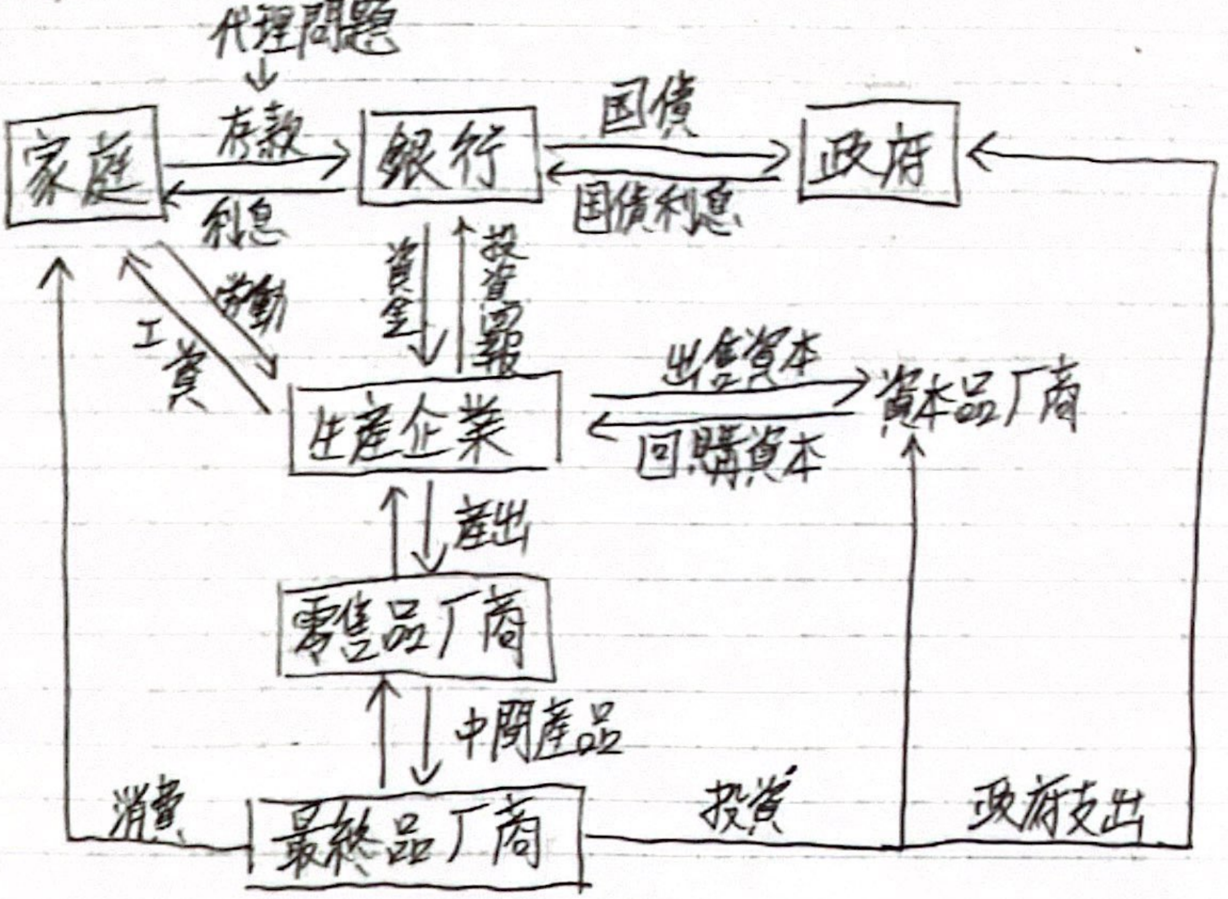
Gertler and Karadi (2011) with soverign default.

1.模型説明



熊琛,金昊 (2018) 中国工业経済

Bocola (2016) Journal of Political Economy.

