Standard Errors and Hypothesis Testing

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Motivation

- Is your portfolio manager skilled?
- How to choose between two investment strategies based on the backtests?
- How should the FDA set its review standard for a new COVID-19 vaccine?
- How to write a scam email?

Outline

1. Standard Errors of Estimators
2. GMM
3. The Delta Method
4. Hypothesis Testing
5. Mini-case: Comparing Portfolio Managers

Ordinary Least Squares

Asymptotic distribution of OLS

When n is large, the LS estimator $\hat{\beta}_{LS}$ is approximately normally distributed,

$$\hat{\beta}_{LS} \overset{a}{\sim} \mathcal{N}\left(\beta, \sigma^2 (X'X)^{-1}\right)$$

- In practice, we estimate σ^2 with $RSS/(n-p)$.

Maximum Likelihood

Asymptotic distribution of MLE

When n is large, the maximum likelihood estimator $\hat{\theta}_{ML}$ is approximately normally distributed,

$$\hat{\theta}_{ML} \overset{a}{\sim} \mathcal{N}\left(\theta, \mathbf{I}(\theta)^{-1}\right)$$

where

$$\mathbf{I}(\theta) = -\mathrm{E}\left[\frac{\partial^2 \ln(p(\mathbf{x}|\theta))}{\partial\theta\partial\theta'}\right]$$

- $p(\mathbf{x}|\theta)$ is the likelihood function.
- In practice, we can estimate $\mathbf{I}$ as

$$\hat{\mathbf{I}}(\hat{\theta}) = -\frac{\partial^2 \ln p(\mathbf{x}|\hat{\theta})}{\partial\hat{\theta}\partial\hat{\theta}'}$$

- Q: Does this imply unbiasedness? Consistency?

Example: Mean of Gaussian distribution

- IID Gaussian observations, mean μ, variance σ^2 (known). Parameter $\theta = \mu$.
- Log likelihood:

$$\ln p(\mathbf{x}|\mu) = \ln \prod_{t=1}^{T} p(x_t|\mu) = \sum_{t=1}^{T} \ln p(x_t|\mu) = \sum_{t=1}^{T} \left(\ln \frac{1}{\sqrt{2\pi\sigma^2}} - \frac{(x_t-\mu)^2}{2\sigma^2} \right)$$

- MLE for μ:

$$\widehat{\mu}_{ML} = \frac{1}{T} \sum_{t=1}^{T} x_t$$

$$\mathbf{I} = -\mathrm{E}\left[\frac{\partial^2 \left(\sum_{t=1}^{T} - \frac{(x_t-\mu)^2}{2\sigma^2} \right)}{\partial \mu^2} \right] = \frac{T}{\sigma^2}$$

- Asymptotic distribution of $\widehat{\mu}_{ML}$:

$$\hat{\mu}_{ML} \overset{a}{\sim} \mathcal{N}\left(\mu, \frac{\sigma^2}{T}\right)$$

Q: What if both μ and σ^2 are unknown?

Example: Exponential distribution

- IID sample x_t, $t = 1, \ldots, T$ from an exponential distribution

$$p(x|\theta) = \theta e^{-\theta x}, \quad \theta > 0$$

- Likelihood function:

$$p(\mathbf{x}|\theta) = \theta^T e^{-\theta \sum_{t=1}^T x_t}$$

- MLE:

$$\widehat{\theta}_{ML} = (\overline{x})^{-1}, \quad \overline{x} = \frac{\sum_{t=1}^T x_t}{T}$$

$$\mathbf{I} = -\mathrm{E}\left[\frac{\partial^2 (\sum_{t=1}^T \ln\theta - \theta x_t)}{\partial\theta^2}\right] = \frac{T}{\theta^2}$$

- Asymptotic distribution of $\widehat{\theta}_{ML}$:

$$\hat{\theta}_{ML} \overset{a}{\sim} \mathcal{N}\left(\theta, \frac{\theta^2}{T}\right)$$

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Generalized Method of Moments (GMM)

- Recall the method of moments estimator, where we estimate p parameters with p moment conditions.
- More generally, suppose a model provides a collection of N moment conditions:

$$E[f(x_i, \theta)] = 0$$

  - ↪ Dimension: f is $N \times 1$; θ is $p \times 1$.
  - ↪ Method of moments: $N = p$. More generally, we can have $N \geq p$.

