Class Notes Topic 1

Introduction and Calculating Yields, Prices and Returns

15.438 Fixed Income Securities and Derivatives

Professor Deborah Lucas

Spring 2021

Course objectives

- Primary: Master fixed income analytics
  - Acquire tools to **value** fixed income securities and their derivatives
  - Acquire tools to **understand, manage and trade on the risk** of interest rate sensitive claims

- Secondary: Become familiar with major fixed income markets and instruments, the determinants of rates, and recent innovations and developments

Uses of skills acquired

- Sales and Trading
- Portfolio Management, PCS
- Commercial and Investment Banking, Hedge Funds
- Insurance Companies, Valuation Practices
- Government Financial Institutions and Central Banks
- Corporate and personal financial decision-making
- Prep for the CFA exams

My own fixed income-related forays

- *Theme of my research*: Governments as the worlds' largest financial institutions, and largest creators of fixed income securities and derivatives. What they do needs to be better understood and made more transparent!
- Former chief economist for the U.S. Congressional Budget Office (CBO), later founding assistant director for CBO's Financial Analysis Division.
- Came to Sloan (previously at Kellogg) to join finance group in 2010, and to start and direct the MIT Golub Center for Finance and Policy (GCFP)
- Past independent director of Anthracite Capital, a REIT
- Independent Director at CME, world's largest futures exchange
- Many research projects related to fixed income (credit subsidies, mortgages and reverse mortgages, student loans, banking…); consulting for central banks, CBO, OECD

Topic outline

Topic 1: Introduction, and Calculating Yields, Prices and Returns

Measuring and Calculating Yields, Prices and Returns

Floating Rate Bonds

Brief Intro to Yield Curves and Discount Functions

Featured Market: The Money Market

Topic 2: Basic Fixed Income Tools I

Duration and Convexity

Duration and Convexity-based Risk Management Strategies

Homework #1 due 2/25

Topic 3: Basic Fixed Income Tools II

More on Yield Curves and Strip Curves

Forward Rates and Forward Curves

Macroeconomic Models of Interest Rates and the Yield Curve

Featured Market: The U.S. Treasury Market

Discussion Case: Deutsche Bank: Finding Relative-Value Trades

Homework #2 due 3/11

Topic outline

Topic 4: Forwards, Futures and Swaps

Valuing Forwards and Futures
Interest Rate Swaps
Hedging Strategies
Speculating on Spreads
Currency Swaps and Covered Interest Rate Parity

Featured Markets: Repurchase Agreements and Interest Rate Futures

Homework #3 due 4/1

MIDTERM 4/8 IN CLASS

Topic 5: Options on Fixed Income Securities

Options Basics
Valuing Callable and Puttable Bonds on Binomial Trees
Mortgage Prepayment Risk
Valuing Caps and Floors
Effective Duration and OAS

Featured Market: Municipal Securities

Topic 6: Introduction to Continuous Time Models

Topic outline

Topic 7: Credit Risk

Understanding and Modeling Credit Risk

Credit Derivatives

Featured Markets: Corporate Bonds and CDS

Discussion case: Structured Credit Index Products and Default Correlation

Homework #4 due 5/6

Topic 8: Securitization

Value at Risk

Securitization Basics

Asset-backed Securities

Mortgage-backed Securities

Featured Market: The Mortgage Market

Discussion case: Orange County, Value at Risk

Homework #5 due 5/13

Texts

- Required:
  - *Fixed Income Securities,* Bruce Tuckman and Angel Serat, 3rd Edition, 2012
  - Class notes
    - You should print these out for class each week
  - Case packet

- Additional reference texts:
  - *Fixed Income Securities*, Pietro Veronesi
  - *The Handbook of Fixed Income Securities*, Frank Fabozzi
  - *Options Futures and other Derivatives*, John Hull

Where this course fits in

- Most complete and in-depth coverage of fixed income topics available in a single class

- Some overlap with other courses, e.g.,
  - 15.437 Futures and Options
  - 15.429 Securitization of Mortgages

Grading and expectations

Midterm (in class)	30%
Final (in class, cumulative)	45%
Homework Assignments (5 total)	20%
Participation	5%

- Class participation matters--**be sure to bring your name card in person and to keep your camera on online.**

- ***Homework:*** *Groups can have up to four people (no exceptions). All group members should contribute meaningfully to products submitted for grading. Group are self-selected and can change (but usually don't). You can form teams across different sections. You also can discuss the questions with any current student and refer to current year class materials, but old answer keys are off limits.*

- ***Exams:*** *must be completed independently and within the allotted time. Only the specifically allowed materials may be used as reference materials for exams.*

Special rules for hyflex and online

- **Zoom interface and laptop use:**
  - As always, electronic communications are for official class activities only
  - Everyone should be on Zoom during the class. Remote students should have cameras turned on
  - You can use Chat to share ideas, preferably with everyone, and for questions
  - You can also raise a Zoom hand, or a physical hand if you’re in a classroom
  - The TA and facilitator will help monitor and pass along who wants to speak
  - You also need to be on Zoom for polls, breakout rooms, etc.

