Lecture 6: Binary Dependent Variable

Big Data Analytics,Spring 2019

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1. Introduction to limited dependent variable
2. The Linear Probability Model(LPM)
3. Nonlinear probability model

Introduction to limited dependent variable

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The Linear Probability Model(LPM)

The linear probability model

- If a outcome variable is **binary**,then the expecation of it is

$$E[Y] = 1 \times Pr(Y = 1) + 0 \times Pr(Y = 0) = Pr(Y = 1)$$

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- Then we have the probability of Y conditional on X

$$E[Y|X_{1i}, ..., X_{ki}] = Pr(Y = 1|X_{1i}, ..., X_{ki})$$

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- The conditional expectation equals the probability that $Y_i = 1$ conditional on $X_{1i}, ..., X_{ki}$:

$$E[Y|X_{1i}, ..., X_{ki}] = Pr(Y = 1|X_{1i}, ..., X_{ki}) = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + ... + \beta_k X_{ki}$$

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- The population coefficient β_j equals the change in the probability that $Y_i = 1$ associated with a unit change in X_j.

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  - R^2 is not a useful statistic now.

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    - Error terms are heteroskedastic

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- What determines whether or not a mortgage application is approved or denied?

An Example: Mortgage applications

- eg.Boston HMDA data: a data set on mortgage(住房贷款) applications collected by the Federal Reserve Bank in Boston.

Variable	Description	Mean	SD
deny	$= 1$ if application is denied	0.120	0.325
pi_ratio	monthly loan payments / monthly income	0.331	0.107
black	$= 1$ if applicant is black	0.142	0.350

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  - More reasonable one: "payments/monthly income ratio increase *0.1(10%)*,then probability being denied will also increase *0.06(6%)*".

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- What is the effect of race on the probability of *denial*, *holding constant the P/I ratio?* To keep things simple, we focus on differences between black applicants and white applicants.

$$\widehat{deny} = \underset{(0.029)}{-0.091} + \underset{(0.089)}{0.559}\ P/I\ ratio + \underset{(0.025)}{0.177} black$$

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- The coefficient on black, *0.177*, indicates that an African American applicant has a *17.7%* higher probability of having a mortgage application denied than a white applicant, holding constant their payment-to-income ratio.
- This coefficient is significant at the 1% level (the t-statistic is 7.11).

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- Always suffer heteroskedasticity.
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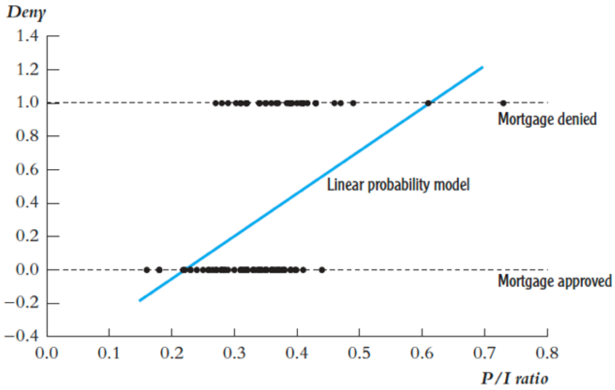
- While in LPM model,the predicted probability can be below 0 or above 1!

Mortgage applications: Predicted value

FIGURE 11.1 Scatterplot of Mortgage Application Denial and the Payment-to-Income Ratio

Mortgage applicants with a high ratio of debt payments to income (*P/I ratio*) are more likely to have their application denied (*deny* = 1 if denied, *deny* = 0 if approved). The linear probability model uses a straight line to model the probability of denial, conditional on the *P/I ratio*.



