Machine Learning and Causal Inference

MIXTAPE TRACK



Imagine you are a life insurance underwriter. You receive an application for life insurance from someone with the following characteristics:

male

- male
- ▶ age 67

- male
- ▶ age 67
- high blood pressure

- ▶ male
- ▶ age 67
- high blood pressure
- high cholesterol

- male
- ▶ age 67
- high blood pressure
- high cholesterol
- family history of heart disease

- male
- ▶ age 67
- high blood pressure
- ▶ high cholesterol
- family history of heart disease
- ▶ and . . .

- ▶ male
- ▶ age 67
- high blood pressure
- high cholesterol
- ► family history of heart disease
- ▶ and . . .
- was admitted to the hospital yesterday



Now imagine you are a loved one of someone with the following characteristics:

male

- male
- age 67

- male
- age 67
- high blood pressure

- male
- ▶ age 67
- high blood pressure
- high cholesterol

- male
- age 67
- high blood pressure
- high cholesterol
- family history of heart disease

- male
- age 67
- high blood pressure
- high cholesterol
- family history of heart disease
- ▶ and . . .

- male
- ▶ age 67
- high blood pressure
- high cholesterol
- ► family history of heart disease
- ▶ and . . .
- is having chest pains.

- male
- ▶ age 67
- high blood pressure
- ▶ high cholesterol
- ► family history of heart disease
- ▶ and . . .
- ▶ is having chest pains.
- Should you take him to the hospital?











Prepare

➤ A loan officer wants to know the likelihood of an individual repaying a loan based on income, employment, and other characteristics.





Prepare

A loan officer wants to know the likelihood of an individual repaying a loan based on income, employment, and other characteristics.



Influence

► A mortgage lender wants to know if direct debit will increase loan repayments







Prepare

► In order to decide whether to invest in a start-up, an investor needs to know how likely the start-up is to succeed, given the entrepreneur's experience and the characteristics of the industry.

Influence





Prepare

► In order to decide whether to invest in a start-up, an investor needs to know how likely the start-up is to succeed, given the entrepreneur's experience and the characteristics of the industry.



Influence

An entrepreneur needs to know what the effect of receiving funding from a private equity investor (rather than getting a loan) is on the ultimate success of an enterprise.







Prepare

A bail hearing judge needs to know how likely a defendant is to flee before trial, given his or her charges, criminal history, and other characteristics

Influence





Prepare

➤ A bail hearing judge needs to know how likely a defendant is to flee before trial, given his or her charges, criminal history, and other characteristics



Influence

▶ A policy maker needs to know the effect of being released on bail (rather than detained) prior to trial on ultimate conviction







Prepare

 A home seller wants to know what price homes with the characteristics of his or her home typically sell for

Influence





Prepare

➤ A home seller wants to know what price homes with the characteristics of his or her home typically sell for



Influence

A home seller wants to know by how much installing new windows will raise the value of his or her home







Prepare

➤ A Harvard admissions officer wants to know how likely an applicant with given credentials is to graduate in 4 years

Influence





Prepare

➤ A Harvard admissions officer wants to know how likely an applicant with given credentials is to graduate in 4 years



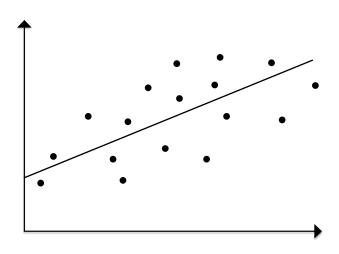
Influence

▶ A labor economist wants to know whether individuals of a certain ethnic background are less likely to get into Harvard than applicants with similar academic credentials



Prediction vs. Causality: Target

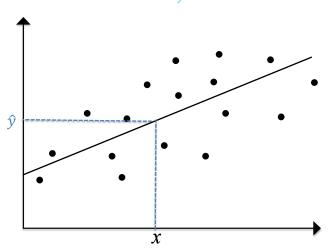
$$y_i = \alpha + \beta x_i + \varepsilon_i$$



Prediction vs. Causality: Target

Prediction

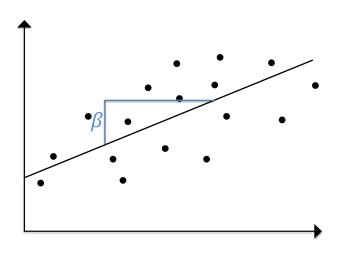
$$y_i = \underbrace{\alpha + \beta x_i}_{\hat{v}} + \varepsilon_i$$



