15.457 Recitation 3

Jonathan Jensen

March 26, 2021

Review: GMM I

- Suppose we are interested in estimating parameters θ_{px1}
- We observe data $x_1, ..., x_T$ where x_t is k-dimensional
- We have an N dimensional function $f(x_t, \theta)$
- Moment Condition: $E[f(x_t, \theta)] = 0$
- Exclusion restriction: $N \geq p$
- Define $g_T(\theta) = \frac{1}{T}\sum_{t=1}^{T} f(x_t, \theta)$ Sample mean of moment condition
- Idea: Minimize quadratic form of the sample mean of the moment condition

Review: GMM II

- Idea: Minimize quadratic form of the sample mean of the moment condition
- Recall OLS minimizes quadratic form of $(Y - X\beta)$
- More generally: $\hat{\theta}_{GMM} = arg\ min_{\theta} g_T(\theta)'\hat{W} g_T(\theta)$
- Why W?
- Because N may not equal P, So in order to take a quadratic form of g we need the weighting matrix to normalize dimensions
- Optimal $W = S^{-1}$

Review: GMM Asymptotics

- Asymptotic distribution: $\sqrt{T}(\hat{\theta} - \theta) \sim^{a} N(0, V)$
- $V = (d' S^{-1} d)^{-1}$
- $d = \frac{\partial E[f(x_t, \theta)]}{\partial \theta'}$ Dimension Nxp
- $S = E[f(x_t, \theta) f(x_t, \theta)']$ Dimension NxN
- We can estimate d, S, and V using $\hat{\theta}$ and sample means in place of expectations

Review: The Delta Method

- A very general way to conduct hypothesis testing
- Allows us to test restrictions on smooth functions of our parameters
- let $h(\theta)$ be a j-dimensional function equal to zero (under the null)
- define $A = \frac{\partial h(\theta)}{\partial \theta'}$ dimension j by p
- $h(\hat{\theta}) - h(\theta) \sim^{a} N(0, \hat{A}\hat{V}\hat{A}')$
- Hypothesis Testing. Under the null hypothesis $h(\hat{\theta})'\hat{A}\hat{V}\hat{A}'h(\hat{\theta}) \sim \chi^2(j)$

Example

We observe returns from 2 managers $x_1, ...x_T$ where $x_t = \begin{pmatrix} x_t^1 \\ x_t^2 \end{pmatrix}$

- Can we use GMM/delta-method to compare sharpe ratios?
- $\theta = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \sigma_1^2 \\ \sigma_2^2 \end{pmatrix}$
- $f(x_t, \theta) = \begin{pmatrix} x_t^1 - \mu_1 \\ x_t^2 - \mu_2 \\ (x_t^1 - \mu_1)^2 - \sigma_1^2 \\ (x_t^2 - \mu_2)^2 - \sigma_2^2 \end{pmatrix}$
- $h(\theta) = \frac{\mu_1}{\sigma_1} - \frac{\mu_2}{\sigma_2}$