}) Q_t CB_t^{FI} + (R_t^{re} - R_t^{s}) RE_t^{FI} + R_t^{s} P_t X_t^{FI}$$

其中，R_t^s为儲蓄利率，R_t^{re}为準備金利率，R_t^b为長期債券收益率.

- 融資約束：

$$Q_t (CB_t^{FI} + K B_{t-1}^{FI}) \leq \theta_t P_t X_t^{FI}$$

即长期債不超過淨資産的一定比例，θ_t为信貸市場沖击.

·銀行利潤最大化

$$\max_{CB_t^{FI}, RE_t^{FI}} P_t \frac{\lambda_{t+1}}{\lambda_t} \frac{D_{t+1}^{FI}}{P_{t+1}} = \Pi_{t+1}^{-1} \frac{\lambda_{t+1}}{\lambda_t} D_{t+1}^{FI}$$

$$s.t.\quad Q_t(CB_t^{FI} + K B_{t-1}^{FI}) \leq \theta_t P_t X_t^{FI} \quad (\theta_t \text{为信贷冲击})$$

$$\mathcal{L}^{Bank} = \Pi_{t+1}^{-1} \frac{\lambda_{t+1}}{\lambda_t} \left[(R_{t+1}^b - R_t^s) Q_t CB_t^{FI} + (R_t^{re} - R_t^s) RE_t^{FI} + R_t^s P_t X_t^{FI} \right]$$
$$+ \Omega_t \left[\theta_t P_t X_t^{FI} - Q_t (CB_t^{FI} + K B_{t-1}^{FI}) \right]$$

$$\frac{\partial \mathcal{L}^{Bank}}{\partial CB_t^{FI}} : \quad \Pi_{t+1}^{-1} \frac{\lambda_{t+1}}{\lambda_t} (R_{t+1}^b - R_t^s) Q_t - Q_t \Omega_t = 0$$

$$\frac{\partial \mathcal{L}^{Bank}}{\partial RE_t^{FI}} : \quad \Pi_{t+1}^{-1} \frac{\lambda_{t+1}}{\lambda_t} (R_t^{re} - R_t^s) = 0 .$$

银行均衡条件：

$$Q_t cb^{FI} + re_t^{FI} = s^{FI} + X_t^{FI} \quad ⑧$$

$$Q_t (cb^{FI} + K \Pi_t^{-1} b_{t-1}^{FI}) = \theta_t X_t^{FI} \quad ⑨$$

$$\Pi_{t+1}^{-1} \frac{\lambda_{t+1}}{\lambda_t} (R_{t+1}^b - R_t^s) = \Omega_t \quad ⑩ \quad (\Omega_t > 0)$$

$$R_t^{re} = R_t^s \quad ⑪$$

5、最終品厂商

$$\underset{Y(f)_t}{Max} \; P_t Y_t - \int_0^1 P(f)_t Y(f)_t \, df$$

$$s.t. \; Y_t = \left[\int_0^1 Y(f)_t^{\frac{\varepsilon-1}{\varepsilon}} df\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$\Rightarrow FOC: \; Y(f)_t = \left(\frac{P(f)_t}{P_t}\right)^{-\varepsilon} Y_t$$

zero-profit-condition: $P_t^{1-\varepsilon} = \int_0^1 P(f)_t^{1-\varepsilon} df$

6、中間品厂商

- $$\underset{L(f)_t}{Min} \; -W_t L(f)_t + MC_t \left[y(f)_t - \left(\frac{P(f)_t}{P_t}\right)^{-\varepsilon} Y_t \right]$$

$$s.t. \; y(f)_t = A_t L(f)_t$$

$$\Rightarrow \; W_t = A_t MC_t \quad (12)$$

- $$\underset{P(f)_t}{Max} \; E_t \sum_{i=0}^{\infty} (\beta\phi)^i \lambda_{t+i} \left(P(f)_t - MC_t\right) Y(f)_t$$

$$s.t. \; Y(f)_t = \left(\frac{P(f)_t}{P_t}\right)^{-\varepsilon} Y_t.$$

$$FOC: \; P(f)_t^* = \frac{\varepsilon}{\varepsilon-1} \frac{\sum_{i=0}^{\infty} (\beta\phi)^i \lambda_{t+i} MC_{t+i} Y_{t+i}}{\sum_{i=0}^{\infty} (\beta\phi)^i \lambda_{t+i} Y_{t+i} / P_{t+i}}$$