- **Class hours:** Classes will start and end 10 minutes from the official time; **all class sessions are 70 minutes**. For the evening class, that will provide a 20 minute break in the middle.
- It is imperative that you arrive on time, and classes will start promptly. Inform me or the TA in advance by e-mail if there are special circumstances requiring late arrival or early departure.
- Lectures and recitations will be recorded and posted

Communications

- E-mail: dlucas@mit.edu
  - Office hours: Wednesdays 4 to 5:30pm and by appointment
- TA: J.R. Scott
  - Office hours online
  - Will hold periodic recitations, and review sessions before midterm and final, online
- Facilitator: Inbar Zilbar
- Canvas is your go-to source for announcements, assignments, supplemental notes, readings, etc.

Topic 1.1

Basic Bond Math

Conceptual Goals for This Topic

- Get a sense of the enormity and variety of the fixed income marketplace *(see BTAS Overview of Global Fixed Income Markets)*
- Understand that cash flows are fundamental. **Bond prices are present values of future cash flows.**
  - Principle of "No Arbitrage" for pricing—identical future cash flows have an identical value today
  - All valuation comes down to projecting cash flows and finding the right discount rate**s**
  - Quotation conventions are just conventions (but important to know how some of them work)
- Understand that the risk associated with fixed income investing often depends on one's investment horizon

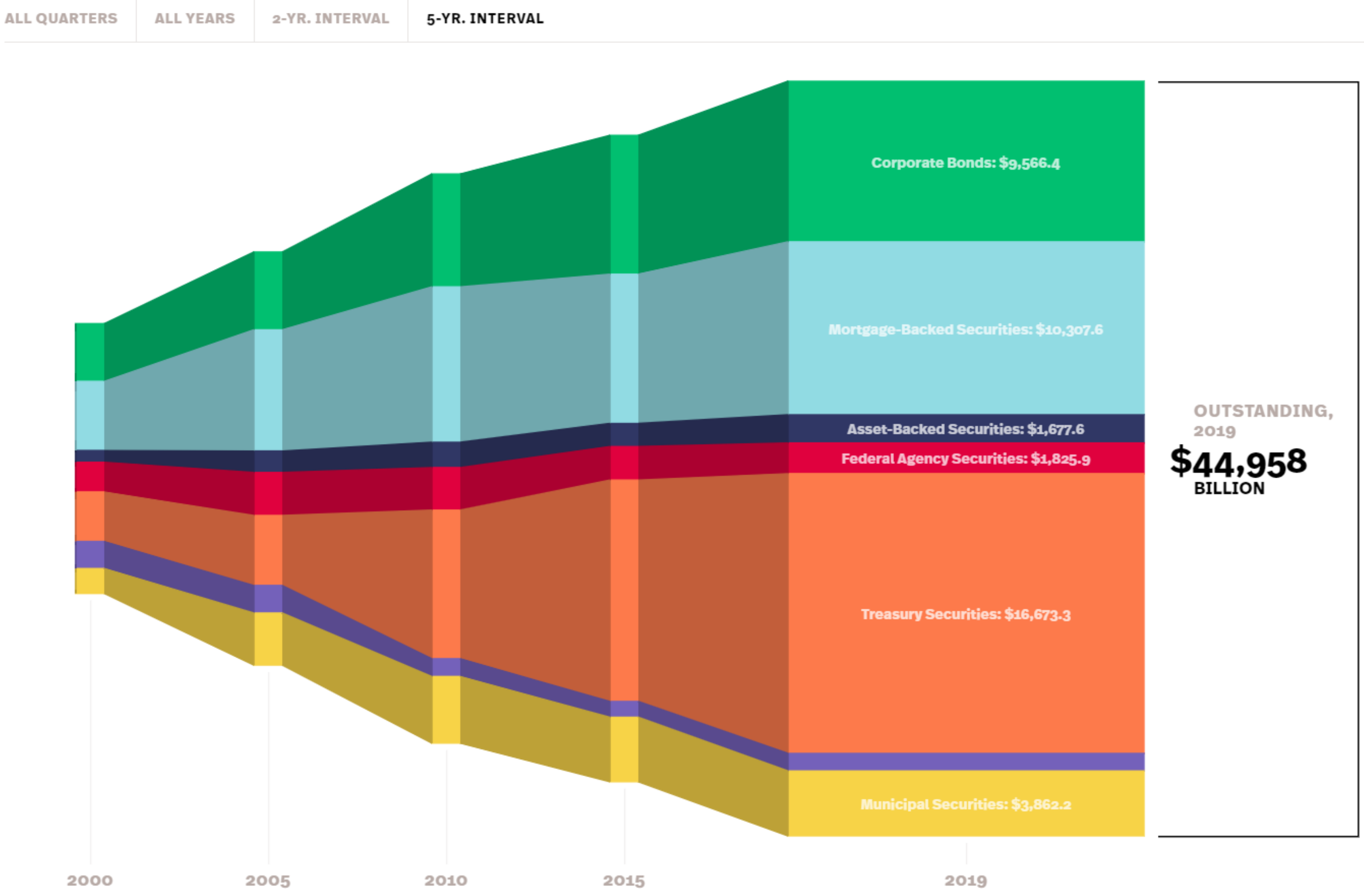
Acquired skills

- Recall how to compute a yield to maturity, but understand the limitations of this measure of return. (recall IRR)
- Learn some of the standard quotation conventions for coupon bonds and money market instruments.
- Become familiar with some of the built-in Excel spreadsheet bond functions.
- Learn how to estimate total returns (also called horizon returns) for coupon bonds. (recall future value)
- Learn basics of pricing floating rate bonds.
- Understand how to derive "discount functions" from bond yields, and vice versa. (another way to represent yield curves)

Fixed Income Outstanding

(United States) $45 trillion!