Prediction vs. Causality: Target

Causality

$$y_i = \alpha + \beta x_i + \varepsilon_i$$



Causality

Causality

► Gold standard: RCT



Causality

► Gold standard: RCT



► Aluminum standard: Regression or IV strategies that approximate controlled experiments

Causality

► Gold standard: RCT



► Aluminum standard: Regression or IV strategies that approximate controlled experiments

Prediction

Causality

► Gold standard: RCT



► Aluminum standard: Regression or IV strategies that approximate controlled experiments

Prediction

Supervised machine learning algorithms

Prediction vs. Causality: Where shall the twain meet?

We've seen that prediction and causality

answer different questions

We've seen that prediction and causality

- answer different questions
- serve different purposes

We've seen that prediction and causality

- answer different questions
- serve different purposes
- seek different targets

We've seen that prediction and causality

- answer different questions
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- seek different targets
- use different methods

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Different strokes for different folks, or complementary tools in an applied economist's toolkit?

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Different strokes for different folks, or **complementary tools in** an applied economist's toolkit?

► Illustrate using the Oregon Health Insurance Experiment (go to python)

Traditional regression strategy:

1. Regress Y_i on X_i and compute the residuals,

$$\begin{aligned} \tilde{Y}_i &= Y_i - \hat{Y}_i^{OLS}, \\ \hat{Y}_i^{OLS} &= X_i' \left(X'X \right)^{-1} X'Y \end{aligned}$$

2. Regress D_i on X_i and compute the residuals,

$$\begin{array}{rcl} \tilde{D}_{i} & = & D_{i} - \hat{D}_{i}^{OLS}, \\ \hat{D}_{i}^{OLS} & = & X_{i}' \left(X'X \right)^{-1} X'D \end{array}$$

3. Regress \tilde{Y}_i on \tilde{D}_i .

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3. Regress \tilde{Y}_i on \tilde{D}_i .

When OLS might not be the right tool for the job:

- \triangleright there are many variables in X_i
- \blacktriangleright the relationship between X_i and Y_i or D_i may not be linear

ML-augmented regression strategy:

1. Predict Y_i using X_i with ML and compute the residuals,

$$egin{array}{lll} ilde{Y}_i &=& Y_i - \hat{Y}_i^{ML}, \\ \hat{Y}_i^{ML} &=& ext{prediction generated by ML} \end{array}$$

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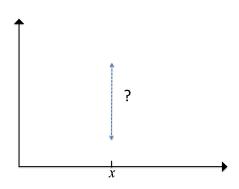
Most common ML methods in applied economics:

- Lasso
- Ridge
- ► Elastic net
- ▶ Random forest



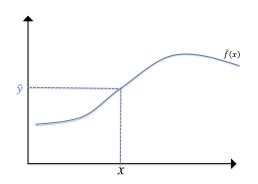
Getting serious about prediction

► **Goal:** Predict an out-of-sample outcome *Y*



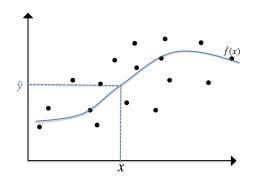
Getting serious about prediction

- ► **Goal:** Predict an out-of-sample outcome *Y*
- ▶ as a function, $\hat{f}(X)$, of **features** $X = (1, X_1, X_2, \dots, X_K)'$.



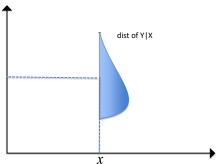
Getting serious about prediction

- ▶ Goal: Predict an out-of-sample outcome Y
- ▶ as a function, $\hat{f}(X)$, of **features** $X = (1, X_1, X_2, \dots, X_K)'$.
- Estimate the function f̂ (aka "train the model") based on training sample {(Y_i, X_i); i = 1, ..., N}



► Want our prediction to be "close," i.e. minimize the expected loss function:

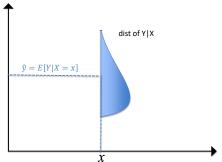
$$\min_{f(x)} E\left[L\left(Y - f\left(x\right)\right)|X = x\right]$$



