15.438 Fixed Income
Prof. Lucas

Problem Set #1

These problems provide practice in calculating and interpreting yields. You will need to do some of the problems using a spreadsheet program or a financial calculator with functions like the ones discussed in class. Be sure to write down (no need to type) a precise representative formula in each case so that the grader can see what you did wrong if the numerical answer is off. Alternatively, you can write down the Excel function and label the inputs, e.g., NPV(rate=10%, CF1=6, CF2=6, CF3=6, CF4=100). Please *do not* hand in a spreadsheet with the answers embedded. The help menu in Excel provides an excellent description of the available financial functions and how to use them.

1. Time Path of a Premium Bond. Consider a 5 year, 14% coupon bond (semiannual payments) selling at a premium of $1,300 per $1,000 face value.

(a) What is the yield to maturity on a bond equivalent basis?

(b) Assume that the yield to maturity remains constant over time. Compute both the clean and dirty price paths of the bond. Specifically, compute the clean and dirty prices at monthly intervals over the life of the bond. Plot both of those price paths over time. Be sure to label the graphs. Notice that the graphs show the characteristic shape of the price path of a premium bond when rates don't change (known as "pull to par").

(c) Is the clean price path or the dirty price path smoother? Briefly explain why there is the difference that you see.

2. Time Path of a Discount Bond. Consider a 6 year, 8% coupon bond (semiannual payments) selling at a discount of $900 per $1,000 face value.

(a) What is the yield to maturity on a bond equivalent basis? What is the yield to maturity on a monthly basis? What is the effective annual rate?

(b) Assume that the yield to maturity on a bond equivalent basis remains constant over time. Compute both the clean and dirty price paths of the bond. Specifically, compute the clean and dirty prices at monthly intervals over the life of the bond. Plot both of those price paths over time. Be sure to label the graphs. This gives the characteristic shape of the price path of a discount bond when rates don't change, and also exhibits "pull to par."

3. Changing basis. A bond is quoted at 3.625% on a monthly basis (i.e., 3.625% is the APR). What is the effective annual rate? What bond equivalent yield results in the same effective annual rate? What APR quoted on a quarterly basis results in the same effective annual rate? What APR quoted on a continuous basis results in the same effective annual rate?

4. **Comparing money market yields.** A 90-day Treasury bill is quoted at a rate of 5.275% on a discount basis. Calculate the rate that would have to be quoted on a 90-

day CD on a simple interest rate basis that would provide the same promised return over the 90 days. To realize the same effective rate over a given maturity, will the quoted discount rate or quote simple interest rate be lower, and why?

5. Total Returns and Scenario Analysis. Consider a 7.5%, 10-year non-callable bond with semiannual payments. Its current yield to maturity is 8%. ***(All yields are given on a bond equivalent basis. You should state yields in your answers on a b.e. basis too.)***

(a) What is the current bond price?

(b) Assume that you plan to hold the bond to maturity, and expect the reinvestment rate on coupon interest to be 6% over the life of the investment. Compute the total expected dollar returns. Decompose those returns into three parts: coupon payments, interest on interest, and capital gains (or losses). What is the expected yield over the holding period? Explain why it is different than the current 8% YTM.

(c) Now assume that you only plan to hold the bond for three years (i.e., you have a three year investment horizon). You anticipate that the yield on a 7-year bond in three years will be 8.5%, and that you will be able to reinvest the coupons received at 7.75% over the three years the bond is held. What is your total return at the end of three years? Decompose these returns into three parts: coupon payments, interest on interest, and capital gains (or losses). *[Hint: the capital gain or loss will depend on the sales price of the bond, which in turn depends on the required yield on the bond at the end of three years.]*

(d) As in part (c), you have a three-year horizon. But now assume you are not so sure about your guess about reinvestment rates and the sales price in three years. To get a better feeling for your exposure, do a "Scenario Analysis." That is, examine total returns under a variety of rate assumptions. Fill out the following Table, doing a calculation similar to part (c) for each cell. *[To create a useful analysis tool, set this up as a spreadsheet in which you can change the maturity, coupon rate, investment horizon, etc. (This, however, is not required)]*

Total Dollar Returns at End of Three Year Horizon as Function of Reinvestment Rate and Yield on Bond Sold:

	Yield at end of 3 years	6%	10%
Reinvestment rate			
6%		?	?
10%		?	?

6. Yield to First Put. In class we discussed the common practice of reporting the yield to first call as well as the YTM, and treating the minimum of the two yields as the better predictor of the realized yield for investors.

A bond may be "puttable," which means investors have the option to sell it back to the issuer for a pre-specified price before maturity. Imagine that the bond in problem (2) above is puttable at par at the end of the fourth year and thereafter.

(a) Calculate the yield to first put and state it on a bond equivalent basis.

(b) Under what market interest rate conditions are investors most likely to exercise a put option on a bond?

(c) Do you think the YTM or the yield to first put is likely to be a better indicator of the realized yield for an investor in this case? Explain your reasoning, and generalize it to comment on whether you think the best rule for evaluating puttable bonds is to focus on (i) YTM or (ii) yield to worst or (iii) yield to best. (Recall yield to worst is the lowest yield across all possible exercise dates. Analogously, yield to best is the highest yield across all possible exercise dates.)