15.438 Fixed Income
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Problem Set #3

1. An inverse floater is a bond that pays interest at a rate that moves inversely with an index market rate. (See the teaching note on Canvas in "Additional Materials")

The following bonds are available:

- 6 year floating rate bond, indexed to 1-year LIBOR, annual resets, priced at par
- 6 year 5.5% coupon bond, annual payments, priced at $101 per $100 face value

The 1-year LIBOR rate is currently 4%.

a. Describe a portfolio consisting of long or short positions in these bonds that has a payoff equivalent to that on an inverse floater with a coupon rate of 11% – LIBOR and a face value of $1000. Specifically, what is the dollar amount of the position (present value) in each security?
b. What is the dollar duration of the portfolio that replicates the inverse floater? What can you conclude is the effective duration of the inverse floater?
c. What types of financial institutions do you think would be natural investors in inverse floaters and why?

2. The par yield curve, which is used in swap pricing, is an artificial construct that gives the coupon rate such that a bond sells at par given the current term structure of interest rates. For instance (assuming everything in this example is annual), if spot rates are $Y_1 = .05$ and $Y_2 = .055$, then the par yield curve for two years gives a coupon rate that solves $100 = c100/1.05 + (1+c)100/(1.055)^2$, or c = .05487. Compute the par yield curve going out 5 years, given the spot yield curve from the Deutsche Bank case below based on Treasury rates quoted on 8/15/2003. Do this for Treasury bonds, so payments are semiannual. (You should report coupon values for 10 time periods: 6 months, 1 year, 1.5 years, etc.)

	Zeros (price per $100)	Bond Equiv Spot Yield
2/15/2004	99.56099	0.8819%
8/15/2004	98.81763	1.1930%
2/15/2005	97.67381	1.5753%
8/15/2005	96.34063	1.8727%
2/15/2006	94.73745	2.1742%
8/15/2006	92.89697	2.4711%
2/15/2007	90.7564	2.7905%
8/15/2007	88.74181	3.0084%
2/15/2008	86.47913	3.2544%
8/15/2008	84.15589	3.4799%

3. The dealers who make a market in Treasury securities are compensated for risk and their labor in part by selling at a higher price (the "ask" price) than they buy at (the "bid" price). Here is a schedule of hypothetical bid and ask prices for zero coupon bonds:

Maturity (in years)	Price per $100 Face Value Bid	Ask
1	92.593	92.623
2	84.633	84.663
3	77.238	77.268

(a) Using this information, explain how an investor (who is not a dealer) can lock in a borrowing rate for one year starting in two years, by buying and selling the appropriate zero coupon bonds. Base the calculation on a purchase/sale of $1000. What is the implied forward borrowing rate implied by these transactions on a bond equivalent basis?

(b) Now explain how an investor (who is not a dealer) can lock in a lending rate for one year starting in two years, by buying and selling zero coupon bonds? What is the implied forward lending rate implied by these transactions?

(c) Explain briefly why the implied forward borrowing and lending rate are different, and why they could not be different in the opposite direction (i.e., flipped).

4. Tomorrow is an expiration date for the Treasury bond futures contract, and today's futures price is $99.0. The following three bonds are among the many that qualify for delivery:

Maturity (years)	Coupon (semiannual)	Price (per $100 face)	Conversion Factor
16	5.4%	$97.140	0.939
20	6.0%	$110.677	1.0
24	7.2%	$112.631	1.152

a. Which of these bonds is most likely to be delivered when the contract matures? Explain.

b. You manage a $300 million fixed income portfolio of with a modified duration of 10 years, and you want to delta hedge the overnight exposure to interest rate risk using this futures contract. What is the dollar size of the short position in the futures contract that you would take to do this (assume there are no constraints due to minimum contract size)? Show a calculation and explain your reasoning.