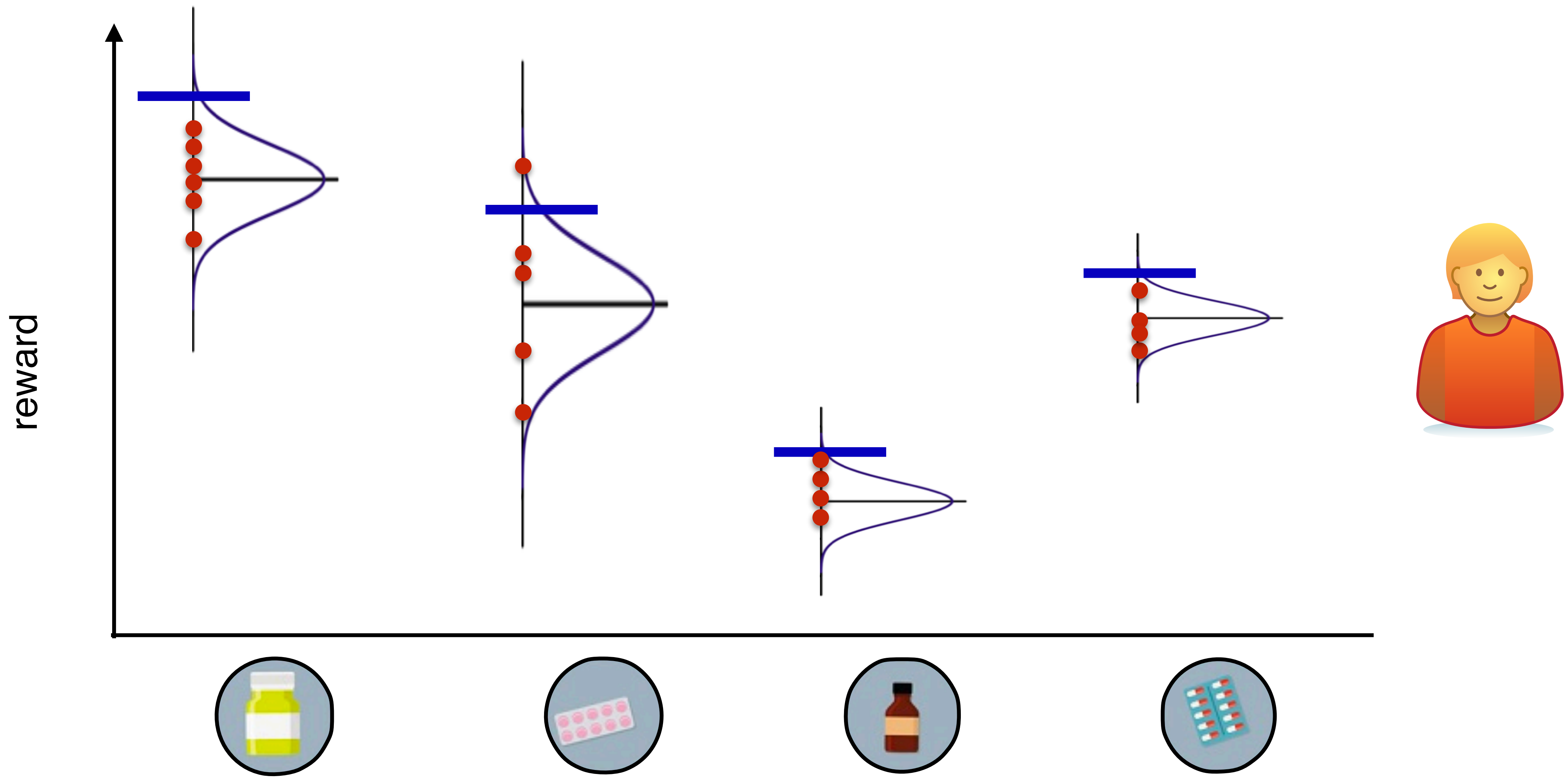
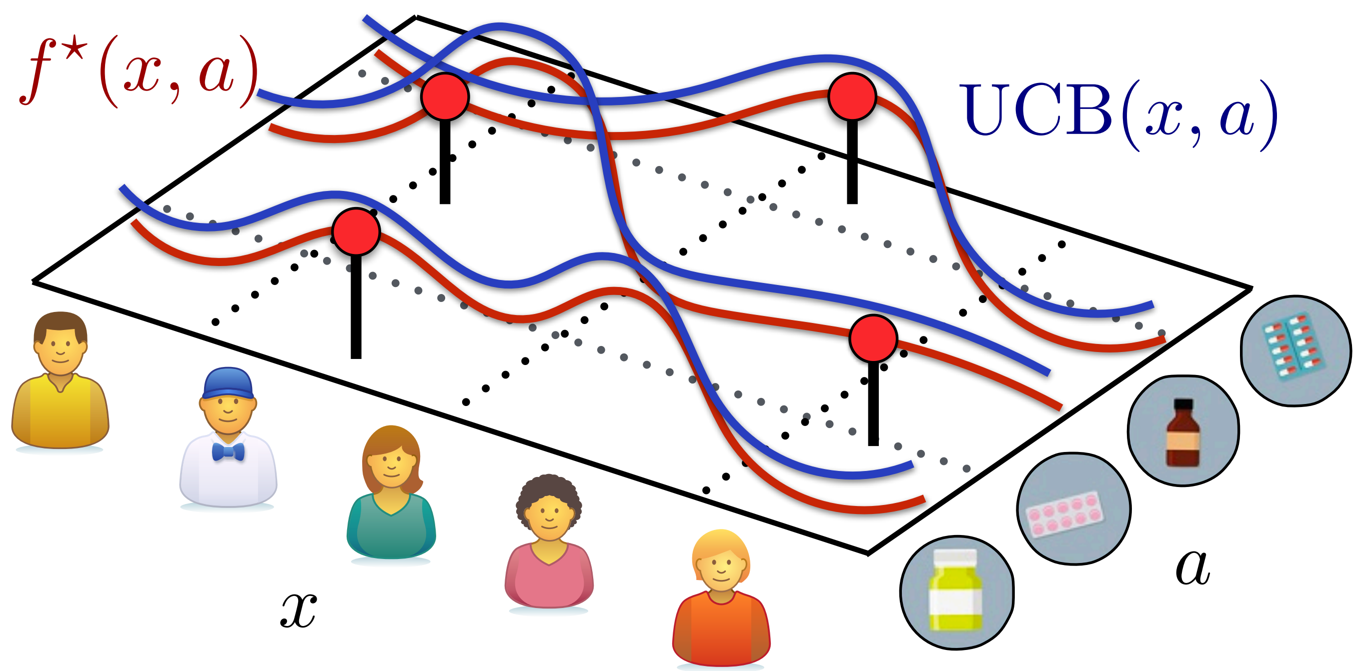
Upper confidence bound (UCB) algorithm

