Lecture 2

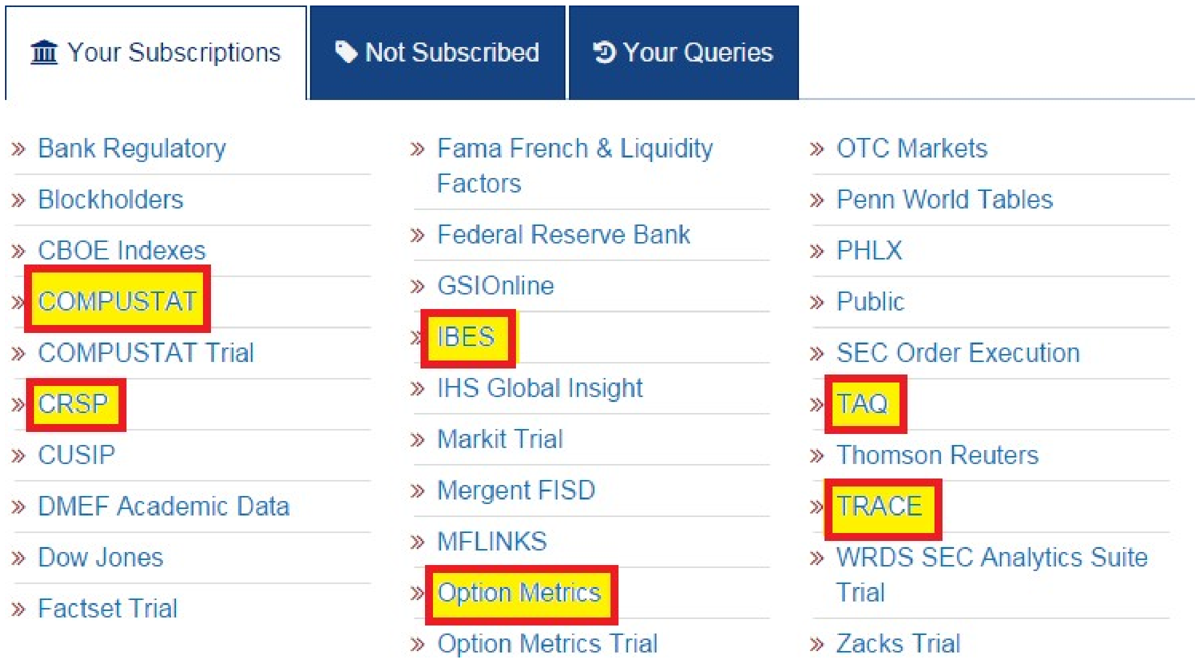
Estimating Expected Returns: Alpha & Beta & the CAPM

Outline for Today

- Where to get financial data?
- Estimating expected stock return using CAPM
- Running regressions to estimate the CAPM alpha and beta
- Testing CAPM

Where to Get Data

- Bloomberg
- Datastream
- WRDS: →



- CRSP:
  - Stock
  - Treasury Bonds
  - Mutual Funds
- Prof. Ken French's Website.

The Expected Return

- For any financial instrument, the single most important number is its **expected** return
- Suppose right now we are in year t, let R_{t+1} denote the stock return to be realized next year. Our investment decision relies on the **expectation**:

$$\mu = E(R_{t+1}) \ .$$

- Just to emphasize, μ is a number, while R_{t+1} is a random variable, drawn from a distribution with mean μ and standard deviation σ
- To estimate this number μ with precision is perhaps the biggest headache in Finance

Estimating the Expected Return μ

- We estimate μ by using historical data:

$$\hat{\mu} = \frac{1}{N} \sum_{t=1}^{N} R_t \, .$$

- It is as simple as taking a sample average.

- How can this sample average of *past* realized returns help us form an expectation of the *future*?

- Under the assumption that history repeats itself (i.i.d). Each R_t in the past was independently drawn from an identical distribution with mean μ and standard deviation σ.