1、家庭（工人+銀行家）

$$\max_{C_t, H_t, D_{t+1}} \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t - \gamma C_{t-1}) + \zeta \ln(1 - H_t) \right]$$

s.t. $P_{t+1} D_{t+1} = R_t (P_t W_t H_t + P_t D_t + P_t J_t - P_t C_t - P_t T_t)$

$$\Leftrightarrow \quad C_t + \frac{D_{t+1}}{R_t} = W_t H_t + \frac{1}{\pi_{t+1}} D_t + J_t - T_t.$$

$$\frac{\partial \mathcal{L}}{\partial C_t} : \quad \frac{1}{C_t - \gamma C_{t-1}} - \lambda_t - \beta \gamma \frac{1}{C_{t+1} - \gamma C_t} = 0 \qquad \underline{(1)}$$

$$\frac{\partial \mathcal{L}}{\partial H_t} : \quad -\zeta \frac{1}{1 - H_t} + \lambda_t W_t = 0 \qquad \underline{(2)}$$

$$\frac{\partial \mathcal{L}}{\partial D_{t+1}} : \quad -\lambda_t \frac{1}{R_t} + \beta \lambda_{t+1} \frac{1}{\pi_{t+1}} = 0. \qquad \underline{(3)}$$

2、銀行 ① 基本設定

- 銀行与家庭之間信息不対称，存在代理人問題。
- 即，銀行每期都面临破産风険，并且如果銀行市值低于資産価値的一定比例，銀行会宣布破産.

Balance sheet： $\underbrace{Q_t^K K_{t+1} + Q_t^B B_{t+1}}_{\text{資産(実質)}} = \underbrace{n_t + \frac{D_{t+1}}{R_t}}_{\text{净資産+負債}}$

銀行的最大化目标(市值)：

$\sum_{k=1} \psi^{k-1}(1-\psi)=1$

ψ：存活率

$$\underline{V_t = E_t\left[\sum_{k=1}^{\infty} \frac{\lambda_{t+k}}{\lambda_t} \psi^{k-1}(1-\psi) n_{t+k}\right]}$$

$$= E_t\left[\frac{\lambda_{t+1}}{\lambda_t}(1-\psi) n_{t+1} + \sum_{k=2}^{\infty} \frac{\lambda_{t+k}}{\lambda_t} \psi^{k-1}(1-\psi) n_{t+k}\right]$$

$\downarrow k \equiv r+1$

$$= E_t\left[\frac{\lambda_{t+1}}{\lambda_t}(1-\psi) n_{t+1} + \sum_{r=1}^{\infty} \frac{\lambda_{t+r+1}}{\lambda_t} \psi^{r}(1-\psi) n_{t+r+1}\right]$$

$$= E_t\left[\frac{\lambda_{t+1}}{\lambda_t}(1-\psi) n_{t+1} + \underline{\psi \frac{\lambda_{t+1}}{\lambda_t} \sum_{r=1}^{\infty} \frac{\lambda_{t+r+1}}{\lambda_{t+1}} \psi^{r-1}(1-\psi) n_{t+r+1}}\right]$$

$$= E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left[(1-\psi) n_{t+1} + \psi V_{t+1}\right]\right\}$$

銀行的净資産迁移式：

$$n_{t+1} = R_{t+1}^K Q_t^K K_{t+1} + R_{t+1}^B Q_t^B B_{t+1} - R_t \frac{D_{t+1}}{R_t}$$

2、銀行②最优化問題

$$V_t(n_t) = \max_{K_{t+1}, B_{t+1}} E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[(1-\psi) n_{t+1} + \psi V_{t+1}(n_{t+1}) \right] \right\}$$

s.t. $Q_t^K K_{t+1} + Q_t^B B_{t+1} = n_t + \frac{D_{t+1}}{R_t}$ 〈1〉

$n_{t+1} = R_{t+1}^K Q_t^K K_{t+1} + R_{t+1}^B Q_t^B B_{t+1} - D_{t+1}$ 〈2〉

$V_t(n_t) \geqslant \lambda^K Q_t^K K_{t+1} + \lambda^B Q_t^B B_{t+1}$ 〈3〉

↑ 銀行市值約束(流動性約束)

