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1. OLS estimator for AR(1) is biased, but consistent.

Consider $X_{t+1} = \phi_1 X_t + \varepsilon_{t+1}$, $\varepsilon_{t+1} \sim N(0, \sigma^2)$

square loss $L = \sum_{t=1}^{T} (\hat{X}_t - X_t)^2$

$$= \sum_{t=1}^{T} (\hat{\phi}_1 X_{t-1} - X_t)^2$$

$$\frac{\partial L}{\partial \hat{\phi}_1} = 0 \Rightarrow \hat{\phi}_1 = \frac{\sum_{t=1}^{T} X_{t-1} X_t}{\sum_{t=1}^{T} X_{t-1}^2}$$

$$= \phi_1 + \sum_{t=1}^{T} \frac{X_{t-1}}{\sum_{t=1}^{T} X_{t-1}^2} \varepsilon_t$$

ε_t is independent on X_{t-1}, but

dependent on $\sum_{t=1}^{T} X_{t-1}^2$.

Thus, $E(\hat{\phi}_1) \neq \phi_1$, so it is biased.

If under the assumption that $|\phi_1| < 1$, which means AR(1) is stationary, by ergodic thm, OLS estimator is consistent.

2. $r_t = 0.01 + 0.1 r_{t-2} + \varepsilon_t$, $\varepsilon_t \sim N(0, 0.02)$

(a) $E(r_t) = 0.01 + 0.1 E(r_{t-2})$

$\Rightarrow 0.9\mu = 0.01 \Rightarrow \mu = \frac{1}{90}$

$Var(r_t) = 0.01 Var(r_{t-2}) + Var(\varepsilon_t)$

$\Rightarrow 0.99 Var(r_t) = 0.02$

$\Rightarrow Var(r_t) = \frac{2}{99}$

(b) $\gamma_1 = Cov(r_t, r_{t-1})$

$= Cov(0.01 + 0.1 r_{t-2} + \varepsilon_t, r_{t-1})$

$= 0$

$= \rho_1$

$$\gamma_2 = Cov(r_t, r_{t-2})$$

$$= Cov(0.01 + 0.1 r_{t-2} + \varepsilon_t, r_{t-2})$$

$$= 0.1\, Cov(r_{t-2}, r_{t-2}) = 0.1 \cdot \frac{2}{99}$$

$$\rho_2 = \gamma_2 / \gamma_0 = 0.1$$

(c) $E_{100}[r_{101}] = E_{100}[0.01 + 0.1 r_{99} + \varepsilon_{101}]$

$$= 0.01 + 0.1 \times 0.02 = 0.012$$

error $= r_{101} - E_{100}[r_{101}] = \varepsilon_{101}$

Std. deviation $= \sqrt{0.02} = 0.1414$

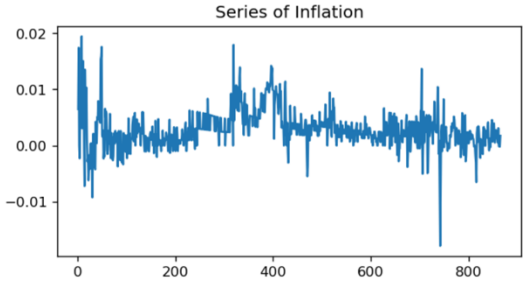
$E_{100}[r_{102}] = E_{100}[0.01 + 0.1 r_{100} + \varepsilon_{102}]$

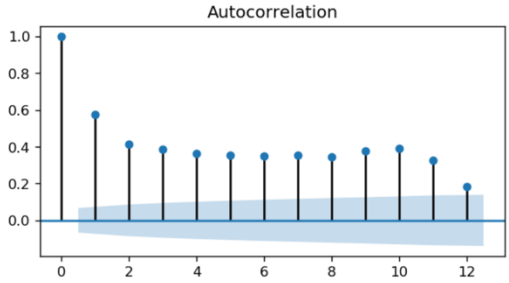
$$= 0.01 + 0.1 \times 0.01 = 0.011$$

error $= r_{102} - E_{100}[r_{102}] = \varepsilon_{102}$

Std deviation $= \sqrt{0.02} = 0.1414$

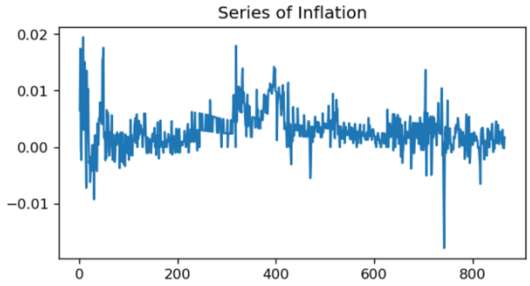
3. (a)

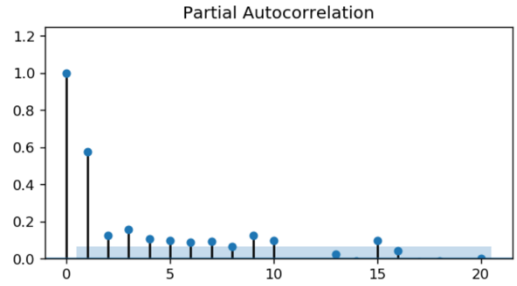




(b)

Plot the PACF for the inflation series:





From the graph, we see that lag-1 is highly significant, so choose an AR(1) model.