The Estimator Has Noise

- We use historical returns to estimate the number μ:

$$\hat{\mu} = \frac{1}{N} \sum_{t=1}^{N} R_t$$

- Recall that R_t is a random variable, drawn every year from a distribution with mean μ and standard deviation σ.

- As a result, $\hat{\mu}$ inherits the randomness from R_t. In other word, it is not really a number: var$(\hat{\mu})$ is not zero.

- If this variance var$(\hat{\mu})$ is large, then the estimator is noisy.

The Standard Error of $\hat{\mu}$

- Let's first calculate $\text{var}(\hat{\mu})$:

$$\text{var}\left(\frac{1}{N}\sum_{t=1}^{N} R_t\right) = \frac{1}{N^2}\sum_{t=1}^{N}\text{var}(R_t) = \frac{1}{N^2} \times N \times \sigma^2 = \frac{1}{N}\sigma^2$$

- The **standard error** of $\hat{\mu}$ is the same as $\text{std}(\hat{\mu})$:

$$\text{standard error} = \frac{\text{std}(R_t)}{\sqrt{N}} = \frac{\sigma}{\sqrt{N}}$$

Estimating μ for the US Aggregate Stock Market

- Using annual data from 1927 to 2014, we have 88 data points.
- The sample average is $\text{avg}(R) = 12\%$. The sample standard deviation is $\text{std}(R) = 20\%$.
- The **standard error** of $\hat{\mu}$:

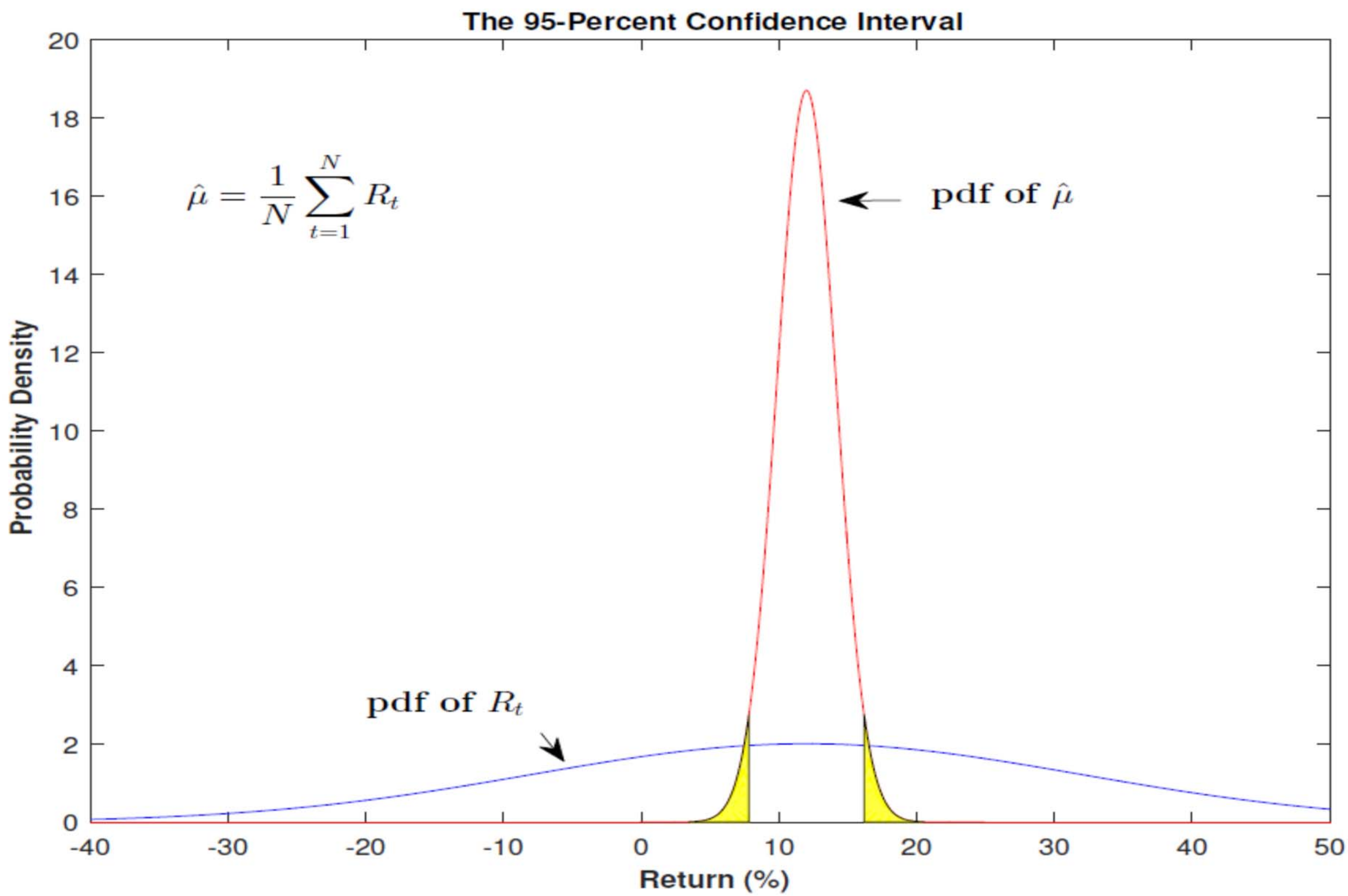
$$\text{s.e.} = \text{std}(R)/\sqrt{N} = 20\%/\sqrt{88} = 2.13\%$$