求解方法：猜解法，由于值函数是 n_{t+1} 的線性式，我们猜測值函数線性，然後用待定系数法求解值函数。

Step 1：猜測值函数 $V_t(n_t) = A_t n_t$

$$\Rightarrow V_{t+1}(n_{t+1}) = A_{t+1} n_{t+1}$$

Step 2：代入貝尔曼方程(目标函数)

$$V_t(n_t) = A_t n_t = \frac{\lambda_{t+1}}{\lambda_t} \left[(1-\psi) n_{t+1} + \psi A_{t+1} n_{t+1} \right]$$

$$= \frac{\lambda_{t+1}}{\lambda_t} (1 - \psi + \psi A_{t+1}) n_{t+1}.$$

Step 3：整合約束〈1〉,〈2〉

由〈1〉得 $D_{t+1} = R_t (Q_t^K K_{t+1} + Q_t^B B_{t+1} - n_t)$

代入〈2〉中 $n_{t+1} = R_{t+1}^K Q_t^K K_{t+1} + R_{t+1}^B Q_t^B B_{t+1} - R_t (Q_t^K K_{t+1} + Q_t^B B_{t+1} - n_t)$

$$= (R_{t+1}^K - R_t) Q_t^K K_{t+1} + (R_{t+1}^B - R_t) Q_t^B B_{t+1} + R_t n_t$$

Step 4：根据包络線定理，将最优解代入目标函数，其一階条件应同樣満足最优化的条件。

提銀行的最优問題転化为：

$$V_t(n_t) = \underset{K_{t+1}, B_{t+1}}{Max} \; E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t}(1-\psi+\psi A_{t+1}) n_{t+1} \right\}$$

$$s.t. \; n_{t+1} = (R^K_{t+1} - R) Q^K_t K_{t+1} + (R^B_{t+1} - R) Q^B_t B_{t+1} + R_t n_t$$

$$V_t(n_t) = A_t n_t \geq \lambda^K Q^K_t K_{t+1} + \lambda^B Q^B_t B_{t+1}$$

$$\mathcal{L} \equiv \frac{\lambda_{t+1}}{\lambda_t}(1-\psi+\psi A_{t+1})\left[(R^K_{t+1} - R) Q^K_t K_{t+1} + (R^B_{t+1} - R) Q^B_t B_{t+1} + R_t n_t\right]$$

$$+ u_t (A_t n_t - \lambda^K Q^K_t K_{t+1} - \lambda^B Q^B_t B_{t+1})$$

FOCS:

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} : \frac{\lambda_{t+1}}{\lambda_t}(1-\psi+\psi A_{t+1}) (R^K_{t+1} - R_t) Q^K_t = u_t \lambda^K Q^K_t \quad \langle 4 \rangle$$

$$\frac{\partial \mathcal{L}}{\partial B_{t+1}} : \frac{\lambda_{t+1}}{\lambda_t}(1-\psi+\psi A_{t+1}) (R^B_{t+1} - R_t) Q^B_t = u_t \lambda^B Q^B_t \quad \langle 5 \rangle$$

$$u_t (A_t n_t - \lambda^K Q^K_t K_{t+1} - \lambda^B Q^B_t B_{t+1}) = 0 \quad \langle 6 \rangle$$

↑ 松馳互補条件：$u_t A_t n_t = u_t \lambda^K Q^K_t K_{t+1} + u_t \lambda^B Q^B_t B_{t+1}$

将⟨4⟩⟨5⟩⟨6⟩的一階条件代入目標函数，求 A_t 的值.

$$V_t(n_t) = A_t n_t = u_t \lambda^K Q^K_t K_{t+1} + u_t \lambda^B Q^B_t B_{t+1} \quad \leftarrow \langle 4 \rangle \langle 5 \rangle \text{ 代入}$$

$$+ \frac{\lambda_{t+1}}{\lambda_t}(1-\psi+\psi A_{t+1}) R_t n_t$$

代入<6>

$$\Rightarrow V_t(n_t) = A_t n_t = u_t A_t n_t + \frac{\lambda_{t+1}}{\lambda_t}(1-\psi+\psi A_{t+1}) R_t n_t.$$

$$\Rightarrow A_t = \frac{1}{1-u_t} \underline{\frac{\lambda_{t+1}}{\lambda_t}(1-\psi+\psi A_{t+1})} R_t$$

$$= \frac{1}{1-u_t} \underline{\Omega_{t+1}} R_t \qquad \text{(若 } u_t<1,\ A_t>0,\ V(n_t)>0\text{)}$$

