MIT Sloan / Fidelity 2021 Finance Lab

Research on Portfolio Insurance Strategies

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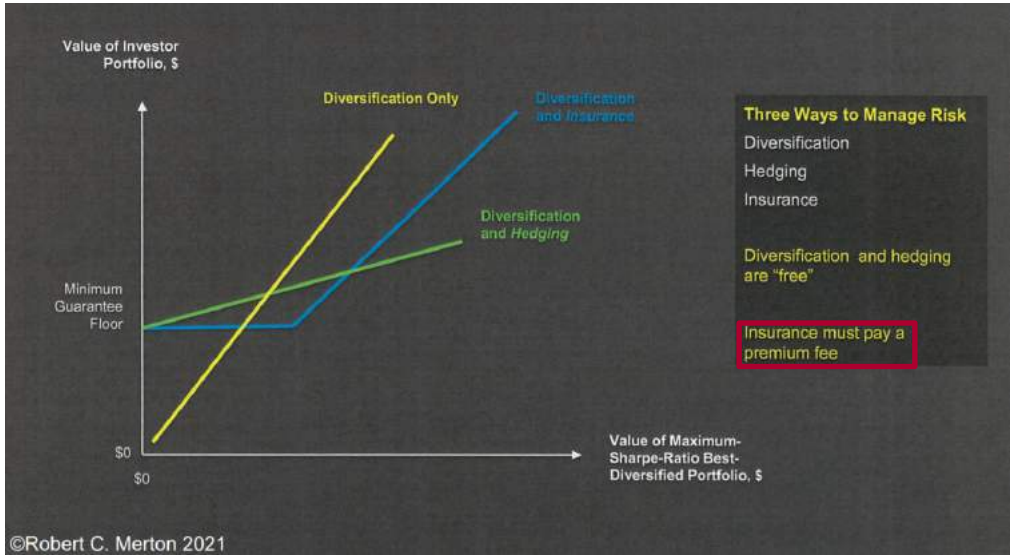
Content

MIT MANAGEMENT SLOAN SCHOOL

Portfolio Insurance: Overview

Portfolio Insurance
Background



Portfolio Insurance:

Minimum guarantee level + upside potential

Why Researching Portfolio Insurance?

1. Unpredicted market downturns: Covid-19
2. Aging population, early retirement: Fidelity customers without income sources
3. Limited choice, high cost, and inefficiency

Portfolio Insurance

Our Methodology & Research Journey

One Methodology: Exploration

New topic for both the student team and sponsor team: start from scratch and explore obstacles and improvements

Two Resourceful Advisors: Sponsors and Faculties

We want to thank Brad, Drew, Andrey, Michael, Prof. Rao, Prof. Vartak, and Pierre for the great advice and enjoyable meetings!

Three Major Parts:

Option Based Portfolio Insurance (**OBPI**):
- Theory & Implement Problems
- Main idea of Adjusted Strategy
- Put-based & Call-based; Hist vol & Implied vol
- Intuitive, conventional, but complex

Constant Proportion Portfolio Insurance (**CPPI**):
- Foundation idea & different types
- Exploration and investment strategies
- Easy to **implement, interpret,** and **individualize**

Monte-Carlo simulation:

- Return Simulation: Higher volatility ⇔ higher strategy return
- Entering Time Simulation: Put-based OBPI's domination
- OBPI vs. CPPI

OBPI

Option-based Portfolio Insurance Strategy

What is OBPI

Option-Based Portfolio Insurance Strategy

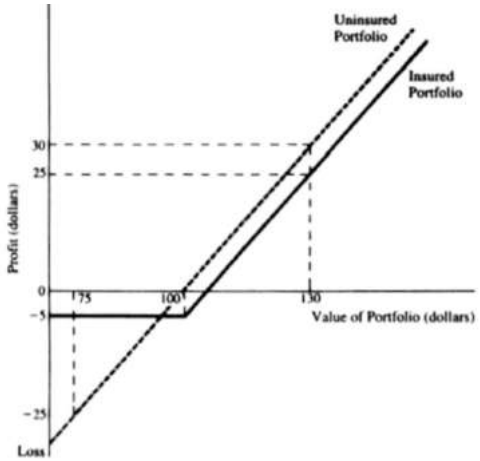
hedge

- Put-based → Buy N shares stock and N shares of European Put options
- Call-based → Invest in bills and purchase N European Call options

Floor Ratio: p
Principal: V_0
Horizon: T

→ Multipler: N
Strike Price: K

Guarantee a floor of $p * V_0$ during the investment horizon [0,T]



What is OBPI
How to implement

- **Put-based OBPI:**
  - *N shares stock and N Put options (maturity T/strike price K)*
  - *Solve strike K and multipler N:*

$$N * \left(S_t + Put(0, S_t, K)\right) = V_t$$
$$N * K = p * V_t$$

where V_t is portfolio value at time t, p is the floor ratio.

  - *The insured amount is $V_T = n(S_T + \max(K - S_T, 0))$.*

- **Call-based OBPI:**
  - *B_0 Bills and N European call options(maturity T/strike price K).*
  - *Solve strike K and multipler N:*

$$N * Call(0, S_t, K) + B_t = V_t$$
$$N * K = p * V_t$$

  - *The insured amount is $V_T = B_0 * e^{rT}$*

$$\frac{C(0,S_0,K,sigma)}{K} = \frac{1-pe^{-rT}}{p}$$
$$N = \frac{V_0}{S_0+P(0,S_0,K(p),sigma)}$$

- **Two kinds of volatility input:**

Hist Vol: *SMA based on stock returns*

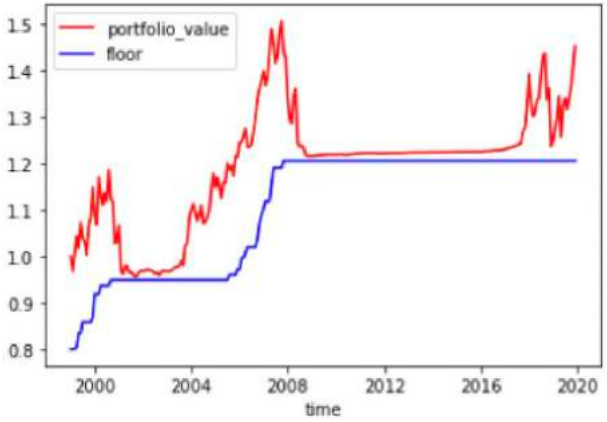
Implied Vol: *Implied by traded index options (strike price K nearest to the equivalent strike price of synthetic put/option)*

- **Replicate option using stock and bond.**

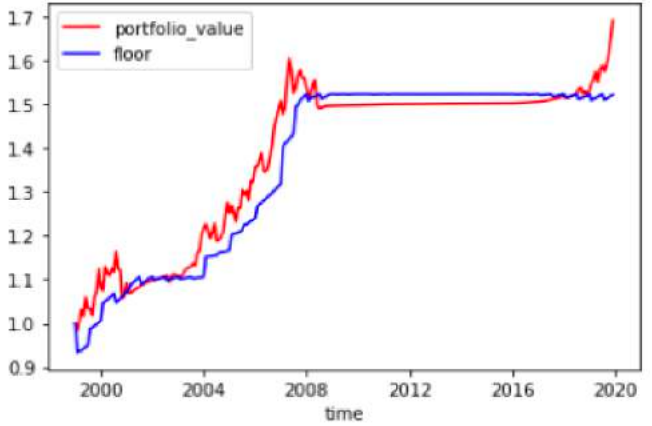
Insight of OBPI

Basic Strategy

- T = 1 month and p = 0.8
- Keep floor as p * peak value of V_t in the past



- T=1 month and p = 0.95
- Once the portfolio value goes below the floor, the calculated strike price will be infinite
- it is too slow to recover