Nonlinear probability model

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- To address this problem,consider nonlinear probability models

$$\begin{aligned} Pr(Y_i = 1|X_1, ...X_k) &= G(Z) \\ &= G(\beta_0 + \beta_1 X_{1,i} + \beta_2 X_{2,i} + ... + \beta_k X_{k,i}) \end{aligned}$$

where $Z = \beta_0 + \beta_1 X_{1,i} + \beta_2 X_{2,i} + ... + \beta_k X_{k,i}$ and $0 \leq g(Z) \leq 1$

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Probit Model

- Probit regression models the probability that $Y = 1$

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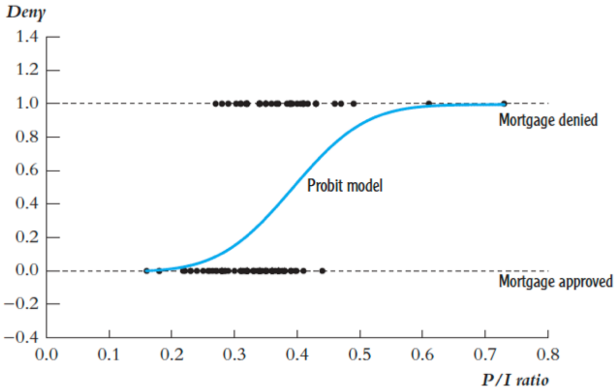
- Using the cumulative standard normal distribution function $\Phi(Z)$ and $0 \leq \Phi(Z) \leq 1$
- Since $\Phi(z) = Pr(Z \leq z)$ we have that the predicted probabilities of the probit model are between 0 and 1.

Probit Model

FIGURE 11.2 Probit Model of the Probability of Denial, Given *P/I Ratio*

The probit model uses the cumulative normal distribution function to model the probability of denial given the payment-to-income ratio or, more generally, to model $\Pr(Y = 1 | X)$. Unlike the linear probability model, the probit conditional probabilities are always between 0 and 1.



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3. taking their difference.

Probit Model with one regression

- Suppose the probit population regression model with only one regressors, X_1

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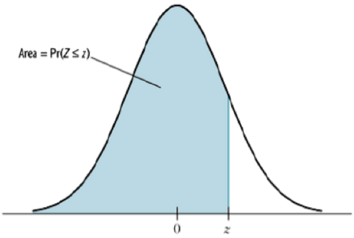
- So the probability $Pr(Y = 1) = Pr(z \leq -0.8) = \Phi(-0.8)$

Probit Model

- $Pr(Y = 1) = Pr(Z \leq -0.8) = \Phi(-.8) = 0.2119$

TABLE 1 The Cumulative Standard Normal Distribution Function, $\Phi(z) = \Pr(Z$ ″ $z)$



z	Second Decimal Value of z: 0	1	2	3	4	5	6	7	8	9
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121

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- Suppose $\beta_0=-1.6, \beta_1=2\ and\ \beta_2=0.5$. And If $X_1=0.4\ and\ X_2=1$ then

$$z=-1.6+2\times 0.4+0.5\times 1=-0.3$$

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$$z = -1.6 + 2 \times 0.4 + 0.5 \times 1 = -0.3$$

- So the probability $Pr(Y = 1) = Pr(z \leq -0.3) = \Phi(-0.3) = 0.38$

Example: Mortgage Applications

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$$\Phi(-2.19 + 2.97 \times 0.3) = \Phi(-1.3) = 0.097$$

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- The estimated change in the probability of denial is $0.159 - 0.097 = 0.062$

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  - *Marginal Effect at Mean* (MEM): ME at $X = \bar{X}$(at the sample mean of the regressors)

Effect of a change in X: Marginal Effects

- For nonlinear models, the ME varies with the point of evaluation
  - *Marginal Effect at a Representative Value* (MER):ME at $X = X^*$ (at representative values of the regerssors)
  - *Marginal Effect at Mean* (MEM): ME at $X = \bar{X}$(at the sample mean of the regressors)
  - *Average Marginal Effect* (AME): average of ME at each $X = X_i$ (at sample values and then average)

Example: Mortgage applications: marginal effect

- Because the probit regression function is nonlinear, the effect of a change in X depends on the starting value of X.

$$\frac{\partial Pr(deny = 1|P/I\ ratio)}{\partial P/I\ ratio} = \Phi(-2.19 + 2.97P/I\ ratio) \times 2.97$$

$$\left.\frac{\partial Pr(deny = 1|P/I\ ratio)}{\partial P/I\ ratio}\right|_{at\ mean} = \Phi(-2.19 + 2.97 \times 0.331) \times 2.97$$

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Special Case: The explanatory variable is discrete.

