Lecture 3

Tests of the CAPM and the First Thoughts on Searching for Convincing for Alpha

Tests of CAPM

- Suppose we randomly chose 100 random stocks and estimated the CAPM alpha for each stock (one regression at a time) using Newey-West standard errors

- How many would we expect to be significant (positive or negative) at the 1% level if CAPM holds? (finding 5 in 100 would be fairly strong evidence against CAPM)

- Suppose that there are 20 main industry groups, and thus our random sample has 5 firms per industry. Suppose firms in industry have very highly correlated returns ➔ if one has alpha significant at 1% level than they probably all will ➔ 5 in 100 is not that rare

- Need to adjust for <u>cross-sectional correlation</u>

Approaches to do this: 1. Estimate or adjust for the underlying the correlation
2. Pick portfolios of stocks deliberately so correlation is close to 0

Regression CAPM Tests Based on Distribution of Alphas

Define $\mathbf{Z}_t$ as an $(N \times 1)$ vector of excess returns for N assets (or portfolios of assets). For these N assets, the excess returns can be described using the excess-return market model:

$$\mathbf{Z}_t = \boldsymbol{\alpha} + \boldsymbol{\beta} Z_{mt} + \boldsymbol{\epsilon}_t$$

$$\mathrm{E}[\boldsymbol{\epsilon}_t] = \mathbf{0}$$

$$\mathrm{E}[\boldsymbol{\epsilon}_t \boldsymbol{\epsilon}_t{}'] = \boldsymbol{\Sigma}$$

$$\mathrm{E}[Z_{mt}] = \mu_m, \qquad \mathrm{E}[(Z_{mt} - \mu_m)^2] = \sigma_m^2$$

$$\mathrm{Cov}[Z_{mt}, \boldsymbol{\epsilon}_t] = \mathbf{0}.$$

$\boldsymbol{\beta}$ is the $(N \times 1)$ vector of betas, Z_{mt} is the time period t market portfolio excess return, and $\boldsymbol{\alpha}$ and $\boldsymbol{\epsilon}_t$ are (Nx1) vectors of asset return intercepts and disturbances, respectively.

Regression CAPM Tests Based on Distribution of Alphas

- Estimating coefficients is same as doing OLS for each security/asset (like earlier), but an OLS system estimation allows us to also estimate these underlying correlations. OLS = MLE under assumptions ➔ good way to go if underlying assumptions are true.

- If we are willing to assume the error terms are normally distributed, we could perform a joint test on whether all of the alphas are zero (i.e., the alpha vector equals 0). This will be akin to an F-test.

- If we are not willing to assume normality and/or we want to use the data more efficiently, we can undertake a Generalized Method of Moments (GMM) estimation and then test whether the alphas are jointly equal to 0.

- Approach is agnostic on slope of Security Market Line and is all about looking at the distribution of intercepts.

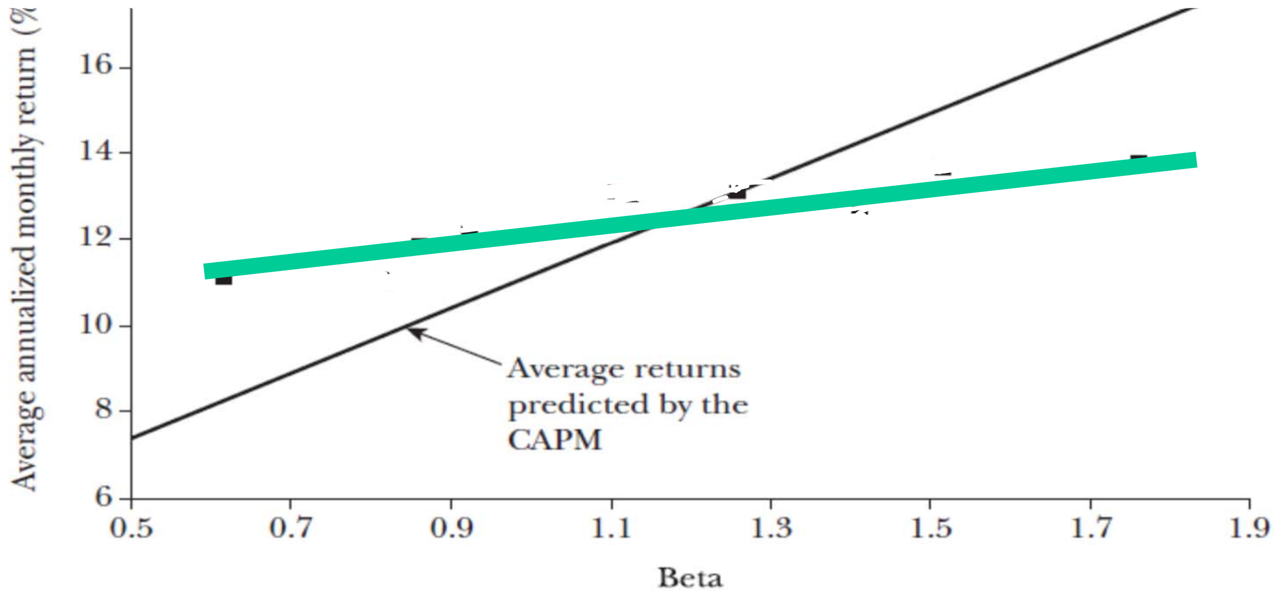
- Evidence from this approach is somewhat cloudy