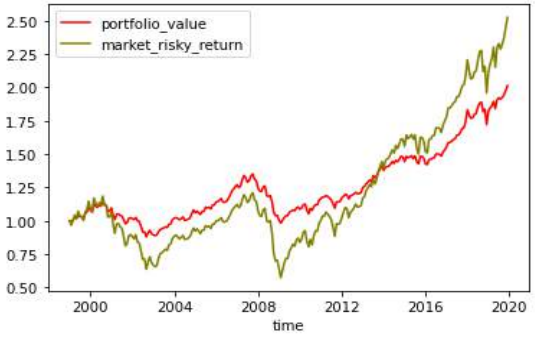


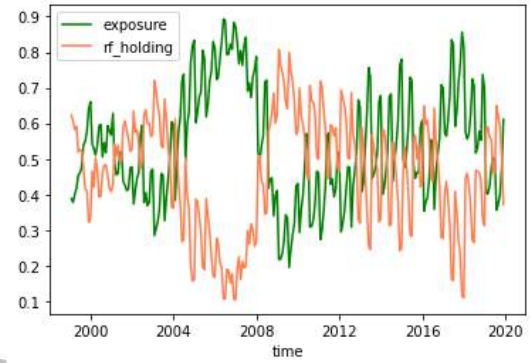
Strategy
Main Idea

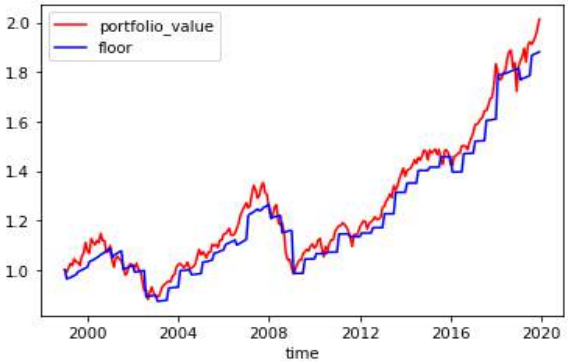
Recover p $*$ V_t in each T
【Keep portfolio value drop by less than (1-p) in each T】

- **T=6 months as sub-investment horizon.**
- **Reinvest current V_t at the end of T and update the floor to be $p * V_t$**
- **Rebalance monthly. Replicate the put/call option using stock and bond.**
- **Implement strategy using Put-based OBPI/Call-based OBPI & hist/implied**

Strategy
Insight







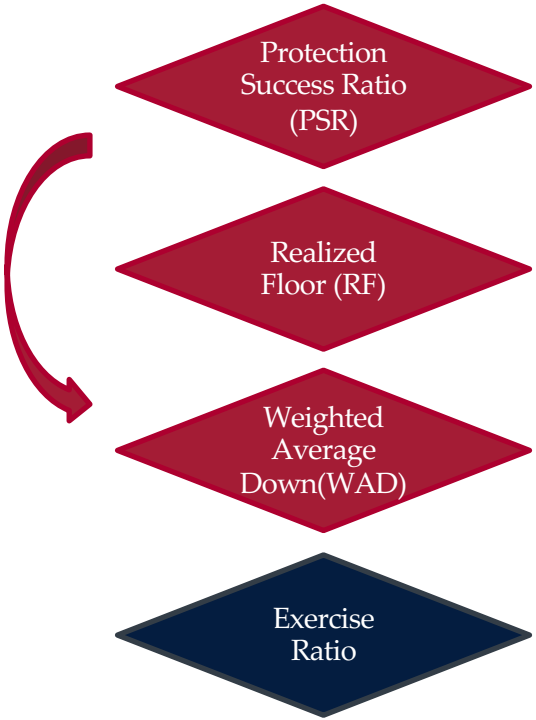
- S&P 500 Index
- Implement: Put-based
- Volatility Input: Hist Vol
- Floor_ratio P: 0.98
- T = 6 months

- When the market goes down, the portfolio value is protected. When the market goes up, the portfolio goes up slowly but still maintains potential.

- In new strategy, the portfolio value recovers faster when market recovers.

Strategy

How to Measure Performance



$$\frac{\sum_{i=1}^{M} 1_{\{if\ successfully\ protect\ at\ the\ end\ of\ T\}}}{total\ Number\ of\ T}$$

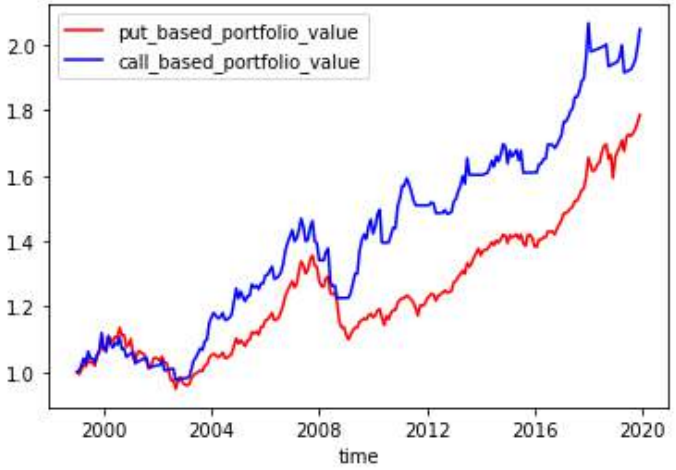
The minimum of portfolio value during the whole investment horizon.

$$\frac{\sum_{i=1}^{M} precentage\ of\ value\ below\ floor}{total\ Number\ of\ T}$$

$$\frac{\sum_{i=1}^{M} 1_{\{if\ option\ exercises\}}}{total\ number\ of\ T}$$

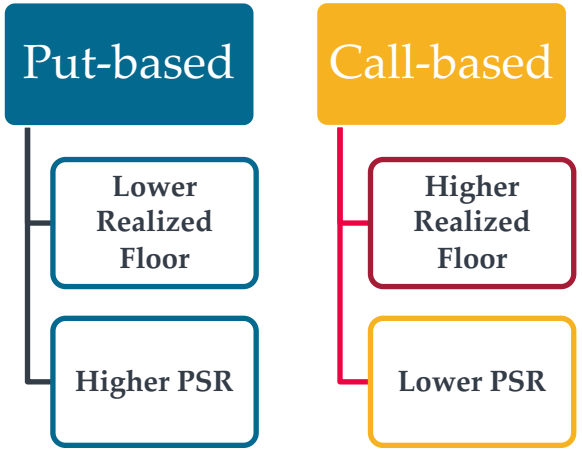
Put-based v.s. Call-based

S&P 500 Hist Data



- S&P 500 Index
- Implement: Put-based v.s. Call-based
- Volatility Input: Hist Vol
- Floor_ratio P: 0.99
- T = 6 months