- Moment conditions in finite sample:

$$g_n(\theta) = \widehat{E}[f(x_i, \theta)] = \frac{1}{n} \sum_{i=1}^{n} f(x_i, \theta) = 0$$

- GMM estimator:

$$\hat{\theta}_{GMM} = \arg\min_{\theta} g_n'(\theta)\, W\, g_n(\theta)$$

  - ↪ W: $N \times N$ weighting matrix

GMM Standard Errors: Uncorrelated Observations

- Under mild regularity conditions, GMM estimators are consistent.
- Assume that $f(x_i, \theta)$ are uncorrelated across observations.
- Define

$$\widehat{d} = \left. \frac{\partial \widehat{\mathrm{E}}(f(x_i, \theta))}{\partial \theta'} \right|_{\widehat{\theta}}, \qquad \widehat{S} = \widehat{\mathrm{E}}[f(x_i, \widehat{\theta}) f(x_i, \widehat{\theta})']$$

- GMM estimators are asymptotically normal. By choosing $W = \hat{S}^{-1}$, we get an efficient estimator.

Asymptotic distribution of efficient GMM

When n is large, the efficient GMM estimator $\widehat{\theta}_{GMM}$ is approximately normally distributed,

$$\widehat{\theta}_{GMM} \overset{a}{\sim} \mathcal{N}\left(\theta, \frac{1}{n}\left(\widehat{d}'\widehat{S}^{-1}\widehat{d}\right)^{-1}\right)$$

- In R, use the 'gmm' package.

Example: Mean and Standard Deviation, Gaussian Distribution

- Compute standard errors for estimators of mean and standard deviation using GMM, with

$$f_1(x_t,\theta) = x_t - \mu$$
$$f_2(x_t,\theta) = (x_t - \mu)^2 - \sigma^2$$

- Then we have

$$\widehat{d} = \left.\frac{\partial \widehat{\mathrm{E}}(f(x_t,\theta))}{\partial \theta'}\right|_{\widehat{\theta}} = \begin{bmatrix} -1 & 0 \\ -2(\widehat{\mathrm{E}}(x_t) - \widehat{\mu}) & -2\widehat{\sigma} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2\widehat{\sigma} \end{bmatrix}$$

$$\widehat{S} = \widehat{\mathrm{E}}[f(x_t,\widehat{\theta})f(x_t,\widehat{\theta})'] = \widehat{\mathrm{E}}\begin{bmatrix} f_1^2 & f_1 f_2 \\ f_1 f_2 & f_2^2 \end{bmatrix}$$

- Thus,

$$\widehat{\theta} - \theta_0 \overset{a}{\sim} \mathcal{N}\left(0, \frac{1}{T}\widehat{V}\right), \qquad \widehat{V} = \left(\widehat{d}'\widehat{S}^{-1}\widehat{d}\right)^{-1}$$

Example: Mean and Standard Deviation, Gaussian Distribution

- Recall that for Gaussian distribution, $\mathrm{E}[(x-\mu_0)^3] = 0$, $\mathrm{E}[(x-\mu_0)^4] = 3\sigma_0^4$.
- Using LLN,

$$\lim_{T\to\infty} \widehat{d} = d \equiv \begin{bmatrix} -1 & 0 \\ 0 & -2\sigma_0 \end{bmatrix}$$

$$\lim_{T\to\infty} \widehat{S} = S \equiv \begin{bmatrix} \sigma_0^2 & 0 \\ 0 & 2\sigma_0^4 \end{bmatrix}$$

$$\widehat{\theta} - \theta_0 \sim \mathcal{N}(0, \frac{1}{T}\widehat{V})$$

$$\lim_{T\to\infty} \widehat{V} = \left(d' S^{-1} d\right)^{-1} = \begin{bmatrix} \sigma_0^2 & 0 \\ 0 & \frac{1}{2}\sigma_0^2 \end{bmatrix}$$

Example: Mean and Standard Deviation, Gaussian Distribution

Results

```
mu =       0.1000     sigma     = 0.2000
mu_hat = 0.1196     sigma_hat = 0.1936
SE_mu =  0.0194     SE_sigma  = 0.0142
```

- 95% confidence intervals for parameter estimates can be constructed as

$$\widehat{CI}(\theta_i) = [\widehat{\theta}_i - 1.96 \times SE(\widehat{\theta}_i), \widehat{\theta}_i + 1.96 \times SE(\widehat{\theta}_i)], \quad i = 1, 2$$

- How good are the CI's in a finite sample?
- Perform a Monte Carlo experiment: Simulate M independent artificial samples and compute the coverage frequency.
- Based on 100,000 simulations, the coverage frequencies for μ and σ are 0.945 and 0.929.

