Handout for Class #4

Asset Management, Lifecycle Investing, and Retirement Finance

Robert C. Merton

15.467

Class #4

Monday, March 1, 2021

<u>Lecture</u>

Management of Non-Linear Asset Risk: Credit Risk

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15.467 Asset Management, Lifecycle Investing and Retirement Finance

II. Measuring Investment Performance and Risks

2:30pm Mon., March 1, 2021 online

TOPIC

Class 4: Management of Non-Linear Asset Risk: Credit Risk

MATERIALS

Required Reading:

On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Continuous-Time Finance,* Chapter 12, pp. 388-412; 361-367 (Textbook)

"Model of Excellence in Financial Risk Management," M. Waters, Treasury & Risk, December 12, 2019.

Explore https://www.moodysanalytics.com/

Review Reading on the Cox-Ross-Rubinstein binomial option model "Further Developments in Option Pricing Theory," *Continuous-Time Finance*, Chapter 10, pp. 337-341 (Class Textbook)

Optional Reading:

"The Economics of Credit Default Swaps," Robert Jarrow, *Annual Review of Financial Economics*, Vol. 3, 2011, pp. 235-258.

Brealey, Myers and Allen, 12th edition; pp. 597-607.

"An Analytical Derivation of the Cost of Deposit Insurance and Loan Guarantees," *Continuous-Time Finance*, Chapter 19, pp. 625-633. (Textbook)

"On the Cost of Deposit Insurance When There are Surveillance Costs, *Continuous-Time Finance*, Chapter 20, pp. 634-648 (Textbook)

'Forecasting Sovereign Default Risk with Merton's Model," J. Duyvesteyn and M. Martens, *Journal of Fixed Income*, Fall 2015.

"Predicting Default - Merton vs. Leland," J. Forssbaeck and A. Vilhelmsson, SSRN, February 9, 2017.

"Predicting default among UK companies: a Merton approach," *Financial Stability Review,* June 2003.

"How Profitable is Capital Structure Arbitrage,?" *Financial Analysts Journal*, September/October 2006, pp. 47-62.

"Envisioning the Future of Securities Analysis," *CFA Magazine,* Susan Trammell, July-August 2004, pp. 24-28.

"Harvard's Financial Scientist," Peter Carr, *Bloomberg Markets,* October 2006, pp 166-169.

"Merton's Back! CDS vs. Puts," Merrill Lynch, August 2007.

"A Model Mind," *CFA Magazine* Roger Mitchell, July-August 2004, pp. 34-37.

Risk Measurement for Credit-Risk Assets

Non-linear risks of being a lender when there is risk of default

RISKY DEBT + GUARANTEE OF DEBT = RISK-FREE DEBT

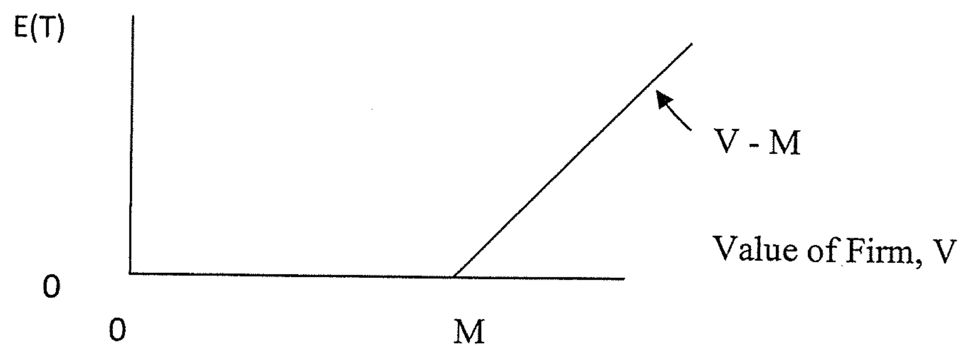
RISKY DEBT = RISK-FREE DEBT – GUARANTEE OF DEBT

<table>
<tr><td colspan="2">Corporation</td></tr>
<tr><td rowspan="2">Operating Assets, A</td><td>Debt (face value B, zero-coupon), D</td></tr>
<tr><td>Common Stock, E</td></tr>
</table>