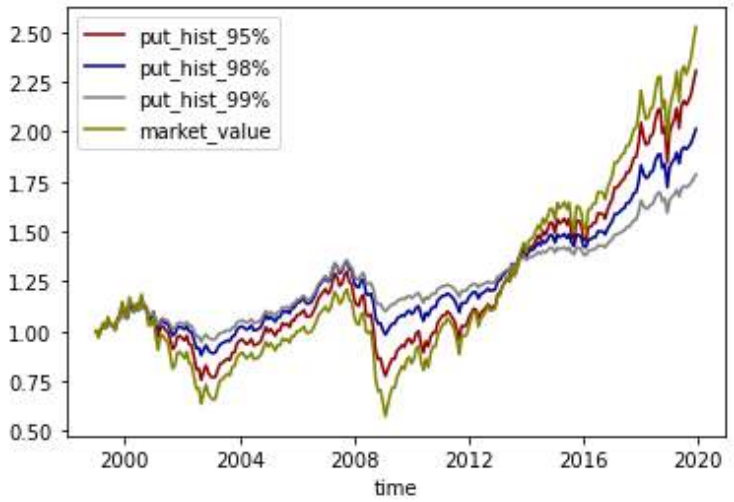
	Protection Success Ratio	Realized Floor	WAD
Put-based	86%	94.78%	-0.018
Call-based	72%	97.6%	-0.02

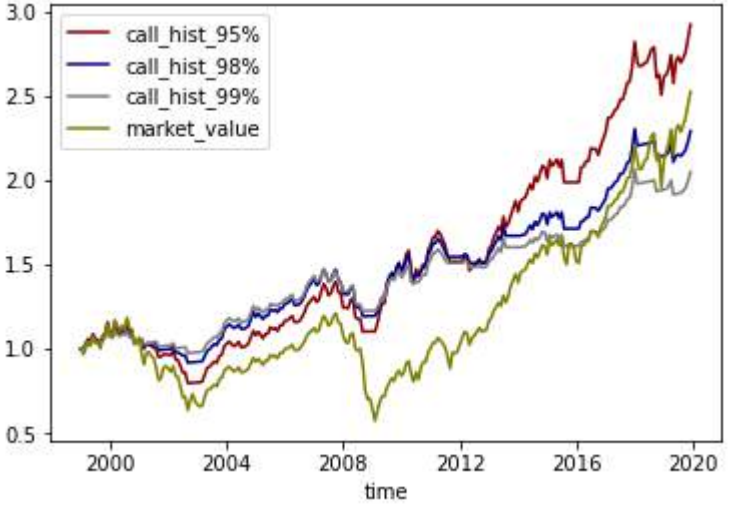


Different Floor

Insight

- Choose floor ratio p as **99%, 98% and 95%,** which means the return during 6-month period will not be less than 1%, 2% and 5%.
- 1999/01-2019/12, choose **M=42** 6-month periods to run the strategy.





Measure Indicator
Protection Success Ratio

Put-based Protection Success Ratio

	99%	98%	95%
Hist vol	83.33%	83.33%	85.71%
Implied vol	78.57%	83.33%	92.86%

Call-based Protection Success Ratio

	99%	98%	95%
Hist vol	64.28%	66.67%	83.33%
Implied vol	66.67%	64.28%	83.33%

- Put-based ——higher protection success ratio, especially when p is **large** (e.g. 99%)
- No obvious difference between implied vol and hist vol.
- But when p is small, PSR (put implied) is **larger** than PSR(put hist)

Measure Indicator
Realized Floor

Put-based Realized Floor

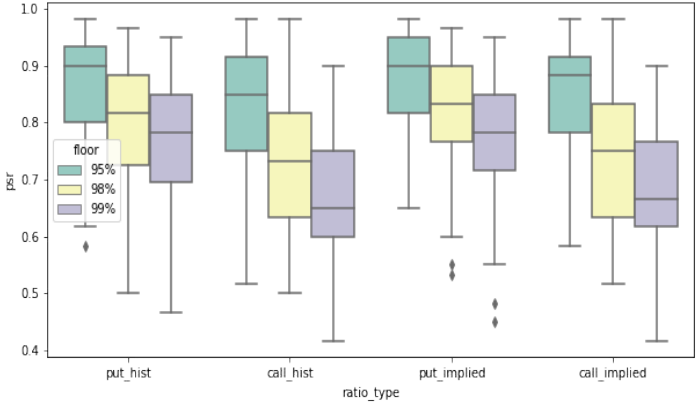
	99%	98%	95%
Hist vol	94.78%	87.95%	75.67%
Implied vol	93.77%	88.36%	82.71%

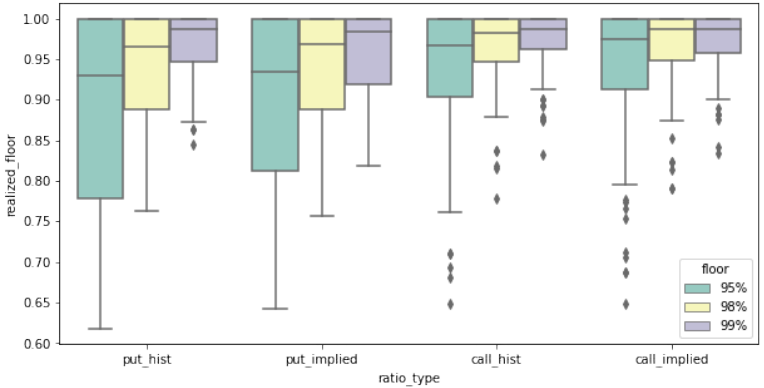
Call-based Realized Floor

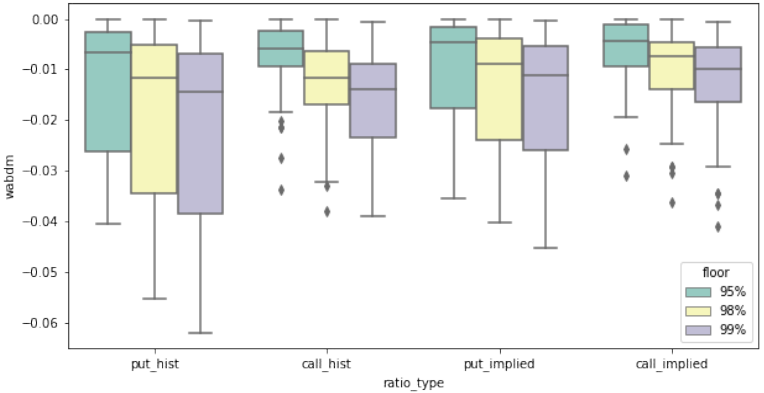
	99%	98%	95%
Hist vol	97.6%	91.79%	79.75%
Implied vol	96.24%	88.96%	82.46%

- Call-based ——higher realized floor, especially when p is **large** (e.g. 99%)
- No obvious difference between implied vol and hist vol.
- But when p is small, RF (implied) is **larger** than RF(hist).