Source: SIFMA



Corporate

Mortgage-backed

Asset-backed

Federal Agency

Treasury

Money Market

Municipal

2000

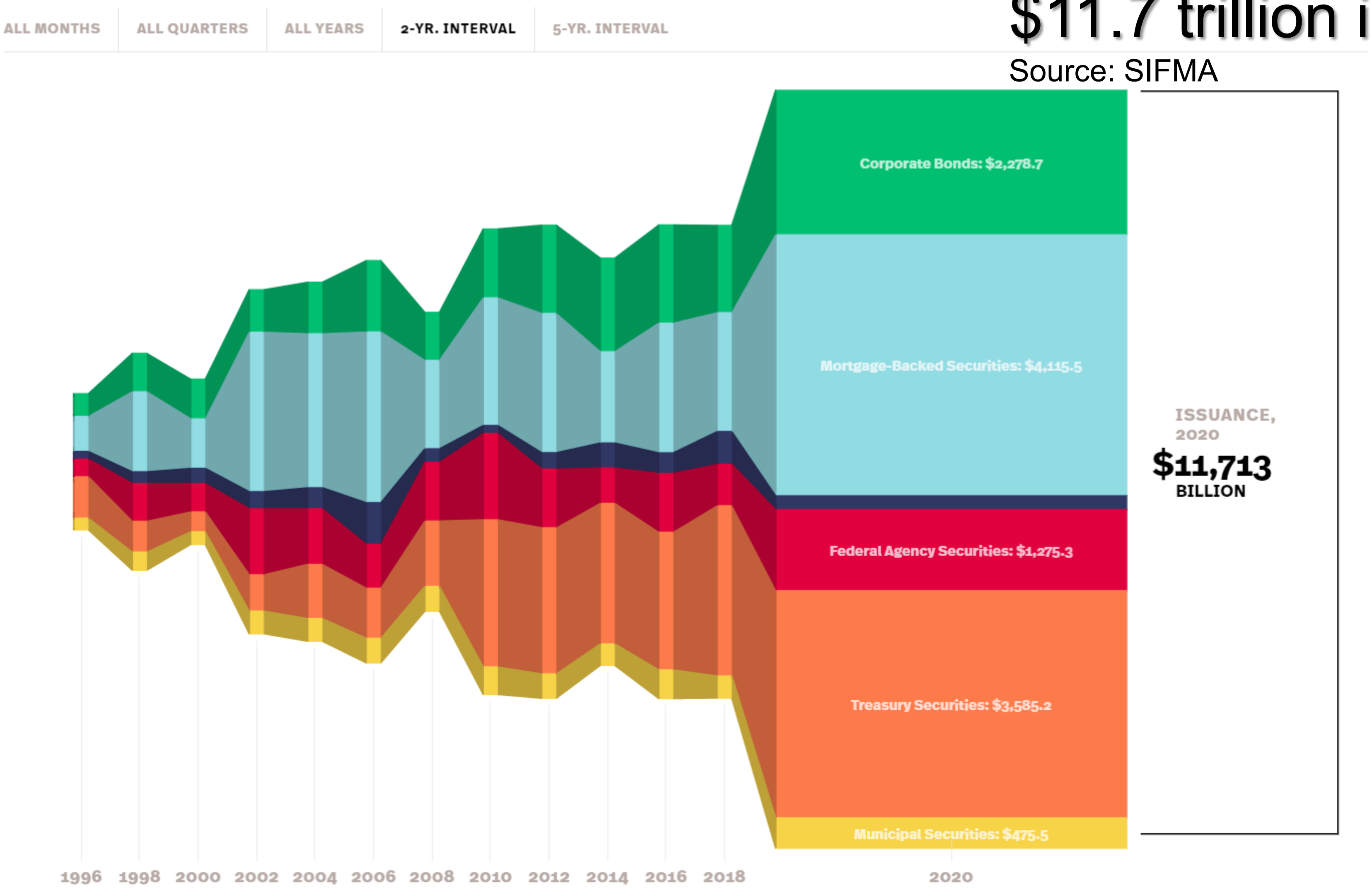
2019

Fixed Income Issuance

(United States)

$11.7 trillion in 2020!