- The 95% confidence interval of our estimator:

$$[12\% - 1.96 \times 2.13\%, 12\% + 1.96 \times 2.13\%] = [7.8\%, 16.2\%]$$

- The **t-stat** of this estimator is (signal-to-noise ratio),

$$\text{t-stat} = \frac{\text{avg}(R)}{\text{std}(R)/\sqrt{N}} = \frac{12\%}{2.13\%} = 5.63\,.$$


The 95-Percent Confidence Interval
$\hat{\mu} = \frac{1}{N}\sum_{t=1}^{N} R_t$
pdf of $\hat{\mu}$
pdf of R_t
Probability Density
Return (%)
20
18
16
14
12
10
8
6
4
2
0
-40
-30
-20
-10
0
10
20
30
40
50

Problems with Precision

- Usually, the time series we are dealing with are much shorter. For example, the average life span of a hedge fund is around 5 years.
- Also, the volatility of individual stocks is much higher than that of the aggregate market. For example, the annual volatility for Apple is 49.16%. For smaller stocks, the number is even higher: around 100%.
- In these cases, approach on prior slides won't be precise enough (and also i.i.d. assumption will be trouble)

Estimating μ using Monthly Returns

- Since the standard error of $\hat{\mu}$ depends on the number of observations, why don't we use monthly returns to improve on our precision?
- Using monthly aggregate stock returns from January 1927 through December 2011, we have 1020 months. So N=1020!
- The mean of the time series is 0.91%, and std is 5.46%.
- So the standard error of $\hat{\mu}$ is:

$$\text{s.e.} = 5.46\%/\sqrt{1020} = 0.1718\%$$

- The signal-to-noise ratio:

$$\text{t-stat} = \frac{0.91\%}{0.1718\%} = 5.30$$

- We increased N by a factor of 12. Yet, the t-stat remains more or less the same as before.

Chopping the Time Series into Finer Intervals?

- It can be shown that when it comes to the precision of estimating μ, it is the length of the time series that matters. Chopping the time series into finer intervals does not help.
- Professor Merton has written a paper on that. See "On Estimating the Expected Return on the Market," *Journal of Financial Economics*, 1980.
- But when it comes to estimating the volatility of stock returns, this approach of chopping does help and is widely used. We will come back to this.