Back-testing Results of ratio
Explanation



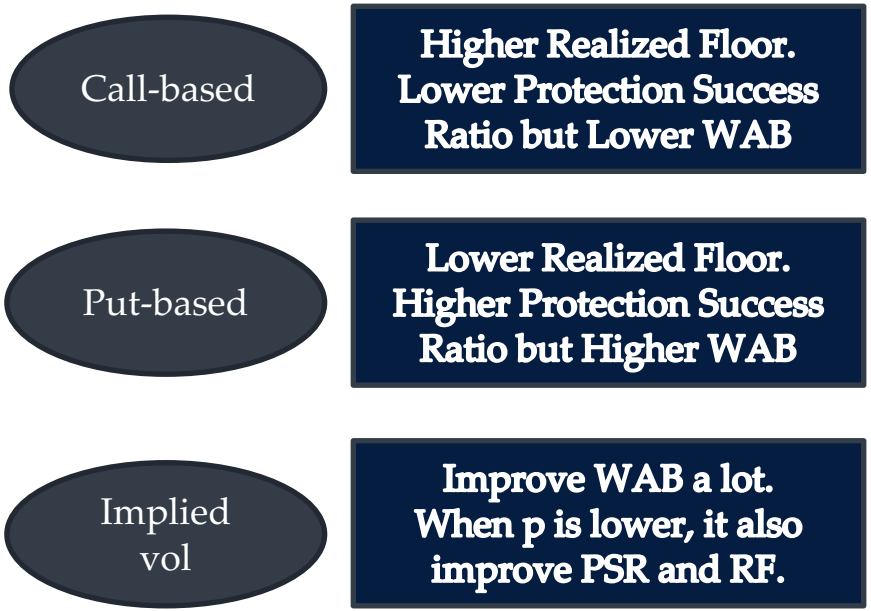




- Results in **PSR** and **RF** are the **same** with previous slides.
- Call-based——**lower** WAD, better than Put-based.
- **Implied vol improves WAD** a lot in both put_based and call_based situation

Take-away of OBPI
Main Idea

- Call-based performs better if you want to **achieve a higher return**.
- Put-based performs better if you are really **risky-averse**.
- However, If you care more about the **magnitude of down** rather than if we protect sucessfully, choose call-based.
- Implied vol improves PSR and RF only when p is lower, maybe because we have protected it very well when p is higher.



CPPI

Constant Proportion Portfolio Insurance Strategy

CPPI
Introduction

CPPI (Constant Proportion Portfolio Insurance)
- Introduced by Black and Jones (1987)
- Dynamically allocate between two assets

I. Abstract allocation:
- **Floor**: lowest acceptable portfolio value
- Cushion: excess value over floor

II. Investment Decision:
Invest **Multiple** * **Cushion** in **risky** asset
Invest the rest in risk-free asset

Example:
Value = 1.1, Floor = 0.8, multiple = 2 ->
Cushion = Value – Floor = 0.3;
*Risky investment = Cushion * multiple = 0.6*
Risk-free investment = Total Value – Risky = 0.5

Interpretation:
If the risky asset falls less by 50% this month:
"survive" above the floor!

CPPI
Introduction

Parameters: **Floor and Multiple**

Family of CPPI:

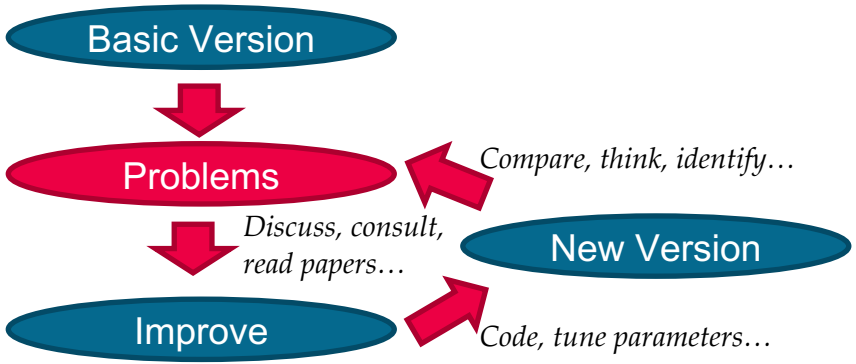
- Floor:

Predetermined value; Grow with risk-free rate; Proportion of highest historical value; two-stage floor;…

- Multiple:

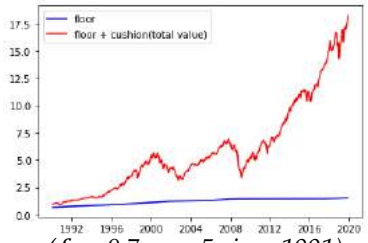
Constant, rule-of-thumb; VaR backed; Dynamic, implied vol/historical vol…

Our approach:

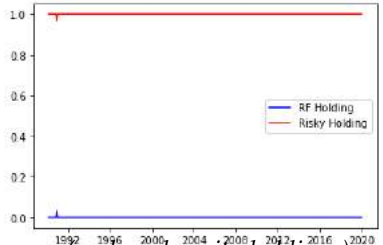


CPPI
Basic CPPI

Basic CPPI:
Floor: pre-determined initial value and grows with risk-free rate
Multiple: pre-determined constant

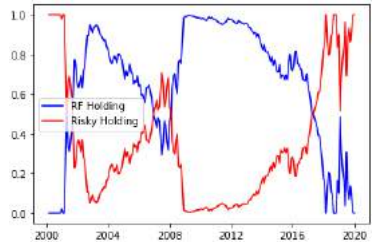


($f_0 = 0.7$, $m = 5$ since 1991)



(cash and equity holdings)

($f_0 = 0.7$, $m = 5$ since 2000)



(cash and equity holdings)

Floor depends extremely on <u>when the strategy started</u>
Start with bull: too **low** floor and too **aggressive** after few years, **<u>"securitized"</u>**
Start with bear: too **high** floor and too **conservative** after few years, **<u>"monetized"</u>**

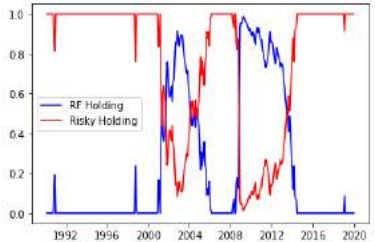
Thoughts:
Floor growing with the risk-free rate is too brutal force

Can the floor "consider" the market dynamism or portfolio performance?

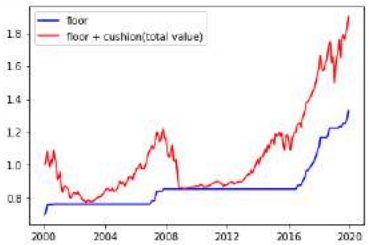
CPPI
Dynamic Floor CPPI

Dynamic Floor CPPI (*TIPP, Brennan and Schwartz, 1988*):
Floor: a <u>proportion</u> of the historical peak portfolio value
Multiple: pre-determined constant

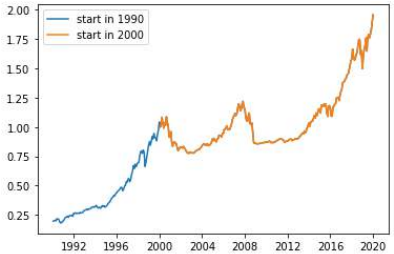
($f_0 = 0.7$, $m = 5$ since 1991)



(*cash and equity holdings*)



($f_0 = 0.7$, $m = 5$ since 2000)



(*Set 01/2000 portfolio value = 1*)

Improvements:
1. The floor does not depend on entering time
2. No substantial drawdown (60% in basic CPPI)

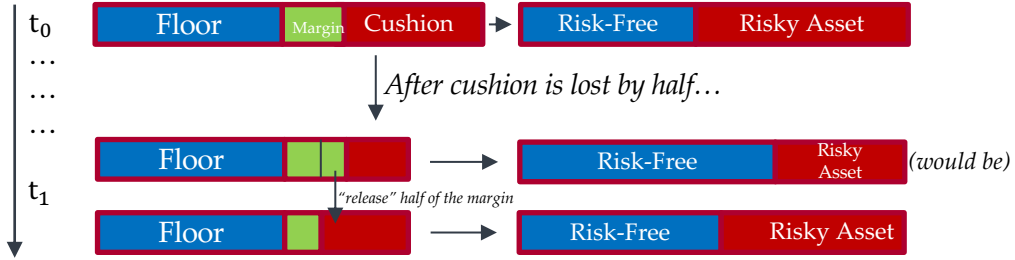
Thoughts:
1. Drawdown from peak -> floor can be too high as a proportion (70%) of the peak.