Source: SIFMA



1996

2020

Other market size stats

- Global debt $215 trillion
  - 325% of world GDP; a third added in the last decade
- Global equity markets $73 trillion
- Global derivatives markets estimated at more than $1 quadrillion(!) in notional outstanding

Cash flow basics

- Fixed income securities promise a series of pre-specified payments at pre-determined points in time.
- Promised cash flows on a bond are determined by:
  - Price, “p”
  - Face value, “F” (also called **par** value)
  - Coupon rate, “c”; coupon payment = cF
  - Payment frequency
  - Amortization schedule
- Realized cash flows are uncertain if a security has attached options, default risk, etc.
  - *We will abstract from those uncertainties for now*

Our prototypical bond for most examples in this lecture:

- **Coupon Bond**
  - coupon rate = fraction "c" of face value paid annually
  - “C” denotes annual dollar coupon
- **Payment Frequency** is semiannual with **Payment Amount** of c/2 x face
- **Face value** is paid as "balloon payment" at maturity (non-amortizing)
- **Price** quoted as a portion of face value (often per $1,000 face)
- **No default or prepayment risk**

Example 1-1: Finding cash flows

- What are the cash flows associated with buying a 6.5% coupon bond with a face value of $1,000 with semi-annual coupon payments, a maturity of 2 years, and a price quoted at $98 per $100 of face value?

- Answer:

-980	32.5	32.5	32.5	1032.5	
0	**.5**	**1**	**1.5**	**2**	**(time in yrs)**

Yield to Maturity (YTM) *or* Internal Rate of Return (IRR)

- A bond's yield "*y*" answers the question: *"What is the constant rate of return that equates the bond price with the present value of promised future payments?"*

$$p = \frac{C_1}{(1+y)} + \frac{C_2}{(1+y)^2} + \ldots + \frac{C_n}{(1+y)^n}$$

or

$$p = \sum_{i=1}^{n} \frac{Ci}{(1+y)^i}$$

- *p = price*
- C_i *= promised cash flow at end of period i*
- *n = number of periods*
- *y = yield **per period***

- Given the values of p, C_i, and n, you can solve for the yield, y, using a financial calculator or spreadsheet program.

- **Example 1-2:** A financial instrument offers the following annual payments:

Year	Promised annual payments
1	\$2,000
2	\$2,000
3	\$2,500
4	\$4,000

Suppose it is selling for \$7,702.

What is the annual yield?

It turns out to be 12% because:

$\$7,702 = 2000/1.12 + 2000/(1.12)^2 + 2500/(1.12)^3 + 4000/(1.12)^4$ (or calculate in Excel)

- **Example 1-3:** (*turning the question around*)

Same promised cash flows as above. Investors demand a 12% annual yield.
What is the bond's price?

Clearly the price is \$7,702, the value of the cash flows discounted at 12% per year. (Or calculate in Excel)

Quotation conventions

- Bond yields are quoted in many different ways. To translate a quoted yield into promised cash flows and ultimately a price, **you have to know which convention you are working with!**

- To meaningfully compare the yields on alternative investments quoted differently, **you need to convert all of the yields to the same convention or "basis."**

- You do not need to memorize the myriad bond market quotation conventions for this class. But **you will need to learn to work with several common conventions**.

Bond Equivalent Yield

- **Bond equivalent yield (b.e.)** is the most common quotation convention for coupon bond yields.
  - E.g., U.S. Treasury bonds and notes
  - It can also be described as an "annual percentage rate (APR) quoted on a semiannual basis."

- **Formal Definition of Bond Equivalent Yield:** The b.e. yield is the single interest rate, "*y*" that equates the price of a bond, "*p*", to the present value of future cash flows, assuming a return of y/2 every six months, and semiannual compounding. It solves*:

$$p = \frac{C_1}{(1+y/2)} + \frac{C_2}{(1+y/2)^2} + \ldots + \frac{C_n}{(1+y/2)^n}$$

where:

n = the number of six month periods until maturity

C_i = the cash flow arriving in the i^{th} six month period

**We assume here that the next payment is in exactly 6 months. We'll look at the more complicated formula for the general case a little later.*

Effective Annual Yields

- The **"effective annual yield"** is the return earned over the year, stated as a **simple interest rate** (i.e., no intermediate opportunities for compounding).
- In general, an **effective yield** over a period is the **simple interest rate** earned over that period, taking into account all compounding opportunities.
- Example: If "Y" is the quoted b.e. yield, investors earn Y/2 every six months. The corresponding effective annual yield is:

$$(1 + Y/2)^2 - 1$$

- **Comprehension Check Question**:

A security is quoted with a bond equivalent yield of 10.5%.

What is the effective annual yield?

a. 10.5% b. $(1+\frac{.105}{2})^2 - 1 = 10.776\%$

What is the effective 6- month yield?

a. 10.5% b. $\sqrt{1.105} - 1 = 5.119\%$ c. 10.5%/2 = 5.25%

Examples of other timing conventions

- **The examples here contrast yields on a bond equivalent basis with yields quoted as an APR on a quarterly or monthly basis.**

- A bond quoted on a **quarterly basis** (i.e., 4 compounding periods per year), but with **semiannual payments**, would be discounted using the formula:

$$p = \frac{C_1}{\left(1 + y_q / 4\right)^2} + \frac{C_2}{\left(1 + y_q / 4\right)^4} + \ldots + \frac{C_n}{\left(1 + y_q / 4\right)^{2n}}$$

  - "n" denotes the total number of 6-month intervals, and cash flows arrive at 6-month intervals
  - Here the APR is Y_q; the <u>effective annual yield</u> is $(1+Y_q/4)^4 - 1$
  - The effective quarterly yield is $Y_q/4$

- A bond quoted on a **monthly basis** (i.e., 12 compounding periods per year), with **semiannual payments**, would be discounted using the formula:

$$p = \frac{C_1}{\left(1 + y_m / 12\right)^6} + \frac{C_2}{\left(1 + y_m / 12\right)^{12}} + \ldots + \frac{C_n}{\left(1 + y_m / 12\right)^{6n}}$$

  - Here the APR is Y_m; the <u>effective annual yield</u> is $(1+Y_m/12)^{12} - 1$.