Recursive:

$$x_{1t} = \lambda_t MC_t Y_t + \beta\phi \, x_{1t+1} \Pi_{t+1}^{\varepsilon} \quad (13)$$

$$x_{2t} = \lambda_t Y_t + \beta\phi \, x_{2t+1} \Pi_{t+1}^{\varepsilon-1} \quad (14)$$

$$\Pi_t^* = \frac{\varepsilon}{\varepsilon-1} \Pi_t \frac{x_{1t}}{x_{2t}} \quad (15)$$

6、央行

央行持有長期債券 B_t^{cb}，使用來自銀行的準備金.

Balance sheet：

$$Q_t B_t^{cb} = RE_t \Rightarrow \underline{re_t = Q_t b_t^{cb}} \quad (16)$$

QE 政策： $T_t = R_{b,t} Q_{t-1} b_{cb,t-1} - R_{re,t-1} re_{t-1}$

$$\underline{QE_t = Q_t b_t^{cb}} \quad (17)$$

$$\underline{\ln(QE_t) = \rho_{QE} \ln(QE_{t-1}) + (1-\rho_{QE}) \ln(\bar{QE}) + \varepsilon_t^{QE}} \quad (18)$$

泰勒規則：

$$\underline{\ln(R_t^s) = \rho_r \ln(R_{t-1}^s) + (1-\rho_r)\left[\ln(R^s) + \phi_\pi \ln\left(\frac{\pi_t}{\pi}\right) + \phi_y \ln\left(\frac{y_t}{y_{t-1}}\right)\right]} \quad (19)$$

7. 均衡与加总

• 価格加总

$$P_t^{1-\varepsilon} = \int_0^1 P(f)_t^{1-\varepsilon} df = \int_0^\phi P(f)_{t-1}^{1-\varepsilon} df + \int_\phi^1 P(f)_t^{*1-\varepsilon} df$$

$$= \phi P_{t-1}^{1-\varepsilon} + (1-\phi) P(f)_t^{*1-\varepsilon}$$

$$\Rightarrow \underline{\pi_t^{1-\varepsilon} = \phi + (1-\phi)\pi_t^{*1-\varepsilon}} \quad (20)$$

• 產出加总

$$\int_0^1 Y(f)_t df = \int_0^1 \left(\frac{P(f)_t}{P_t}\right)^{-\varepsilon} df \, Y_t$$

$$\Rightarrow \int_0^1 A_t L(f)_t df = V_t Y_t$$

↑価格離散核

$$\Rightarrow v_t Y_t = A_t L_t \quad (21)$$

$$v_t = \int_0^1 \left(\frac{P(f)_t}{P_t}\right)^{-\varepsilon} df = \int_0^{\zeta} \left(\frac{P(f)_{t-1}}{P_t}\right)^{-\varepsilon} df + \int_{\zeta}^1 \left(\frac{P(f)_t^*}{P_t}\right)^{-\varepsilon} df$$

$$v_t = \phi \Pi_t^{\varepsilon} v_{t-1} + (1-\phi) \Pi_t^{*\,-\varepsilon} \quad (22)$$

- Aggregate resources

$$Y_t = C_t + C_{b,t} \quad (23)$$

- 劳動家庭对非劳家庭的転移支付規則：

$$P_t X_t^b = (1 + KQ_t) B_{t-1}$$

$$\Rightarrow X_t^b = (1 + KQ_t) \Pi_t^{-1} b_{t-1} \quad (24)$$

即非劳家庭所必需支付的 coupon 都由転移支付填补.