2. CPPI strategy implies selling low and buy high

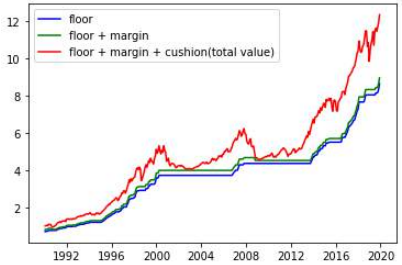
CPPI

Dynamic Double Floor CPPI

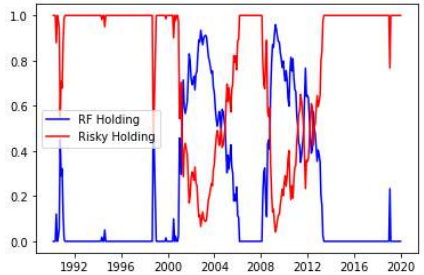
Dynamic Double Floor CPPI (*margin CPPI, Boulier & Kanniganti, 2005*):
Floor: two stage, an extra "margin" that can be released
Multiple: pre-determined constant



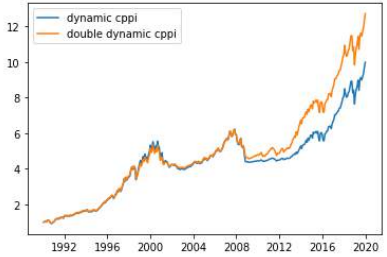
Released half of the margin to cushion, and thus increase risky asset holding

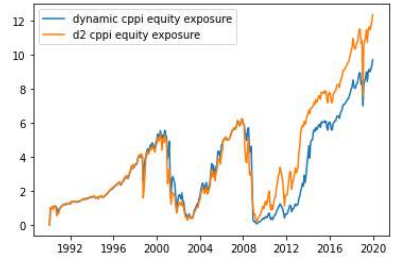


($f_0 = 0.7$, margin = 0.1,$m = 5$)



(cash and equity holdings)





Compare with Dynamic CPPI: performance and exposures

Improvements:
Extra holdings when you lose cushion by half:
Prevents strategy being monetized + buy "some" when cheap

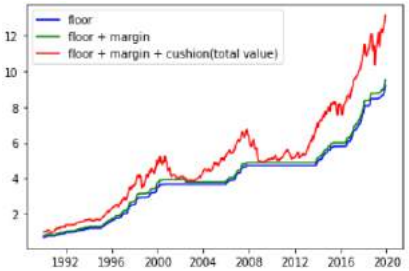
Thoughts:
Now it is the time to improve the constant multiple!

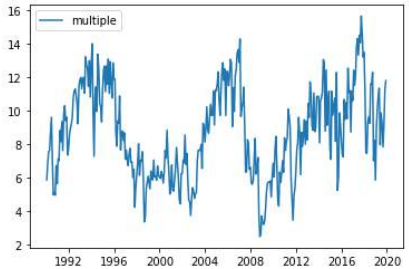
CPPI
VaR (Implied Vol) CPPI

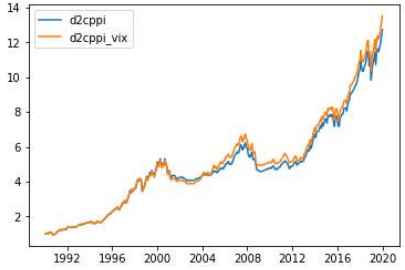
Value-at-Risk CPPI (*Hamidi, Maillet, & Prigent, 2008*)
Floor: Double Dynamic (Two Stage)
Multiple: Dynamic, such that VaR = Cushion

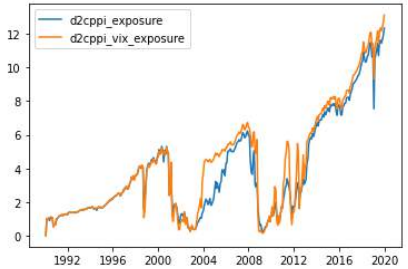
Example: next month worst 1% case of S&P return believed to be -18% - set multiple = 1/0.18 = 5.56

Using VIX as S&P 500 30-day volatility predictor -> assume normal, pick VaR level -> multiple









(Compare with constant-multiple CPPI: performance and exposures)

Improvements:

1. **Low-vol** market – **high** multiple & equity exposure
   High-vol market – **low** multiple & equity exposure

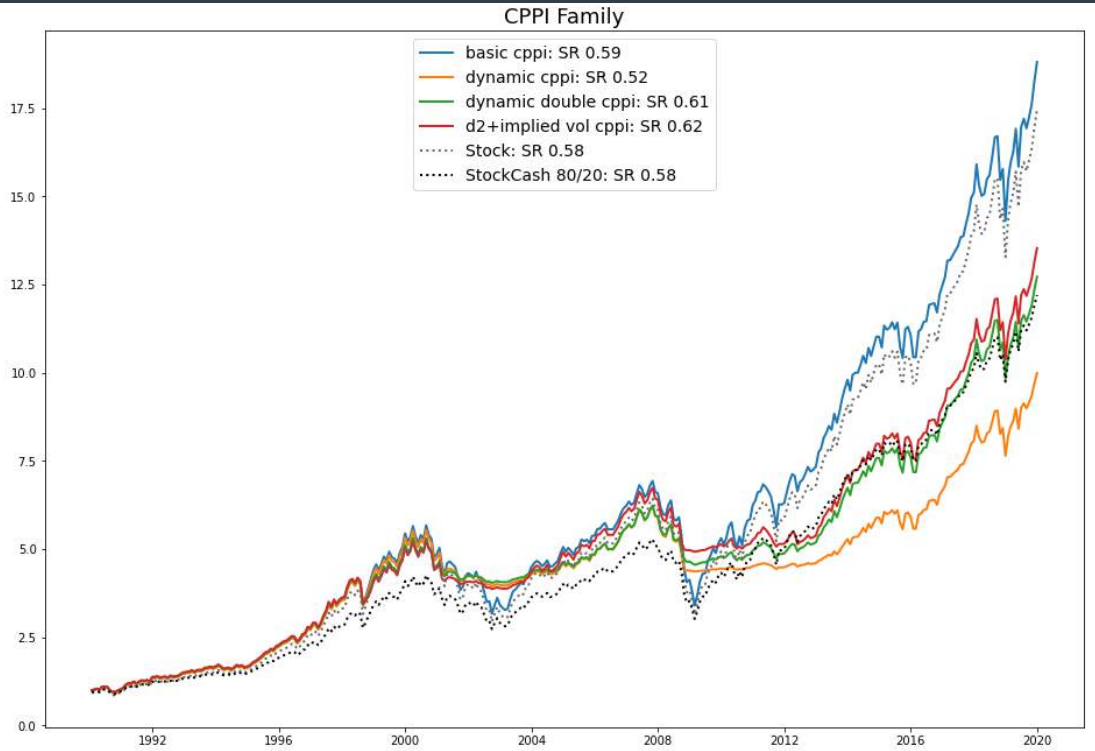
2. Better risk management interpretation

CPPI
Comparison

Implied Volatility CPPI: different VaR levels

Takeaways of CPPI:

A powerful portfolio insurance strategy family that is easy to **implement, interpret,** and **individualize**



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Exploration – commodity CPPI

Motivations, Objectives, and Warning

Inspired by the CPPI-type dynamic allocation rule, but replacing risk-free cash with commodities

Motivations

1. Some clients prefer stable equity exposures & do not want cash in their portfolio
2. Commodities are believed to be **natural hedge** against equities

Objectives

1. Limit minimum equity exposure (80%) – invest **20%** in strategy and **80%** in equity
2. Choose the right commodity: correlation, post-crisis performance...