GMM Standard Errors: Correlated Observations

- When $f(x_t, \theta)$ are correlated over time, formulas for standard errors must be adjusted to account for autocorrelation.
- Correlated observations affect the effective sample size.
- The relation
$$\mathrm{Var}[\widehat{\theta}] = \frac{1}{T}\left(\widehat{d}^{-1}\widehat{S}\left(\widehat{d}'\right)^{-1}\right) = \frac{1}{T}\left(\widehat{d}\widehat{S}^{-1}\widehat{d}'\right)^{-1}$$
is still valid. But need to modify the estimator $\widehat{S}$.
- In an infinite sample,
$$S = \sum_{j=-\infty}^{\infty} \mathrm{E}\left[f(x_t, \theta_0) f(x_{t-j}, \theta_0)'\right]$$

Estimating S: Newey-West

- Newey-West procedure for computing standard errors prescribes

$$\widehat{S} = \sum_{j=-k}^{k} \frac{k-|j|}{k} \frac{1}{T} \sum_{t=1}^{T} f(x_t, \widehat{\theta}) f(x_{t-j}, \widehat{\theta})' \qquad \text{(Drop out-of-range terms)}$$

- k is the bandwidth parameter.
- In a finite sample, need k to be small compared to T, but large enough to cover the intertemporal dependence range.
- The larger the sample size, the larger k should be. Suggested value is $k \propto T^{1/3}$.
- Consider several values of k and compare the results.

Example: Confidence Interval for Sample Mean

- Perform a Monte Carlo experiment.
- Generate data according to an AR(1) process

$$x_t = \theta x_{t-1} + u_t$$

  u_t are IID, $\mathcal{N}(0,1)$ random variables.
- Simulate 10,000 artificial samples, $T = 200$.

θ	0.0	0.1	0.5	0.9
k	0	5	10	20
Coverage frequency, NW-adjusted	0.95	0.93	0.91	0.80
Coverage frequency, unadjusted ($k = 0$)	0.95	0.92	0.75	0.33

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Review: Vector Notation

- Suppose θ is a vector. We always think of θ as a column:

$$\theta = \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_N \end{pmatrix}, \qquad \theta' = \begin{pmatrix} \theta_1 & \dots & \theta_N \end{pmatrix}$$

- Partial derivatives of a smooth scalar-valued function $h(\theta)$:

$$\frac{\partial h(\theta)}{\partial \theta} = \begin{pmatrix} \frac{\partial h(\theta)}{\partial \theta_1} \\ \vdots \\ \frac{\partial h(\theta)}{\partial \theta_N} \end{pmatrix}, \qquad \frac{\partial h(\theta)}{\partial \theta'} \equiv \begin{pmatrix} \frac{\partial h(\theta)}{\partial \theta_1} & \dots & \frac{\partial h(\theta)}{\partial \theta_N} \end{pmatrix}$$

- If $h(\theta)$ is a vector of functions, $(h_1(\theta), ..., h_M(\theta))'$,

$$\frac{\partial h(\theta)}{\partial \theta'} = \begin{bmatrix} \frac{\partial h_1(\theta)}{\partial \theta_1} & \dots & \frac{\partial h_1(\theta)}{\partial \theta_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial h_M(\theta)}{\partial \theta_1} & \dots & \frac{\partial h_M(\theta)}{\partial \theta_N} \end{bmatrix}$$

Multi-variate Normal Distribution

- Density function of $x \sim \mathcal{N}(\mu, \Omega)$:

$$\phi(x) = \left((2\pi)^N |\Omega|\right)^{-1/2} e^{-\frac{1}{2}(x-\mu)'\Omega^{-1}(x-\mu)}$$

- Linear combinations of jointly normal random variables are normally distributed:

$$x \sim \mathcal{N}(\mu, \Omega) \Rightarrow Ax \sim \mathcal{N}(A\mu, A\Omega A')$$

