15.415 Foundations of Modern Finance

Leonid Kogan and Jiang Wang
MIT Sloan School of Management

Lecture 10: Options, Part 2

Key concepts

- Binomial model: risk-neutral pricing
- State prices
- Exotic options
- American options
- Empirical implementation of the binomial model
- The Black-Scholes-Merton model
- Option Greeks
- Implementing the BSM model
- Brief history of option pricing

Binomial option pricing model

- Consider the binomial model in a general form.
- The stock and bond prices are given by the following processes:

$$S_0 \begin{cases} uS \\ dS \end{cases} \qquad\qquad 1 \begin{cases} 1+r \\ 1+r \end{cases}$$

  where $u > 1 + r > d$ to avoid arbitrage between the stock and the risk-free asset.

  - Consider a call option on the stock with a strike of *K*. Its payoff will be

$$S_0 \begin{cases} C_u = \max[0, uS - K] \\ C_d = \max[0, dS - K] \end{cases}$$

- We price the call by replication.

Binomial option pricing model

- Form a replication portfolio with the stock and bond:
  - δ shares of the stock,
  - b dollars in the riskless bond.

  such that:

$$\delta u S + b(1+r) = C_u$$

$$\delta d S + b(1+r) = C_d$$

- Unique solution:

$$\delta = \frac{C_u - C_d}{(u-d)S}, \qquad b = \frac{1}{1+r}\frac{uC_d - dC_u}{(u-d)}$$

- We then have:

$$C = \delta S + b = \frac{C_u - C_d}{u-d} + \frac{1}{1+r}\frac{uC_d - dC_u}{(u-d)}$$

Risk neutral probability

- Define:

$$q_u = \frac{(1+r)-d}{u-d}, \qquad q_d = \frac{u-(1+r)}{u-d}$$

- We can then write:

$$C = \frac{q_u C_u + q_d C_d}{1+r}$$

- Since $0 < q_u, q_d < 1$ and $q_u + q_d = 1$, we can interpret $q_u = q$ and $q_d = 1 - q$ as probabilities for the up- and down-states.

- We then have:

$$C = \frac{q C_u + (1-q) C_d}{1+r} = \frac{\mathrm{E}^Q[C_T]}{1+r}$$

where $\mathrm{E}^Q[\cdot]$ is the expectation under probability $Q = (q, 1-q)$, which is called the risk-neutral probability.

Risk neutral probability

- With the risk neutral probability, we can price any asset easily.
- Consider the example from Part 1:
  - Parameters are $S = 50$ and $u = 1.5, d = 0.5, r = 1.1$. Then,
$$q = \frac{1.1 - 0.5}{1.5 - 0.5} = 0.6$$
  - The stock price is:
$$S = \frac{(0.6)(75) + (0.4)(25)}{1 + 0.1} = 50$$
  - The bond price is:
$$B = \frac{(0.6)(1.1) + (0.4)(1.1)}{1 + 0.1} = 1$$
  - The price of a call option on the stock with a strike of $50 is:
$$C = \frac{(0.6)(25) + (0.4)(0)}{1 + 0.1} = 13.64$$

Risk neutral probability

A two-period call on the stock with a strike $K = 50$:

$$
\begin{aligned}
C &= \frac{\mathrm{E}^Q[C_2]}{(1+r)^2} \\
&= \frac{(0.6)^2(62.5) + (0.6)(0.4)(0) + (0.4)(0.6)(0) + (0.4)^2(0)}{(1+0.1)^2} \\
&= \frac{22.5}{1.1^2} = 18.60
\end{aligned}
$$

A put on the stock with a strike $K = 50$:

$$
\begin{aligned}
P &= \frac{(0.6)^2(0) + 2(0.6)(0.4)(12.5) + (0.4)^2(37.5)}{(1+0.1)^2} \\
&= \frac{12.0}{1.1^2} = 9.92
\end{aligned}
$$

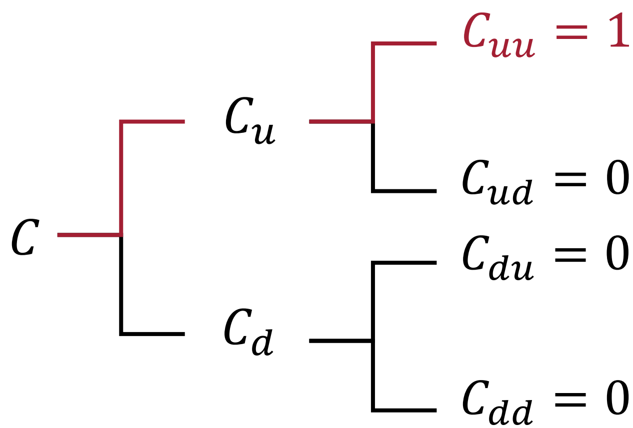
Key concepts

- Binomial model: risk-neutral pricing
- State prices
- Exotic options
- American options
- Empirical implementation of the binomial model
- The Black-Scholes-Merton model
- Option Greeks
- Implementing the BSM model
- Brief history of option pricing

State prices

- We can consider the following "digital option": it pays off \$1 only in a given future state.

- A digital option that pays \$1 at $t = 2$ only if stock price goes up in both periods:



- Denote the price of this option by ϕ_{uu}. Similarly, we have ϕ_{ud}, ϕ_{du}, ϕ_{dd}.