- In default, the holder of the guarantee receives promised value of the debt minus value of assets recovered from defaulting entity = max [0, B-A]
- Value of guarantee = put option on the assets of borrower
- Credit default swaps are guarantees of debt and therefore are put options on the assets of the borrower

The Relation Between Risky Debt, Leveraged Equity, and Options

The payoff to equity, E, is like a call option on the firm

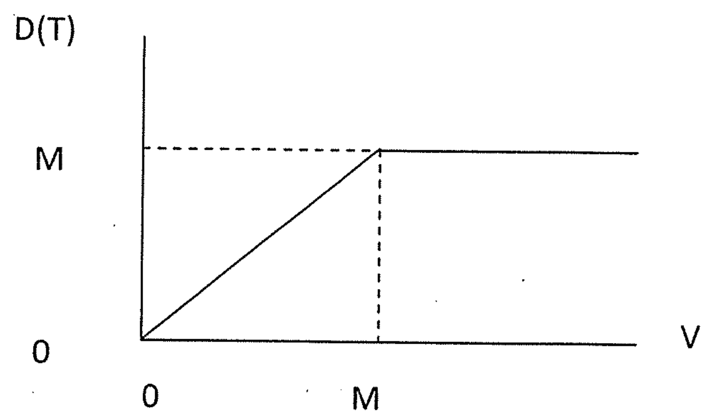


The payoff to debt is like: i) **"fully-covered call write,"** (long the firm and short a call option on the firm),

$$D(T) = V - E(T) = V - \max[0, V - M],$$

or ii) "**fully-covered put write**," (long a T-year, UST Zero-coupon bond with principal amount M and short a put option on the firm),