► Want our prediction to be "close," i.e. minimize the expected loss function:

$$\min_{f(x)} E\left[L\left(Y - f\left(x\right)\right)|X = x\right]$$

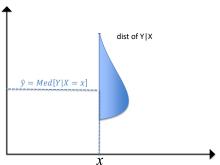
▶ Squared loss: $L(d) = d^2 \implies f^*(x) = E[Y|X = x]$



Want our prediction to be "close," i.e. minimize the expected loss function:

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- ▶ Squared loss: $L(d) = d^2 \implies f^*(x) = E[Y|X = x]$
- ▶ Absolute loss: $L(d) = |d| \implies f^*(x) = Med[Y|X = x]$

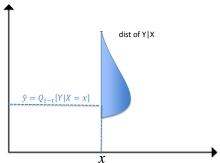


Want our prediction to be "close," i.e. minimize the expected loss function:

$$\min_{f(x)} E[L(Y - f(x))|X = x]$$

- ▶ Squared loss: $L(d) = d^2 \implies f^*(x) = E[Y|X = x]$
- ▶ Absolute loss: $L(d) = |d| \implies f^*(x) = Med[Y|X = x]$
- ► Asymmetric loss:

$$L_{\tau}(d) = d(\tau - 1(d < 0)) \implies f^{*}(x) = Q_{1-\tau}[Y|X = x]$$



▶ Prediction problem solved if we knew $f^*(x) = E[Y|X = x]$

- ▶ Prediction problem solved if we knew $f^*(x) = E[Y|X = x]$
- ▶ But we have to settle for an estimate: $\hat{f}(x)$;

$$E\left[\left(Y-\hat{f}\left(x\right)\right)^{2}\middle|X=x\right]$$
 becomes:

$$\left(E\left[\hat{f}(x) - f^*(x)\right]\right)^2 + E\left[\left(\hat{f}(x) - E\left[\hat{f}(x)\right]\right)^2\right] + E\left[(Y - f^*(x))^2 | X = x\right]$$

prediction bias squared prediction variance irreducible error.

- ▶ Prediction problem solved if we knew $f^*(x) = E[Y|X = x]$
- ▶ But we have to settle for an estimate: $\hat{f}(x)$;

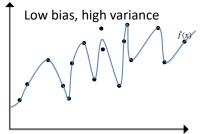
$$E\left[\left(Y-\hat{f}\left(X\right)\right)^{2}\middle|X=X\right]$$
 becomes:

$$\left(E\left[\hat{f}\left(x\right)-f^{*}\left(x\right)\right]\right)^{2} \qquad \text{prediction bias square}$$

$$+E\left[\left(\hat{f}\left(x\right)-E\left[\hat{f}\left(x\right)\right]\right)^{2}\right] \qquad \text{prediction variance}$$

$$+E\left[\left(Y-f^{*}\left(x\right)\right)^{2}|X=x\right] \qquad \text{irreducible error.}$$

prediction bias squared irreducible error.

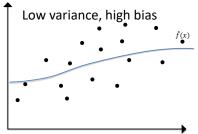


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prediction bias squared prediction variance irreducible error.



Python example: predicting earnings in the $\ensuremath{\mathsf{NLSY}}$

Penalized Regression: Lasso

- When is it the right tool for the job:
 - When you have a large number of potential regressors (including powers or other transformations), maybe even more than the sample size!
 - Out of these, only a relatively few (but you don't know which) really matter (what do we mean by "matter?"). We call this approximate sparsity
- Theoretical definition:

$$\arg\min_{b} \sum_{i=1}^{n} (y_i - x_i'b)^2 + \lambda \sum_{j=1}^{k} |b_j|$$

What does λ do and how do we choose it?

- Caveats and considerations:
 - Important to standardize regressors pre-lasso
 - ► Can give unexpected results with dummy variables
 - Resist the temptation to interpret coefficients or the included variables as the "true model!"
- Let's give it a go in python!



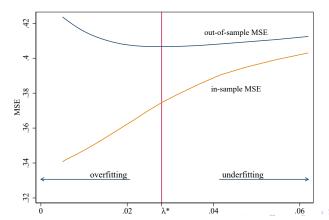
Choosing Tuning Parameters: Cross-Validation

All supervised ML methods have tuning parameters:

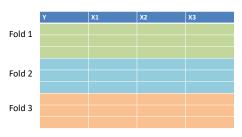
Lasso: λ Ridge: α

► Random forests: tree depth, etc.

