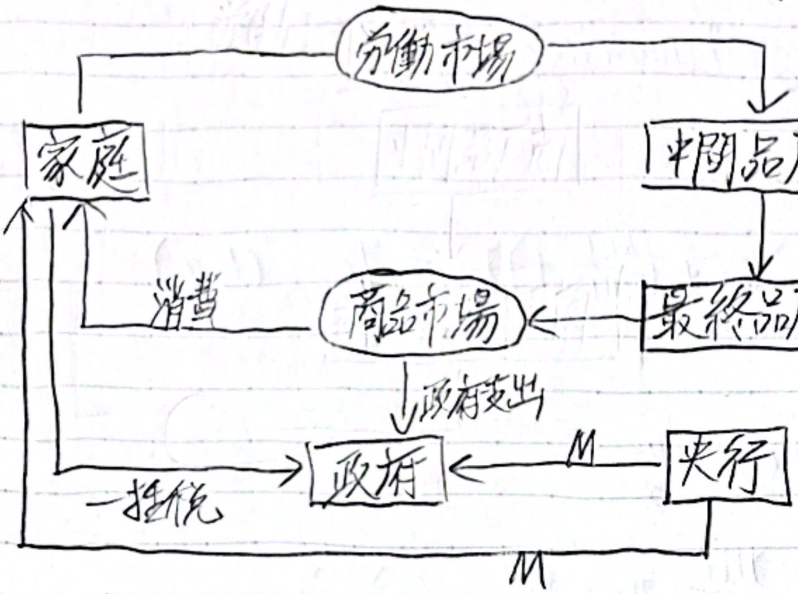
Galí (2020) JME "The effects of a money-financed fiscal stimulus"



1、政策部門(政府、央行)

a、Government budget

$$P_tG_t + B_{t-1}R_{t-1} = P_tT_t + B_t + M_t^s - M_{t-1}^s$$

$$\Rightarrow G_t + b_{t-1}\frac{R_{t-1}}{\Pi_t} = T_t + b_t + m_t^s - m_{t-1}^s\frac{1}{\Pi_t} \quad (1)$$

b、Lump-sum tax rule:

In Galí (2020): $\hat{t}_t = \psi_b \hat{b}_{t-1} + \hat{t}_t^*$ (where $\hat{x}_t \equiv \frac{x_t - x}{Y}$)

$$\Rightarrow \frac{T_t - T}{Y} = \psi_b \frac{b_{t-1} - b}{Y} + \frac{T_t^* - T^*}{Y} \quad (2)$$

c、Exogenous process for expenditure and Tax.

$$\frac{G_t - G}{Y} = \delta \frac{G_{t-1} - G}{Y} + \varepsilon_t^G \quad (3)$$

$$\frac{T_t^* - T^*}{Y} = \delta \frac{T_{t-1}^* - T^*}{Y} + \varepsilon_t^{T^*} \quad (4)$$

d. Policy Regimes for Central Bank.

d1. Regime of Money-Financing.

$b_t = \bar{b}$.

$$\Rightarrow m_t^s - m_{t-1}^s \frac{1}{\pi_t} = G_t + \left(\frac{R_{t-1}}{\pi_t} - 1\right)\bar{b} - T_t \quad (5a)$$

d2. Regime of Debt-Financing

$$\frac{M_t^s}{M_{t-1}^s} = g_t^m \Rightarrow \frac{m_t^s}{m_{t-1}^s}\pi_t = g_t^m \quad (6)$$

$$\pi_t = 1 \quad (5b)$$

2. Household

$$E_0 \sum_{t=0}^{\infty} \beta^t Z_t U(C_t, m_t, N_t)$$

s.t. $C_t + b_t + m_t = \frac{R_{t-1}}{\pi_t} b_{t-1} + m_{t-1}\frac{1}{\pi_t} + W_t N_t + \Pi_t - T_t$.

