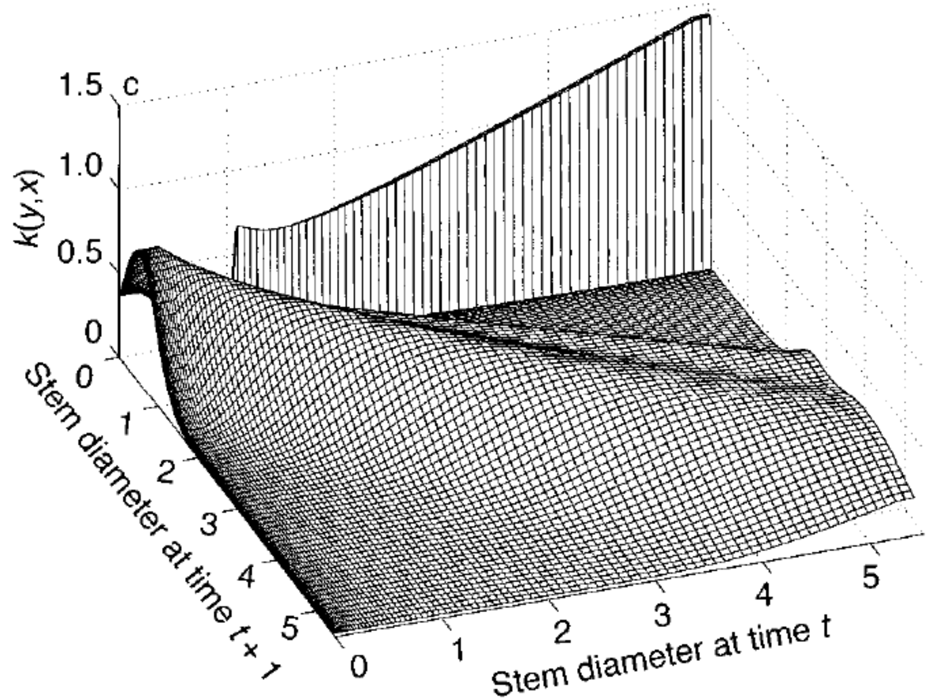
Basics of plant population modelling and its application



Pieter Zuidema, Pieter.zuidema@wur.nl

Goals

Learning outcomes

- Basic knowledge of matrix models and Integral Projection Models (IPMs)
- Able to construct IPMs
- Able to run matrix models & IPMs
- Able to interpret model output
- Able to use 3 relevant R-packages

Programme

Monday**: Matrix models**

Tuesday**: Integral Projection Models: construction**
Plus first paper discussion

Wednesday**: Integral Projection Models: output**
Plus: second paper discussion

Thursday**: Integral Projection Models: more applications**
Plus: preparing presentations

Friday**: Presentations**

Programme

Monday February 10th: Matrix models

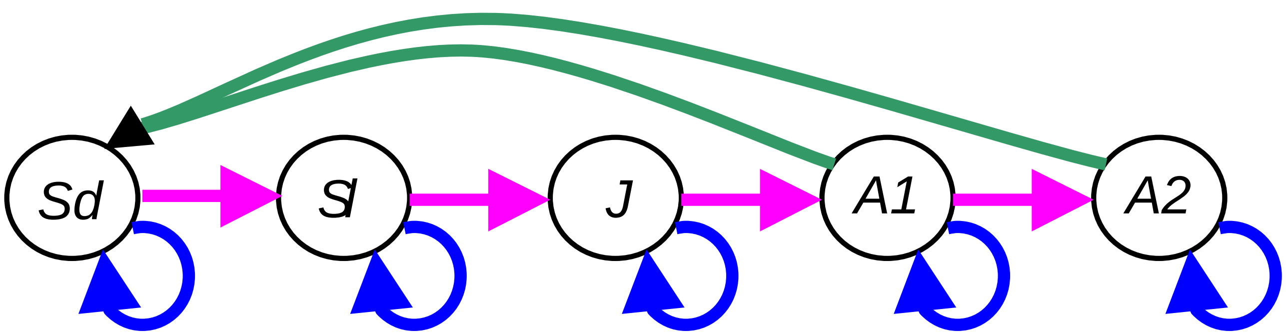
9-10.30	Lectures: matrix models
10.30-12	Exercises: Matrix model general output acai
12	Lunch
2-4	Exercises: Matrix model mahogany, logging, compadre
4-5	Discuss results of exercises

Computer practical

- Rstudio
- Three packages: popbio, popdemo & ipmr
- Example data provided
- Work from R-code
- Make sure you understand the code you are running

Lecture: Intro matrix models

- Applications
- Basics
- Asymptotic growth rate
- Elasticity analysis
- Introduction to example species



Applications matrix models

- Fundamental questions
  - Critical life stages?
  - Evolutionary advantage of life history strategy
  - Importance of seed input for survival?
- Conservation of threatened species
  - What are critical life stages?
  - What is extinction risk?

