15.438
Fixed Income
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Answers to HW#3

1. a .The portfolio is found by finding face value amounts X and Y to match the target portfolio payouts at all future dates. To match the interest payments:

X(.055) + Y(LIBOR) = (.11 – LIBOR)$1000

And to match the face value at maturity:

X + Y = $1000.

Solving the two equations in two unknowns yields X = $2000 and Y=-$1000.The price paid for the coupon bond is (1.01)*$2000 = $2020. The amount sold of the floating rate bond is $1000.

b. The dollar duration of the portfolio = $2020Dm(fixed) - $1000Dm(floating)

The 5.5% bond has an effective annual yield of 5.301%. (Calculated in Excel using RATE(6,5.5,-101,100).) The yield on a bond equivalent basis is 5.233%. (Calculated to use as input into Convexity.xls to get duration.)

Dm(fixed) = 5.009
Dm(floating) = 1/1.04 = .9615

Dollar duration of portfolio is: $2020(5.009) - $1000(.9615) = $9156.7. For every one percentage point change in interest rates the value of the portfolio changes by approximately $91.56.

The effective duration of the inverse floater is the same as for the portfolio that replicates its cash flows. The effective duration is (dP/dr)/P, which in this case can be found as the dollar duration divided by the price of the inverse floater. Its price is $2020-$1000 = $1020. The effective duration is $9156/$1020 = 8.977 years.

c. Since inverse floaters have high duration, financial institutions that face long-term liabilities (e.g. pension funds) might be interested in buying these instruments without having to use large amount of cash today.

Investors who are concerned about their cash flow reinvestment might also want to buy inverse floaters for hedging purpose.

2. The par coupon reported below is based on the formula:

$$c = \frac{1 - \frac{1}{(1+Y_N)^N}}{\frac{1}{(1+Y_1)^1} + \frac{1}{(1+Y_2)^2} + \ldots + \frac{1}{(1+Y_N)^N}}$$

where Y_i is the effective 6 month rate (bond equivalent rate divided by 2), and N is the number of six month periods until maturity. This comes from generalizing and rearranging the equation describing the definition of a par yield curve given in the problem.

Par Yield Curve	Maturity
0.008819	2/15/2004
0.01192	8/15/2004
0.015715	2/15/2005
0.018652	8/15/2005
0.021606	2/15/2006
0.024492	8/15/2006
0.027561	2/15/2007
0.029645	8/15/2007
0.031964	2/15/2008
0.034067	8/15/2008

3a. Simultaneously buy 1000 dollars of the two year bond and sell 1000 dollars of the three year bond. The face value of the two year bond at time 2 is (100/84.663)*$1000=$1181.154. The face value of the 3 year bond at time 3 is (100/77.238)*$1000= $1294.7 The forward borrowing rate on an effective annual basis is: ($1294.7-$1181.154)/$1181.154 = 9.613%. Convert this to a bond equivalent yield: $(1.09613^{.5}-1)2 = 9.393\%$. Notice that the calculations assume that the investor buys at the ask and sells at the bid. Also note that you could have computed a bid and ask yield curve, and used them to find the implied forward rates according to the formula $(1+Y_3)^3/(1+Y_2)^2 = 1+f(0,1,2)$.

b. Simultaneously buy X dollars of the three year bond and sell X dollars of the two year bond. You could also base the calculation on buying and selling $1000 of each as above, but to demonstrate an alternative approach here I solve it using the transaction price of the three year bond as the basis for the calculation: you buy and sell 77.268. Then the time 2 cash outflow is 100(77.268/84.633)=91.2977. The effective annual forward lending rate is (100-91.2977)/91.2977 = 9.5318%. Convert this to a bond equivalent yield: $(1.095318^{.5}-1)2 = 9.3149\%$.

c. If the implied borrowing and lending rates were reversed, it would represent an arbitrage opportunity because the implied borrowing rate would be lower than the implied lending rate.

4. a.
16 year bond: 99(0.939) – 97.140 = -4.169
20 year bond: 99(1) – 110.677 = -11.68
24 year bond: 99(1.152)-112.631 = 1.417

so the 24 year bond is most likely to be delivered.

b. YTM of the 24 year bond can be found by RATE(48,3.6,-112.631,100)*2 = 6.183%. Use the duration calculator to find that Dm(F) = 12.055. The dollar amount in the futures contract is given by 300M*10/12.055 = $248.86M.