$$D(T) = M - \max[0, M - V]$$

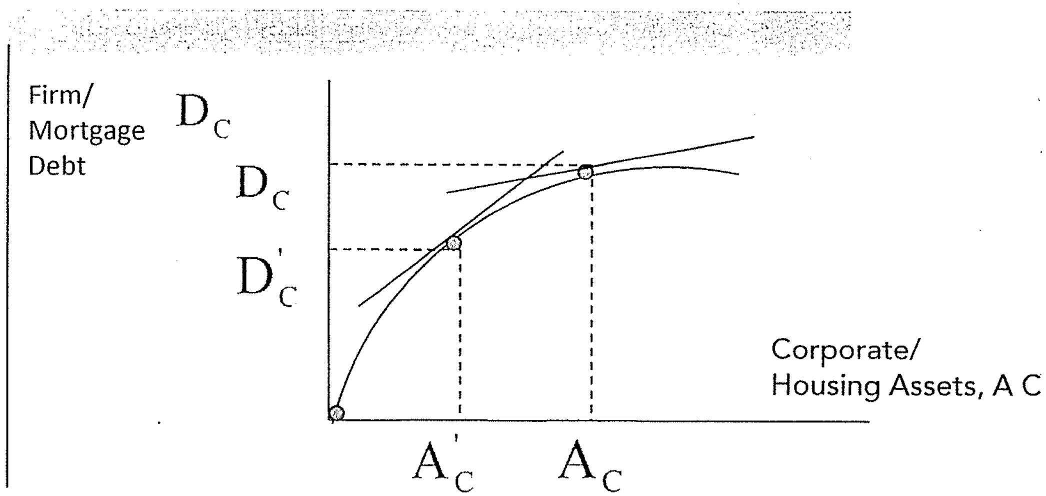


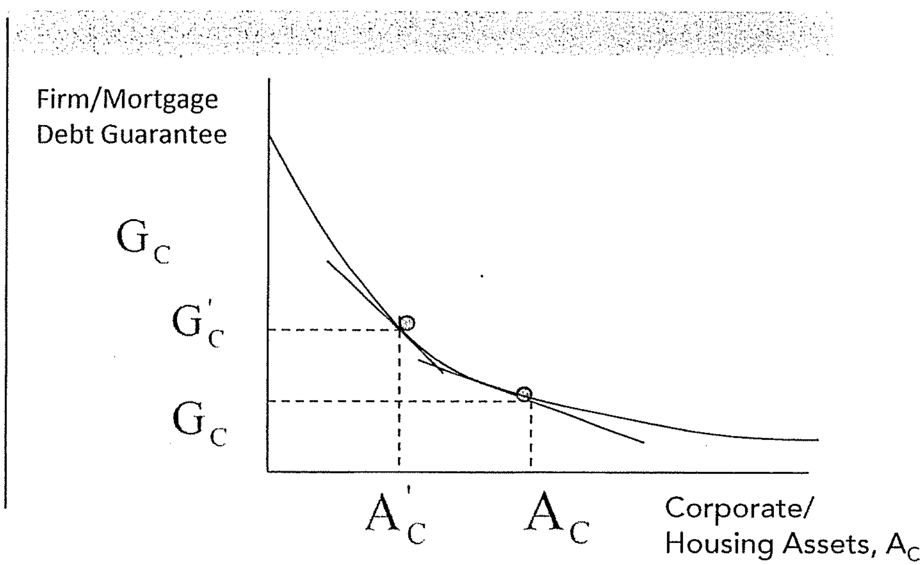
Debt guarantee, G, = Riskless debt – Risky debt

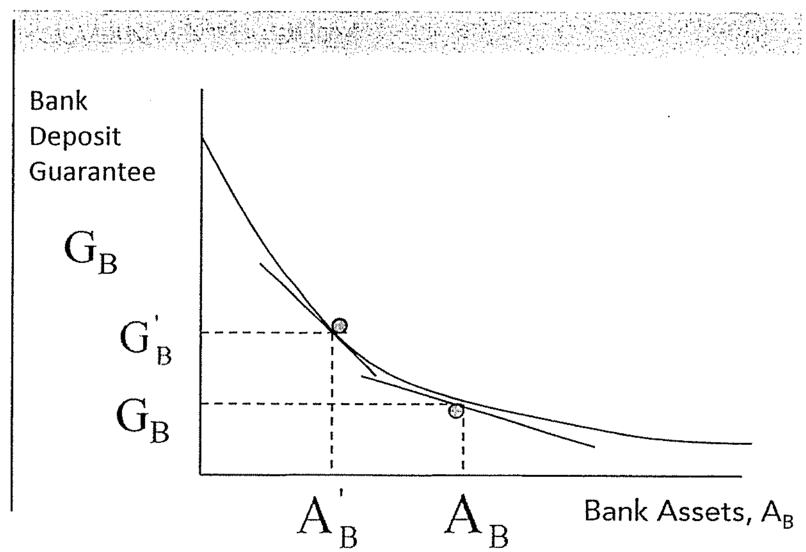
$$G(T) = M - D(T)$$
$$= \max[0, M - V]$$

so payoff on guarantee of debt is like a put option on the firm's value.

Non-linear Credit Risk Buildup







The Payoff Structure of Debt with Default Risk

Let DistressCorp be a limited-liability firm with total assets whose mark-to-market value at time t is V(t) and with two classes of liabilities: (i) zero-coupon debt which matures at time T and which has aggregate face value (promised payment) of $M and (ii) equity, which is entitled to the assets of the firm, once the obligations to the debt are discharged. If at time T, the promised payment to the debt is not made, then the debtholders seize the assets. Let D(t) denote the market value of the debt at time t and E(t) denote the market value of the equity at time t. As a value-identity, $V(t) = D(t) + E(t)$, always. With no costs of seizure and exact-priority rules, we have that:

If at time T, $V(T) = V$ and:

if $V \leq M$, then

$$D(T) = V$$
$$E(T) = O$$

if $V > M$, then

$$D(T) = M$$
$$E(T) = V\text{-}M$$

That is,

$$D(T) = \min [V,M]$$
$$E(T) = \max [O, V\text{-}M]$$

Example: Suppose that Distress Corp's total assets consist of 1000 shares of XYZ corporation and terms of the debt are that it matures in (T =) 2 years and the aggregate promised payment, on the debt (M =) $100,000. The current price of XYZ shares is $100, and so V(O) = $100,000.