- 观察非劳家庭的予算制約，合并可得

$$C_{b,t} = Q_t b_t \quad (25)$$

- 金融市場出清：

$re_t = re_t^{FI}$ 準備金

$s_t = s_t^{FI}$ 儲蓄

$$b_t = b_t^{FI} + b_t^{cb} \quad 長期債 \quad (26)$$

8、外生冲击

$$\ln(\theta_t) = \rho_\theta \ln(\theta_{t-1}) + (1-\rho_\theta)\ln(\theta) + \varepsilon_t^\theta \quad (27)$$

$$\ln(A_t) = \rho_a \ln(A_{t-1}) + (1-\rho_a)\ln(A) + \varepsilon_t^a \quad (28)$$

KOKUYO LOOSE LEAF ノ-836BT 6 mm ruled x 36 lines

均衡条件整理

$$C_t^{-\sigma} = \lambda_t \quad ①$$

$$\psi L_t^{\chi} = \lambda_t W_t \quad ②$$

$$\lambda_t = \beta \lambda_{t+1} \frac{R_t^s}{\pi_{t+1}} \quad ③$$

労働型家庭

$$C_{bt} = b_t - K \pi_t^{-1} b_{t-1} \quad ④$$

$$\lambda_{b,t} = C_{b,t}^{-\sigma} \quad ⑤$$

$$R_t^b = \frac{1+KQ_t}{Q_{t-1}} \quad ⑥$$

$$\lambda_t^b = \beta^b \lambda_{t+1}^b \pi_{t+1}^{-1} R_{t+1}^b \quad ⑦$$

非労働型

仮設 対銀行的投資固定.

$$Q_t (b_t^{FI} - K \pi_t^{-1} b_{t-1}^{FI}) + re_t = S_t + X^{FI} \quad ⑧$$

$$Q_t b_t^{FI} = \theta_t X^{FI} \quad ⑨$$

$$\pi_t^{-1} \frac{\lambda_{t+1}}{\lambda_t} (R_{t+1}^b - R_t^s) = \Omega_t \quad ⑩$$

$(\Omega > 0)$

$$R_t^{re} = R_t^s \quad ⑪$$

銀行

$$W_t = A_t mC_t \quad ⑫$$

$$x_{1,t} = \lambda_t mC_t Y_t + \beta\phi x_{1,t+1} \pi_{t+1}^{\varepsilon} \quad ⑬$$

$$x_{2,t} = \lambda_t Y_t + \beta\phi x_{2,t+1} \pi_{t+1}^{\varepsilon-1} \quad ⑭$$

$$\pi_t^{*} = \frac{\varepsilon}{\varepsilon-1} \pi_t \frac{x_{1t}}{x_{2t}} \quad ⑮$$

粘性価格+cost

$$re_t = Q_t b_t^{cb} \quad ⑯$$

$$QE_t = Q_t b_t^{cb} \quad ⑰$$

$$\ln(QE_t) = \rho_{QE}\ln(QE_{t-1}) + (1-\rho_{QE})\ln(QE) + \varepsilon_t^{QE} \quad ⑱$$

$$\ln(R_t^S) = \rho_r \ln(R_{t-1}^S) + (1-\rho_r)\left[\ln(R^S) + \phi_\pi \ln\left(\frac{\pi_t}{\pi}\right) + \phi_y \ln\left(\frac{y_t}{y_{t-1}}\right)\right] + \varepsilon_t^r \quad ⑲$$

央行.