AutoReg Model Results

Dep. Variable:	pai	No. Observations:	865
Model:	AutoReg(1)	Log Likelihood	3852.184
Method:	Conditional MLE	S.D. of innovations	0.003
Date:	Mon, 26 Apr 2021	AIC	-11.748
Time:	20:43:25	BIC	-11.731
Sample:	1	HQIC	-11.742
	865		

	coef	std err	z	P>|z|	[0.025	0.975]
intercept	0.0012	0.000	9.723	0.000	0.001	0.001
pai.L1	0.5755	0.028	20.704	0.000	0.521	0.630

Roots

	Real	Imaginary	Modulus	Frequency
AR.1	1.7376	+0.0000j	1.7376	0.0000

Using AIC:

AutoReg Model Results

Dep. Variable:	pai	No. Observations:	865
Model:	AutoReg(15)	Log Likelihood	3919.851
Method:	Conditional MLE	S.D. of innovations	0.002
Date:	Mon, 26 Apr 2021	AIC	-12.021
Time:	20:43:42	BIC	-11.926
Sample:	15	HQIC	-11.985
	865		

	coef	std err	z	P>|z|	[0.025	0.975]
intercept	0.0004	0.000	3.191	0.001	0.000	0.001
pai.L1	0.4414	0.034	13.123	0.000	0.376	0.507
pai.L2	0.0763	0.036	2.098	0.036	0.005	0.148
pai.L3	0.0621	0.036	1.710	0.087	-0.009	0.133
pai.L4	0.0153	0.036	0.427	0.669	-0.055	0.085
pai.L5	0.0547	0.035	1.542	0.123	-0.015	0.124
pai.L6	0.0118	0.035	0.335	0.738	-0.057	0.081
pai.L7	0.0575	0.035	1.641	0.101	-0.011	0.126
pai.L8	0.0279	0.034	0.812	0.417	-0.039	0.095
pai.L9	0.0601	0.034	1.752	0.080	-0.007	0.127
pai.L10	0.0766	0.034	2.229	0.026	0.009	0.144
pai.L11	0.0702	0.034	2.039	0.041	0.003	0.138
pai.L12	-0.1683	0.034	-4.897	0.000	-0.236	-0.101
pai.L13	-0.0068	0.035	-0.197	0.844	-0.075	0.061
pai.L14	-0.0350	0.034	-1.025	0.305	-0.102	0.032
pai.L15	0.0948	0.031	3.010	0.003	0.033	0.156

Using BIC:

AutoReg Model Results

Dep. Variable:	pai	No. Observations:	865
Model:	AutoReg(12)	Log Likelihood	3907.258
Method:	Conditional MLE	S.D. of innovations	0.002
Date:	Mon, 26 Apr 2021	AIC	-11.966
Time:	20:55:49	BIC	-11.888
Sample:	12	HQIC	-11.936
	865		

	coef	std err	z	P>|z|	[0.025	0.975]
intercept	0.0005	0.000	3.771	0.000	0.000	0.001
pai.L1	0.4287	0.034	12.742	0.000	0.363	0.495
pai.L2	0.0509	0.036	1.396	0.163	-0.021	0.122
pai.L3	0.0472	0.036	1.301	0.193	-0.024	0.118
pai.L4	0.0596	0.036	1.658	0.097	-0.011	0.130
pai.L5	0.0507	0.035	1.438	0.150	-0.018	0.120
pai.L6	-0.0126	0.035	-0.358	0.720	-0.082	0.056
pai.L7	0.1013	0.035	2.878	0.004	0.032	0.170
pai.L8	0.0230	0.035	0.651	0.515	-0.046	0.092
pai.L9	0.0749	0.035	2.132	0.033	0.006	0.144
pai.L10	0.1126	0.035	3.227	0.001	0.044	0.181
pai.L11	0.0484	0.035	1.403	0.161	-0.019	0.116
pai.L12	-0.1759	0.032	-5.519	0.000	-0.238	-0.113

Python ar_select_order function selects AR(15) under AIC, and selects AR(12) under BIC.