Replication Portfolios

At Year 0

	Debt	Equity	Firm
Shares XYZ	296	704	1000
Riskless Debt	$54,785	$(54,785)	0
Value	$84,385	$15,615	$100,000

At Year 1

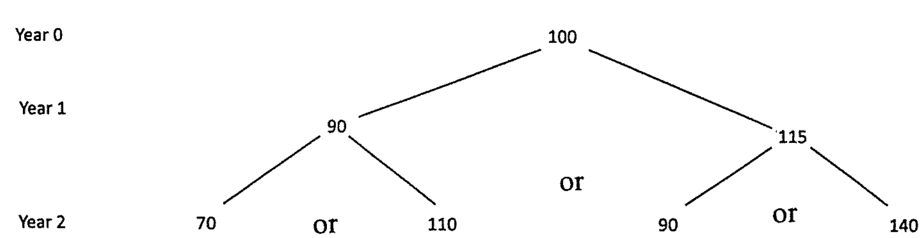
If XYZ = $90

	Debt	Equity	Firm
Shares XYZ	750	250	1000
Riskless Debt	$16,667	$(16,667)	0
Value	$84,167	$5,833	$90,000

If XYZ = $115

	Debt	Equity	Firm
Shares XYZ	200	800	1000
Riskless Debt	$68,567	$(68,567)	0
Value	$91,567	$23,433	$115,000

Future Possible Prices of XYZ



See Continuous-Time Finance, pp. 337-341 for the formula to derive the replicating portfolio allocations above in the Cox-Ross-Rubinstein binomial option pricing model. For the calculations in this example, see the pages at the back of the handout.

Relation Among Asset, Equity, and Debt Expected Return

CAPM

$$(3a)\quad \bar{R}_V(t) = R_f(t) + \beta_V(t)\left[\bar{R}_M(t) - R_f(t)\right]$$

$$(3b)\quad \bar{R}_D(t) = R_f(t) + \beta_D(t)\left[\bar{R}_M(t) - R_f(t)\right]$$

$$(3c)\quad \bar{R}_E(t) = R_f(t) + \beta_E(t)\left[\bar{R}_M(t) - R_f(t)\right]$$

Firm Relations In Terms of Betas

$$(4a)\quad \bar{R}_D(t) = R_f(t) + \left[\beta_D(t)/\beta_V(t)\right]\left[\bar{R}_V(t) - R_f(t)\right]$$

$$(4b)\quad \bar{R}_E(t) = R_f(t) + \left[\beta_E(t)/\beta_V(t)\right]\left[\bar{R}_V(t) - R_f(t)\right]$$

Firm Relations in Terms of Replicating Portfolio Weights

$$(5a)\quad \bar{R}_D(t) = R_f(t) + w_D(t)\left[\bar{R}_V(t) - R_f(t)\right]$$

$$(5b)\quad \bar{R}_E(t) = R_f(t) + w_E(t)\left[\bar{R}_V(t) - R_f(t)\right]$$

$$(5c)\quad \bar{R}_D(t) = R_f(t) + \left[w_D(t)/w_E(t)\right]\left[\bar{R}_E(t) - R_f(t)\right]$$

Estimating the Return Performance on a Credit Risk Portfolio

$$R_p - R_f = a + b\left(R_m - R_f\right) - c\ \text{Call Return} + e_p$$

Expected Returns and Betas for Debt and Equity: Distress Corp

Suppose $\bar{R}_M = 10\%$; $\bar{R}_f = 5\%$; $\beta_V = 2.0$ for $t = 0, 1$

(6) $\bar{R}_V(t) = .05 + 2\ (.10 - .05) = 15\%$

Debt

Replicating Portfolio	Year (0)	Year 1 (V = 90)	Year 1 (V = 115)
% in Firm Asset $[w_D(t)]$	35%	80%	25%
% in Riskless Debt $[1 - w_D(t)]$	65%	20%	75%
% of Total Firm Value	84.4%	93.5%	79.6%
βeta $(w_D(t)\beta_V)$	0.70	1.60	0.50
Expected Return $[\bar{R}_D(t)]$	8.5%	17.0%	7.5%
Promised Yield to Maturity	8.9%	18.8%	9.2%