$\frac{\partial \mathcal{L}}{\partial C_t}$: $\lambda_t = Z_t U_{c,t}$ ①

$\frac{\partial \mathcal{L}}{\partial m_t}$: $\lambda_t - \beta \lambda_{t+1} \frac{1}{\pi_{t+1}} = Z_t U_{m,t}$ ②

$\frac{\partial \mathcal{L}}{\partial b_t}$: $-\lambda_t + \beta \lambda_{t+1} \frac{R_t}{\pi_{t+1}} = 0$. ③

$\frac{\partial \mathcal{L}}{\partial N_t}$: $\lambda_t W_t = Z_t U_{N,t}$ ④

combine ②, ③ and substitude ①

$$\lambda_t - \frac{\lambda_t}{R_t} = Z_t U_{m,t}$$

$$\Rightarrow \left(\frac{R_t - 1}{R_t}\right) Z_t U_{c,t} = Z_t U_{m,t}$$

$$\Rightarrow \frac{U_{m,t}}{U_{c,t}} = \frac{R_t - 1}{R_t} \qquad (7)$$

substitude ① to ②

$$\frac{Z_{t+1} U_{c,t+1}}{Z_t U_{c,t}} = \left(\beta \frac{R_t}{\Pi_{t+1}}\right)^{-1} \qquad (8)$$

substitude ① to ④

$$\frac{U_{N,t}}{U_{c,t}} = W_t \qquad (9)$$

of course, The transfersary condition should be satisfied.

$$\lim_{t \to \infty} \frac{\lambda_t}{\lambda_0} \left(\frac{R_{t-1} b_{t-1} + M_{t-1}}{\Pi_t}\right) = 0$$

3. Final good firm

$$\max_{y_{j,t}} \ P_t Y_t - \int_0^1 P_{j,t} y_{j,t} \, dj$$

$$\text{s.t. } Y_t = \left(\int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$\text{FOC: } y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\varepsilon} Y_t$$

$$\text{ZPC: } P_t^{1-\varepsilon} = \int_0^1 P_{j,t}^{1-\varepsilon} \, dj.$$

4. Intermediate Firms

$$\min_{N_t} \ w_t N_t + mc_t \left[y_{j,t} - A_t N_{j,t}^{1-\alpha} \right]$$

$$\Rightarrow \underline{w_t = mc_t A_t (1-\alpha) N_{j,t}^{-\alpha}} \quad (10)$$

$$\max_{P_{j,t}} \ E_t \sum_{k=0}^{\infty} (\beta\xi)^k \frac{\lambda_{t+k}}{\lambda_t} \left[\frac{\Pi_{n=1}^{k} \tilde{\pi}_{t+n} P_{j,t}}{P_{t+k}} - mc_{t+k} \right] y_{j,t+k}.$$

$$\text{s.t. } Y_{t+k} = \left(\frac{\Pi_{n=1}^{k} \tilde{\pi}_{t+k} P_{j,t}}{P_{t+k}} \right)^{-\varepsilon} Y_t.$$

$$\Rightarrow \begin{cases} \frac{P_{j,t}}{P_t} \equiv \pi_t^* = \frac{\varepsilon}{\varepsilon-1} \frac{X_t^1}{X_t^2}. & (11) \\ X_t^1 = \lambda_t mc_t Y_t + \beta\xi \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{-\varepsilon} X_{t+1}^1 & (12) \\ X_t^2 = \lambda_t Y_t + \beta\xi \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{1-\varepsilon} X_{t+1}^2 & (13) \\ \tilde{\pi}_t = \pi_{t-1}^{\iota} \pi^{1-\iota} & (14) \\ 1 = \xi \tilde{\pi}_t^{1-\varepsilon} \pi_t^{\varepsilon-1} + (1-\xi) \pi_t^{*\,1-\varepsilon} & (15) \end{cases}$$

5. Aggregate, Market clear and Exogenous process.

$$\int_0^1 y_{j,t}\,dj = \int_0^1 A_t N_{j,t}^{1-\alpha}\,dj$$

$$\Rightarrow \underbrace{\int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon} dj}\, Y_t = A_t N_t^{1-\alpha}$$

$$\Rightarrow \underline{D_t Y_t = A_t N_t^{1-\alpha}} \quad (16)$$

where $\underline{D_t = (1-\xi)\,\pi_t^{*\,-\varepsilon} + \xi_p\, \tilde{\pi}_{t-1}^{-\varepsilon}\, \pi_t^{\varepsilon}\, D_{t-1}}$ (17).