- ϕ_{uu}, ϕ_{ud}, ϕ_{du}, and ϕ_{dd} are the (Arrow-Debreu) state prices.

- Each gives the price of a "state-contingent claim", which pays off one unit only in a given state.

State prices and risk-neutral probabilities

- State prices are proportional to risk-neutral probabilities:

$$\phi_u = \frac{q}{1+r}, \quad \phi_d = \frac{1-q}{1+r}$$

$$\phi_{uu} = \frac{q^2}{(1+r)^2}, \quad \phi_{ud} = \frac{q(1-q)}{(1+r)^2}, \quad \phi_{du} = \frac{(1-q)q}{(1+r)^2}, \quad \phi_{dd} = \frac{(1-q)^2}{(1+r)^2}$$

- Knowing the state prices, we can price any security whose payoff is given by the path of the stock price.

Key concepts

- Binomial model: risk-neutral pricing
- State prices
- Exotic options
- American options
- Empirical implementation of the binomial model
- The Black-Scholes-Merton model
- Option Greeks
- Implementing the BSM model
- Brief history of option pricing

Exotic options: risk-neutral pricing

- Payoff of exotic options is path-dependent.
- For example, payoff of a look-back option may depend on the maximum stock price observed over the life of the option contract.
- Original pricing by replication is not practical: the number of nodes on the binomial tree grows exponentially with the number of time periods.
  - Without path dependence, the tree *recombines*: “ud” node = “du” node, etc.
- Can use risk-neutral pricing for exotic options.
- Estimate the option price by Monte Carlo simulation: sample from the set of terminal nodes according to their risk-neutral probabilities.
- Replicating portfolio can be computed at each node once the option prices are known.

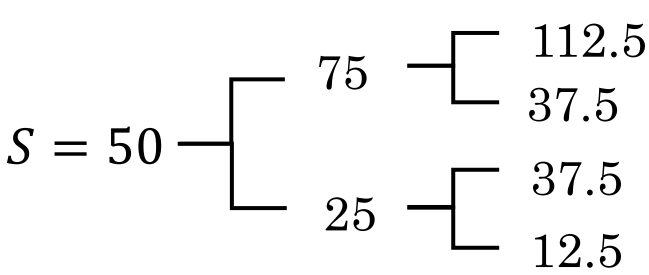
Example: Asian option

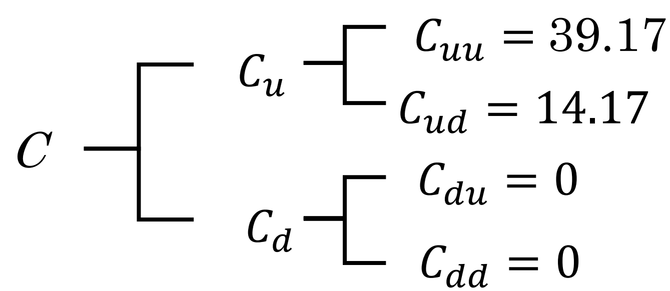
- Two-period ($T = 3$) Asian call option with a strike of \$40. Its payoff is:

$$C_3 = \max[0, \bar{S}_3 - 40]$$

where $\bar{S}_3$ is the average price between $t = 1$ and 3.

- Then





- The price of the call is therefore:

$$C = \frac{(0.6)^2(39.17) + (0.6)(0.4)(14.17) + (0.4)(0.6)(0) + (0.4)^2(0)}{(1+0.1)^2}$$

$$= \frac{17.50}{1.1^2} = 14.46$$

Example: Asian option

- Compute the replicating portfolio as needed, for each visited node.
- For example, to compute the replicating portfolio at node "u" at $t = 1$, need to know only the prices of the option in nodes "uu" and "ud".
  - Buy δ shares of stock, and invest b at the risk-free rate, where

$$\delta = \frac{39.17 - 14.17}{112.5 - 37.5} = 0.333,$$

$$b = C_u - \delta u S = \frac{0.6 \times 39.17 + 0.4 \times 14.17}{1.1} - 0.333 \times 75 = 1.52$$

Key concepts

- Binomial model: risk-neutral pricing
- State prices
- Exotic options
- American options
- Empirical implementation of the binomial model
- The Black-Scholes-Merton model
- Option Greeks
- Implementing the BSM model
- Brief history of option pricing

American options: pricing

- The holder of the option may decide to exercise at any point before maturity.

- Option value P satisfies:

$$P_t = \max\left(\text{Payoff}_t, \frac{1}{1+r_f} \mathrm{E}_t^Q [P_{t+1}]\right)$$

American put option

Strike	100
u	1.2
d	0.8
r_f	0.1
q	0.75
Physical probability	
p	0.5

	t=0	t=1	t=2	Option Payoff
Stock Tree			144.00	0.00
		120.00	96.00	4.00
	100.00			
		80.00	96.00	4.00
			64.00	36.00
Option Tree			0.00	
		0.91	4.00	
	5.17			
		$\boxed{20.00}$	4.00	
No-exercise value		*10.91*	36.00	