Regression CAPM Tests Based on Slope of SML

- Estimate beta for securities using monthly returns over recent past
- Form 10 decile-based value-weighted portfolios each month based on these beta estimates. Estimate portfolio beta over same past data.
- Regress portfolio excess return in subsequent month against portfolio beta estimate. This is a single regression each month with 10 observations.
- If CAPM is valid, in each monthly regression, the constant (alpha) should be 0 and the slope of line should be the market risk premium
- Cross-sectional correlation will be a large issue in each monthly regression
- Aggregate across all months by taking mean of all of the constants as the alpha estimate and mean of all slopes as the slope estimate, calculate significance from dispersion of annual estimates
- This is legitimate under fairly weak statistical assumptions (not necessarily the best way to do it, but robust, quick and dirty, and now fairly standard) – Called the Fama-MacBeth Approach

CAPM Tests Based on Slope of SML

Graphical Representation of Evidence from this Procedure



<u>CAPM clearly is missing something, relation is too flat</u>

Univariate Sort Portfolio Tests vs. Regression Tests

- Fama-MacBeth approach still assumes linearity between expected returns and beta (very useful when we start considering multiple factors)
- Alternative that does not require linearity: create decile portfolios sorted by recent beta estimates. For each month, record the beta of the portfolio (weighted average of individual betas) and the excess return of the portfolio over the next month (weighted average of individual excess returns)
- Tabulate mean excess returns and mean betas for each portfolio
- Tabulate standard errors/t-stats of these means using Newey-West standard errors with six lags (Practical way to do this, regress against constant)
- If CAPM is true, the excess returns and betas should more or less line up with the SML
- Often consider both equal-weighted and value-weighted versions of these tests

Univariate Sort Portfolio Tests vs. Regression Tests

TABLE 8.4 Univariate Portfolio Analysis—Equal-Weighted

This table presents the results of univariate portfolio analyses of the relation between each of measures of market beta and future stock returns. Monthly portfolios are formed by sorting all stocks in the CRSP sample into portfolios using decile breakpoints calculated based on the given sort variable using all stocks in the CRSP sample. The table shows the average sort variable value, equal-weighted one-month-ahead excess return (in percent per month), and the CAPM alpha (in percent per month) for each of the 10 decile portfolios as well as for the long-short zero-cost portfolio that is long the 10th decile portfolio and short the first decile portfolio. Newey and West (1987) *t*-statistics, adjusted using six lags, testing the null hypothesis that the average portfolio excess return or CAPM alpha is equal to zero, are shown in parentheses.

Sort Variable	Coefficient	1	2	3	4	5	6	7	8	9	10	10-1
β^{5Y}	β^{5Y}	0.07	0.47	0.66	0.82	0.98	1.14	1.31	1.53	1.84	2.56	2.49
	Excess return	0.83	0.76	0.86	0.89	0.91	0.92	0.84	0.81	0.78	0.75	−0.08
		(3.53)	(3.45)	(3.72)	(3.58)	(3.31)	(3.24)	(2.68)	(2.40)	(2.04)	(1.69)	(−0.27)
	CAPM α	0.57	0.43	0.48	0.47	0.43	0.40	0.28	0.20	0.11	−0.03	−0.60
		(3.05)	(3.18)	(3.76)	(3.63)	(3.02)	(2.83)	(1.74)	(1.18)	(0.52)	(−0.11)	(−2.35)