Example 1-4: Yield Calculations

- 20 year bond, 9% coupon, semiannual payments, P=$1,346.72. **What is the yield on a bond equivalent basis?**

  (in general we'll assume F=$1000 unless otherwise stated)

$$1346.72 = \frac{45}{(1+y/2)} + \frac{45}{(1+y/2)^2} + ... + \frac{1045}{(1+y/2)^{40}}$$

y = 6% (use a financial calculator or calculate in Excel)

- **What is the effective annual yield?**

  effective annual yield = $(1.03)^2 - 1 = 6.09\%$

- **What is the yield, *r*, on a continuous basis?**

  $(1/1.0609) = e^{-r} \Rightarrow r = 5.912\%$

- ***In general notice that:***
  - *When the price is greater than the par value, then the coupon rate is greater than the b.e. yield.*
  - *When the price is less than the par value, then the coupon rate is less than the b.e. yield.*

Example 1-5: Yield when there is no coupon

- Zero coupon (or "pure discount") bond. Matures in 2 years.
  F = $1,000. p = $850. y = yield on b.e. basis
- **What is the yield on a bond equivalent basis?**

$$850 = \frac{1000}{(1+\frac{y}{2})^4}$$

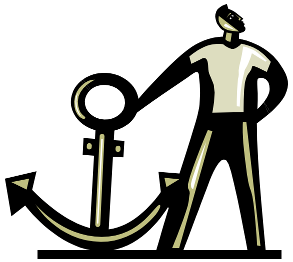
y/2 = 4.145% and y = 8.29% (or calculate in Excel)

- **What is the effective annual yield?**

  effective annual yield = $(1.04145)^2 - 1 = 8.46\%$

- **What is the yield on a monthly basis?**

$(1+y_{mon}/12)^{12} - 1 = 8.46\%$, which implies $y_{mon} = 8.149\%$



- *Notice that the price, or equivalently the **effective annual yield, serves as an anchor** in all these calculations.*

Computing yields and prices when the settlement date falls between coupon payments

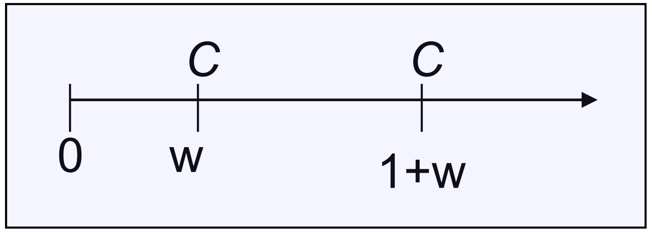
- **What happens when a bond is sold between coupon periods?**
  - A formula is used to calculate how the upcoming coupon payment is to be split between the buyer and seller
  - In general, compensation for the seller's share of the next coupon payment is included in the purchase price paid
  - A variety of formulas exist; different types of securities use slightly different formulas

- More precisely, to compute the transaction price we need to know:
  - Days to next coupon payment. (There are a variety of **day count conventions**)
  - How to compute present value of cash flows received over fractional periods
  - How much the seller is paid for interest earned since the last coupon payment.

The "dirty" price is the present value of the remaining cash flows. It is the full price the buyer actually pays, including accrued interest:

$$tp = \frac{C}{(1+\frac{y}{2})^{w}} + \frac{C}{(1+\frac{y}{2})^{1+w}} + \ldots + \frac{C+F}{(1+\frac{y}{2})^{n-1+w}}$$

You should think of this equation as defining the b.e. yield from market transaction prices



tp = Dirty price (transaction price)

C = semiannual coupon payment

y = the b.e. yield

w = (# days between settlement and next coupon) ÷ (# days in coupon period)

n = number of remaining coupon payments

F = face value

Computing yields and prices when the settlement date falls between coupon payments

- **The quoted price is the “clean” or “flat” price.**
  - It can be thought of as the price “as if” the next coupon were six months away. *(This is how it is often described; but it’s not literally correct.)*

- **<u>Definition</u>: clean price = dirty price - accrued interest**
- accrued interest = (1-w)C
  - w = (# days between settlement and next coupon) ÷ (# days in coupon period)

- **In Excel:**
  - "**PRICE**" returns the clean price, given the quoted yield and other information.
  - "**YIELD**" returns the yield, given the clean price and other information.
  - "**ACCRINT**" returns the accrued interest, given dates and other information.