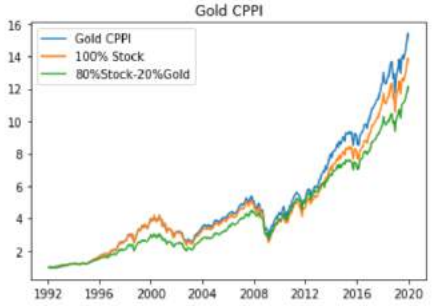
Warning:

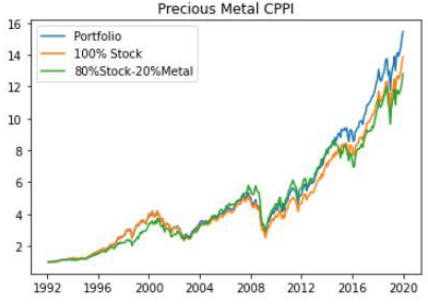
Minimum equity + risky commodities: no longer "insurance"

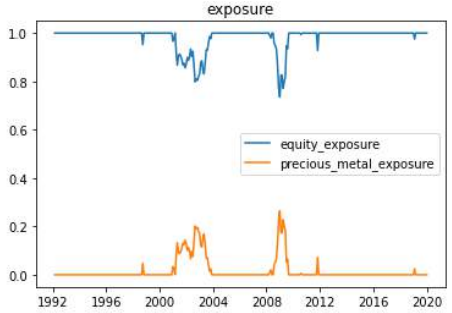
Exploration – commodity CPPI

Some Performances

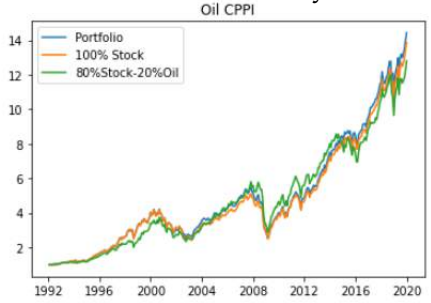
Commodities that work…

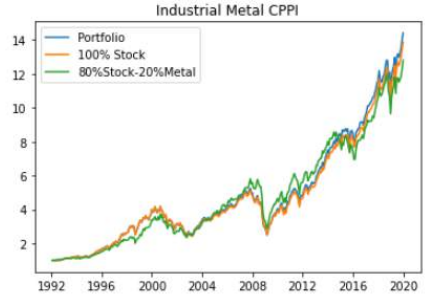


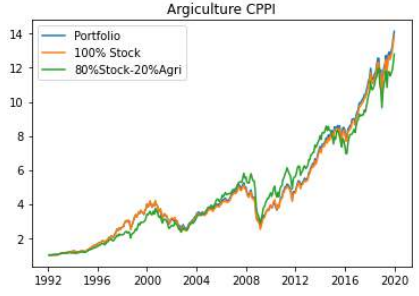




Commodities that barely work…



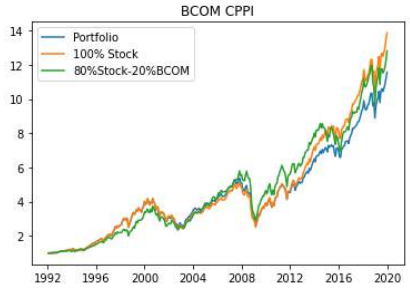


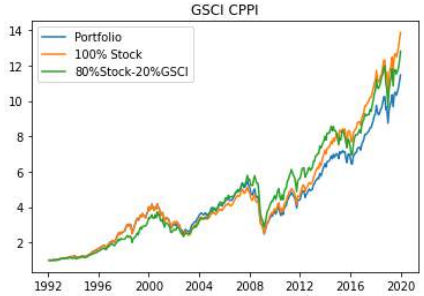


Exploration – commodity CPPI

Some Performances

Commodities that do not work…





BCOM, GSCI: commodity indices (Bloomberg Commodity Index, Goldman Sachs Commodity Index)

Takeaways of commodity CPPI Strategies:

1. Reasonable & functional dynamic allocation rule that maintains high equity exposure and enhances equity performance
2. Not all commodity works: a good filtering method can be analyzing correlation & looking into post-crisis performance

Back Tests and Strategy Comparison

Monte Carlo Simulation

Monte Carlo 1 – Return Simulation

Assumptions and Scenario Settings

Assumptions:

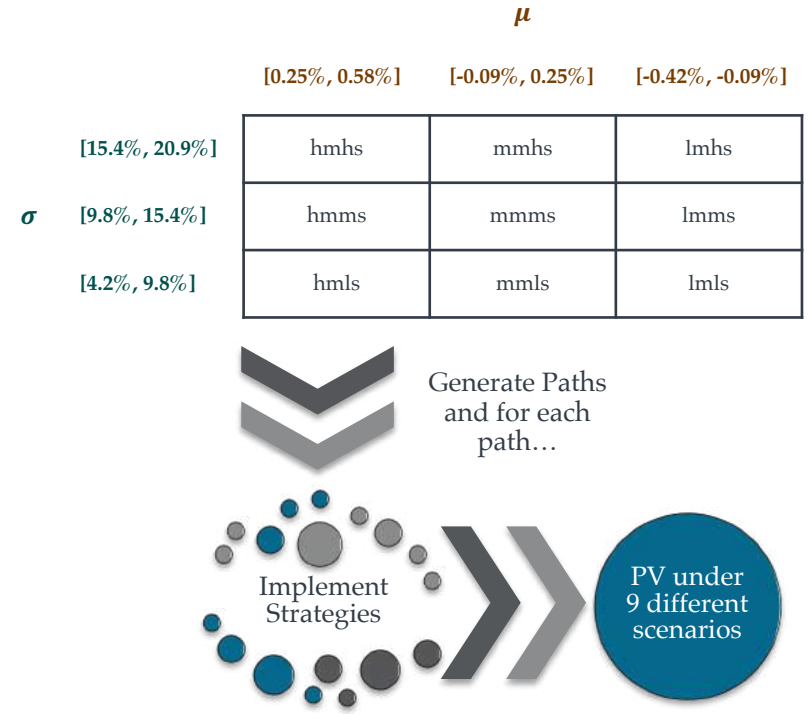
1) Pre-known mean and volatility of returns over investment horizon
2) Returns follow normal distribution

Step 1 From two uniform distribution based on parameters from historical returns, a monthly mean return and a volatility of **risky asset** are generated.

Step 2 Randomly Sample Parameters: Generate 9 scenarios.

Step 3 Implement 4 strategies over T = 360 observations (30-year horizon). Summarize performance and risk.

Step 4 New run repeated from **Step 1** on, for a total of N = 1,000 simulated paths under each scenario.