American options: dynamic replication

- Replicate the option using the same algorithm as for European options: compute the option's delta from option prices and stock prices:

$$\delta = \frac{C_u - C_d}{(u-d)S}$$

$$b = \frac{1}{1+r}\frac{uC_d - dC_u}{(u-d)}$$

$$\frac{0.91 - 20.00}{120.00 - 80.00} = -0.48$$

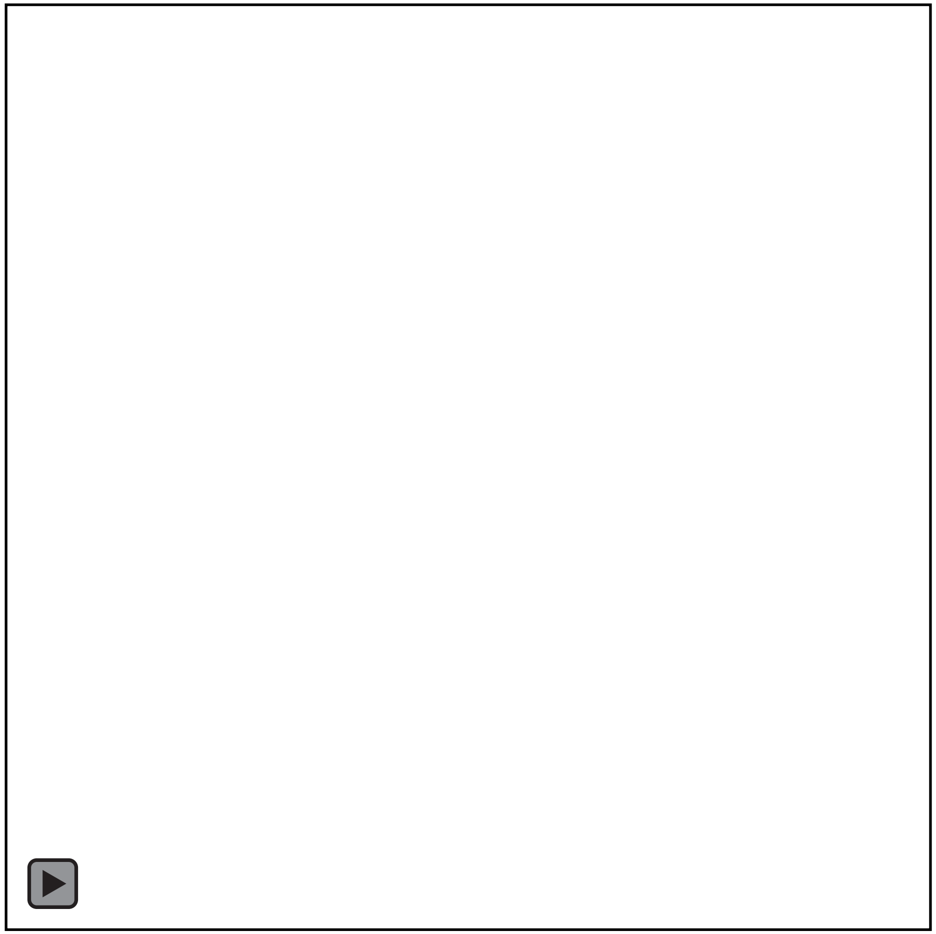
t=0	t=1	t=2
Stock Tree		144.00
	120.00	96.00
100.00		
	80.00	96.00
		64.00
Option Tree		
		0.00
	0.91	4.00
5.17		
	20.00	4.00
No-exercise value	*10.91*	36.00
Delta		
	-0.08	
$\boxed{-0.48}$		
	-1.00	
Risk-free investment		
	10.91	
52.89		
	100.00	

Key concepts

- Binomial model: risk-neutral pricing
- State prices
- State prices
- Exotic options
- Empirical implementation of the binomial model
- The Black-Scholes-Merton model
- Option Greeks
- Implementing the BSM model
- Brief history of option pricing

Implementing binomial model

- Binomial distribution of log returns converges to Normal distribution.



Implementing binomial model

- Key model parameters u and d need to be chosen to reflect the distribution of the stock return.

- One possible choice is:

$$u = \exp\left(\sigma\sqrt{\frac{T}{n}}\right), \qquad d = 1/u, \qquad p = \frac{1}{2} + \frac{1}{2}\left(\frac{\mu}{\sigma}\right)\sqrt{\frac{T}{n}}$$

  where μ and σ are the mean and standard deviation of the stock's annual return, and n is the number of steps in a year.

- We refer to σ as the stock's volatility.

Key concepts

- Binomial model: risk-neutral pricing
- State prices
- State prices
- Exotic options
- Empirical implementation of the binomial model
- The Black-Scholes-Merton model
- Option Greeks
- Implementing the BSM model
- Brief history of option pricing

Black-Scholes-Merton formula

- If we let the period-length get smaller and smaller, in the limit we obtain the option pricing formula:

$$C(S,K,T) = SN(x) - KR^{-T}N(x - \sigma\sqrt{T})$$

- x is defined by:

$$x = \frac{\ln\left(\frac{S}{KR^{-T}}\right)}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T}$$

- T is time to option maturity, in units of a year;
- $R = 1 + r$, where r is the annual riskless interest rate;
- σ is the volatility of annual returns on the underlying asset;
- $N(\cdot)$ is the cumulative normal density function;
- S is log-normally distributed (i.e., $\ln S$ is normally distributed).