Equity

Replicating Portfolio	Year (0)	Year 1 (V = 90)	Year 1 (V = 115)
% in Firm Asset $[w_E(t)]$	451%	386%	393%
% in Riskless Debt $[1 - w_E(t)]$	(351%)	(286%)	(293%)
% of Total Firm Value	15.6%	6.5%	20.4%
βeta $(w_E(t)\beta_V)$	9.02	7.72	7.86
Expected Return $[\bar{R}_E(t)]$	50.1%	43.6%	44.3%

Estimating the Return Performance on a Credit Risk Portfolio

$$R_p - R_f = a + b\left(R_m - R_f\right) - c\ \text{Call Return} + e_p$$

Promised Bond Yield and Credit Guarantee Value

	Debt Price With Guarantee (5% yield)	Debt Price without Guarantee	Promised Yield to Maturity	Value of Guarantee	Percentage of No-G Price
Year 0	$90,700	$84,385	8.86%	$6,315	7.48%
Year 1					
$90	$95,235	$84,167	18.81%	$11,068	13.15%
$115	$95,235	$91,567	9.21%	$3,668	4.00%
Year 2					
$70	$100,000	$70,000	–	$30,000	42.9%
$90	$100,000	$90,000	–	$10,000	11.1%
$110	$100,000	$100,000	–	0	0
$140	$100,000	$100,000	–	0	0

Promised Bond Yield and Realized Returns on Bonds

	Promised Yield to Maturity	Realized Compound Return from Year O	Realized Return From Year 1
Year 0	8.86%	—	—
Year 1			
$90	18.81%	(0.26)%	—
$115	9.21%	8.51%	—
Year 2			
$70	—	(8.92)%	(16.83)%
$90	—	3.27%	(1.71)%
$110	—	8.86%	18.81%
$140	—	8.86%	9.21%

Valuing Risky Debt Using Black-Scholes Call-Option-Pricing Model

Black-Scholes Model

$$E(T) = C(V,T) = VN(d) - Me^{-rT}\ N(d - (\sigma\sqrt{T})$$

where

$$d = [\log(V/M) + (r + \tfrac{1}{2}\sigma^2)T] / \sigma\sqrt{T},$$

r = (continuously-compounded) riskless interest rate, σ^2 is the variance of the total rate of return on the firm; N( ) is the cumulative normal density function

If V = value of the firm at time 0, then

$$\begin{aligned} D(0) &= V - C(V,T) \\ &= V[1 - N(d)] + Me^{-rT} N(d - \sigma\sqrt{T}) \end{aligned}$$

$$\begin{aligned} G(0) &= Me^{-rT} - D(0) = Me^{-rT} - V + C(V,T) \\ &= Me^{-rT}[1 - N(d - \sigma\sqrt{T})] - V[1 - N(d)] \end{aligned}$$

