Project 0: Inaugural Project

Vision: The inaugural project teaches you to solve a simple economic model and present the results.

- **Objectives:** In your inaugural project, you should show that you can:
  1. Apply simple numerical solution and simulation methods
  2. Structure a code project
  3. Document code
  4. Present results in text form and in figures
- **Content:** In your inaugural project, you should:
  1. Solve and simulate a pre-specified economic model (see next page)
  2. Visualize results

  Example of structure: See this repository.
- **Structure:** Your inaugural project should consist of:
  1. A README.md with a short introduction to your project
  2. A single self-contained notebook (.ipynb) presenting the analysis
  3. Fully documented Python files (.py)
- **Hand-in:** On GitHub by uploading it to the subfolder *inaugralproject*, which is located in:

  github.com/NumEconCopenhagen/projects-YEAR-YOURGROUPNAME
- **Deadline:** See Calendar.
- **Exam:** Your inaugural project will be a part of your exam portfolio.
  You can incorporate feedback before handing in the final version.

Housing demand and taxation

The following exercise is a very simplified emulation of the tax reform that will be implemented in Denmark in 2024. The numbers are made up, but you can read more here and here. Unfortunately only in Danish.

We consider a household looking to buy a home. The household is endowed with an amount of cash-on-hand, m, that may spent on housing as well as other consumption c. The household derives utility from housing by a measure of its quality, h. A home of quality h has the price p_h and is subject to progressive taxation and mortgage costs, summarized by the function $\tau(\cdot)$. The objective of the household is therefore to choose optimal amounts of housing and consumption

$$
\begin{aligned}
c^{\star}, h^{\star} &= \arg\max_{c,h} c^{1-\phi} h^{\phi} && (1) \\
&s.t. \\
\widetilde{p}_h &= p_h \varepsilon && (2) \\
m &= \tau(p_h, \widetilde{p}_h) + c && (3) \\
\tau(p_h, \widetilde{p}_h) &= r p_h + \tau^g \widetilde{p}_h + \tau^p \max\{\widetilde{p}_h - \overline{p}, 0\} && (4)
\end{aligned}
$$

To clarify the restrictions (2)-(4): homes are not taxed according to their market value, p_h, but instead according to a public assessment, denoted $\widetilde{p}_h$. The public assessments are politically set below market value by a factor of ε in (2). Equation (3) states that cash-on-hand is divided between housing costs and consumption. The first element of equation (4) states that the household will purchase its home with no down-payment and an interest-only mortgage carrying the interest r. The second element is the base housing tax levied on the assessment, while the last element is a progressive housing tax. That is, home values above the cutoff $\overline{p}$ are taxed at an additional rate of τ^p.

Questions

1) Construct a function that solves household's problem above. We let the market price of a home be equal to it's quality:

$$p_h = h$$

and assume that the household in question has cash-on-hand $m = 0.5$. Notice that the monetary units is in millions DKK. In addition, use the following parameters

$$\phi = 0.3,\ \varepsilon = 0.5,\ r = 0.03,\ \tau^g = 0.012,\ \tau^p = 0.004,\ \overline{p} = 3$$

tip: you do not have to use bounded optimization (but you can of course).
tip: use the built-in function max() to model equation (4). Do not use np.max().
suggestion: use a dictionary to store all the parameters by their names.

2) Plot $c^{\star}$ and $h^{\star}$ as functions of m in the range 0.4 to 1.5.

Now consider a whole population of $N = 10,000$ households looking to buy homes. The households differ by their cash-on-hand. Assume that the distribution of cash-on-hand is given by

$$m_i \sim \texttt{Lognormal}(-0.4, 0.35)$$

NOTE: when generating random numbers throughout the assignment, use the seed 1.

We let the public assessment associated with the optimal housing choice of household i, $h_i^\star$, be denoted $\widetilde{p}_{h,i}^\star$. The total tax revenue T collected under this parameterization is then

$$T = \sum_{i=1}^{N} \tau^g \widetilde{p}_{h,i}^\star + \tau^p \max\left\{\widetilde{p}_{h,i}^\star - \overline{p}, 0\right\}$$

3) Calculate the average tax burden pr household T/N.

   tip: create a function to calculate T
   Bonus question (optional): try and plot the distributions of cash-on-hand and $h^\star$. Notice something odd about the distributions - what do you think may be the cause?

4) The policy maker decides to reform the tax system on housing. Calculate the average taxes that will occur with the following changes to parameters:

   $$\varepsilon = 0.8,\ \tau^g = 0.01,\ \tau^p = 0.009,\ \overline{p} = 8$$

   while the other parameters remain the same.

   tip: create a new dictionary with updated parameters, such that you can easily switch between the new set of parameters and the old one. Careful with copying!

5) Now, instead of implementing the reform in 4) directly, the policy maker wants to ensure that the average home owner does not pay more in housing taxes. The policy maker therefore imposes that reform changes to $\varepsilon, \tau^p, \overline{p}$ will be offset by lowering τ^g such that average tax payments are unchaged from before the reform. Calculate the new τ^g that meets this requirement.

   tip: if this takes a long time on your computer, use a lower N and re-run 3)-4).