Black-Scholes-Merton formula

- An interpretation of the Black-Scholes-Merton formula:
  - The call is equivalent to a levered long position in the stock;
  - $SN(x)$ is the amount invested in the stock;
  - $KR^{-T}N(x-\sigma\sqrt{T})$is the dollar amount borrowed;
  - The option's delta is $N(x)=\frac{\partial C}{\partial S}$. It is the limit of the binomial formula as the time step converges to zero, and single-period stock price movements become infinitesimal:

$$\frac{C_u - C_d}{uS - dS} \to \frac{\partial C}{\partial S}$$

Black-Scholes-Merton formula

- The Black-Scholes-Merton formula has a convenient scaling property: all prices can be re-scaled in terms of multiples of the stock price:

$$\frac{C(S,K,T)}{S} = N(x) - \frac{KR^{-T}}{S} N\left(x - \sigma\sqrt{T}\right)$$

$$x = \frac{\ln\left(\frac{S}{KR^{-T}}\right)}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T}$$

  - Also, only cumulative stock return volatility matters: $\sqrt{\sigma^2 T}$.

Example: applying the BSM formula

- Consider a European call option on a stock with the following data.
- $S = 50, K = 50, T = 30/365$ (30 calendar days to maturity).
- The volatility σ is 30% per year;
- The current annual interest rate is 5.895% (simple rate).

Then:

$$x = \frac{\ln\left(\frac{50}{50(1.05895)^{-\frac{30}{365}}}\right)}{(0.3)\sqrt{\frac{30}{365}}} + \frac{1}{2}(0.3)\sqrt{\frac{30}{365}} = 0.0977$$

$$C = 50N(0.0977) - 50(1.05895)^{-\frac{30}{365}}N\left(0.0977 - 0.3\sqrt{\frac{30}{365}}\right)$$

$$= 50(0.53890) - 50(0.99530)(0.50468) = 1.83$$

Properties of option prices

- The stock price follows a geometric Brownian motion (lognormal returns).
- The interest rate is constant.
- The option price obtained by the binomial model converges to the Black-Scholes price.
- The BSM option prices satisfy:

Increase in	Call	Put
Stock price S	Increase	Decrease
Strike price K	Decrease	Increase
Volatility σ	Increase	Increase
Time to maturity T	Increase	Increase
Interest rate r	Increase	Decrease

Key concepts

- Binomial model: risk-neutral pricing
- State prices
- State prices
- Exotic options
- Empirical implementation of the binomial model
- The Black-Scholes-Merton model
- Option Greeks
- Implementing the BSM model
- Brief history of option pricing

Option Greeks

- Option Greeks measure sensitivity of option prices to small changes in various inputs: the underlying price and model parameters

- Delta: $\delta = \frac{\partial C}{\partial S}$

- Gamma: $\Gamma = \frac{\partial \delta}{\partial S} = \frac{\partial^2 C}{\partial S^2}$

- Theta: $\Theta = \frac{\partial C}{\partial T}$

- Vega: $\nu = \frac{\partial C}{\partial \sigma}$

- Omega: $\Omega = \frac{\partial C}{\partial S}\frac{S}{C}$

Option Greeks: an empirical example

US Budget Impasse Threatened Default in August 2011:
Stocks plummeted, calls dropped sharply, puts surged

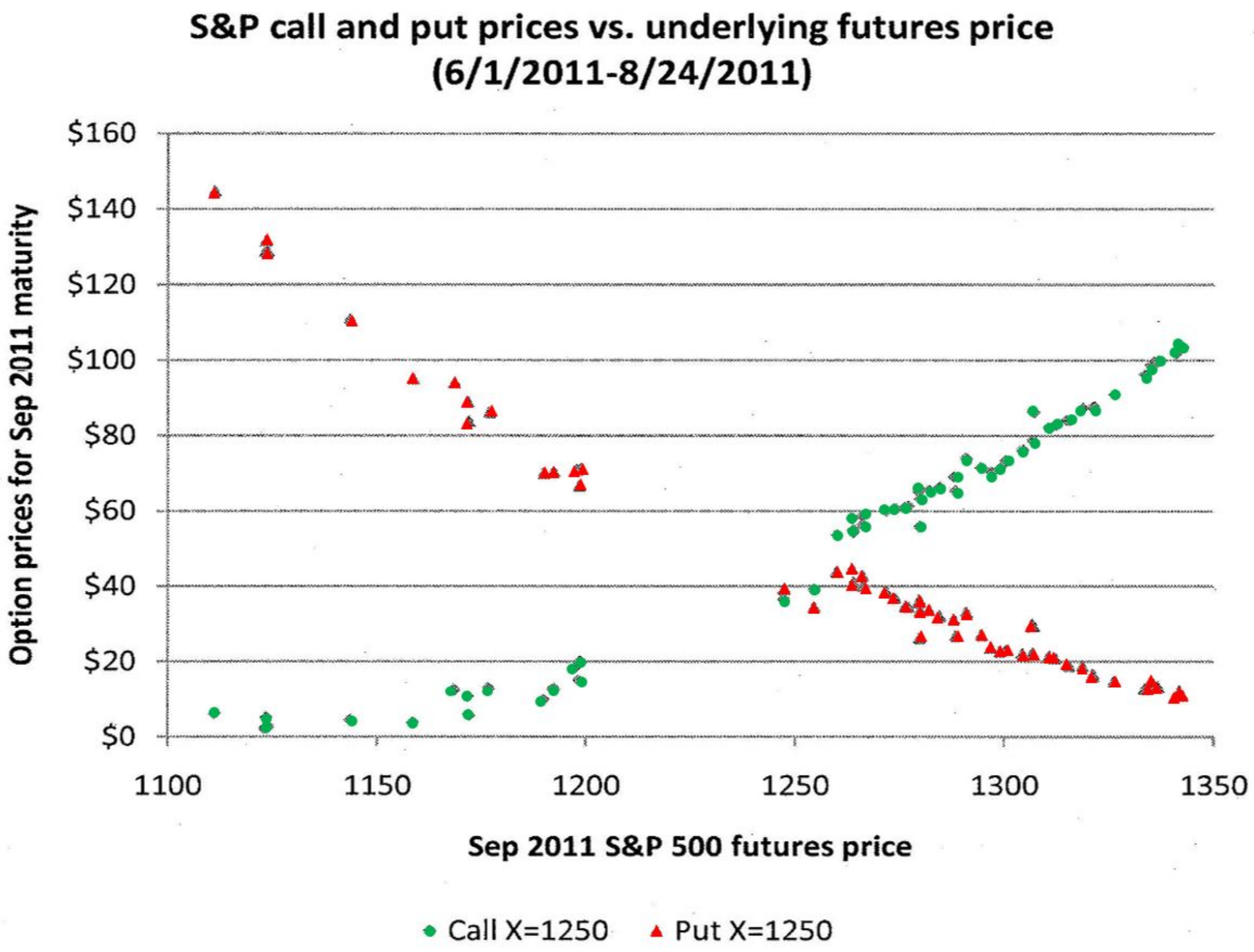
S&P futures, Sep 2011 contract, call options