- The distribution of the sum of squares of n independent $\mathcal{N}(0,1)$ variables is called χ^2 with n degrees of freedom:

$$\varepsilon \sim \mathcal{N}(0, I) \Rightarrow \varepsilon'\varepsilon \sim \chi^2(\dim(\varepsilon))$$

- Distribution of a common quadratic function of a normal vector

$$x \sim \mathcal{N}(0, \Omega) \Rightarrow x'\Omega^{-1}x \sim \chi^2(\dim(x))$$

The Delta Method

- Given the estimator $\widehat{\theta}$, want to derive the asymptotic distribution of the vector of smooth functions $h(\widehat{\theta})$.
- Locally, a smooth function is approximately linear (in $\widehat{\theta}$):

$$h(\widehat{\theta}) \approx h(\theta_0) + \left.\frac{\partial h(\theta)}{\partial \theta'}\right|_{\theta_0} (\widehat{\theta} - \theta_0)$$

- Let $\widehat{\theta} - \theta_0 \sim \mathcal{N}(0, \Omega)$, $\Omega = \mathrm{Var}(\widehat{\theta})$ *is small* ($\propto 1/T$), then

$$h(\widehat{\theta}) - h(\theta_0) \sim \mathcal{N}\left(0, A\Omega A'\right)$$
$$A = \left.\frac{\partial h(\theta)}{\partial \theta'}\right|_{\theta_0}$$

- In estimation, replace A and Ω with consistent estimates $\widehat{A} = \left.\frac{\partial h(\theta)}{\partial \theta'}\right|_{\widehat{\theta}}$ and $\widehat{\Omega}$:

$$h(\widehat{\theta}) - h(\theta_0) \sim \mathcal{N}\left(0, \widehat{A}\widehat{\Omega}\widehat{A}'\right)$$

Example: Sharpe Ratio Distribution by Delta Method

Uncertainty about Sharpe Ratio

You have estimated the mean and standard deviation of monthly excess returns $(\widehat{\mu}, \widehat{\sigma})$. The implied Sharpe ratio is then $\widehat{SR} = h(\widehat{\theta}) = \widehat{\mu}/\widehat{\sigma}$. What is the 95% confidence interval for SR?

- Suppose the asymptotic variance-covariance matrix of parameter estimates $\widehat{\theta} = (\widehat{\mu}, \widehat{\sigma})'$ is estimated to be $\widehat{\Omega}$.
- Compute

$$\widehat{A} = \left.\frac{\partial h(\theta)}{\partial \theta'}\right|_{\widehat{\theta}} = \begin{pmatrix} \frac{1}{\widehat{\sigma}} & -\frac{\widehat{\mu}}{\widehat{\sigma}^2} \end{pmatrix}$$

- Variance of the Sharpe ratio estimate is

$$\begin{pmatrix} \frac{1}{\widehat{\sigma}} & -\frac{\widehat{\mu}}{\widehat{\sigma}^2} \end{pmatrix} \widehat{\Omega} \begin{pmatrix} \frac{1}{\widehat{\sigma}} \\ -\frac{\widehat{\mu}}{\widehat{\sigma}^2} \end{pmatrix}$$

- Q: Does the covariance between $\widehat{\mu}$ and $\widehat{\sigma}$ matter?

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Hypothesis Tests

- Sample of independent observations $x_1, \ldots, x_T$ with distribution $p(x, \theta_0)$.
- Want to test the null hypothesis H_0, which is a set of restrictions on the parameter vector θ_0, e.g., $b'\theta_0 = 0$.
- Statistical test is a decision rule: rejecting the null if some conditions are satisfied by the sample, i.e.,

$$\text{Reject if } \ (x_1, \ldots, x_T) \in \mathbb{A}$$

Type I and Type II error

- Type I error: False rejection of a true null (false positive).
- Type II error: Failure to reject a false null (false negative).
- Test size: Upper bound on the probability of rejecting the null hypothesis over all cases in which the null is correct.

Examples: Type I and Type II errors

- Fire alarm
  - ↪ Fire alarm does not go off when there is a fire.
  - ↪ Fire alarm goes off when there is no fire.
- FDA drug testing
  - ↪ A beneficial drug being rejected.
  - ↪ A harmful drug getting approved.
- The "Nigerian Prince" e-mail scams
  - ↪ Why do these scams seem "obvious"?