Step 5：求解拉格朗日乘子 u_t

根据KKT条件<6>，

a、若 $u_t>0$，则 $A_t n_t - \lambda^K Q_t^K K_{t+1} - \lambda^B Q_t^B B_{t+1} = 0$

代入 A_t 可得：

$$\frac{1}{1-u_t}\Omega_{t+1} R_t n_t = \lambda^K Q_t^K K_{t+1} + \lambda^B Q_t^B B_{t+1}$$

$$\Rightarrow u_t = 1 - \frac{\Omega_{t+1} R_t n_t}{\lambda^K Q_t^K K_{t+1} + \lambda^B Q_t^B B_{t+1}}$$

b、若不等式約束并不bind，则 $u_t = 0$.

綜上，$u_t = \max\left(1 - \frac{\Omega_{t+1} R_t n_t}{\lambda^K Q_t^K K_{t+1} + \lambda^B Q_t^B B_{t+1}},\ 0\right) < 1$.

整理均衡条件：

$$\Omega_{t+1} \equiv \frac{\lambda_{t+1}}{\lambda_t}(1-\psi+\psi A_{t+1}) \tag{4}$$

$$\Omega_{t+1}(R^K_{t+1}-R_t) = \mu_t \lambda^K \tag{5}$$

$$\Omega_{t+1}(R^B_{t+1}-R_t) = \mu_t \lambda^B \tag{6}$$

$$A_t = \frac{1}{1-\mu_t}\Omega_{t+1}R_t \tag{7}$$

$$\mu_t = 1-\frac{\Omega_{t+1}R_t n_t}{\lambda^K Q^K_t K_{t+1}+\lambda^B Q^B_t B_{t+1}} \tag{8}$$

$$Q^K_t K_{t+1}+Q^B_t B_{t+1} = n_t+\frac{D_{t+1}}{R_t} \tag{9}$$

$$L_t \equiv \frac{Q^K_t K_{t+1}+Q^B_t B_{t+1}}{n_t} \tag{10}$$

$$S^K_t \equiv R^K_{t+1}-R_t \tag{11}$$

$$S^B_t \equiv R^B_{t+1}-R_t \tag{12}$$

銀行在无限期中破産概率：$\sum_{k=0}^{\infty}\psi^k(1-\psi)=1$

因此需要家庭出資新的銀行以保証运作：

$$n_{t+1} = \psi n_t+\eta(Q^K_t K_{t+1}+Q^B_t B_{t+1}) \tag{13}$$

3、生產企業

从家庭雇佣労働，以 $P_{m,t}$ 価格生產商品

每期生產結束，向銀行支付上期貸款本息。

生產結束後，将減耗的資本品卖給資本品厂商。

$$\max_{H_t} \; P_{m,t} Y_t^e - w_t H_t - R_t^K Q_{t-1} K_t + Q_t^K (1-\delta) K_t$$

$$\text{s.t.} \quad Y_t^e = Z_t K_t^{\alpha} H_t^{1-\alpha} \qquad (14)$$

FOC： $$w_t = P_{m,t} Z_t K_t^{\alpha} (1-\alpha) H_t^{-\alpha}$$

$$w_t H_t = (1-\alpha) P_{m,t} Y_t^e \qquad (15)$$

假設生產企業完全競争，zero-profit-condition：

$$0 = P_{m,t} Y_t^e - (1-\alpha) P_{m,t} Y_t^e - R_t^K Q_{t-1} K_t + Q_t^K (1-\delta) K_t$$

$$\Rightarrow R_t^K = \frac{\alpha P_{m,t} Y_t^e}{Q_{t-1}^K K_t} + \frac{Q_t^K (1-\delta)}{Q_{t-1}^K} \qquad (16)$$