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- Assume X_2 is a dummy variable, then partial effect of X_2 changing from 0 to 1:

$$G(\beta_0+\beta_1 X_{1,i}+\beta_2\times 1+\ldots+\beta_k X_{k,i})-G(\beta_0+\beta_1 X_{1,i}+\beta_2\times 0+\ldots+\beta_k X_{k,i})$$

Example: Mortgage applications: Race

```
##
## Call:
## glm(formula = hmda_small$deny ~ hmda_small$pi_rat + hmda_sm
##     family = binomial(link = "probit"), data = hmda_small)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.1208  -0.4762  -0.4251  -0.3550   2.8799
##
## Coefficients:
##                   Estimate Std. Error z value Pr(>|z|)
## (Intercept)       -2.25879    0.13669 -16.525  < 2e-16 ***
## hmda_small$pi_rat  2.74178    0.38047   7.206 5.75e-13 ***
## hmda_small$black   0.70816    0.08335   8.496  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '
```

Example: Mortgage applications: Race

- Mortgage denial (deny) and the payment-toincome ratio (P/I ratio) and race

$$\widehat{Pr(deny = 1|P}/I\ ratio) = \Phi(-2.26 + 2.74P/I\ ratio + 0.71black)$$

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- so the difference between whites and blacks at $P/Iratio = 0.3$ is $0.233 - 0.075 = 0.158$, which means probability of denial for blacks is 15.8% higher than that for whites.

Logit Model

- Logit regression models the probability that $Y = 1$

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- evaluated at $Z = \beta_0 + \beta_1 X_{1,i} + \beta_2 X_{2,i} + ... + \beta_k X_{k,i}$
- since $F(z) = Pr(Z \leq z)$ we have that the predicted probabilities of the probit model are between 0 and 1.

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Logit Model

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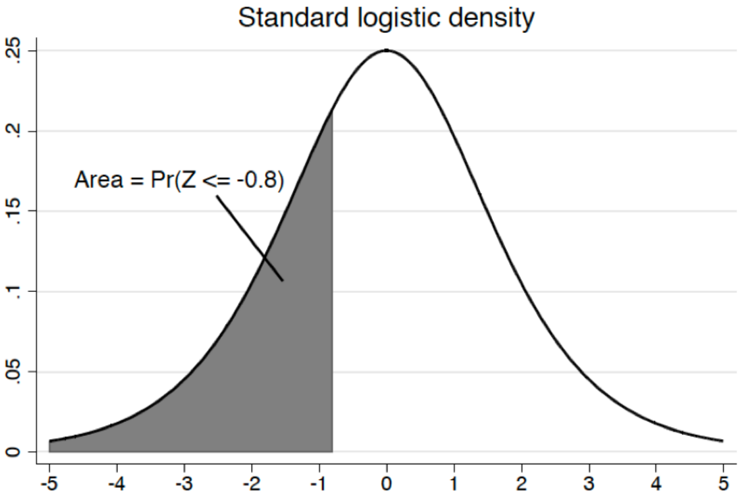

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$$\widehat{Pr(deny = 1|P/I\ ratio)} = F(\underset{(0.35)}{-4.13} + \underset{(0.96)}{5.37} P/I\ ratio + \underset{(0.15)}{1.27} black)$$

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$$\frac{1}{1 + e^{-(-4.13+5.37\times 0.3+1.27\times 0)}} = 0.074$$

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- the difference is

$$0.222 - 0.074 = 0.148 = 14.8\%$$

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- While Logit and Probit models are nonlinear in the coefficients $\beta_0, \beta_1, ..., \beta_k$
  - these models can NOT be estimated by OLS
- The method used to estimate logit and probit models is **Maximum Likelihood Estimation** (MLE).

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- The MLE is consistent and normally distributed in large samples.

Statistical inference based on the MLE

- Because the MLE is normally distributed in large samples, statistical inference about the probit and logit coefficients based on the MLE proceeds in the same way as inference about the linear regression function coefficients based on the OLS estimator.

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- If $Y_i = 1$ and the predicted probability exceeds 50% or if $Y_i = 0$ and the predicted probability is less than 50%, then Y_i is said to be correctly predicted.

