15.438 Fixed Income
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Problem Set #2

1. Assume the yield curve is flat at 3% (effective annual rates). A pension fund wants to immunize a deferred annuity with the following terms: The annuity pays $20,000 per year for 5 consecutive years, with the first payment made 3 years from today.

The pension fund is considering between two alternatives to immunize the liability:
(i) Invest in a combination of 1-year and 10-year zero-coupon bonds
(ii) Invest in a combination of 4-year and 7-year zero-coupon bonds
(*Work with yields as effective annual rates, and do the calculations on an annual basis*.)

(a) What is the modified duration of the liability? (Optional for extra credit: derive a general formula relating the modified duration of a stream of deferred payments to the duration of the same stream of payments if they start immediately. A hint is to start by writing down the present value formula and differentiating.)

(b) How much of each bond (in dollars) would you buy under alternative (i) to immunize the portfolio? And how much of each bond under alternative (ii)?

(c) Imagine that immediately after making this initial investment in the two bonds, the yield curve makes a large shift. The new yield curve is Y(1) = .04; Y(2) = .0425, Y(3) = .045, Y(4) = .0475, Y(5) = .05, Y(6) = .0525, Y(7) = .055, Y(8) = .0575, Y(9) = .06, Y(10) = .0625. Recalculate the value of the liability and the value of each of the alternative portfolios.

(d) In theory, which of the two strategies would you generally have expected to do better when yields change significantly, and why? Which performs better here in practice? Explain the intuition behind which turns out to be better in this case, and how it relates to the theoretical case.

(e) After the yield curve shifts the liability is no longer immunized. Using the same two bonds as in alternative (i) explain the trades you would need to make to restore immunization. That is, calculate the new amounts that would be invested in each of the bonds. (Hint: In recalculating the new duration of the liability, first find its new price and then calculate its YTM.)

2. In this question, state all of the results on a <u>bond equivalent basis</u>.
a) Use the Treasury bond data from the Deutsche Bank case (tools for calculation in *bootstrap.xls*, available on the class web page) to construct a spot yield curve going out five years.
b) Plot the spot rate curve. Also plot a yield curve based on each coupon bonds' yields to maturity.
c) Is the discrepancy between the two curves in (b) large? Under what yield curve conditions would you expect it to be smaller than what you see here?

3. The Deutsche Bank case also has spot rates based on the bank's proprietary model estimates.
a) The rates given are at an annual interval, so you have to interpolate to fill in the six month model spot rates. Do this using the Nelson Siegel model and write down the

parameter estimates that give a good fit. You can do this using the Nelson Siegel spreadsheet program provided and iterating by hand on the assumed parameters, or if you are ambitious you can write code that automates the process of fitting the parameters (not required).
b) Use your interpolated proprietary spot yield curve from (a) to find the theoretically correct price of the 1-year and 3-year Treasury bonds that you used to do the bootstrapping in problem 2. Make sure to convert bond equivalent rates to effective rates to use for discounting.
b) Based on these price estimates and how they compare to the actual bond prices, propose a trade involving a long position in one of the bonds and a short position in the second bond that will profit if actual prices move closer in line with the theoretical prices. Explain your logic in a few sentences.
c) Since the long and short positions involve bonds of different durations, the proposed trade involves exposure to movements in the overall level of interest rates. Describe how you could protect against this risk using the idea of delta hedging. Specifically, describe the relative size of the long and short position that best protects against the risk that the yield curve shifts.

4. Use the spot yield curve calculated in question 3 to compute the implied 1 year forward rates as of August 2003, for August 2004, August 2005, and August 2006. You will have to think about the right way to go from rates that cover six-month intervals to forward rates that cover one-year periods. Express the forward rates on a bond equivalent basis, and also as effective annual yields. Compare these forward rates to the realized one-year rates for those years listed below. In your opinion, is the forecast error large or small (explain briefly)?

Actual One-Year Treasury Rates	
August 16, 2004	2.01%
August 15, 2005	3.91%
August 15, 2006	5.11%