4、最終産品企業

$$\max_{Y_t^i} \; P_t Y_t - \int_0^1 P_t^i Y_t^i \, di$$

$$\text{s.t.} \quad Y_t = \left[\int_0^1 (Y_t^i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$\Rightarrow \text{FOC}: \; Y_t^i = (P_t^i / P_t)^{-\varepsilon} Y_t$$

zero-profit-condition:

$$P_t = \left(\int_0^1 P_t^{i\,1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$$

5、零售商

$$\max_{P_t^{i*}} \; E_t \sum_{k=0}^{\infty} (\theta\beta)^k \frac{\lambda_{t+k}}{\lambda_t} \left(\frac{P_t^{i*}}{P_{t+k}} - P_{m,t+k} \right) Y_{t+k}^{i*}$$

$$\text{s.t.} \quad Y_{t+k}^{i*} = \left(\frac{P_t^{i*}}{P_t} \right)^{-\varepsilon} Y_{t+k}$$

$$\Rightarrow \quad \Pi_t^* = \frac{\varepsilon}{\varepsilon-1} \frac{X_t^1}{X_t^2} \qquad (17)$$

$$X_t^1 = \lambda_t P_{m,t} Y_t + \theta\beta \, \Pi_{t+1}^{\varepsilon} X_t^1 \qquad (18)$$

$$X_t^2 = \lambda_t Y_t + \theta\beta \, \Pi_{t+1}^{\varepsilon-1} X_t^2 \qquad (19)$$

$$1 = \theta \Pi_t^{\varepsilon-1} + (1-\theta) \Pi_t^{*\,1-\varepsilon} \qquad (20)$$

6、政府部門

每期有δ比例的国債到期，剩余的国債支付ι的cupon

①无风険情況：

$$R^B_{t+1} Q^B_t B_{t+1} = \delta B_{t+1} + (1-\delta)(\iota + Q^B_{t+1}) B_{t+1}$$

$$\Rightarrow R^B_{t+1} = \frac{\delta + (1-\delta)(\iota + Q^B_{t+1})}{Q^B_t}$$

②假設国債有default的风険，default後進行清算，清算成本为Γ.

破産概率：

$$PD_{t+1} = P(\ell_{t+1}=1) = \frac{\exp(M_{t+1})}{1+\exp(M_{t+1})}$$

$$M_{t+1} = (1-\rho_S)\overline{M} + \rho_S M_t + \varepsilon^S_{t+1}.$$

变量替换：$S_t \equiv \exp(M_t) \Leftrightarrow M_t = \ln(S_t)$

那么、

$$PD_{t+1} = \frac{S}{1+S} \qquad (21)$$

$$\ln(S_{t+1}) = (1-\rho_S)\ln(\bar{S}) + \rho_S \ln(S_t) + \varepsilon^S_{t+1} \qquad (22)$$

有风险的情况下，国债回报率成为预期值：

$$R^B_{t+1} = (1-PD_{t+1})\left(\frac{\delta+(1-\delta)(1+Q^B_{t+1})}{Q^B_t}\right)$$

$$+ PD_{t+1}(1-D)\left(\frac{\delta+(1-\delta)(1+Q^B_{t+1})}{Q^B_t}\right)$$

$$= [(1-D)PD_{t+1} + (1-PD_{t+1})]\left(\frac{\delta+(1-\delta)(1+Q^B_{t+1})}{Q^B_t}\right) \quad (23)$$

财政规则：

$$T_t = \gamma_t B_{t-1} \quad (24)$$

政府予算：

$$Q^B_t B_{t+1} = [\delta + (1-\delta)(1+Q^B_t)]B_t + G_t - T_t \quad (25)$$

政府支出：

$$\ln(G_t) = \rho_g \ln(G_{t-1}) + (1-\rho_g)\ln(\bar{G}) + \varepsilon_t \quad (26)$$