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- f^{max}_{probit} is the value of the maximized probit likelihood (which includes the X's)
- $f^{max}_{bernoulli}$ is the value of the maximized Bernoulli likelihood (the probit model excluding all the X's).

Comparing the LPM,Probit and Logit

- All three models: *linear probability, probit, and logit* are just approximations to the unknown population regression function $E(Y|X) = Pr(Y = 1|X)$.

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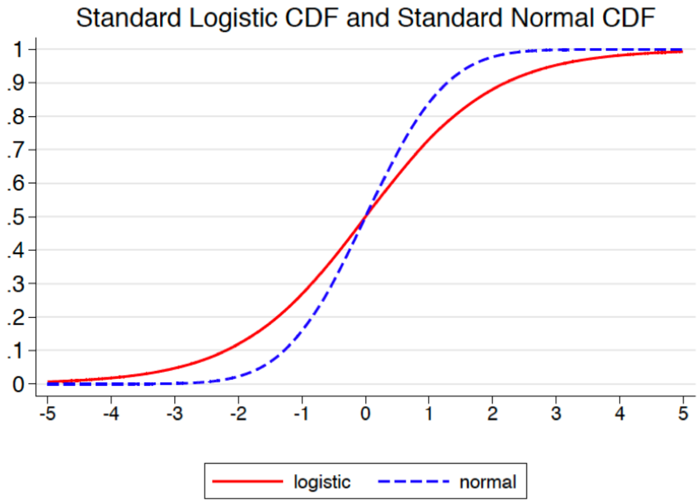
- So which should you use in practice?
  - *There is no one right answer*, and different researchers use different models.
  - *Probit and logit regressions frequently produce similar results*.

Logit v.s. Probit



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Comparing the LPM,Probit and Logit

- The marginal effects and predicted probabilities are much more similar across models.
- Coefficients can be compared across models, using the following rough conversion factors (Amemiya 1981)

$$\hat{\beta}_{logit} \simeq 4\hat{\beta}_{ols}$$
$$\hat{\beta}_{probit} \simeq 2.5\hat{\beta}_{ols}$$
$$\hat{\beta}_{logit} \simeq 1.6\hat{\beta}_{probit}$$

Example: Mortgage Applications(short regression)

Dependent variable: *deny* = 1 if mortgage application is denied, = 0 if accepted			
regression model	LPM	Probit	Logit
black	0.177***	0.71***	1.27***
	(0.025)	(0.083)	(0.15)
P/I ratio	0.559***	2.74***	5.37***
	(0.089)	(0.44)	(0.96)
constant	-0.091***	-2.26***	-4.13***
	(0.029)	(0.16)	(0.35)
difference Pr(*deny*=1) between black and white applicant when *P/I ratio*=0.3	17.7%	15.8%	14.8%

Example: Mortgage Applications(long regression)

TABLE 11.1 Variables Included in Regression Models of Mortgage Decisions

Variable	Definition	Sample Average
Financial Variables		
P/I ratio	Ratio of total monthly debt payments to total monthly income	0.331
housing expense-to-income ratio	Ratio of monthly housing expenses to total monthly income	0.255
loan-to-value ratio	Ratio of size of loan to assessed value of property	0.738
consumer credit score	1 if no "slow" payments or delinquencies 2 if one or two slow payments or delinquencies 3 if more than two slow payments 4 if insufficient credit history for determination 5 if delinquent credit history with payments 60 days overdue 6 if delinquent credit history with payments 90 days overdue	2.1
mortgage credit score	1 if no late mortgage payments 2 if no mortgage payment history 3 if one or two late mortgage payments 4 if more than two late mortgage payments	1.7
public bad credit record	1 if any public record of credit problems (bankruptcy, charge-offs, collection actions) 0 otherwise	0.074

Example: Mortgage Applications(long regression)

Additional Applicant Characteristics

denied mortgage insurance	1 if applicant applied for mortgage insurance and was denied, 0 otherwise	0.020
self-employed	1 if self-employed, 0 otherwise	0.116
single	1 if applicant reported being single, 0 otherwise	0.393
high school diploma	1 if applicant graduated from high school, 0 otherwise	0.984
unemployment rate	1989 Massachusetts unemployment rate in the applicant's industry	3.8
condominium	1 if unit is a condominium, 0 otherwise	0.288
black	1 if applicant is black, 0 if white	0.142
deny	1 if mortgage application denied, 0 otherwise	0.120

图 2: pic

Example: Mortgage Applications(long regression)

TABLE 11.2 Mortgage Denial Regressions Using the Boston HMDA Data

Dependent variable: *deny* = 1 if mortgage application is denied, = 0 if accepted; 2380 observations.

