Financial Markets - 15.433 – Assignment #1
Professor Charles Hadlock

You should complete this assignment in a group of 3-6 students. The group should email a single copy of the assignment to (sloan15433ta@gmail.com) by **Tuesday, September 8th at 10 a.m.**. Exercises #3 and #4 require computer programming and you are free to use a language of your choice (Python, R, Matlab, Stata, or other). Your assignment should include a single pdf document that contains your answers to Exercises #1 and #2 and a summary of your tabulation from the computer/statistics work related to Exercises #3 and #4. You should then also include a document that includes your programming work and associated output. All questions related to the assignment should be directed to the course TAs. Please list all student names as they appear on Canvas along with your MIT ID numbers on the fisrt page of your PDF submission

Exercise #1

You are 35 years old. You just received a $500,000 bonus. You would like to buy a new car, and you are deciding between a $100,000 Porsche, a $200,000 Ferrari, and a $300,000 Bentley. At the same time, you would like to invest a sufficiently large part of your bonus in a retirement account (which earns continuously compounded annual returns with a mean of 10% and a 20% standard deviation) so that you have a 75% probability of having $1 million available from this source for retirement at the age of 50. Which car(s) can you afford to buy? Assume that continuously compounded returns are i.i.d. normally distributed.

Exercise #2

Consider a bond portfolio whose continuously compounded monthly return has a mean of 0.3% and standard deviation of 1.5%. Assume that continuously compounded returns are i.i.d. normally distributed.
(a) What is the probability that, over a horizon of 10 years, the bond portfolio will outperform a riskless T-bill earning the continuously compounded return of 0.3% per month?
(b) How does your answer in part (a) depend on the length of the investment horizon? How does it depend on the volatility of the bond portfolio?

Exercise #3

The purpose of this exercise is to examine the distributions of asset returns at different frequencies (daily, monthly, annual). Download three datasets, *returns daily.txt*, *returns monthly.txt*, and *returns annual.txt*, from Canvas. Each dataset contains three columns: calendar date, returns on the Center for Research in Security Prices (CRSP) value-weighted stock market portfolio, and returns on a portfolio of long-term government bonds.

A. Estimate the means, variances, and standard deviations of returns, at all three frequencies. What are the covariances and correlations between the returns on stocks and bonds, at various frequencies?

B. Estimate the skewness and kurtosis of returns. Based on these statistics, do returns appear to be drawn from a normal distribution?

C. Create the histograms of stock and bond returns at different frequencies. How close are the empirical distributions to the normal distribution?

In the remainder of the exercise, simply assume that the return distribution is normal at each return frequency, even if such an assumption seems imperfect.

D. Compute the 95% confidence intervals for the stock (bond) return over the next period (day, month, year). Compute the 95% confidence interval for the arithmetic average stock (bond) return over the following 30 periods (days, months, years).

E. Compute the absolute shortfall probabilities for stocks and bonds. That is, for a cutoff level k, compute the probability that the return over the next period (day, month, year) will be lower than k. Plot this probability as a function of the threshold level k, for $k = -20\%, -10\%, 0, 10\%, 20\%$.

F. Assuming that returns are i.i.d., compute the probability that the stock return over the next period (day, month, year) will be lower than the bond return.

Exercise #2

The purpose of this exercise is to apply simulation techniques to make predictions about future asset returns. This exercise extends Exercise 1, using the same data.

A. Assume that the stock and bond returns are normally distributed, at each return frequency. Recompute the answers to part E of Exercise 1 by simulation. Simulate 10,000 stock (bond) returns from the appropriate normal distribution and compute the fraction of the 10,000 simulations in which the return is less than k, for k = -20%, -10%, 0, 10%, 20%. Do you get the same answers as in Exercise 1? If not, what is the reason behind the difference?

B. Drop the assumption of normality and recompute the answers to part E of Exercise 1 using the resampling method. That is, randomly resample 10,000 stock (bond) returns from their empirical (i.e., actual, or historical) distributions, and compute the fraction of the 10,000 draws in which the return is less then k, for k = -20%, -10%, 0, 10%, 20%. Do your answers differ from those obtained in part A? How good or bad is the assumption of return normality?

The above two questions deal with predicting one-period returns. The following questions deal with multiperiod returns. Unless you are asked to use the resampling method, assume that the continuously compounded returns of stocks and bonds are i.i.d. normally distributed.

C. Compute analytically the probability that stocks (bonds) will yield a total cumulative return larger than 20% over the following 5 periods (days, months, years).

D. Now answer the same question using the known-distribution simulation approach. That is, simulate 10,000 5-consecutive-period paths of returns, compute the cumulative 5-consecutive-period return for each path, and compute the fraction of the 10,000 simulations in which the cumulative stock (bond) return is larger than 20%. Do you get the same answers? If not, what is the reason behind the difference?

E. Now answer the same question using the resampling approach. That is, randomly resample 10,000 5-consecutive-period paths of returns, compute the cumulative 5-period return for each path, and compute the fraction of the 10,000 draws in which the cumulative stock (bond) return is larger than 20%. Do you get the same answers as in part C? If not, what is the reason behind the difference?

F. Compute analytically the probability that stocks will underperform bonds over the following 30 periods (days, months, years).