	Strike	8/1/2011	8/2/2011	8/3/2011	8/4/2011	8/5/2011	8/8/2011	8/9/2011	8/10/2011	8/11/2011	8/12/2011
	SPU1 Index	**1279.7**	**1247.3**	**1254.5**	**1198.7**	**1197.8**	**1111.3**	**1171.7**	**1123.5**	**1168.5**	**1176.8**
	1000	280.90	249.00	255.90	204.60	204.50	137.40	180.50	143.00	180.20	185.90
	1050	232.00	200.80	207.20	158.80	158.90	99.30	136.40	102.80	136.70	141.50
	1100	184.00	153.80	159.70	116.10	116.10	65.70	95.50	67.40	96.30	100.00
	1150	137.80	109.70	114.70	77.30	78.20	38.20	59.10	37.80	60.80	63.10
Strike price	1200	94.30	69.50	73.50	44.10	43.70	17.80	29.40	16.60	31.90	33.20
	1250	55.80	36.50	39.00	20.10	18.80	6.20	10.70	5.20	12.60	13.00
	1300	25.80	13.60	14.70	6.20	5.30	2.05	2.50	1.20	3.50	3.50
	1350	7.40	3.00	3.15	1.40	1.05	0.60	0.70	0.45	0.70	0.75
	1400	1.20	0.55	0.65	0.45	0.40	0.20	0.30	0.15	0.25	0.20
	1450	0.25	0.10	0.15	0.10	0.05	0.05	0.10	0.05	0.05	0.05

S&P futures, Sep 2011 contract, put options

	Strike	8/1/2011	8/2/2011	8/3/2011	8/4/2011	8/5/2011	8/8/2011	8/9/2011	8/10/2011	8/11/2011	8/12/2011
	SPU1 Index	**1279.7**	**1247.3**	**1254.5**	**1198.7**	**1197.8**	**1111.3**	**1171.7**	**1123.5**	**1168.5**	**1176.8**
	1000	1.50	1.95	1.60	6.10	6.90	26.20	9.00	19.60	11.90	9.30
	1050	2.55	3.65	2.90	10.30	11.30	38.10	14.80	29.40	18.30	14.80
	1100	4.45	6.70	5.40	17.50	18.40	54.40	23.90	43.90	27.90	23.30
	1150	8.20	12.50	10.30	28.70	30.50	76.90	37.40	64.30	42.30	36.30
Strike price	1200	14.70	22.30	19.10	45.40	45.90	106.40	57.70	93.00	63.40	56.40
	1250	26.10	39.20	34.50	71.30	70.90	144.80	88.90	131.60	94.00	86.10
	1300	46.10	66.20	60.10	107.40	107.40	190.60	130.70	177.60	134.90	126.60
	1350	77.60	105.60	98.60	152.60	153.10	239.10	178.90	226.80	182.10	173.80
	1400	121.40	153.10	146.00	201.60	202.50	288.80	228.50	276.50	231.60	223.30
	1450	170.40	202.70	195.50	251.30	252.30	338.70	278.30	326.50	281.50	273.20

Option Greeks: an empirical example



Call options and Greeks

Call Option Price Changes ($ and %) Depend on Strike Price
In the money: big $ moves, modest % moves
Out of the money: smaller $ moves, bigger % moves

