15.438 Fixed Income
Prof. D. Lucas

Problem Set #2

1.

(a)

The modified duration of the liability is 4.797. The modified duration D_m of a L-period deferred cash steam is related to the modified duration D_m^0 of the identical cash stream starting immediately by $D_m = D_m^0 + \frac{L}{1+y}$.

(b)

Note that PV of the liability is 86336.27.

(i): Solve x*(1/1.03) + (1-x)*(10/1.03) = 4.797 and x = 0.5621. Multiply PV of liability to get the dollar value:

Invest $48,531.49 in 1 year zero and $37,804.78 in 10 year zero.

(ii) Solve x*(4/1.03) + (1-x)*(7/1.03) = 4.797 and x = 0.6864.

Invest $59,258.20 in 4 year zero and $27,078.07 in 7 year zero.

(c)

PV of liability is $20{,}000 \cdot \sum_{t=3}^{7} \frac{1}{(1+Y(t))^t} = 78{,}269.75.$

PV of portfolio (i) is $48{,}531.49 \cdot \frac{1.03}{1.04} + 37{,}804.78 \cdot \frac{1.03^{10}}{1.0625^{10}} = 75{,}774.39.$

PV of portfolio (ii) is $59{,}258.20 \cdot \frac{1.03^4}{1.0475^4} + 27{,}078.07 \cdot \frac{1.03^7}{1.055^7} = 78{,}289.80.$

(d)

Using Excel solver, YTM = 5.0737%. The Macaulay duration of the liability can be determined by

Time	CF	PV (@YTM)	t*PV/sum(PV)
3	20000	17240.44431	0.660808694

4	20000	16407.96069	0.838533924
5	20000	15615.67493	0.997554893
6	20000	14861.646	1.139263548
7	20000	14144.02663	1.264961025

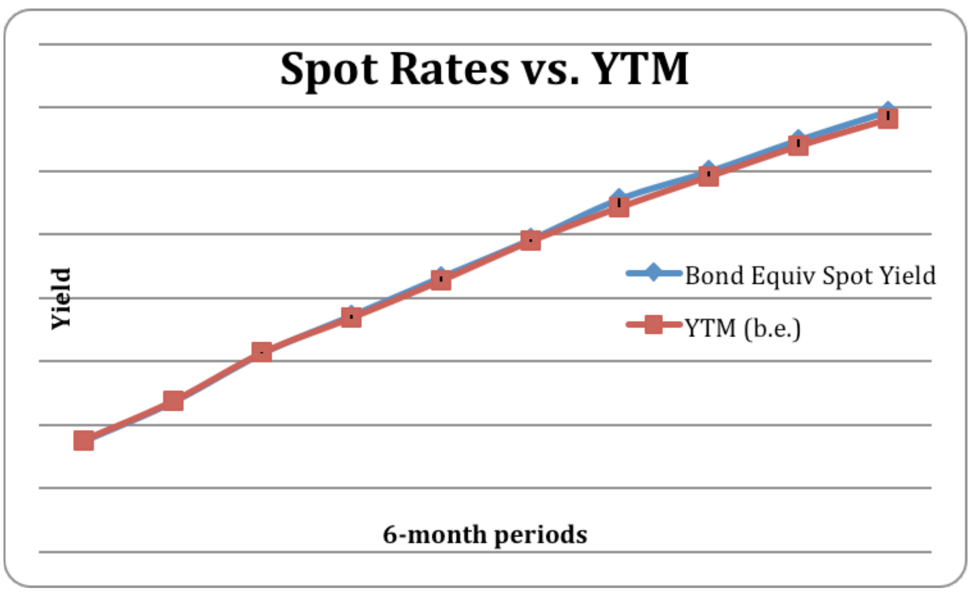
sum(PV)	78269.75256
Macaulay D	4.901122084
Modified D	4.664463225

The new portfolio weight should satisfy x*(1/1.04)+(1-x)*(10/1.0625) = 4.664, so x = 0.5618. Multiply by 78269.75 to get dollar values.

The new portfolio would be: invest $43,971.62 in 1 year zero and $34,298.14 in 10-year zero.

2. Output of bootstrap.xls, based on Treasury bond data from Deutsche Bank case going out five years:

	Zeros	Bond Equiv Spot Yield	YTM (b.e.)
2/15/2004	99.56099	0.8819%	0.8819%
8/15/2004	98.81763	1.1930%	1.1913%
2/15/2005	97.67381	1.5753%	1.5716%
8/15/2005	96.34063	1.8727%	1.8478%
2/15/2006	94.73745	2.1742%	2.1407%
8/15/2006	92.89697	2.4711%	2.4498%
2/15/2007	90.7564	2.7905%	2.7175%
8/15/2007	88.74181	3.0084%	2.9606%
2/15/2008	86.47913	3.2544%	3.1997%
8/15/2008	84.15589	3.4799%	3.4098%