$$\underline{m_t^s = m_t^d}$$

$$\underline{\ln\left(\frac{A_t}{A}\right) = \delta \ln\left(\frac{A_{t-1}}{A}\right) + \varepsilon_t^A} \quad (18)$$

$$\underline{\ln\left(\frac{Z_t}{Z}\right) = \delta \ln\left(\frac{Z_{t-1}}{Z}\right) + \varepsilon_t^Z \quad (19)}$$

$$\underline{Y_t = C_t + G_t \quad (20)}$$

Equilibrium.

$$G_t + b_{t-1}\frac{R_{t-1}}{\pi_t} = T_t + b_t + m_t - m_{t-1}\frac{1}{\pi_t} \qquad (1)$$

$$\frac{T_t - T}{Y} = \psi_b \frac{b_{t-1} - b}{Y} + \frac{T_t^* - T^*}{Y} \qquad (2)$$

$$\frac{G_t - G}{Y} = \delta \frac{G_{t-1} - G}{Y} + \varepsilon_t^G \qquad (3)$$

$$\frac{T_t^* - T^*}{Y} = \delta \frac{T_{t-1}^* - T^*}{Y} + \varepsilon_t^{T^*} \qquad (4)$$

$$m_t - m_{t-1}\frac{1}{\pi_t} = G_t + \left(\frac{R_{t-1}}{\pi_t} - 1\right)\bar{b} - T_t \qquad (5a)$$

$$\pi_t = 1 \qquad (5b)$$

$$\frac{m_t}{m_{t-1}}\pi_t = g_t^m \qquad (6)$$

$$\frac{U_{m,t}}{U_{c,t}} = \frac{R_t - 1}{R_t} \qquad (7)$$

$$\frac{Z_{t+1}}{Z_t}\frac{U_{c,t+1}}{U_{c,t}} = \frac{\pi_{t+1}}{\beta R_t} \qquad (8)$$

$$\frac{U_{N,t}}{U_{c,t}} = W_t \qquad (9)$$

$$W_t = A_t mc_t (1-\alpha) N_t^{-\alpha} \qquad (10)$$

$$\pi_t^* = \frac{\varepsilon}{\varepsilon - 1}\frac{X_t^1}{X_t^2} \qquad (11)$$

$$X_t^1 = \lambda_t mc_t Y_t + \beta\xi\left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}}\right)^{-\varepsilon} X_{t+1}^1 \qquad (12)$$

$$X_t^2 = \lambda_t Y_t + \beta \zeta \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}}\right)^{1-\varepsilon} X_{t+1}^2 \quad (13)$$

$$\tilde{\pi}_t = \pi_{t-1}^{\iota} \pi^{1-\iota} \quad (14)$$

$$1 = \zeta \tilde{\pi}_t^{1-\varepsilon} \pi_t^{\varepsilon-1} + (1-\zeta) \pi_t^{*1-\varepsilon} \quad (15)$$

$$D_t Y_t = A_t N_t^{1-\alpha} \quad (16)$$

$$D_t = (1-\zeta)\pi_t^{*-\varepsilon} + \zeta \tilde{\pi}_{t-1}^{-\varepsilon} \pi_t^{\varepsilon} D_{t-1} \quad (17)$$

$$Y_t = C_t + G_t \quad (18)$$

$$\ln\left(\frac{A_t}{A}\right) = \delta \ln\left(\frac{A_{t-1}}{A}\right) + \varepsilon_t^A \quad (19)$$

$$\ln\left(\frac{Z_t}{Z}\right) = \delta \ln\left(\frac{Z_{t-1}}{Z}\right) + \varepsilon_t^Z \quad (20)$$

$$\lambda_t = Z_t U_{c,t} \quad (21)$$

Define $U(C_t, m_t, N_t) = \ln(C_t) + \frac{\chi}{1-\delta}\left(\frac{m_t}{C_t^a} - \bar{x}\right)^{1-\delta} - \frac{N_t^{1+\varphi}}{1+\varphi}$

$$U_{N,t} = \psi N_t^{\phi} \quad (22)$$

$$U_{c,t} = \frac{1}{C_t} \quad (23)$$

$$U_{m,t} = \chi\left(\frac{m_t}{C_t^a} - \bar{x}\right)^{-\delta} \frac{1}{C_t^a} \quad (24)$$

G, b, R, π, T	D, Y, C, λ
m, T^{*}, g^m, U_m, U_c	
Z, U_N, w, A, mc	
$N, \pi^{*}, X^1, X^2, \tilde{\pi}$	