7、資本品生產企業

$$K_{t+1} = (1-\delta)K_t + I_t - \frac{\chi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t \quad (27)$$

$$\max_{I_t} \quad Q_t^K\left[I_t - \frac{\chi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2\right]K_t - I_t$$

$$\text{FOC:} \quad Q_t^K = \left[1 - \chi\left(\frac{I_t}{K_t} - \delta\right)\right]^{-1} \quad (28)$$

8、央行

$$i_t = \frac{R_t}{\pi_{t+1}} \quad (29)$$

$$\ln(i_t) = \phi_\pi \ln\left(\frac{\pi_t}{\pi}\right) + \varepsilon_t^i \quad (30)$$

9、市場出清和加总

$$Y_t^e = \int_0^1 Y_t^i \, di = \int_0^1 \left(\frac{P_t^i}{P_t}\right)^{-\varepsilon} di \, Y_t = d_t Y_t \quad (31)$$

$$d_t = \theta \pi_t^{\varepsilon} d_{t-1} + (1-\theta)\pi_t^{*\,-\varepsilon} \quad (32)$$

$$\ln(Z_t) = \rho_Z \ln(Z_{t-1}) + \varepsilon_t^Z \quad (33)$$

$$Y_t = C_t + I_t + G_t + \frac{\chi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t \quad (34)$$

10. Summary

(1) $\frac{1}{C_t - \gamma C_{t-1}} - \lambda_t - \beta\gamma \frac{1}{C_{t+1} - \gamma C_t} = 0$

(2) $\lambda_t w_t - \zeta \frac{1}{1-H_t} = 0$

(3) $\beta \lambda_{t+1} \frac{1}{\pi_{t+1}} - \lambda_t \frac{1}{R_t} = 0$

(4) $\Omega_{t+1} = \frac{\lambda_{t+1}}{\lambda_t}(1 - \psi + \psi A_{t+1})$

(5) $\Omega_{t+1}(R_{t+1}^K - R_t) = \mu_t \lambda^K$

(6) $\Omega_{t+1}(R_{t+1}^B - R_t) = \mu_t \lambda^B$

(7) $A_t = \frac{1}{1-\mu_t} \Omega_{t+1} R_t$

(8) $\mu_t = 1 - \frac{\Omega_{t+1} R_t n_t}{\lambda^K Q_t^K K_{t+1} + \lambda^B Q_t^B B_{t+1}}$

(9) $\frac{D_{t+1}}{R_t} = Q_t^K K_{t+1} + Q_t^B B_{t+1} - n_t$

(10) $L_t = \frac{Q_t^K K_{t+1} + Q_t^B B_{t+1}}{n_t}$

(11) $S_t^K = R_{t+1}^K - R_t$

(12) $S_t^B = R_{t+1}^B - R_t$

(13) $n_t = \psi n_{t-1} + \eta(Q_{t-1}^K K_t + Q_{t-1}^B B_t)$

(14) $Y_t^e = Z_t K_t^\alpha H_t^{1-\alpha}$

(15) $w_t H_t = (1-\alpha) P_{m,t} Y_t^e$

(16) $R_t^K = \frac{\alpha P_{m,t} Y_t^e}{Q_{t-1}^K K_t} + \frac{Q_t^K (1-\delta)}{Q_{t-1}^K}$

(17) $\Pi_t^* = \frac{\varepsilon}{\varepsilon-1} \frac{X_t^1}{X_t^2}$

(18) $X_t^1 = \lambda_t P_{m,t} Y_t + \theta\beta \Pi_{t+1}^{\varepsilon} X_{t+1}^1$

(19) $X_t^2 = \lambda_t Y_t + \theta\beta \Pi_{t+1}^{\varepsilon-1} X_{t+1}^2$

(20) $1 = \theta \Pi_{t-1}^{\varepsilon-1} + (1-\theta) \Pi_t^{1-\varepsilon}$

(21) $PD_{t+1} = \frac{S}{1+S}$

(22) $\ln(S_{t+1}) = \rho^s \ln(S_{t-1}) + (1-\rho^s)\ln(\bar{S}) + \varepsilon_{t+1}^s$

(23) $R_{t+1}^B = [(1-\Gamma) PD_{t+1} + 1 - PD_{t+1}] \left(\frac{\delta + (1-\delta)(1+Q^B)}{Q_t^B} \right)$

(24) $T_t = \gamma^\tau B_{t-1}$

(25) $Q_t^B B_t = [\delta + (1-\delta)(1+Q_t^B)] B_{t-1} + G_t - T_t$.