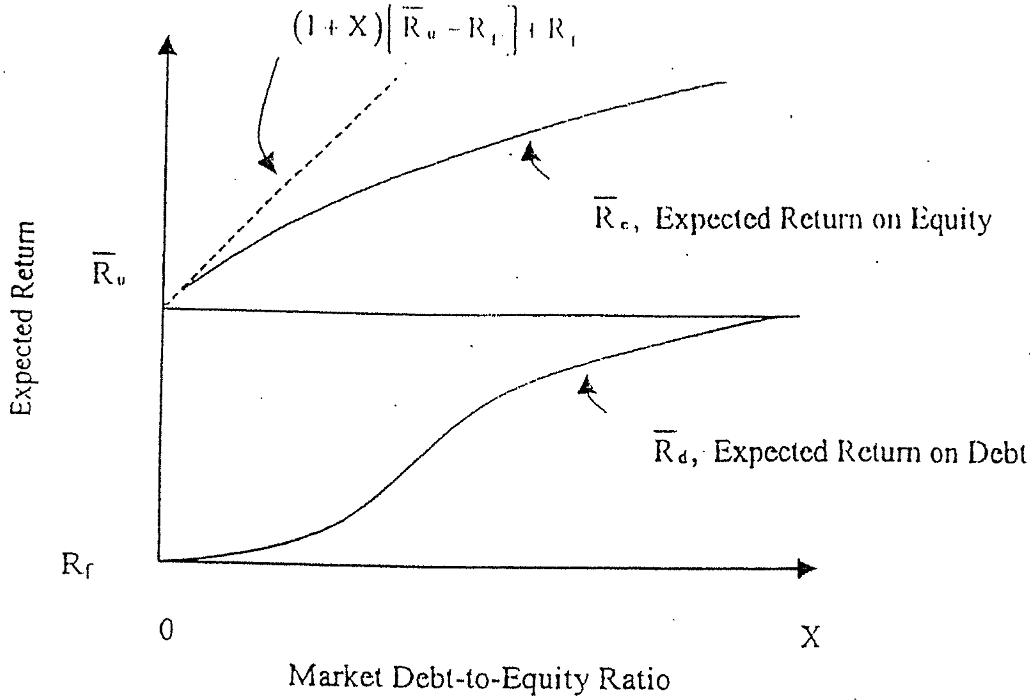
Promised-Yield (to maturity) on debt, R

$$(1 + R)^T = M/D(0)$$

$$R = [M/D(0)]^{\frac{1}{T}} - 1$$

There are more-complicated versions of the Contingent-Claims analysis/Option-Pricing Model for coupon debt, sinking funds, call provisions, multiple debt issues with different seniority, and different target financing and payout policies.

Expected Return, Yield-to-Maturity on Debt with Risk of Default



Expected Return on Debt and Equity vs. Market Debt-to-Equity Ratio.

If M = promised principal payment on Zero-coupon debt that matures at time T and D is the market price of the debt today (time 0), then the (promised) Yield-to-Maturity on the dept, R, is given by $R = [\frac{M}{D}]^{1/T} - 1$.

If $\bar{R}_d$ = expected rate of return (per period) until maturity, then

$$\bar{R}_d = \left\{\frac{[1-\text{probability of default}]XM + (\text{probability of default})X\text{ Expected Recovery}}{D}\right\}^{\frac{1}{T}} - 1$$

Factors that Influence (or don't) the Debt and Equity Valuation

Holding as unchanged all other factors in this list, a positive change in	Direction of the change in Value of D	Direction of the change in Value of E
1) Riskfree Interest rate, r	↓	↑
2) Value of firm, V	↑	↑
3) Time to Maturity, T	↓	↑
4) Principal Amount, M	↑	↓
5) Variance of firm Value, σ^2	↓	↑
6) Expected return on the firm, $E(V(T))/V$	none →	none →
7) Beta of the firm, β	none →	none →
8) Investors' taste for risk-bearing	none →	none →

Note: changes in items (6-8) may cause changes in items (1-5) and thereby, indirectly affect the value of debt and equity, but 1-5 are the "channels."

Review Study Questions on Credit Risky Debt:

1. What essential information about the underlying firm does the "option based" [aka "CCA"] model of risky debt require to do valuation and risk exposure analysis?

2. One of the chief advantages claimed for this type of risky bond pricing model over others is that it reflects rapidly deteriorating credit-worthiness for individual firms much more quickly than the other model types. Explain why that is the case.

3. Can you explain the difference between "risk-neutral" and actual default probabilities? Which of the two is generally larger? What additional information is needed to estimate actual default probabilities over risk-neutral ones? What purpose do "risk-neutral" probabilities serve?

4. Is the yield-to-maturity on a risky bond equal to the expected return on that bond?

5. Suppose there are two bonds, each with identical promised payments, as to maturity and timing. Is it true that the one with the lower market price(higher yield to maturity) has a higher expected return? Why?

6. Suppose there are two bonds, each with identical promised payments, as to maturity and timing, and suppose that we know that the probability of default is the same for each bond. Is it true that the one with the lower market price(higher yield to maturity) has a higher expected return? Why?