補足：为什么設定 $\frac{\chi}{1+\sigma}\left(\frac{m_t}{c_t^a}-\bar{x}\right)^{1+\sigma}$？

$\Rightarrow$ 我们希望 $h\left(\frac{m_t}{c_t}\right) \equiv \frac{u_{m,t}}{u_{c,t}}$ 满足几个条件

① $\frac{u_{m,t}}{u_{c,t}}$ 是 $\frac{m_t}{c_t}$ 的减函数

② 当経済处于零利率下限，即 $R_t=1$ 時，

即，$\frac{u_{m,t}}{u_{c,t}} = \frac{R_t-1}{R_t} = 0 \Rightarrow \frac{m_t}{c_t} = \bar{x}$.

显然，$\frac{u_{m,t}}{u_{c,t}} = \chi\left(\frac{m_t}{c_t}-\bar{x}\right)^{\sigma}$ 可以满足以上要求。

So, what is the value of χ, $\bar{x}$ and σ?

Let $h\left(\frac{m_t}{c_t}\right) = \frac{R_t-1}{R_t}$

$$h'\frac{m}{c}\hat{m}_t - h'\frac{m}{c}\hat{c}_t = -\beta\hat{R}_t \qquad \left(\hat{x}_t = \frac{x_t-x}{x}\right)$$

$$\Rightarrow \hat{m}_t = \hat{c}_t - \beta\frac{1}{h'}\frac{c}{m}\hat{R}_t$$

where $h' = -\sigma\chi\left(\frac{m}{c}-\bar{x}\right)^{-\sigma-1}$

We will set $Y=1$, $G=0.2$, so that $C=0.8$.

Moreover, we calibrate $\bar{x}$, so that $\frac{m}{Y} = \frac{m}{C}\frac{C}{Y} = 3$.

In this case, $\frac{m}{c} = 3 \div 0.8$.

$\beta\sigma^{-1}\chi^{-1}\left(\frac{m}{c}-\bar{x}\right)^{\sigma+1} \in [6, 8]$, so we can calibrate χ.

Steady state

(6) $\underline{\pi=1}$ ① $\underline{g^m=1}$ ② (14) $\underline{\tilde{\pi}=\pi^{\iota}\pi^{1-\iota}=1}$ ③

(15) $\underline{\pi^{*}=1}$ ④ (17) $\underline{D=1}$ ⑤

(16) calibrate A s.t. $\underline{Y=1}$ ⑥ $\boxed{1}$✓

(3) $\underline{G=0.2}$ 外生 ⑦ (18) $\underline{C=Y-G}$ ⑧.

(8) $1=\frac{1}{\beta R} \Rightarrow \underline{R=\frac{1}{\beta}}$ ⑨

calibrate ψ s.t. $\underline{N=\frac{1}{3}}$ ⑩ $\boxed{1}$✓

(23) $\underline{u_c=\frac{1}{C}}$ ⑪

calibrate $\bar{x}$ s.t. $\frac{m}{Y}=3 \Rightarrow \underline{m=3Y}$ ⑫ $\boxed{1}$✓

(7) $\underline{u_m=\frac{R-1}{R}u_c}$ ⑬

Let $\underline{\sigma=1}$,

$$\begin{cases} \beta x^{-1}\left(\frac{m}{c}-\bar{x}\right)^{\sigma+1}=7 \\ u_m \times C = x\left(\frac{m}{c}-\bar{x}\right)^{-\sigma} \end{cases} \Rightarrow \text{求解得到 } \underline{x,\ \bar{x}}$$

(16) $\underline{A=\frac{Y}{N^{1-\alpha}}}$ ⑭

(11), (12), (13) $\underline{mc=\frac{\varepsilon-1}{\varepsilon}}$ ⑮.

(10) $W = mc(1-\alpha)\frac{Y}{N}$ 16

(9) $u_N = u_c \times W$ 17

(22) $\psi = u_N N^{-\phi}$

(20) $z = 1$ 18 (21) $\lambda = z u_c$ 19

(1) $b = 1.7$ 外生 20

$T = G + (R-1)b$ 21

$T^* = G$ 22

(12) $X^1 = \frac{\lambda \cdot mc \cdot Y}{1-\beta\xi}$ 23

(13) $X^2 = \frac{\lambda Y}{1-\beta\xi}$ 24