Day Count Conventions for Accrued Interest (AI)*

Actual/365	AI = C x days/365
Actual/360	AI = C x days/360
Actual/actual	AI = C x days/actual days in the year
30/360	All months are assumed to have 30 days (e.g. there are 30 days between Feb. 9 and Mar. 9). If the first date is on the 31st change it to the 30th. If the 2nd date falls on the 31st and the first date is on the 30th or 31st, change the 2nd date to the 30th.
30E/360	Like 30/360 except that if the 2nd date is on the 31st it is always changed to the 30th.

*from *Bond Market Securities* by Moorad Choudhry, Prentice Hall

Example 1-6: Calculating Accrued Interest

Buy 8 1/2% Treasury note quoted at clean price P=$1010 (per $1000 face), actual/actual

Maturity Date: July 1, 2018

Settlement Date: May 1, 2016

- ***How much accrued interest must you pay per $1000 face value?***

	Date 1/1	5/1	7/1	2017 1/1	7/1	2018 1/1	7/1
Cash flows:	42.5		**42.5**	42.5	42.5	42.5	1042.5

1/1/16 - 5/1/16 = 121 days

1/1/16 - 7/1/16 = 182 days

Accrued interest = (121/182)$42.5 = $28.26 in interest.

(Exercise: Check this in EXCEL! **A useful trick:** Always use the last coupon date before the transaction, which is 1/1/2016, as the issue date. Then use 7/1/2016 as the first interest date.)

- **Total payment is $1038.26, the dirty price.**
- **Yield is 7.98%,** also calculated in EXCEL.

Yield to First Call, and Yield to Worst

- Callable bonds give the issuer the right (but not the obligation) to repurchase the bond at a fixed price at a specified future date or dates.
- The repurchase price is the “call price.”
- The call option will be exercised when it is to the advantage of the issuer: when the present value of the remaining payments exceeds the call price.
- **Definition:** The **yield to first call** (on a b.e. basis) is the rate that solves:

$$p = \frac{C}{(1+y/2)} + \frac{C}{(1+y/2)^2} + \ldots + \frac{C+Q}{(1+y/2)^n}$$

  - C = semiannual coupon payment
  - Q = first call date strike price
  - n = number of six month periods until the bond is first callable

- There may be a declining schedule of call prices over time.
- **Definition**: **Yield to worst is the lowest yield based on all call prices and possible call dates.**

Examples, Yield to First Call

- Example 1-7: 20 year bond, 10% coupon. Callable anytime after 5 years at par (for $1000 per $1000 face value).
  - If the yield in 5 years is less than 10%, the bond will be worth more than $1000 and it is likely to be called.
  - That is because a new bond can be issued to replace it at a lower interest rate.

- Example 1-8: Say p = 105, C = 6, F = 100. The bond is callable in 5 years at par, makes semi-annual coupon payments and matures in 20 years.

  YTM (b.e.y.) = 5.58% (calculate in Excel)

  Yield to first call (b.e.y.) = 4.86% (calculate in Excel)

- *When the yield to first call is lower than the yield to maturity, it offers a more conservative (and probably more accurate) estimate of returns.*

Prices vs. yields – discussion questions

- We have seen that:
  - Given a yield, (the quotation convention), and the promised cash flows, you can always find the price.
  - Given the price and the promised cash flows, you can always find the yield (for any quotation convention).

1. When and for what are prices more useful than yields?

2. When and for what are yields more useful than prices?

3. What is required for yield comparisons to be meaningful?

4. When is the quoted yield a sure thing?

Topic I.2

Horizon Returns

Thinking About Risk in the Fixed Income Marketplace

- Market commentators point to a long list of risk factors that affect prices:
  - * **Market *or* price risk**
  - * **Reinvestment risk**
    - Related to both are:
      - Yield curve or maturity risk
      - Inflation or purchasing power risk
      - Exchange rate or currency risk
  - **Credit or default risk**
    - Political or legal risk
    - Event risk
    - Sector risk
  - **Marketability or liquidity risk**
  - **Timing or call risk**
  - **Volatility risk**

"Interest rate risk"

- We begin by focusing on various aspects of **interest rate risk**
  - "Price Risk" arises from uncertainty about bond prices at the time of sale
  - "Reinvestment Risk" arises from the uncertain reinvestment rate on coupon income
  - Both price and reinvestment risk are primarily due to changes in market interest rates over time (pure rate of time preference + inflation expectations).

Risk and Horizon

- **"Risk" refers to the uncertainty associated with the return on an investment.**
- **In each example below:**

  (1) Is it riskier for you to invest in a 15-year government coupon bond, or roll over one-year government bonds?

  (2) Are you exposed to price risk, reinvestment risk or both?

  - You manage a pension fund, with the bulk of liabilities coming due in 15 years.
  - You are a portfolio manager, and mark to market every day. Large negative shocks to portfolio value result in a loss of investors.
  - You are the CFO of a large company. You just completed a 10-year bond issue to finance an expansion. The funds will be needed in 6 months, and you need to invest them in the interim.
- **What general conclusions can you draw from this?**

Because interest rates change and holding periods vary, ***quoted*** yields rarely correspond to ***realized*** yields!