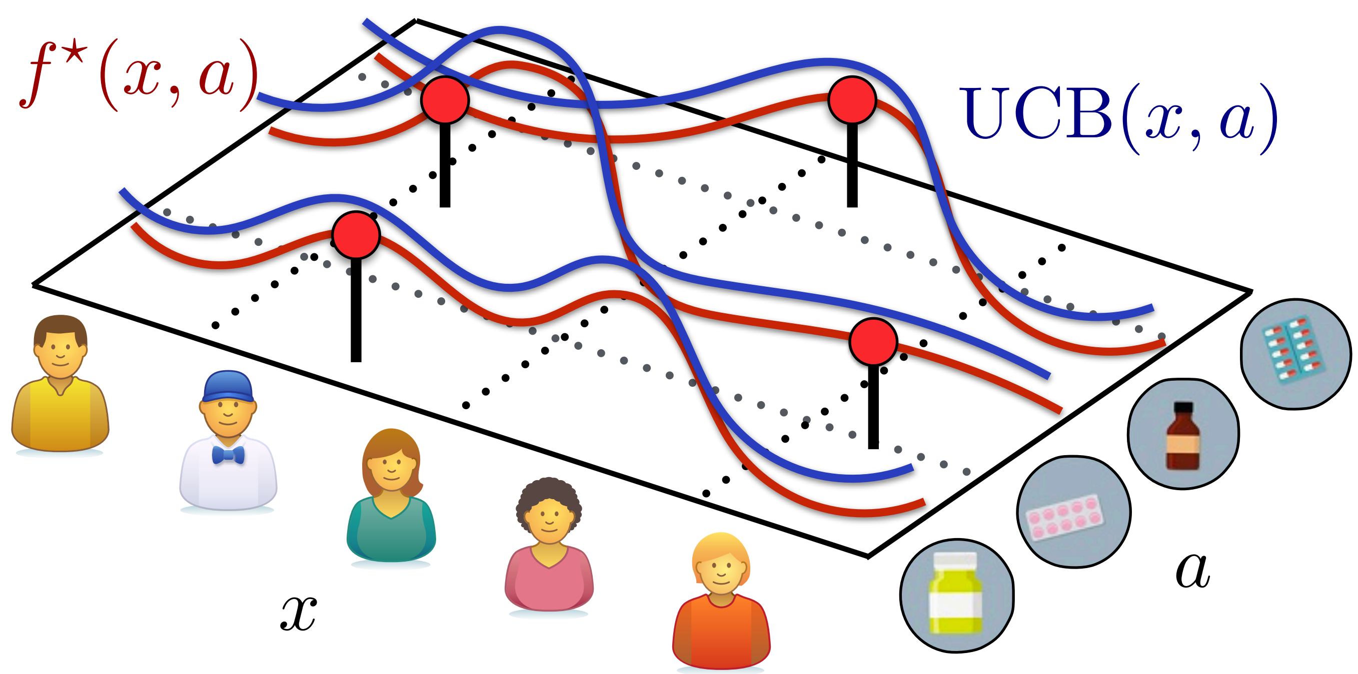


[Lai & Robbins ’85, Agrawal ’95, Auer et al. ’02]

General-purpose methods: Challenges



General-purpose methods: Challenges



- In general, no hope of constructing valid/shrinking confidence intervals for all (x, a).
- Exceptions:
  - Linear, generalized linear models (restrictive)
  - Nonparametric models (curse of dimensionality)

Lower bounds/counterexamples: [Russo & Van Roy '13], [**F**, Rakhlin, Simchi-Levi, Xu '20]

The SquareCB algorithm [F and Rakhlin'20]

SquareCB

For $t = 1, \ldots, T$:

- Receive context x_t.

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- Sample $a_t \sim p$ and learning algorithm with $(x_t, a_t, r_t(a_t))$.

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For $t = 1, \ldots, T$:

- Receive context x_t.
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- Inverse Gap Weighting (**IGW**):
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  with $p_b =$ remaining probability.

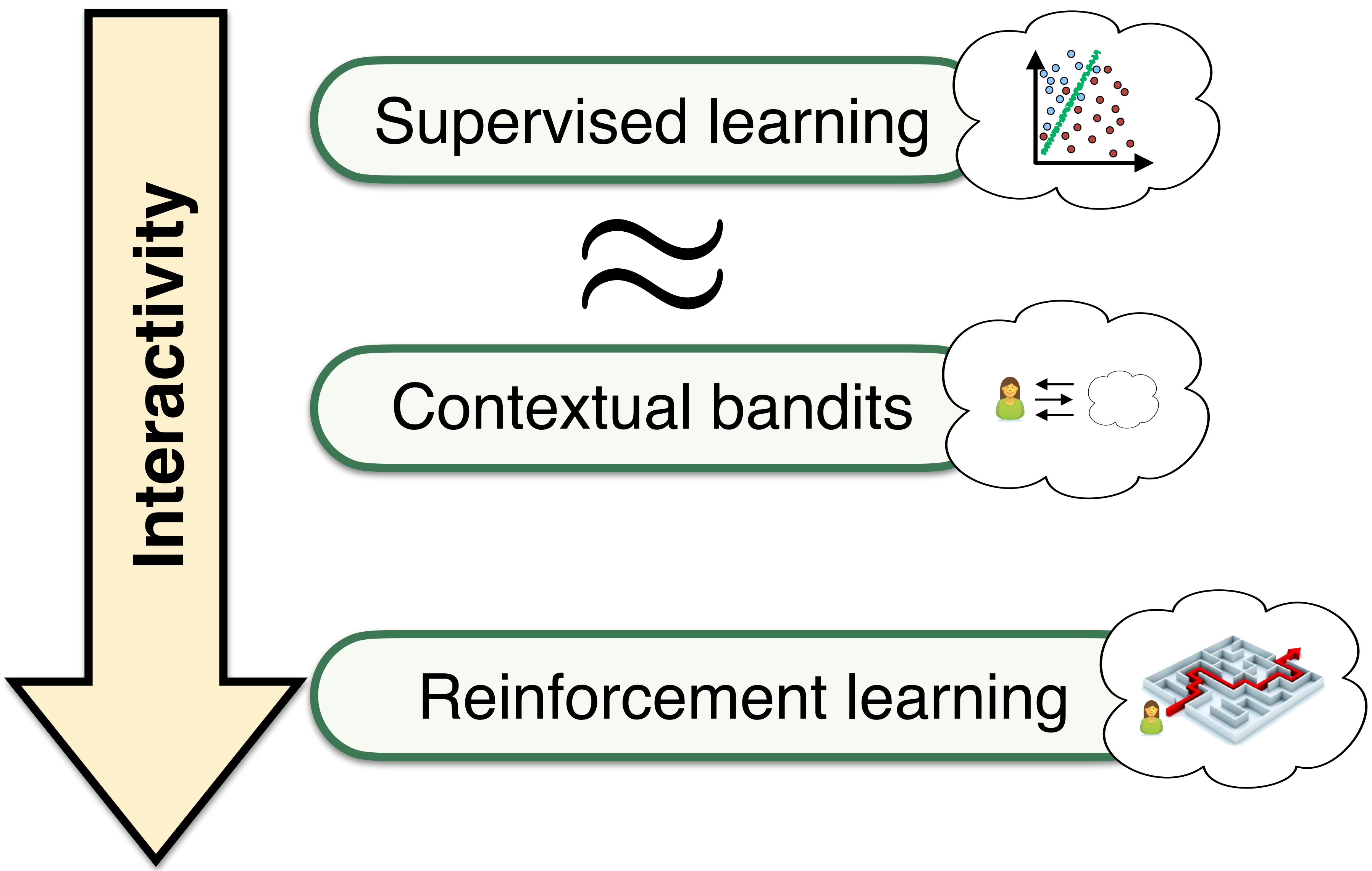
- Sample $a_t \sim p$ and learning algorithm with $(x_t, a_t, r_t(a_t))$.