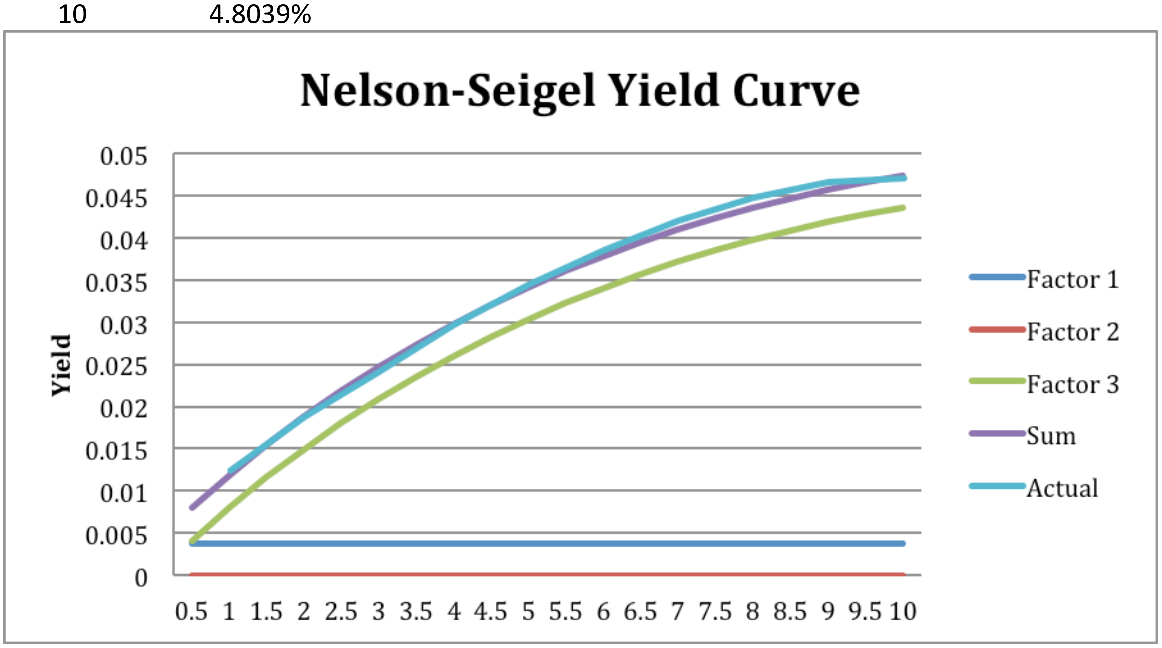


The discrepancy between the two curves is small. It is true in general that the difference between the spot curve and yield curve will be larger when the yield curve is steeper, with the spot curve above the yield curve when the yield curve slopes up. There will also be a large discrepancy when high-coupon bonds are used to calculate the YTM curve.

3.
(a)
The parameter estimates of the Nelson-Siegel using the least square method yields

theta 0	theta 1	theta 2	lambda
0.003803082	3E-05	0.161234489	9.373235454

Maturity	N-S (b.e.)
0.5	0.7999%
1	1.1879%
1.5	1.5494%
2	1.8858%
2.5	2.1985%
3	2.4887%
3.5	2.7578%
4	3.0070%
4.5	3.2373%
5	3.4500%
5.5	3.6459%
6	3.8261%
6.5	3.9916%
7	4.1431%
7.5	4.2816%
8	4.4078%
8.5	4.5224%
9	4.6262%
9.5	4.7198%
10	4.8039%



The resulting theoretical price of the two bonds, discounting coupons at the corresponding **effective** model spot rate, is given in the table below. Also listed is the actual price, the yield to maturity (based on the actual price, and the modified duration, calculated with the convexity4.xls calculator.

	Actual Price	Model Price	YTM (b.e.)	modified duration
1-Year	100.9254	100.931	1.186%	0.9889
3-Year	99.7848	99.736	2.467%	2.878

b. Based on these calculations, the 1-year bond is cheap and the 3-year bond is expensive. The trading strategy that represents a bet that prices move more in line with the model price is to go long the 1-year and short the 3-year.

c. Even if relative rates move to be closer in line with the model predictions, you are exposed to price risk if the sensitivity of the two positions to overall changes in the level of interest rates is not hedged. By equating the hedge ratios of the long and short positions, you are protected against parallel shifts of the yield curve because the positions have offsetting sensitivity. For example, per $1000 of the 1-year bond bought, short X of the 3-year such that $1000(.9889) = X(2.878). Then X = $343.61. This is not a perfect hedge against interest rate changes. For instance, if the yield curve flattens you will lose money.

4. Note that in computing forward rates to use for discounting, you always must do the calculations using effective rates for the relevant time period. Otherwise the formulas will not give answers that correspond to the rates you can actually lock in using a synthetic forward (that is, going long and short at the same time in the cash market). The effective 1 year forward rates reported below are the result of compounding two adjacent 1 six-month effective forward rates.

Maturity		N-S yield (b.e.)	Forward rates (effective 6-mo)	Forward rates (effective annual)	Forward rates (b.e.)
2/15/04	1	0.800%	0.788%		
8/15/04	2	1.188%	1.137%	2.603%	2.586%
2/15/05	3	1.549%	1.449%		
8/15/05	4	1.886%	1.727%	3.734%	3.700%
2/15/06	5	2.198%	1.973%		
8/15/06	6	2.489%	2.190%	4.622%	4.570%
2/15/07	7	2.758%	2.380%		
8/15/07	8	3.007%			

The implied forward rates are compared to the realized 1-year rates for the corresponding period (as given in the assignment) in the last two columns. (So compare the prediction of 2.59% based on forward rates with the 2.01% actual rate for 2004; 3.70% with actual 3.91% for 2005; 4.57% with actual 5.11% for 2006.) Although the model-implied forward rates do not equal the realized rates, the errors are not large and do not appear to be systematic.