Univariate Sort Portfolio Tests vs. Regression Tests

TABLE 8.5 Univariate Portfolio Analysis—Value-Weighted

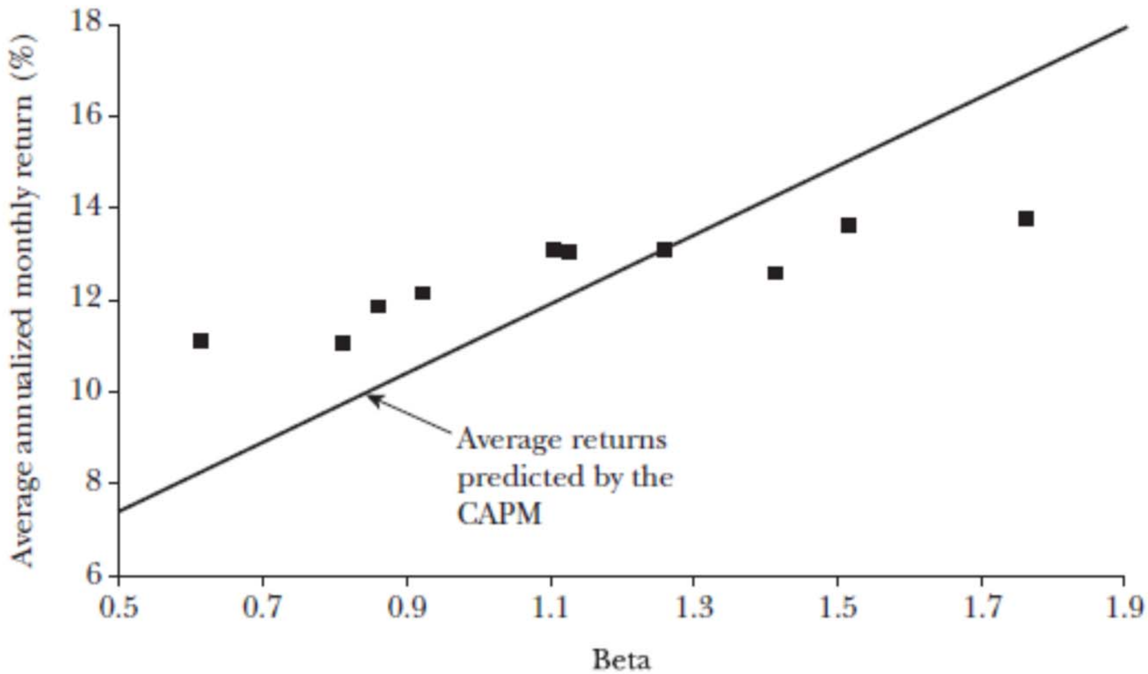
This table presents the results of univariate portfolio analyses of the relation between each of measures of market beta and future stock returns. Monthly portfolios are formed by sorting all stocks in the CRSP sample into portfolios using decile breakpoints calculated based on the given sort variable using all stocks in the CRSP sample. The table shows the value-weighted one-month-ahead excess return and CAPM alpha (in percent per month) for each of the 10 decile portfolios as well as for the long–short zero-cost portfolio that is long the 10th decile portfolio and short the first decile portfolio. Newey and West (1987) t-statistics, adjusted using six lags, testing the null hypothesis that the average portfolio excess return or CAPM alpha is equal to zero, are shown in parentheses.

Sort Variable	Coefficient	1	2	3	4	5	6	7	8	9	10	10-1
β^{5Y}	Excess return	0.43	0.43	0.59	0.44	0.47	0.54	0.49	0.45	0.51	0.50	0.08
		(2.46)	(2.61)	(3.55)	(2.36)	(2.32)	(2.43)	(1.94)	(1.59)	(1.53)	(1.23)	(0.21)
	CAPM α	0.16	0.14	0.25	0.05	0.03	0.05	−0.06	−0.16	−0.17	−0.31	−0.47
		(1.22)	(1.36)	(2.83)	(0.65)	(0.43)	(0.61)	(−0.77)	(−1.73)	(−1.10)	(−1.64)	(−1.73)

Univariate Sort Findings in Pictures

From Fama-French (2004)