$$\pi_t^{1-\varepsilon} = \phi + (1-\phi)\pi_t^{*\,1-\varepsilon} \quad ⑳$$

$$V_t Y_t = A_t L_t \quad ㉑$$

$$V_t = \phi \pi_t^{\varepsilon} V_{t-1} + (1-\phi)\pi_t^{*-\varepsilon} \quad ㉒$$

$$Y_t = C_t + C_{b,t} \quad ㉓$$

$$X_t^b = (1+\kappa Q_t)\pi_t^{-1} b_{t-1} \quad ㉔$$

$$C_{b,t} = Q_t b_t \quad ㉕$$

$$b_t = b_t^{FI} + b_t^{cb} \quad ㉖$$

$$\ln(\theta_t) = \rho_\theta \ln(\theta_{t-1}) + (1-\rho_\theta)\ln(\theta) + \varepsilon_t^{\theta} \quad ㉗$$

$$\ln(A_t) = \rho_a \ln(A_{t-1}) + (1-\rho_a)\ln(A) + \varepsilon_t^{a} \quad ㉘$$

变量：C, λ, L, w, R^S, π, C_b, b, λ_b, C_b,
R^b, Q, b^{FI}, re, S, QE, θ, V, Ω, R^{re},
A, mc, x_{1t}, x_{2t}, Y, π^*, b^{cb}, X^b. 28

㉗ $\underline{\bar{\theta}=5}$ (1) $\underline{X^{FI}=0.046}$ $\underline{\bar{\pi}=1}$ (2)

⑳ $\underline{\pi^*=1}$ (3) ㉘ $\underline{A=1}$ (4) ㉒ $\underline{V=1}$ (5)

$$\left.\begin{array}{l} \left.\begin{array}{l} ⑬\ (1-\beta\phi)x_1=\lambda mcY \\ ⑭\ (1-\beta\phi)x_2=\lambda Y \end{array}\right\} mc=\frac{x_1}{x_2} \\ ⑮\ 1=\frac{\varepsilon}{\varepsilon-1}\frac{x_1}{x_2} \end{array}\right\} \underline{mc=\frac{\varepsilon-1}{\varepsilon}}\ (6)$$

Calibrate ψ s.t. $\underline{L=1}$ (7) $\boxed{\checkmark}$

$\underline{K=1-40^{-1}}$ ← 衰減率(10年)

$\underline{\beta=0.995}$ $\underline{\beta_b=0.99}$ $\underline{\chi=1}$ $\underline{X^{FI}=0.046}$

③ $\underline{R^s=\frac{1}{\beta}\ (8)}$

⑦ $\underline{R^b=\frac{1}{\beta_b}\ (9)}$

⑥ $R^b=\frac{1+KQ}{Q} \Rightarrow \underline{Q=\frac{1}{R^b-K}}$ (10)

⑩ $\underline{\Omega=R^b-R^s}$ (11) ⑪ $\underline{R^{re}=R^s}$ (12)

⑱ $\underline{QE=0.1}$ (13) ⑰ $\underline{b^{cb}=\frac{QE}{Q}\ (14)}$

⑯ $\underline{re=QE}$ (15) ⑨ $\underline{b^{FI}=\theta X^{FI}/Q}$ (16)

㉖ $\underline{b=b^{cb}+b^{FI}\ (17)}$ ㉕ $\underline{c_b=Qb}$ (18)

(21) $\underset{=1}{v}Y = \underset{=1}{A}\,\underset{=1}{L} \Rightarrow \underline{Y = 1}$ (19)

(23) $\underline{C = Y - C_b}$ (20) (1) $\underline{\lambda = C^{-\sigma}}$ (21)

(5) $\underline{\lambda_b = C_b^{-\sigma}}$ (22)

(8) $\underline{S = Q(1-K)\,b^{FI} + re - X^{FI}}$ (23)

(13) $\underline{X_1 = \frac{1}{1-\beta\phi}\lambda\, mc\, Y}$ (24)

(14) $\underline{X_2 = \frac{1}{1-\beta\phi}\lambda\, Y}$ (25)

(12) $\underline{w = mc}$ (26)

(24) $\underline{X^b = (1+KQ)\,b}$ (27)

(2) $\underline{\Psi = \lambda w}$ (calibrate for $L=1$)

(4) $\underline{C_b = b - Kb}$ (28)