SP500 Call Options			"Delta" $\left(\frac{\partial C}{\partial S}\right)$		"Omega" $\left(\frac{\partial C}{\partial S}\frac{S}{C}\right)$	
SP500	8/1/2011	8/10/2011	Change	Chg/ChgU	%Change	Elasticity
Underlying	1279.70	1123.50	-156.20	1.00	-12.2	1.0
1000	280.90	143.00	-137.90	0.88	-49.1	4.0
1050	232.00	102.80	-129.20	0.83	-55.7	4.6
1100	184.00	67.40	-116.60	0.75	-63.4	5.2
1150	137.80	37.80	-100.00	0.64	-72.6	5.9
1200	94.30	16.60	-77.70	0.50	-82.4	6.8
1250	55.80	5.20	-50.60	0.32	-90.7	7.4
1300	25.80	1.20	-24.60	0.16	-95.3	7.8
1350	7.40	0.45	-6.95	0.04	-93.9	7.7
1400	1.20	0.15	-1.05	0.01	-87.5	7.2
1450	0.25	0.05	-0.20	0.00	-80.0	6.6

Put options and Greeks

Put Option Price Changes ($ and %) Depend on Strike Price
In the money: big $ moves, modest % moves
Out of the money: smaller $ moves, bigger % moves

SP500 Call Options			**"Delta"** $\left(\frac{\partial C}{\partial S}\right)$		**"Omega"** $\left(\frac{\partial C}{\partial S}\frac{S}{C}\right)$	
SP500	8/1/2011	8/10/2011	Change	Chg/ChgU	%Change	Elasticity
Underlying	1279.70	1123.50	-156.20	1.00	-12.2	1.0
1000	1.5	19.6	18.10	-0.12	1206.7	-98.9
1050	2.6	29.4	26.85	-0.17	1052.9	-86.3
1100	4.5	43.9	39.45	-0.25	886.5	-72.6
1150	8.2	64.3	56.10	-0.36	684.1	-56.1
1200	14.7	93	78.30	-0.5	532.7	-43.6
1250	26.1	131.6	105.50	-0.68	404.2	-33.1
1300	46.1	177.6	131.50	-0.84	285.2	-23.4
1350	77.6	226.8	149.20	-0.96	192.3	-15.8
1400	121.4	276.5	155.10	-0.99	127.8	-10.5
1450	170.4	326.5	156.10	-1.00	91.6	-7.5

Key concepts

- Binomial model: risk-neutral pricing
- State prices
- State prices
- Exotic options
- Empirical implementation of the binomial model
- The Black-Scholes-Merton model
- Option Greeks
- Implementing the BSM model
- Brief history of option pricing

Implementing Black-Scholes

- To implement the Black-Scholes formula, we need to estimate volatility σ.
- Two possible estimates.
- Historic volatility: develop statistical estimates using past returns on the underlying asset.
  - E.g., use daily returns over a given period to estimate daily volatility (standard deviation);
  - Annualize by multiplying daily volatility by $\sqrt{252}$.
- Implied volatility: price options relative to each other.
  - Use the market price of another option (e.g., ATM);
  - Assume that it is given by the BSM formula to solve for σ, which gives the implied volatility.

Example: implied volatility

- Need to price a call on a stock with a strike price of \$110 and a maturity of 1 year. Suppose that the current stock price is \$100 and the one-year interest rate 6.18%.

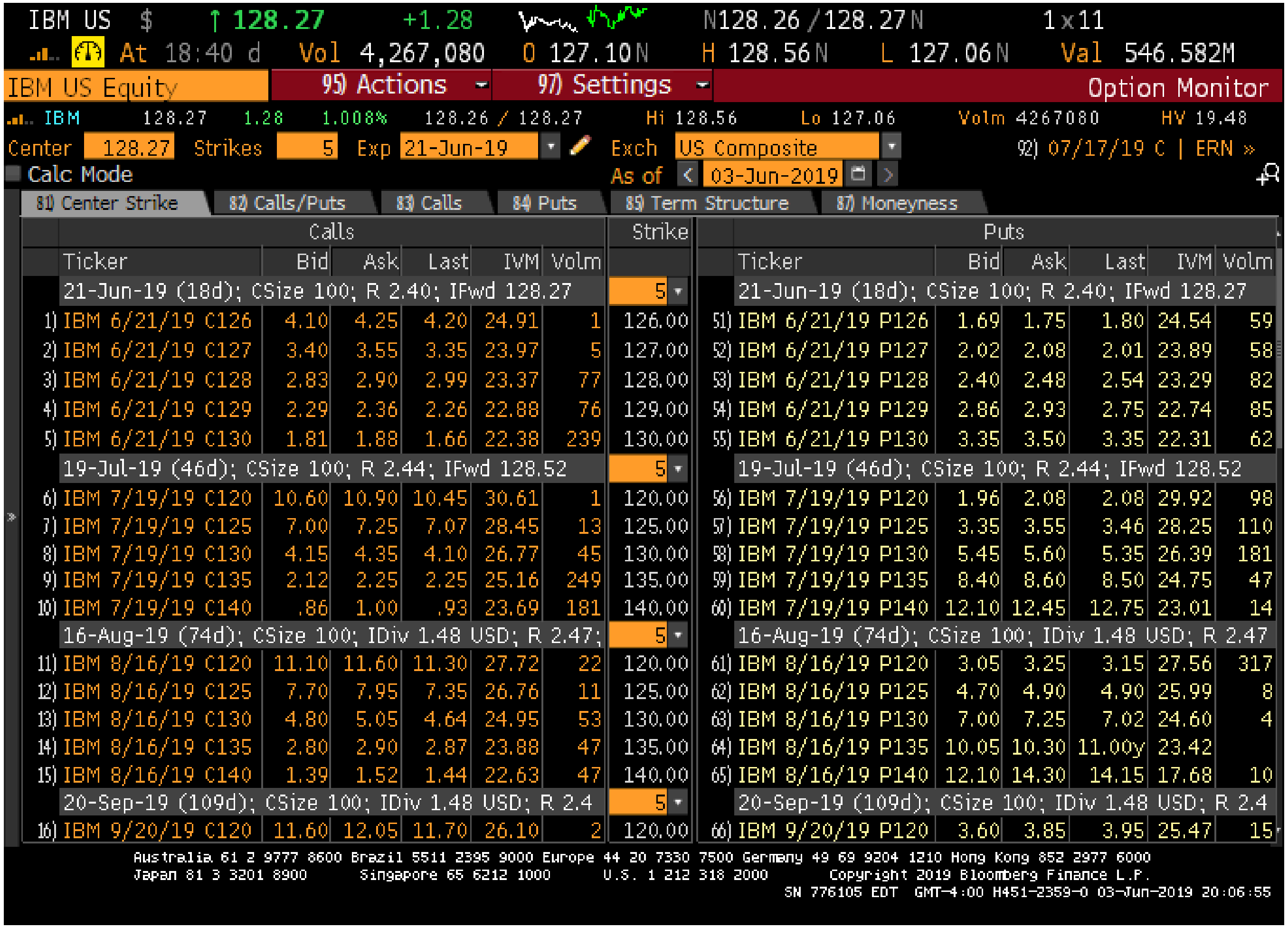
- Suppose that another call with a strike price of \$120 is trading at a market price of \$3.16. The volatility that makes the Black-Scholes price of this call equal to its market price is σ = 19%. This is the implied volatility.