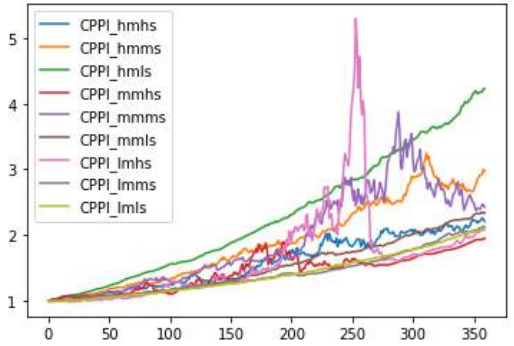
References *Benchmarking, portfolio insurance and technical analysis: a Monte Carlo comparison of dynamic strategies of asset allocation (Cesaria & Cremoninib, 2002)*

Monte Carlo 1 – Return Simulation

Results and Comparison Between Different Scenarios

Note

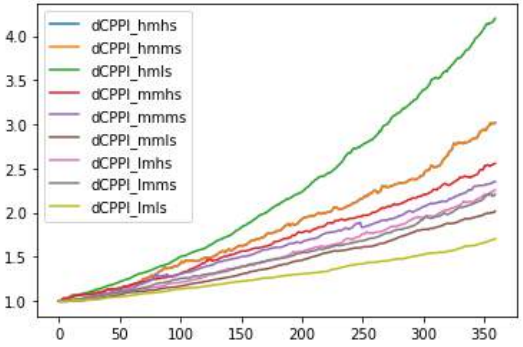
hmhs - high mu high sigma
hmms - high mu medium sigma
hmls - high mu low sigma
mmhs - medium mu high sigma
mmms - medium mu medium sigma
mmls - medium mu low sigma
lmhs - low mu high sigma
lmms - low mu medium sigma
lmls - low mu low sigma

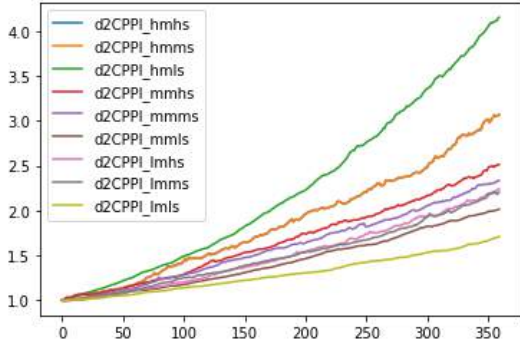


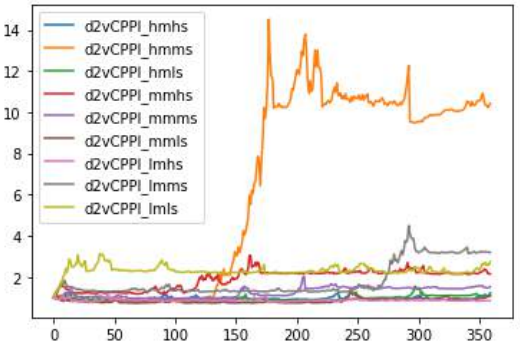
Basic CPPI: high volatility along the paths

Dynamic / Dynamic Double CPPI: higher stability over time, but larger scenario-wise divergence

Time-Variant Multiple CPPI: higher expected end-of-time portfolio value along with higher volatility, highest scenario-wise divergence







Monte Carlo 1 – Return Simulation

Results and Comparison Between Different Scenarios

As the preset floor ratio gets higher, OBPI strategy tends to offer **higher stability over time** as well as **smaller scenario-wise divergence**. Put-based OBPI generally offers higher stability over time than the correspondent call-based OBPI.

Note

p95 – put-based with floor ratio 95%
p98 – put-based with floor ratio 98%
p99 – put-based with floor ratio 99%
c95 – call-based with floor ratio 95%
c98 – call-based with floor ratio 98%
c99 – call-based with floor ratio 99%

hmhs - high mu high sigma
hmms - high mu medium sigma
hmls - high mu low sigma
mmhs - medium mu high sigma
mmms - medium mu medium sigma
mmls - medium mu low sigma
lmhs - low mu high sigma
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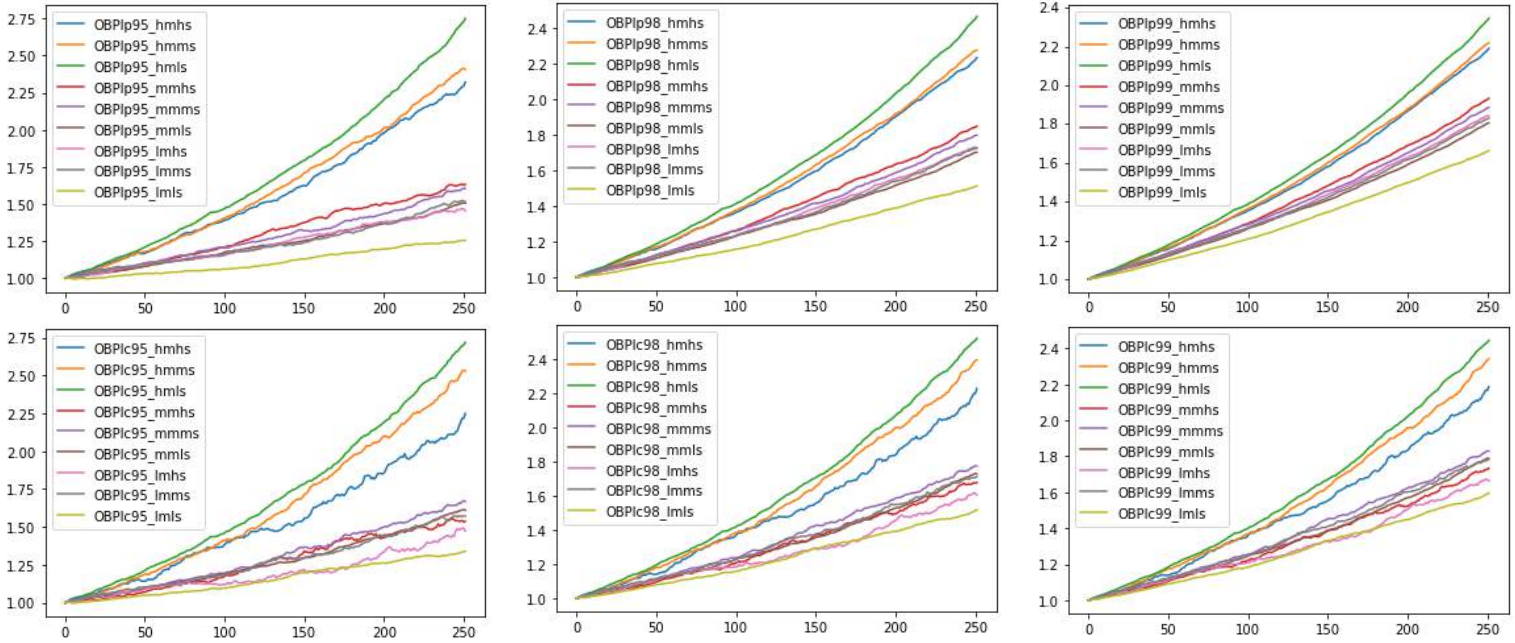
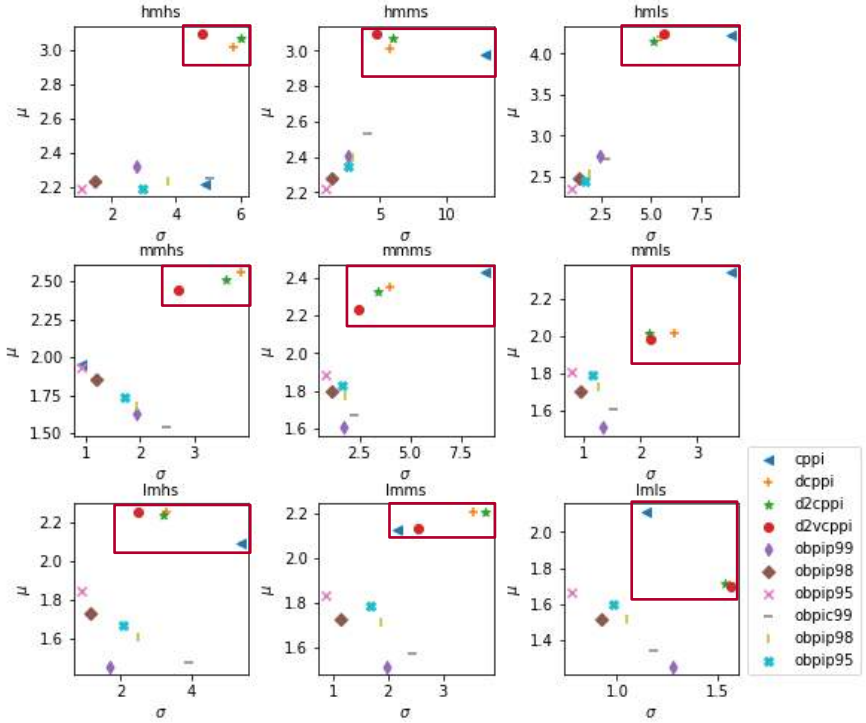


