

**Brief Answers to the Practice Midterm Questions**

1. Dollar return:  $\$1(1 + .052(183/360)) = \$1.02643$

Hedged yen return  $\$1(1+.0017(183/360))116.65/113.74 = \$1.02647$

The returns are the same to 4 decimal places, suggesting no arbitrage opportunity, especially considering the transactions costs associated with the two currency contracts. (I also accepted the answer that the covered yen return is slightly higher.)

2a.

Term	Effective 6-mo	Yield (b.e.b.)
6 months	2.80%	5.60%
1 year	3.30%	6.60%
1.5 years	3.60%	7.20%
2 years	3.80%	7.60%

b.  $p = 5/(1.033)^2 + 105/(1.038)^4 = 95.13384$

effective annual yield solves  $95.13384 = 5/(1+y) + 105/(1+y)^2$

y (effective annual) = 7.718%

y (b.e.b.) = 7.575%

c. coupon payment =  $.067(\$1\text{mm})/2 = \$33,500$

To price bond need  ${}_2f_1$  and  ${}_3f_1$ :

$(1+{}_2f_1)=(1.036)^3/(1.033)^2 = 1 + .04203$

$(1+{}_3f_1)=(1.038)^4/(1.036)^3 = 1 + .04402$

Forward price =  $\$33,500/(1.04203) + \$1,033,500/[(1.04203)(1.04402)] = \$982,144.80$

3a. All are true.

b. A barbell strategy will generally do worse. This is because the relatively large increase in long-term rates, in combination with the greater compounding period, reduces the value of the payments at the long end of the barbell by a disproportionately large amount compared to the intermediate payment of the bullet.

4a. Notice that the yield input on a b.e. basis is 7.846%, not 8%

Mod. Duration 5-yr: 3.916                      Convexity 5-yr: 20.484

Mod. Duration 8 yr: 7.407                      Convexity 8-yr: 61.728

Mod. Duration port:  $(2/5)3.916+(3/5)7.407 = 6.011$

Convexity portfolio:  $(2/5)20.484+(3/5)61.728 = 45.23$

Mod. Duration 4-yr: 3.33                      Convexity 4-yr: 14.857

Mod. Duration 12-yr: 7.865                      Convexity 12-yr: 83.208

b. Short position in either: with 4-yr bond,  $(5)(6.011)/3.33 = \$9.0255$  million; with 12-yr bond  $(5)(6.011)/7.865 = \$3.8214$  million

c. To create a synthetic short, enter a reverse RP with a principal value of \$9.0255 million. You lend this amount, receive the 4-year bond as collateral, and sell the 4-year bond received for \$9.0255 million. The next day you receive  $\$9.0255(1+.072/360) = \$9.0273$  million from repayment of the RP loan, and must return the collateral. If interest rates have gone up, the cost of buying the bond to be returned will be lower than the price locked in on the reverse RP, by an amount approximately equal to the decrease in value of your \$5 million portfolio. For instance, if rates go up by 1%, your portfolio value falls by approximately  $\$5m(6.011)(.01)$ , and the value of the collateral bond falls by  $\$9.0255m(3.33)(.01)$ , an equal amount.

d. The 4-year bond may be preferred on the grounds that it has a lower convexity, (less negative convexity from the perspective of the short). This makes it probable that the long position will gain relative to the short position if rates change.

5. a.  $((1+.0315(119/360))/(1+.0275(29/360)) - 1)(360/90) = .03272$ , or 3.272%

b. The June forward contract locks in a 90-day rate with similar underlying risk starting at the same time.  $100-96.73 = 3.27$

c. The rates are essentially the same (certainly within any small transaction cost), so there is no arbitrage opportunity. If there were a discrepancy, you would set up a short position using the security with the lower rate, and an offsetting long position based on the same principal amount to be delivered in June in the security with the higher rate. The difference would represent an arbitrage profit.

6. 4.a.

M. duration of 3 year bond: 2.861

Convexity of 3 year bond: 9.778

M. Duration of 7 year bond: 6.8898

Convexity of 7 year bond: 50.859

b. Steepening means the yield will go up on the 7-year bond relative to the 3-year bond, and its price will fall. Hence you want to short the 7-year and long the 3-year. More precisely, take a long position of \$1m in the 3-year bond. Delta hedge by taking a short position that solves:

$$\$1m(2.861) = P(6.8898) \Rightarrow P = \$.415252m$$

c. The convexity of your short position is larger than that of your long position, which means that volatility works against you—your next position has negative convexity.