Expected Returns for Individual Securities

- Above approach is hopeless – data too noisy, sparse, and i.i.d. assumptions surely violated
- Must use a model – the most fundamental one is the CAPM
- According to the CAPM, investors are only rewarded for bearing *systematic risk*, the type of risk that *cannot* be diversified away.
- They should not be rewarded for bearing *idiosyncratic risk*, since this uncertainty can be mitigated through appropriate diversification.
- With market risk serving as an anchor, we can now talk about the pricing of individual stocks or other portfolios.

Getting into details

- Which portfolio do you pick as the market portfolio? [S&P 500 or broader index]
- How do you calculate beta? [Regression coefficient]
- What is the risk-free rate? [T-bill rate]
- What regression do you estimate and how do you calculate the standard error of the estimates?

Test of CAPM in the time series

- For a given stock or portfolio i, run the regression

$$r_{it} - r_{ft} = \alpha_i + \beta_{iM}\left(r_{Mt} - r_{ft}\right) + \epsilon_{it}$$

- Take expectations:

$$E(r_{it}) - r_{ft} = \alpha_i + \beta_{iM}\left(E(r_{Mt}) - r_{ft}\right)$$

- CAPM implies $\alpha_i = 0$. If $\alpha_i \neq 0$ for many stocks/portfolios ➔ evidence against CAPM

- Common procedure is to estimate simple linear regression using 5 years of monthly data on an individual stock

- How do we calculate the precision and statistical significance of the estimates?

- Depends on underlying statistical assumptions regarding ϵ_{it}

A Detour on Standard Errors

- Simplest assumption is that ϵ_{it} are i.i.d. This is what “standard” linear regression would assume in the output.

- Problem 1: Heteroskedasticity – suppose ϵ_{it} tends to be larger when r_{Mt} is larger (plausible) ➔ more noise in observations where r_{Mt} is large ➔ should adjust for increased uncertainty from relying on these observations. Procedure for doing this is known as calculating Robust/White/Heteroskedasticity consistent standard errors.

- Problem 2: Serial correlation – suppose abnormally high returns one period tend to predict abnormally high returns next period ➔ ϵ_{it} are not independent ➔ should adjust for fact that each observation partially reflects old information and only partially reflects new information.   Procedure for doing his is known as calculating Newey-West standard errors (with a selected lag).

Alpha and beta of Amazon

1997-2018	Estimate	Std. Error	t-stat
(Intercept)	0.028839	0.009882	2.918**
Mkt.RF	1.899542	0.220272	8.624***
1997-2008	Estimate	Std. Error	t-stat
(Intercept)	0.04407	0.0166	2.655**
Mkt.RF	2.60975	0.35016	7.453***
2009-2018	Estimate	Std. Error	t-stat
(Intercept)	0.022111	0.007437	2.973**
Mkt.RF	0.876381	0.178176	4.919***

Alpha and beta of Amazon

	i.i.d errors				
	Estimate	Std. Error	t-stat	Pr(>|t|)	
(Intercept)	0.028839	0.009882	2.918	0.00383	**
Mkt.RF	1.899542	0.220272	8.624	6.79E-16	***

	Adjusting for heteroskedasticity				
	Estimate	Std. Error	t-stat	Pr(>|t|)	
(Intercept)	0.028839	0.009442	3.0543	0.002493	**
Mkt.RF	1.899542	0.237997	7.9814	4.82E-14	***

	Newey-West with 6 lags				
	Estimate	Std. Error	t-stat	Pr(>|t|)	
(Intercept)	0.028839	0.011351	2.5405	0.01166	*
Mkt.RF	1.899542	0.306846	6.1905	2.36E-09	***

Some Issues To Consider

- If the α of Amazon is positive and significant, does that imply that the CAPM is incorrect?

- What about if lots of firm's alphas were significantly different from zero

- How do we test statistical significance of this? What is the null hypothesis? Is correlation across stocks/portfolios a concern?

- Can a firm's beta change over time?

- Many different tests/approaches to get around some of these challenges and obtain accurate estimates of precision