Dear Mr. Sir,

REQUEST FOR ASSISTANCE-STRICTLY CONFIDENTIAL

I am Dr. Bakare Tunde, the cousin of Nigerian Astronaut, Air Force Major Abacha Tunde. He was the first African in space when he made a secret flight to the Salyut 6 space station in 1979. He was on a later Soviet spaceflight, Soyuz T-16Z to the secret Soviet military space station Salyut 8T in 1989. He was stranded there in 1990 when the Soviet Union was dissolved. His other Soviet crew members returned to earth on the Soyuz T-16Z, but his place was taken up by return cargo. There have been occasional Progrez supply flights to keep him going since that time. He is in good humor, but wants to come home.

In the 14-years since he has been on the station, he has accumulated flight pay and interest amounting to almost $ 15,000,000 American Dollars. This is held in a trust at the Lagos National Savings and Trust Association. If we can obtain access to this money, we can place a down payment with the Russian Space Authorities for a Soyuz return flight to bring him back to Earth. I am told this will cost $ 3,000,000 American Dollars. In order to access the his trust fund we need your assistance.

Consequently, my colleagues and I are willing to transfer the total amount to your account or subsequent disbursement, since we as civil servants are prohibited by the Code of Conduct Bureau (Civil Service Laws) from opening and/ or operating foreign accounts in our names.

Needless to say, the trust reposed on you at this juncture is enormous. In return, we have agreed to offer you 20 percent of the transferred sum, while 10 percent shall be set aside for incidental expenses (internal and external) between the parties in the course of the transaction. You will be mandated to remit the balance 70 percent to other accounts in due course.

Kindly expedite action as we are behind schedule to enable us include downpayment in this financial quarter.

Please acknowledge the receipt of this message via my direct number 234 (0) 9-234-2220 only.

Yours Sincerely, Dr. Bakare Tunde
Astronautics Project Manager
tip@nasrda.gov.ng

http://www.nasrda.gov.ng/

χ^2 Test

- Want to test the Null Hypothesis regarding model parameters:

$$h(\theta) = 0$$

- Construct a χ^2 test:
  - ↪ Estimate the var-cov of $h(\widehat{\theta})$, $\widehat{V}$.
  - ↪ Construct the test statistic

$$\xi = h(\widehat{\theta})' \widehat{V}^{-1} h(\widehat{\theta}) \sim \chi^2(\dim h(\widehat{\theta}))$$

- Reject the Null if the test statistic ξ is sufficiently large. Rejection threshold is determined by the desired test size and the distribution of ξ under the Null.

Example: OLS

- Suppose we run a regression of y_t on a vector of predictors x_t:

$$y_t = \beta_0 + x_t'\beta_1 + \varepsilon_t$$

- Examples of tests:
  - $\hookrightarrow$ $\beta_0 = 0$: Can the portfolio manager generate alpha?
  - $\hookrightarrow$ $\beta_1 = 0$: Can x_t (signal) predict y_t (returns)?

- Test statistic:

$$\xi = \widehat{\beta}' \left[\mathrm{Var}(\widehat{\beta})\right]^{-1} \widehat{\beta} \sim \chi^2(\dim(\beta))$$

- Test of size α: reject the Null if $\xi \geq \overline{\xi}$, where

$$\mathrm{CDF}_{\chi^2(\dim(\beta))}(\overline{\xi}) = 1 - \alpha$$

Bringing in the Economics

- A hypothesis test is typically just an intermediate step that guides us in making certain (financial) decisions.
- Example: FDA can take the following actions at the end of a clinical trial for a new COVID-19 vaccine.

$$\text{Decision} = \begin{cases} \text{Deny} & \text{if efficacy is below } 90\% \\ \text{Approve} & \text{otherwise} \end{cases}$$

- How good is this decision rule?
- The efficacy threshold effectively determines the test size α:

Economic Design of Hypothesis Testing

- A large test size α raises the probability of Type I error. A small test size likely raises the probability of Type II error. How should we set α?
- Loss function $L(m)$, $m \in \{I, II\}$: economic losses of Type I and Type II errors
- Decision theory:

$$\alpha^*(x) = \arg\min_{\alpha} p(m = I|x, \alpha) L(m = I) + p(m = II|x, \alpha) L(m = II)$$

- Example: FDA
  - ↪ A new drug for a life-threatening disease
  - ↪ A new drug for athlete's foot
- Q: How to determine the relative magnitudes of losses for Type I and Type II errors?