(26) $\ln(G_t) = \rho^g \ln(G_{t-1}) + (1-\rho^g)\ln(\bar{G}) + \varepsilon_t^g$

(27) $K_{t+1} = (1-\delta) K_t + I_t - \frac{\chi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$

(28) $Q_t^K = \left[1 - \chi\left(\frac{I_t}{K_t} - \delta\right)\right]^{-1}$

(29) $i_t = \frac{R_t}{\Pi_{t+1}}$

(30) $\ln\left(\frac{i_t}{i}\right) = \phi_\Pi \ln\left(\frac{\Pi_t}{\Pi}\right) + \varepsilon_t^i$

(31) $Y_t^e = d_t Y_t$

(32) $d_t = \theta \Pi_t^\varepsilon d_{t-1} + (1-\theta) \Pi_t^{*-\varepsilon}$

(33) $\ln(Z_t) = \rho^Z \ln(Z_{t-1}) + \varepsilon_t^Z$

(34) $Y_t = C_t + I_t + G_t + \frac{\chi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$

变量：

$$\left.\begin{array}{l} C_t,\ \lambda_t,\ H_t,\ W_t,\ R_t \\ \Omega_t,\ A_t,\ R_t^K,\ R_t^B,\ u_t \\ n_t,\ Q_t^K,\ Q_t^B,\ K_t,\ B_t \\ D_t,\ L_t,\ S_t^K,\ S_t^B,\ Y_t^e \\ p_{m,t},\ \Pi_t^*,\ X_t^1,\ X_t^2,\ Z_t \\ \Pi_t,\ PD_t,\ S_t,\ T_t,\ G_t \\ I_t,\ i_t,\ Y_t,\ d_t \end{array}\right\} (34)$$

11、穩態求解

$\pi = 1$ 外生 ①　　$z = 1$ ②　　$PD = 0.05$ 外生 ③

$\pi^* = 1$ 根据(20) ④　　$P_m = \frac{\varepsilon - 1}{\varepsilon}$ ⑤　　$S = \frac{PD}{1-PD}$ 根据(21) ⑥

$d = 1$ 根据(32) ⑦

(27) $\frac{I}{K} = \delta$

(3) $\beta\lambda\frac{1}{\pi} - \lambda\frac{1}{R} = 0 \Rightarrow R = \frac{1}{\beta}$ ⑧

(28) $Q^K = \left[1 - \chi\left(\frac{I}{K} - \delta\right)\right]^{-1} \Rightarrow Q^K = 1$ ⑨

外生設定 $\frac{G}{Y} = 0.2$

調整 λ^K 使得 $S^K = 0.5\%$ ⑩ ✓

$\Rightarrow R^K = R + S^K$ ⑪

調整 λ^B 使得 $S^B = 0.3\%$ ⑫ ✓

$\Rightarrow R^B = R + S^B$ ⑬

調整 L 使得 $Q^B = 1$ ⑭ ✓

(16) $R^K = \frac{\alpha P_m}{Q^K}\frac{Y}{K} + (1-\delta)$

$\Rightarrow \frac{Y}{K} = (\alpha P_m)^{-1}(R^K - 1 + \delta) \Rightarrow \frac{Y^e}{K} = \frac{Y}{K}$

(114) $Y^e = K^\alpha H^{1-\alpha}$

$\Rightarrow \frac{Y^e}{K} = \left(\frac{H}{K}\right)^{1-\alpha}$

$\Rightarrow \underline{\frac{H}{K} = \left(\frac{Y^e}{K}\right)^{\frac{1}{1-\alpha}}}$