7. We know that if a firm increases its financial leverage, the risk of the (leveraged) equity of the firm goes up; also the risk of its debt goes up. We also know, as an identity, that the value of the firm equals the value of its debt plus the value of its equity. Does it thus follow that if a firm increases its leverage, the risk of the firm goes up? Why?

8. How can CDS, corporate debt prices, and equity-option implied volatilities be used to derive forward-looking market-implied probability distributions for future firm values?

9. To apply a Moody's-KMV-like model with implied values and parameters is it necessary that the debt of the firm is traded? That the equity of the firm is traded?

DERIVING THE HEDGING RATIOS FOR DYNAMIC REPLICATION PORTFOLIO STRATEGIES: BACKWARDS INDUCTION

Case 1: At time t = 1 and stock price of XYZ = $90. Determine the value needed for the replication portfolio, V, the number of shares of *XYZ* to be held, N, and the amount of cash, C. The requirement is that the portfolio has a value at time t = 2, which is $100,000 if the stock price is $70 and $110,000 if the stock price is $110. If S is the stock price and R is the risk-free rate, then the portfolio value and payout at time t =2, P, satisfies:

$$N \times S + (1+R) \times C = P$$

(1) $$N \times 70 + (1.05) \times C = 100{,}000$$

(2) $$N \times 110 + (1.05) \times C = 110{,}000$$

Subtracting (1) from (2), we have that N x 40 = 10,000 or

(3) $$N = 250 \text{ shares}$$

Substituting from (3) into (1) and rearranging terms, we have that C = [100,000 – (250 x 70)]/(1.05) = $82,500/1.05 or

(4) $$C = \$78{,}571$$

The required value of the portfolio V = N x 90 + C = 250 x 90 + 78,571 or

(5) $$V = \$101{,}071$$

DERIVING THE HEDGING RATIOS FOR DYNAMIC REPLICATION PORTFOLIO STRATEGIES: BACKWARDS INDUCTION

Case 2: At time t = 1 and stock price of XYZ = \$115. Determine the value needed for the replication portfolio, V, the number of shares of *XYZ* to be held, N, and the amount of cash, C. The requirement is that the portfolio has a value at time t = 2, which is \$100,000 if the stock price is \$90 and \$140,000 if the stock price is \$140. If S is the stock price and R is the risk-free rate, then the portfolio value and payout at time t =2, P, satisfies:

$$N \times S + (1+R) \times C = P$$

(1) $$N \times 90 + (1.05) \times C = 100{,}000$$

(2) $$N \times 140 + (1.05) \times C = 140{,}000$$

Subtracting (1) from (2), we have that N x 50 = 40,000 or

(3) *N = 800 shares*

Substituting from (3) into (1) and rearranging terms, we have that C = [100,000 – (800 x 90)]/(1.05) = \$28,000/1.05 or

(4) *C = \$26,667*

Required value of the portfolio V = N x 115 + C = 800 x 115 + 26,667 or

(5) *V = \$118,667*

DERIVING THE HEDGING RATIOS FOR DYNAMIC REPLICATION PORTFOLIO STRATEGIES: BACKWARDS INDUCTION

Case 3: At time t = 0 and stock price of XYZ = \$100. Determine the value needed for the replication portfolio, V, the number of shares of *XYZ* to be held, N, and the amount of cash, C. The requirement is that the portfolio has a value at time t = 1, which is \$101,071 if the stock price is \$90 and \$118,667 if the stock price is \$115. If S is the stock price and R is the risk-free rate, then the portfolio value and payout at time t =1, P, satisfies:

$$N \times S + (1+R) \times C = P$$

(1) $$N \times 90 + (1.05) \times C = 101{,}071$$

(2) $$N \times 115 + (1.05) \times C = 118{,}667$$

Subtracting (1) from (2), we have that N x 25 = 17,596 or

(3) $$N = 704 \text{ shares}$$

Substituting from (3) into (1) and rearranging terms, we have that
C = [101,071 – (704 x 100)]/(1.05) = \$37,711/1.05 or

(4) $$C = \$35{,}915$$

Required value of the portfolio V = N x 100 + C = 704 x 100 + 35,915 or

(5) $$V = \$106{,}315$$