Regression Model	LPM	Logit	Probit	Probit	Probit	Probit
Regressor	(1)	(2)	(3)	(4)	(5)	(6)
black	0.084** (0.023)	0.688** (0.182)	0.389** (0.098)	0.371** (0.099)	0.363** (0.100)	0.246 (0.448)
P/I ratio	0.449** (0.114)	4.76** (1.33)	2.44** (0.61)	2.46** (0.60)	2.62** (0.61)	2.57** (0.66)
housing expense-to-income ratio	−0.048 (0.110)	−0.11 (1.29)	−0.18 (0.68)	−0.30 (0.68)	−0.50 (0.70)	−0.54 (0.74)
medium loan-to-value ratio (0.80 ≤ loan-value ratio ≤ 0.95)	0.031* (0.013)	0.46** (0.16)	0.21** (0.08)	0.22** (0.08)	0.22** (0.08)	0.22** (0.08)
high loan-to-value ratio (loan-value ratio > 0.95)	0.189** (0.050)	1.49** (0.32)	0.79** (0.18)	0.79** (0.18)	0.84** (0.18)	0.79** (0.18)
consumer credit score	0.031** (0.005)	0.29** (0.04)	0.15** (0.02)	0.16** (0.02)	0.34** (0.11)	0.16** (0.02)
mortgage credit score	0.021 (0.011)	0.28* (0.14)	0.15* (0.07)	0.11 (0.08)	0.16 (0.10)	0.11 (0.08)
public bad credit record	0.197** (0.035)	1.23** (0.20)	0.70** (0.12)	0.70** (0.12)	0.72** (0.12)	0.70** (0.12)
denied mortgage insurance	0.702** (0.045)	4.55** (0.57)	2.56** (0.30)	2.59** (0.29)	2.59** (0.30)	2.59** (0.29)
self-employed	0.060** (0.021)	0.67** (0.21)	0.36** (0.11)	0.35** (0.11)	0.34** (0.11)	0.35** (0.11)

Example: Mortgage Applications(long regression)

single				0.23** (0.08)	0.23** (0.08)	0.23** (0.08)
high school diploma				–0.61** (0.23)	–0.60* (0.24)	–0.62** (0.23)
unemployment rate				0.03 (0.02)	0.03 (0.02)	0.03 (0.02)
condominium					–0.05 (0.09)	
black × *P/I ratio*						–0.58 (1.47)
black × *housing expense-to-income ratio*						1.23 (1.69)
additional credit rating indicator variables	no	no	no	no	yes	no
constant	–0.183** (0.028)	–5.71** (0.48)	–3.04** (0.23)	–2.57** (0.34)	–2.90** (0.39)	–2.54** (0.35)

Example: Mortgage Applications(long regression)

(Table 11.2 continued)

F-Statistics and p-Values Testing Exclusion of Groups of Variables

	(1)	(2)	(3)	(4)	(5)	(6)
applicant single; high school diploma; industry unemployment rate				5.85 (< 0.001)	5.22 (0.001)	5.79 (< 0.001)
additional credit rating indicator variables					1.22 (0.291)	
race interactions and black						4.96 (0.002)
race interactions only						0.27 (0.766)
difference in predicted probability of denial, white vs. black (percentage points)	8.4%	6.0%	7.1%	6.6%	6.3%	6.5%

These regressions were estimated using the $n = 2380$ observations in the Boston HMDA data set described in Appendix 11.1. The linear probability model was estimated by OLS, and probit and logit regressions were estimated by maximum likelihood. Standard errors are given in parentheses under the coefficients, and p-values are given in parentheses under the F-statistics. The change in predicted probability in the final row was computed for a hypothetical applicant whose values of the regressors, other than race, equal the sample mean. Individual coefficients are statistically significant at the *5% or **1% level.

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