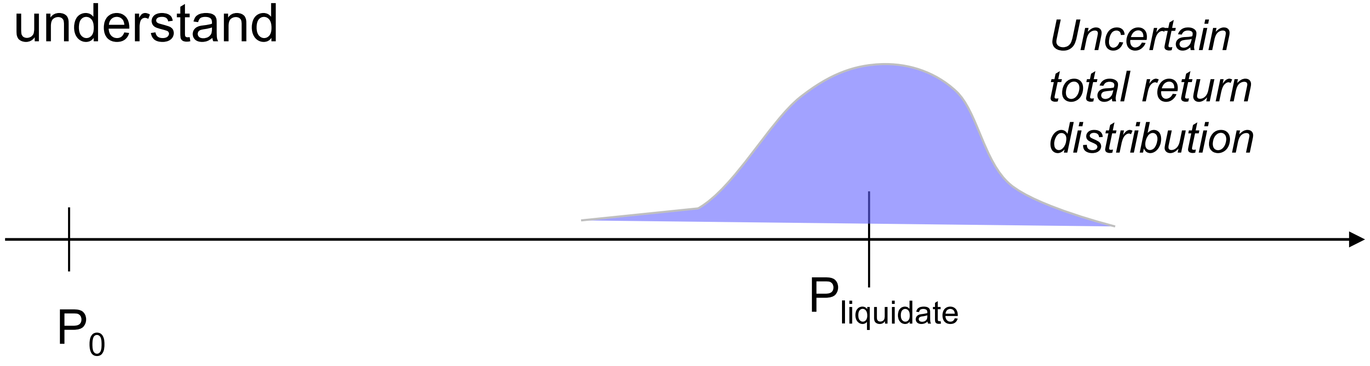
- *Three main components affect **realized yield** over the investment horizon (on default-free, option-free bonds):*
  1. Periodic interest payments (coupon payments)
  2. Income from reinvestment of periodic interest payments
  3. Any capital gains or losses when the bond is sold or matures

- 2. is the source of **reinvestment risk**
- 3. is the source of **price risk**

- The yield-to-maturity abstracts from these risks by assuming:
  - the bond is held to maturity
  - the coupon interest payments will be reinvested at the yield to maturity

Total Return Analysis

- Also called “horizon return” or “realized compound yield.”
- A calculation of **future value**, experimenting with different interest rate changes.
- A popular method that avoids some of the shortcomings of traditional yield calculations and that helps to identify sources of risk.
- Low tech, easy to understand



Decomposing Dollar Returns

First **find future value of coupon interest** using future value of an annuity formula:

$$C\left[\frac{(1+r)^n - 1}{r}\right]$$

C = semiannual coupon payment
n = # of six month periods to maturity or sale
r = effective reinvestment rate on a six month basis

This accounts for coupon interest plus interest on interest from reinvesting.

Part of this, the total coupon interest, is nC.

Thus **interest on interest** is equal to:

$$C\left[\frac{(1+r)^n - 1}{r}\right] - nC$$

Finally, **compute the capital gain,** which equals:

$$P_n - P_0$$

P_0 = purchase price
P_n = sales price or face value

Example 1-9 Decomposing Returns -- Par Bond Held to Maturity

Invest $1,000 in a 7-year bond with 9% semiannual coupon, selling at par.

We know that:

yield = 9% (bond equiv. basis)

earns 4.5% semiannually

If this yield were a sure thing, at maturity the accumulated payments would be:

$$\$1,000(1.045)^{14} = \$1,852$$

Thus the total dollar return at maturity would be $852 ($1,852 - $1,000).

Let's decompose this:

1. Coupon interest plus interest on interest =

$$\$45\left[\frac{(1.045)^{14}-1}{.045}\right] = \$852$$

2. Interest on interest = \$852 - 14(\$45) = \$222

3. Capital gain = \$0 *(why?)*

So interest on interest is \$222/\$852 = 26% of the total return. This is the portion exposed to reinvestment risk.

Example 1-10 Decomposing Returns (Discount Bond)

Invest in a 20 year bond with a 7% semiannual coupon, selling at $816 (per $1,000 face) to yield 9%.

We know that:

yield = 9% (bond equiv. basis)

4.5% semiannually

If this yield were a sure thing, at maturity the accumulated payments would be:

$$\$816(1.045)^{40} = \$4,746$$

Thus the total dollar return at maturity would be $4,746-816 = $3,930.

Let's decompose this:

1. Coupon interest plus interest on interest =

$$\$35\left[\frac{(1.045)^{40}-1}{.045}\right] = \$3,746$$

2. Interest on interest = \$3,746 - 40(\$35) = \$2,346

3. Capital gain = \$1,000-\$816 = \$184

In sum:

Total coupon interest =	\$1,400
Interest on Interest =	\$2,346
Capital gain =	\$ 184
Total =	\$3,930

Example 1-11 Sensitivity Analysis on Total Returns

Again invest in a 20 year bond with a 7% semiannual coupon, selling at $816 (per $1,000 face) to yield 9%.

But now assume that the reinvestment rate will only be 6% (i.e., 3% semiannually).

What is the effect on total returns?