Tuning parameters are the rudder by which we navigate the bias-variance tradeoff.



Choosing Tuning Parameters: Cross-Validation



Cross-validation procedure: Divide sample in K folds

- lacktriangle Choose some value of the tuning parameter, λ
- For each fold $k = 1, \dots, K$
 - 1. Train model leaving out fold k
 - 2. Generate predictions in fold k
 - 3. Compute MSE for fold k: $MSE_k = \frac{1}{n_k} \sum_{i \in k} (Y_i \hat{Y}_i)^2$
- ► Compute overall MSE correponding to the current choice of λ : $MSE(\lambda) = \frac{1}{K} \sum_{k=1}^{K} MSE_k$

Repeat the above for many values of λ , and choose the value λ^* with the lowest cross-validated MSE—time for python!

Penalized Regression: Ridge

- When is it the right tool for the job:
 - When you have a large number of regressors including highly collinear ones
- Theoretical definition:

$$\arg\min_{b} \sum_{i=1}^{n} (y_i - x_i' b)^2 + \alpha \sum_{j=1}^{k} b_j^2$$
$$= (X'X + \alpha I)^{-1} X'Y$$

- Caveats and considerations:
 - Important to standardize regressors pre-ridge
 - Shrinks (biases) coefficients towards zero, but not all the way (unlike lasso)
- Let's give it a go in python!

Penalized Regression: Elastic Net

- Combines lasso and ridge approaches
- Theoretical definition:

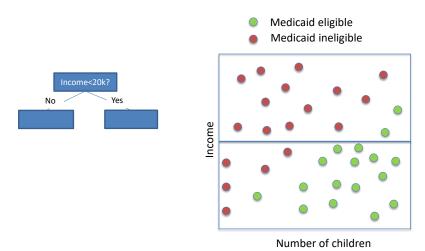
$$\arg\min_{b} \sum_{i=1}^{n} (y_{i} - x_{i}'b)^{2} + \alpha \gamma \sum_{j=1}^{k} |b_{j}| + .5\alpha (1 - \gamma) \sum_{j=1}^{k} b_{j}^{2}$$

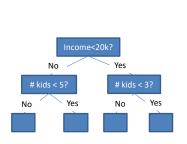
- Caveats and considerations:
 - ightharpoonup Two tuning parameters: α and γ
 - Important to standardize regressors pre-ridge
 - Zeros out many regressors, shrinks (biases) remaining coefficients towards zero
- Let's give it a go in python!

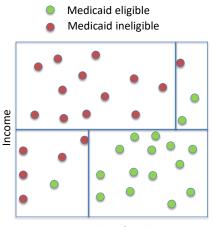
Initial node

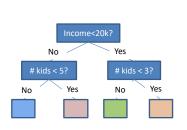
Medicaid eligible Medicaid ineligible Income

Number of children

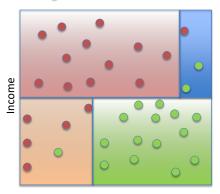




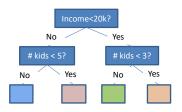




- Medicaid eligible
- Medicaid ineligible



Number of children

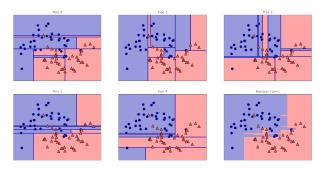


- ▶ Where to split: Choose the feature from $\{x_1, ..., x_p\}$ and the value of that feature to minimize MSE in the resulting child nodes
- Tuning parameters
 - Max depth
 - Min training obs per leaf
 - Min improvement in fit in order to go ahead with a split
- Let's try it in python!

Wisdom of the crowd: predict my father's age!



Forest for the Trees



- Value proposition: reduce variance by averaging together multiple predictions
- The catch: individual trees need to be de-correlated
- Algorithm:
 - ► Grow *B* trees, each on a different bootstrapped sample
 - At each split, consider only a random subset of features
 - Average together the individual predictions
- Let's grow some trees in python!