- We can then use 19% in the Black-Scholes formula to obtain the price of the first call.

- Potential problem: Implied volatility may be different for options with different strikes and maturities (smile and smirk patterns in implied volatility).

Implied volatility

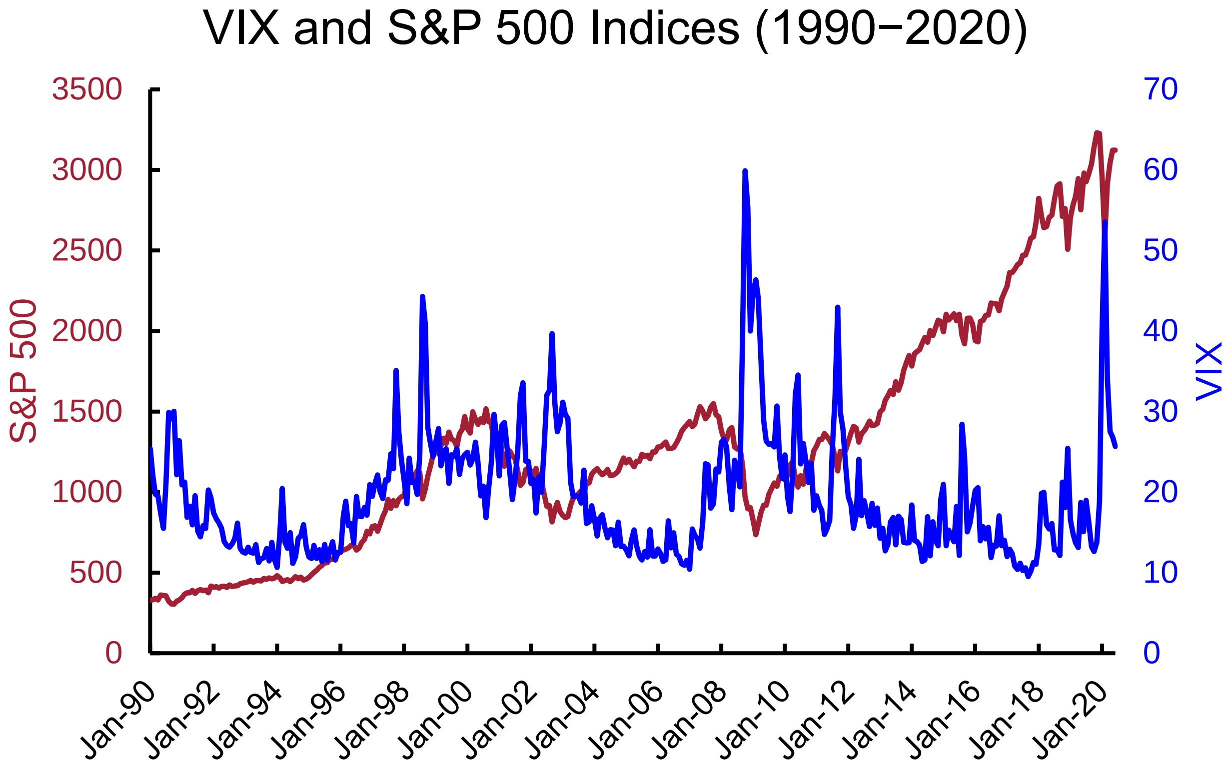
Implied volatility may be different for options with different strikes and maturities (smile/smirk)