SquareCB: Features

Makes decision making as easy as supervised learning!

- Use any out-of-the-box learning algorithm for $\mathcal{F}$.

  $\Longrightarrow$ **SquareCB** takes care of the rest.



Main theorem for SquareCB

Theorem [F & Rakhlin'20]

SquareCB guarantees that w.h.p.,

$$\mathbf{Reg}_{\mathsf{CB}} \leq C \cdot \sqrt{AT \cdot \mathbf{Reg}_{\mathsf{Sq}}(\mathcal{F})},$$

w/ $O(A)$ overhead in runtime and memory, where $A = \#$actions.

Optimality

More examples:

- Linear models (OLS, Ridge)
  [Abe-Long '99], [Auer '02],
  [Chu et al. '11]
- Kernels [Valko et al.'13, Zhou et al.'19]
- Generalized linear models
  [Filippi et al. '10, Li et al. '17]
- Finite classes
  [Auer et al. '01, Agarwal et al.'12]
- Sparse linear (Lasso, Elastic net)
  [Bastani & Bayati'20]
- Nonparametrics
  [Rigollet-Zeevi'10, Perchet-Rigollet'13],
  [Gur, Momeni, Wager'19]

Optimality / Universality

Theorem [F & Rakhlin'20]

For every function class $\mathcal{F}$, **SquareCB** attains the minimax optimal rate for CBs with $\mathcal{F}$.

To prove this, had to characterize *what* the optimal rate is [**F** & Rakhlin'20].

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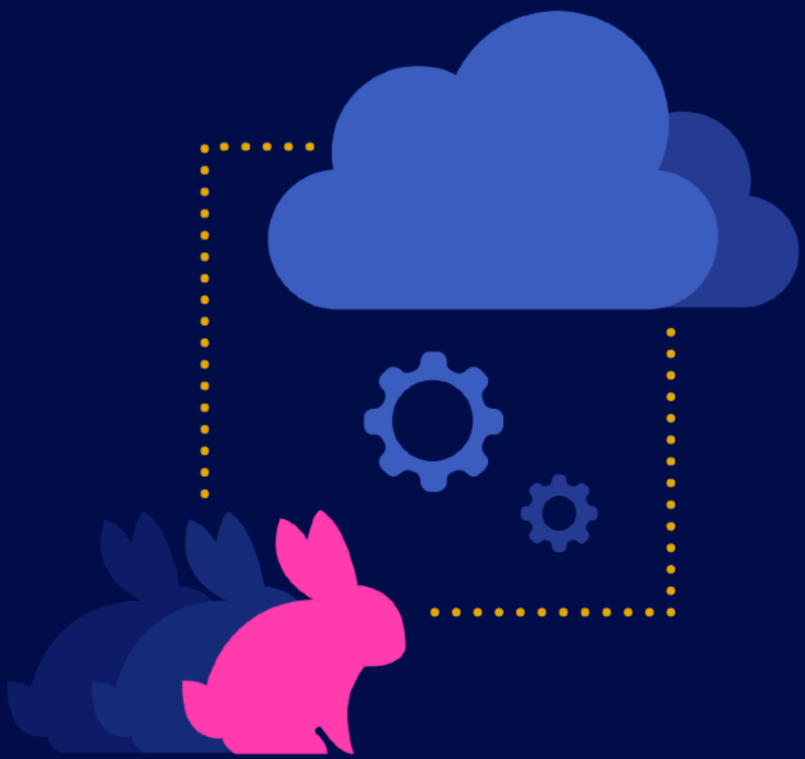

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