Monte Carlo 1 – Return Simulation

Results and Comparison Between Different Strategies



Features of end-of-time portfolio value

CPPI family:

- Higher end-of-time portfolio values
- Along with higher volatilities over time (especially with highly volatile risky asset returns)

OBPI family:

- Less satisfactory average end-of time portfolio value
- Outperforms CPPI in terms of portfolio value volatility

Monte Carlo 2 – Entering Time Simulation

Results and Comparison Between Different Strategies

Parameters

Number of Simulation = 100 times
Strategies Last = 60 months
Simulation Horizon = Recent 30 years

Strategies

CPPI Family

CPPI # basic
dCPPI # dynamic floor
d2CPPI # dynamic double floor
d2vCPPI # time-varying multiple

OBPI Family

OBPI_Put # put-based historical vol
OBPI_Call # call-based historical vol
OBPI_Put_imp # put-based implied vol
OBPI_Call_imp # call-based implied vol

Monte Carlo 2 – Entering Time Simulation

Rankings, Comparison and Client Preference

Using End of Time Portfolio Value and SR as criteria, we would also like to know how our clients might want to choose from different family of strategies.

Utility Function A	$U = \text{mean}(\text{ Ptval }) - \gamma \text{ std }(\text{ Ptval })$
Utility Function B	$U = \text{mean}(SR) - \gamma\, \text{std}(SR)^2$
Utility Function C	$U = \text{mean}(SR) - \gamma\, \text{std}(SR)$

Best Strategy

Gamma (Risk Averse Param)	-5~-1	-0.8~-0.2	0	0.2	0.4	0.6	0.8	1	1.2	1.4~3.4	3.6	3.8	4	4.2	4.4	4.6	4.8	5
Utility Function A	cppi	d2vcppi	obpi_put				d2cppi	obpi_call		obpi_call(imp)								
Utility Function B	cppi	obpi_put										obpi_put(imp)						
Utility Function C	cppi	obpi_put										obpi_put(imp)						

Monte Carlo 2 – Entering Time Simulation

Adding Commodities

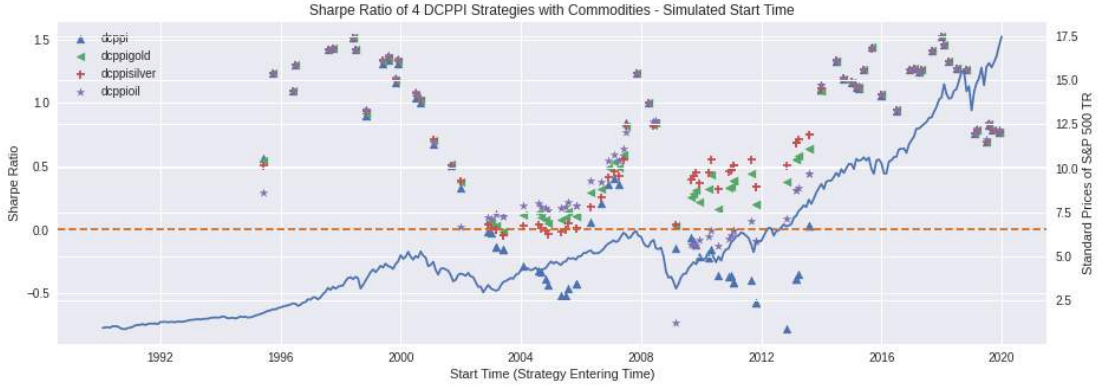
Parameters

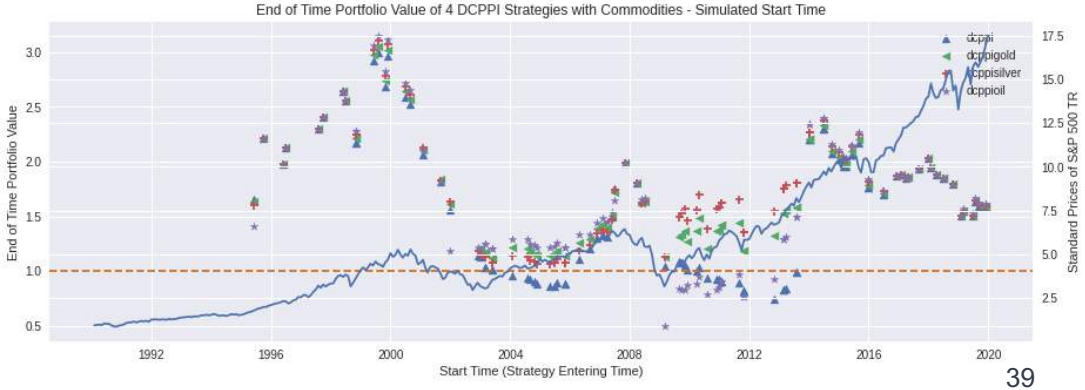
Number of Simulation = 100 times
Strategies Last = 60 months
Simulation Horizon = Recent 30 years

Strategies

CPPI Family

dCPPI
    # dynamic floor
dCPPIgold
    # dynamic floor with gold hedging
dCPPIsilver
    # dynamic floor with silver hedging
dCPPIoil
    # dynamic floor with crude oil hedging





Monte Carlo 2 – Entering Time Simulation

Adding Commodities

Key Takeaways:

	Pros	Cons
CPPI	○ Higher End-of-time Portfolio values as compared to OBPI ○ Flexibility in adding commodities that prevent portfolio from crashing	○ Extensions on basic CPPI **diverge more** across different return scenarios ○ Bring along **higher volatilities**, especially when the underlying risky asset returns have high volatility in themselves
OBPI	○ **Higher stability over time** as well as **smaller scenario-wise divergence** ○ Put-based OBPI covers **the main risk preference** of our clients' according to utility models	○ Less satisfactory average End-of time Portfolio value ○ Less variation in form or flexibility in underlying assets

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THANK YOU

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