(115) $wH = (1-\alpha) P_m Y^e$

$\Rightarrow \underline{w = (1-\alpha) P_m \left(\frac{Y^e}{K}\right)\left(\frac{H}{K}\right)^{-1}}$ ⑮

調整 z 使得 $\underline{H = \frac{1}{3}}$ ⑯ ✓

$\underline{K = H \setminus \frac{H}{K}}$ ⑰　　$\underline{Y^e = \frac{Y^e}{K} \times K}$ ⑱

$\underline{Y = \frac{Y}{K} \times K}$ ⑲　　$\underline{G = \frac{G}{Y} \times Y}$ ⑳

(34) $\underline{C = Y - I - G}$ ㉑　　$\underline{I = \frac{I}{K} \times K}$ ㉒

(1) $\frac{1}{C - \gamma C} - \lambda - \beta\gamma \frac{1}{C - \gamma C} = 0$

$\Rightarrow \underline{\lambda = \frac{1-\beta\gamma}{1-\gamma} \frac{1}{C}}$ ㉓

(118) $\underline{X^1 = \frac{P_m \lambda Y}{1-\theta\beta}}$ ㉔

(119) $\underline{X^2 = \frac{\lambda Y}{1-\theta\beta}}$ ㉕

(2) $z = \lambda w(1-H)$

(23) $R^B = [(1-P)PD + 1 - PD]\left(\dfrac{\delta + (1-\delta)(l + Q^B)}{\underbrace{Q^B}_{=1}}\right)$

$\Rightarrow l = \left(\dfrac{R^B}{(1-P)PD + 1 - PD} - \delta\right)\dfrac{1}{1-\delta} - 1$

(25) $\underbrace{Q^B}_{=1} B = [\delta + (1-\delta)(l + Q^B)]B + G - T$

(24) $T = \gamma^{\tau} B$

$\Rightarrow B = [\delta + (1-\delta)(l+1)]B + G - \gamma^{\tau} B$

$\Rightarrow B = \dfrac{G}{1 + \gamma - \delta - (1-\delta)(l+1)}$ ㉖

$\Rightarrow T = \gamma^{\tau} B$ ㉗

調整 η 使得 $L = 1.5$ ㉘ ✓

(10) $L = \dfrac{K+B}{n} \Rightarrow n = \dfrac{K+B}{L}$ ㉙

(13) $n = \psi n + \eta(K+B) \Rightarrow \eta = (1-\psi)\dfrac{n}{K+B}$

(6) $\Omega(R^B - R) = u\lambda^B \Rightarrow \lambda^B = \frac{1}{u}\Omega(R^B - R)$

(5) $\Omega(R^K - R) = u\lambda^K \Rightarrow \lambda^K = \frac{1}{u}\Omega(R^K - R)$

(8) $u = 1 - \frac{\Omega R n}{\lambda^K K + \lambda^B B}$

$\Rightarrow \frac{1}{1-u} = \frac{\lambda^K K + \lambda^B B}{\Omega R n}$

代入(5),(6)

$\hookrightarrow \frac{1}{1-u} = \frac{\frac{1}{u}\Omega(R^K - R)K + \frac{1}{u}\Omega(R^B - R)B}{\Omega R n}$

$\Rightarrow \frac{u}{1-u} = \frac{(R^K - R)K + (R^B - R)B}{Rn}$

$\Rightarrow u = \left(1 + \frac{(R^K - R)K + (R^B - R)B}{Rn}\right)^{-1}\left(\frac{(R^K - R)K + (R^B - R)B}{Rn}\right)$ (30)

(14) $\Omega = 1 - \psi + \psi A$

(7) $A = \frac{1}{1-u}\Omega R$

$A = \frac{1-\psi}{1-u-\psi R}R$ (31)

$\Omega = 1 - \psi + \psi A$ (32)

$\lambda^B = \frac{1}{u}\Omega(R^B - R)$

$\lambda^K = \frac{1}{u}\Omega(R^K - R)$

(29) $i = \frac{R}{\pi}$ (33)

(9) $D = R(K + B - n)$ (34)