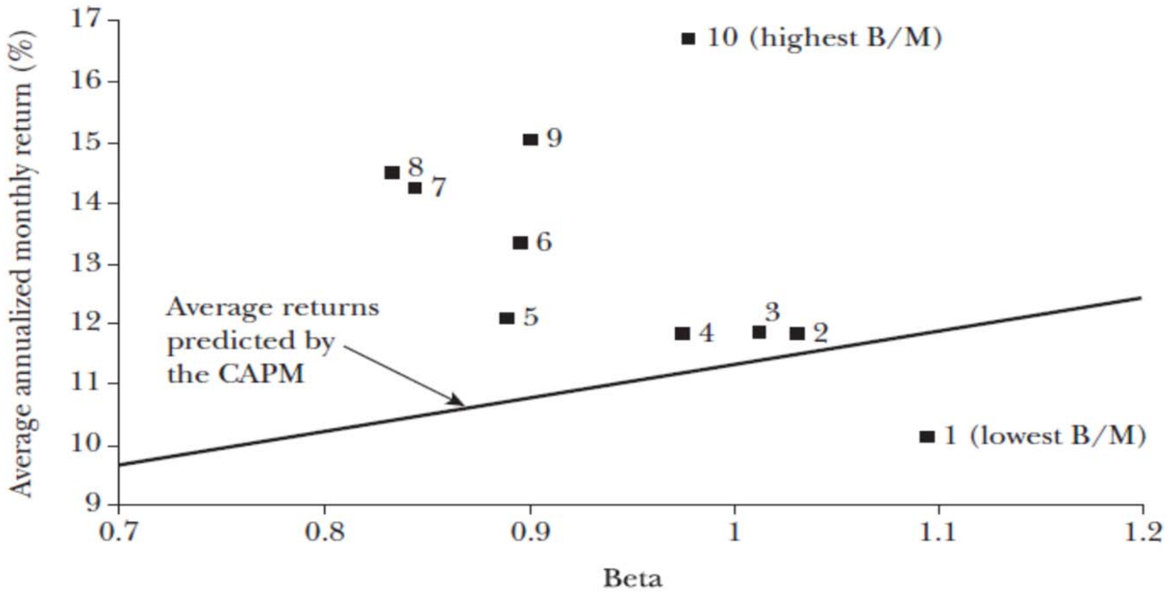
Average Annualized Monthly Return versus Beta for Value Weight Portfolios Formed on Prior Beta, 1928–2003



If CAPM is imperfect, what else explains returns?

General Strategy – Form portfolios sorting based on observable characteristics and look for positive alpha after controlling for market risk. These portfolios will tend to lie above the line on prior slide when following the same procedure but sorting by the characteristics of interest.

Average Annualized Monthly Return versus Beta for Value Weight Portfolios Formed on B/M, 1963–2003



The Zoo of Return Predictability

- If Factor A and Factor B both seem to predict returns above and beyond beta, how do we choose between them?
- If Factor set A and Factor set B both seem to predict returns above and beyond beta, how do we choose?
- More than 350 factors have been published
- Often it is hard to determine whether a new factor offer statistically significant marginal explanatory power in predicting returns
- What is the relationships change over time (implicit assumption in almost models that the coefficients are somewhat stable over time)

Playing With Data

Suppose we consider the **equally-weighted excess return on all stocks** and regress than against the value weighted-return of all stocks. Use Newey-West standard errors. Monthly data from 1926-2019.

Prediction for alpha?

Predictions for beta?

Playing With Data

```
. newey exewret mktrf if permco==20792, lag(6)

Regression with Newey-West standard errors          Number of obs  =       216
maximum lag: 6                                      F(  1,     214)  =    682.11
                                                    Prob > F       =    0.0000
```

exewret	Coef.	Newey-West Std. Err.	t	P>|t|	[95% Conf.	Interval]
mktrf	1.088287	.0416694	26.12	0.000	1.006152	1.170422
_cons	.0430498	.1690872	0.25	0.799	-.2902398	.3763394

Playing With Data

Suppose we sort stocks each month and choose **the smallest decile firms by market cap**. as of the start of the month. Calculate the equally-weighted excess return of this portfolio and regress against value-weighted market return. Use Newey-West standard errors. Monthly data from 1926-2019.

Prediction for alpha?

Predictions for beta?

Playing With Data

```
. newey ewsmall mktrf if keepsmall==1, lag(6)

Regression with Newey-West standard errors          Number of obs =      1122
maximum lag: 6                                      F(  1,   1120) =   117.93
                                                    Prob > F      =   0.0000
```

ewsmall	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
mktrf	1.454775	.133965	10.86	0.000	1.191924	1.717625
_cons	1.183154	.2258208	5.24	0.000	.7400747	1.626234

Playing With Data

Suppose we sort stocks each month and choose **the largest decile firms by market cap**. As of the start of the month. Calculate the equally-weighted excess return of this portfolio and regress against value-weighted market return. Use Newey-West standard errors. Monthly data from 1926-2019.

Prediction for alpha?

Predictions for beta?

Playing With Data

```
. newey ewlarge mktrf if keeplarge==1, lag(6)

Regression with Newey-West standard errors          Number of obs  =      1122
maximum lag: 6                                      F(  1,   1120) =  29882.99
                                                    Prob > F       =    0.0000
```

ewlarge	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
mktrf	1.001077	.005791	172.87	0.000	.9897148	1.01244
_cons	-.0104147	.0214427	-0.49	0.627	-.0524872	.0316577