1. Coupon interest plus interest on interest =

$$\$35\left[\frac{(1.03)^{40}-1}{.03}\right] = \$2,639$$

2. Interest on interest = $2,639 - 40($35) = $1,239

3. Capital gain = $1,000-$816 = $184

In sum:

Total coupon interest =	\$1,400
Interest on Interest =	\$1,239
Capital gain =	\$ 184
Total =	\$2,823

Expected yield solves: $\$816 = \dfrac{\$2{,}823 + \$816}{(1+\frac{y}{2})^{40}}$

y = 7.6%, significantly less than the 9% yield to maturity!

Practice Problem 1.1:

Redo Example 1-11: assume that you plan to sell the bond at the end of four years (at which time its yield will have fallen to 6%). Explain why the effect on realized yield is so different.

Hint: Be sure to account for the new sales price as the present value of remaining payments.

Take-away Points on Horizon Returns

- Estimated total returns are sensitive to both the assumed reinvestment rate and the assumed holding period.
- Calculating total returns under a variety of interest rate assumptions provides a simple, concrete illustration of performance sensitivity to rate changes.
- “Duration” (next week) will provide a useful tool for explaining these effects.

Topic I.3

Floating Rate Bonds

Valuing a Floating Rate Security

- **Definition**: A floating rate security has a variable interest rate that is linked to some market interest rate or other external indicator.
  - E.g., a three-year note with an interest rate equal to 1-year LIBOR + 1%, with a rate reset annually. The 1% is the "credit spread."
- **Fact: Floating rate bonds are priced at par on reset dates.**
  - *This assumes a zero credit spread and no change in credit risk over time. A fixed spread has to be valued separately using yield curve.*
- **Proof**: Let r_i be the one-period reset rate realized at time i. The bond matures in N periods.
- We find the price at time 0 by working backwards.
- At time N-1 there is one remaining payment of principal and interest, equal to $F(1 + r_{N-1})$. Its value at time N-1, P_{N-1}, is $F(1 + r_{N-1})/ (1 + r_{N-1}) = F$.
- Stepping back to time N-2, $P_{N-2} = (F(r_{N-2}) + P_{N-1})/ (1 + r_{N-2}) =$

  $$F(1 + r_{N-2})/ (1 + r_{N-2}) = F.$$
- Continuing in this way, it is clear that the price equals the face value on all reset dates, including at time 0.

Topic I.4

Intro to the Yield Curve and Discount Functions

The Yield Curve

- **<u>Definition</u>**: The **yield curve** gives the yield (or rate of return) on fixed income securities as a function of their time to maturity.

- It is also known as the "**term structure of interest rates**."

- The "**spot**" or "**zero**" yield curve is the most useful for pricing because it provides a **discount rate specific to each future date**. However, the phrase "term structure" is often used more loosely (e.g., based on YTM).

The Yield Curve

- There are different yield curves for different types of securities (gov’t, AA, subprime, muni, ...)
- When people refer to "***The*** yield curve", they mean the yield curve for government securities, which in the U.S. is constructed using Treasury bill and Treasury bond price data. (link to Bloomberg)
- As a basis for pricing, we will focus on the **spot yield curve** for zero coupon (also called “pure discount”) bonds.
  - Yield curves for risky securities generally lie above the Treasury curve, with the yield spread representing compensation for risk and expected losses.
- **Definition**: The **spot yield curve** is based on the YTMs of pure discount bonds (or their synthetic equivalent).

An Essential Distinction: YTM vs. Spot Yields

Say that the 3-year spot yield curve implied by bond market prices is:
Y(0,1) = 11%, Y(0,2) = 12%, Y(0,3) = 12.14%

The price of a 3-year, 10% coupon bond is $95 per $100 face value:

$$95 = 10/1.11 + 10/(1.12)^2 + 110/(1.1214)^3$$

The **yield to maturity** of this bond is 12.08%, which solves

$$95 = \frac{10}{1+YTM} + \frac{10}{(1+YTM)^2} + \frac{110}{(1+YTM)^3}$$

The yield to maturity (YTM) gives some indication of the return over the life of this investment. However, it is not useful for pricing other similar bonds
(e.g., A 3-year bond with a 30% coupon rate would have a different YTM. Would it be higher or lower? Why intuitively?).

The Discount Function

- **Definition**: The **discount function** assigns a price at time t to a certain \$1 cash flow arriving at a future time, T. It is denoted by Z(t,T).
  - The time interval (T-t) is generally in units of years (e.g., 3 months is .25, 10 days is 10/365 = .0274).

- Features
  - Conveys the same information as the spot yield curve
  - Because it is a continuous function, it is convenient to work with even when cash flows do not arrive at even intervals
  - Provides a unifying notation applicable across all comparable securities, thereby abstracting from quotation conventions.

Discount Function (example)

- Assume the spot yield curve derived from zero coupon bonds is given by:

Time To Maturity (in years)	Price (P) (per $100)	Yield (b.e.b.)
1	$96	4.12%
2	$90	5.34%
3	$85	5.49%
4	$80	5.57%

- Example 1-11: What is Z(0,3)? What is the price today of $200,000 in 3 years?

$$Z(0,3) = .85. \quad P = .85(\$200{,}000) = \$170{,}000$$