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Comparing Portfolio Managers

- Suppose you work for TIMCo, a college endowment fund. You are trying to choose between two portfolio managers.
- We observe two series of historical excess returns from the two managers, generated over the same period of time by their proprietary trading strategies:

$$(x_1^1, x_2^1, ..., x_T^1) \text{ and } (x_1^2, x_2^2, ..., x_T^2)$$

- We do not know the exact distribution behind each strategy, but we will assume that these returns are IID over time.
- Contemporaneously, x_t^1 and x_t^2 may be correlated.
- How do we decide which PM is better?
  - $\hookrightarrow$ We want to test the *null hypothesis* that these two strategies have the same Sharpe ratio.

Sharpe Ratio Comparison

- Stack together the two return series to create a new observation vector

$$x_t = (x_t^1, x_t^2)'$$

- The parameter vector is

$$\theta_0 = (\mu_1^0, \sigma_1^0, \mu_2^0, \sigma_2^0)$$

- The null hypothesis is

$$H_0 : \left\{ \frac{\mu_1^0}{\sigma_1^0} - \frac{\mu_2^0}{\sigma_2^0} = 0 \right\}$$

- To construct the rejection region for H_0, estimate the asymptotic distribution of $\frac{\widehat{\mu}_1}{\widehat{\sigma}_1} - \frac{\widehat{\mu}_2}{\widehat{\sigma}_2}$.

Sharpe Ratio Comparison

- Using standard GMM formulas, estimate the asymptotic variance-covariance matrix of the parameter estimates $\widehat{\theta}$, $\widehat{\Omega}$.
- Define

$$h(\theta) = \frac{\mu_1}{\sigma_1} - \frac{\mu_2}{\sigma_2}$$

- Compute

$$\widehat{A} = \left.\frac{\partial h(\theta)}{\partial \theta'}\right|_{\widehat{\theta}} = \begin{pmatrix} \frac{1}{\widehat{\sigma}_1} & -\frac{\widehat{\mu}_1}{(\widehat{\sigma}_1)^2} & -\frac{1}{\widehat{\sigma}_2} & \frac{\widehat{\mu}_2}{(\widehat{\sigma}_2)^2} \end{pmatrix}$$

- Asymptotically, variance of $h(\widehat{\theta})$ is

$$\widehat{\mathrm{Var}}\left[h(\widehat{\theta})\right] = \begin{pmatrix} \frac{1}{\widehat{\sigma}_1} & -\frac{\widehat{\mu}_1}{(\widehat{\sigma}_1)^2} & -\frac{1}{\widehat{\sigma}_2} & \frac{\widehat{\mu}_2}{(\widehat{\sigma}_2)^2} \end{pmatrix} \left[\widehat{\Omega}\right] \begin{pmatrix} \frac{1}{\widehat{\sigma}_1} \\ -\frac{\widehat{\mu}_1}{(\widehat{\sigma}_1)^2} \\ -\frac{1}{\widehat{\sigma}_2} \\ \frac{\widehat{\mu}_2}{(\widehat{\sigma}_2)^2} \end{pmatrix}$$

Sharpe Ratio Comparison

- Under the null hypothesis, $h(\theta_0) = 0$, and therefore

$$\frac{h(\widehat{\theta})}{\sqrt{\widehat{\mathrm{Var}}\left[h(\widehat{\theta})\right]}} = \frac{h(\widehat{\theta}) - h(\theta_0)}{\sqrt{\widehat{\mathrm{Var}}\left[h(\widehat{\theta})\right]}} \sim \mathcal{N}(0,1)$$

- Define the rejection region for the test of the null $h(\theta_0) = 0$ as

$$\mathbb{A} = \left\{ \left| \frac{h(\widehat{\theta})}{\sqrt{\widehat{\mathrm{Var}}\left[h(\widehat{\theta})\right]}} \right| \geq z \right\}$$

- A 5% test is obtained by setting $z = 1.96 = \Phi^{-1}(0.975)$, where Φ is the Standard Normal CDF.
- Q: How is the z test different from a χ^2 test?

Readings

Supplementary readings:

- Campbell, Lo, MacKinlay, 1997, Chapter 5, 6, Sections A.2-4.
- Cochrane, "New facts in finance."