Calls							Strike	Puts						
	Ticker	Bid	Ask	Last	IVM	Volm			Ticker	Bid	Ask	Last	IVM	Volm
	21-Jun-19 (18d); CSize 100; R 2.40; IFwd 128.27						5		21-Jun-19 (18d); CSize 100; R 2.40; IFwd 128.27					
1)	IBM 6/21/19 C126	4.10	4.25	4.20	24.91	1	126.00	51)	IBM 6/21/19 P126	1.69	1.75	1.80	24.54	59
2)	IBM 6/21/19 C127	3.40	3.55	3.35	23.97	5	127.00	52)	IBM 6/21/19 P127	2.02	2.08	2.01	23.89	58
3)	IBM 6/21/19 C128	2.83	2.90	2.99	23.37	77	128.00	53)	IBM 6/21/19 P128	2.40	2.48	2.54	23.29	82
4)	IBM 6/21/19 C129	2.29	2.36	2.26	22.88	76	129.00	54)	IBM 6/21/19 P129	2.86	2.93	2.75	22.74	85
5)	IBM 6/21/19 C130	1.81	1.88	1.66	22.38	239	130.00	55)	IBM 6/21/19 P130	3.35	3.50	3.35	22.31	62
	19-Jul-19 (46d); CSize 100; R 2.44; IFwd 128.52						5		19-Jul-19 (46d); CSize 100; R 2.44; IFwd 128.52					
6)	IBM 7/19/19 C120	10.60	10.90	10.45	30.61	1	120.00	56)	IBM 7/19/19 P120	1.96	2.08	2.08	29.92	98
7)	IBM 7/19/19 C125	7.00	7.25	7.07	28.45	13	125.00	57)	IBM 7/19/19 P125	3.35	3.55	3.46	28.25	110
8)	IBM 7/19/19 C130	4.15	4.35	4.10	26.77	45	130.00	58)	IBM 7/19/19 P130	5.45	5.60	5.35	26.39	181
9)	IBM 7/19/19 C135	2.12	2.25	2.25	25.16	249	135.00	59)	IBM 7/19/19 P135	8.40	8.60	8.50	24.75	47
10)	IBM 7/19/19 C140	.86	1.00	.93	23.69	181	140.00	60)	IBM 7/19/19 P140	12.10	12.45	12.75	23.01	14
	16-Aug-19 (74d); CSize 100; IDiv 1.48 USD; R 2.47;						5		16-Aug-19 (74d); CSize 100; IDiv 1.48 USD; R 2.47					
11)	IBM 8/16/19 C120	11.10	11.60	11.30	27.72	22	120.00	61)	IBM 8/16/19 P120	3.05	3.25	3.15	27.56	317
12)	IBM 8/16/19 C125	7.70	7.95	7.35	26.76	11	125.00	62)	IBM 8/16/19 P125	4.70	4.90	4.90	25.99	8
13)	IBM 8/16/19 C130	4.80	5.05	4.64	24.95	53	130.00	63)	IBM 8/16/19 P130	7.00	7.25	7.02	24.60	4
14)	IBM 8/16/19 C135	2.80	2.90	2.87	23.88	47	135.00	64)	IBM 8/16/19 P135	10.05	10.30	11.00y	23.42	
15)	IBM 8/16/19 C140	1.39	1.52	1.44	22.63	47	140.00	65)	IBM 8/16/19 P140	12.10	14.30	14.15	17.68	10
	20-Sep-19 (109d); CSize 100; IDiv 1.48 USD; R 2.4						5		20-Sep-19 (109d); CSize 100; IDiv 1.48 USD; R 2.4					
16)	IBM 9/20/19 C120	11.60	12.05	11.70	26.10	2	120.00	66)	IBM 9/20/19 P120	3.60	3.85	3.95	25.47	15

Implied volatility: VIX

- VIX is a composite summary of implied vols across call and put options with different strike prices.
- VIX is an indicator of future market volatility over the next 30 days.



Key concepts

- Binomial model: risk-neutral pricing
- State prices
- State prices
- Exotic options
- Empirical implementation of the binomial model
- The Black-Scholes-Merton model
- Option Greeks
- Implementing the BSM model
- Brief history of option pricing

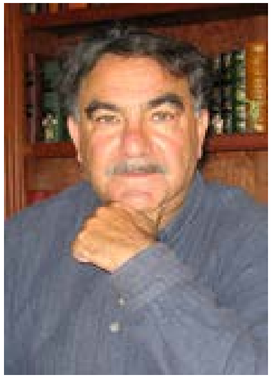
Brief history of option pricing

- Paul Samuelson, “Rational theory of warrant pricing,” 1965.
- Fischer Black and Myron Scholes, “The pricing of options and corporate liabilities,” 1973.
- Robert Merton, “Theory of rational option pricing,” 1973.

Brief history of option pricing

- John Cox, Stephen Ross, and Mark Rubinstein, “Option pricing: a simplified approach,” 1979 (binomial model).
- John Cox and Stephen Ross, “The valuation of options for alternative stochastic processes,” 1976 (risk neutral valuation).

Key concepts

- Binomial model: risk-neutral pricing
- State prices
- Exotic options
- American options
- Empirical implementation of the binomial model
- The Black-Scholes-Merton model
- Option Greeks
- Implementing the BSM model
- Brief history of option pricing

Scratch pad