Applications matrix models

- Exploited species
  - What is effect on future populations?
  - What is available for next harvest?
- Fragmented species
  - What is viability of fragmented species
  - How important is seed input?
- Invasive species
  - What is the invasion speed?
  - What phases contribute most to this?

https://compadre-db.org

HOME DATA EDUCATION RESOURCES CONTRIBUTE DATA ABOUT HELP LINKS

	Taxonomic Species	Studies	Matrix Population Models
Plants	760	643	8851
Animals	415	395	3317

Species Map

https://www.compadre-db.org/

Lecture: Intro matrix models

- Applications
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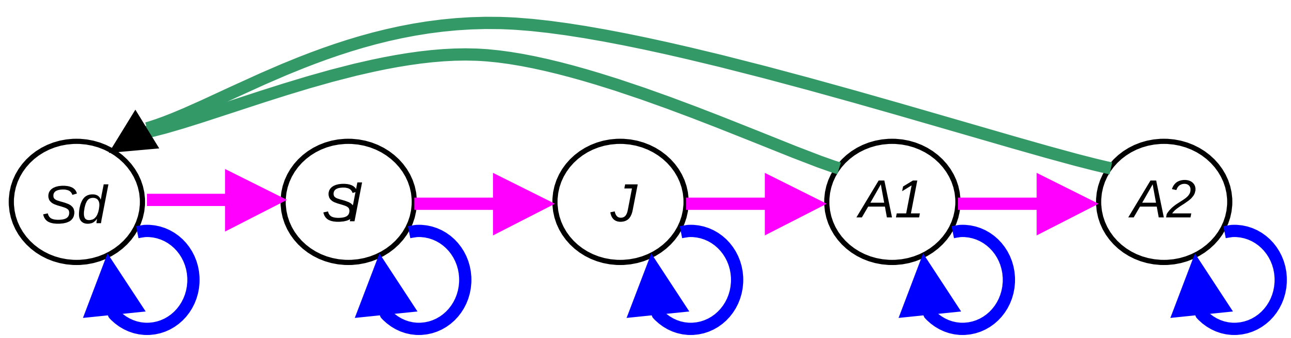
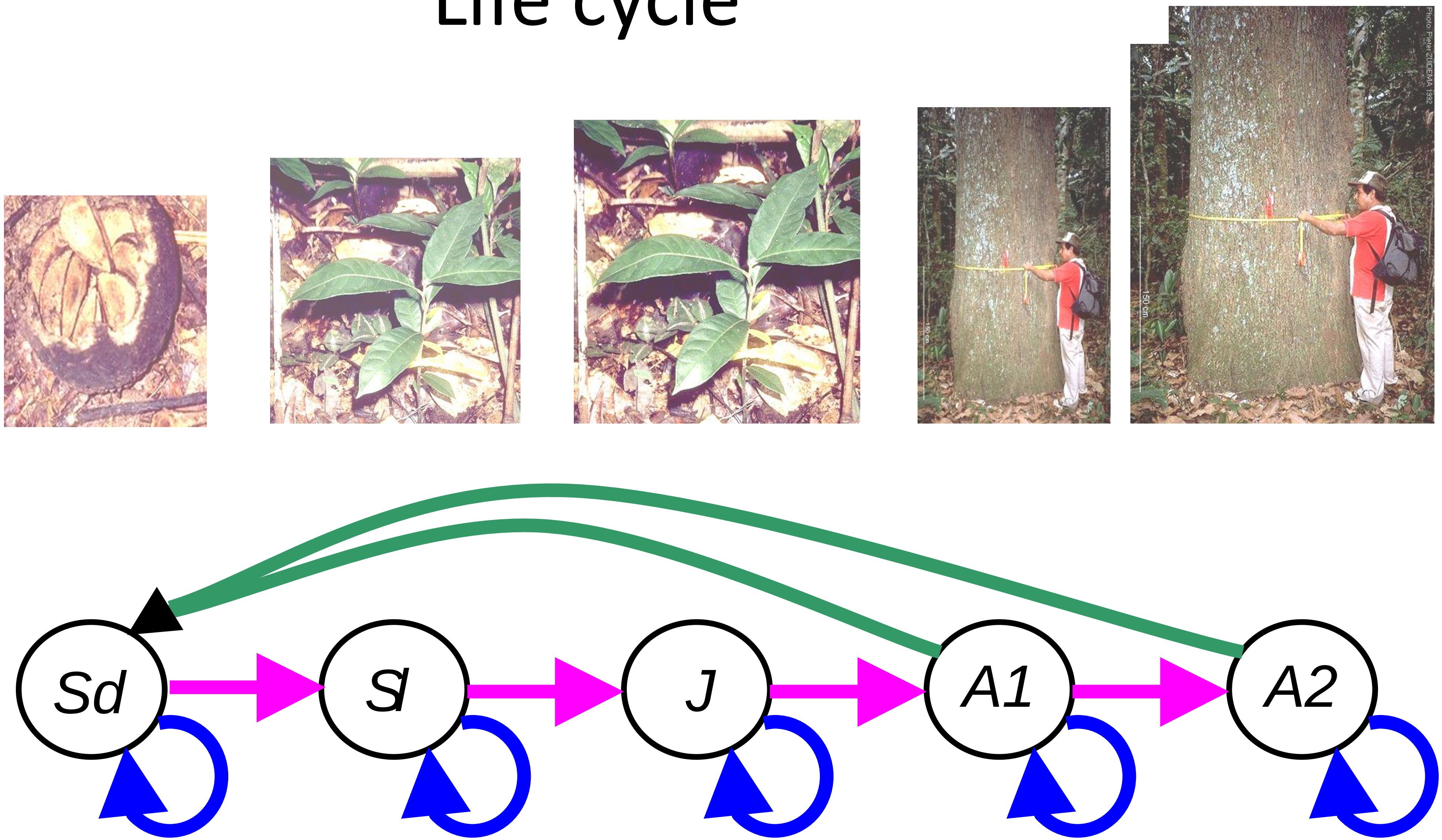
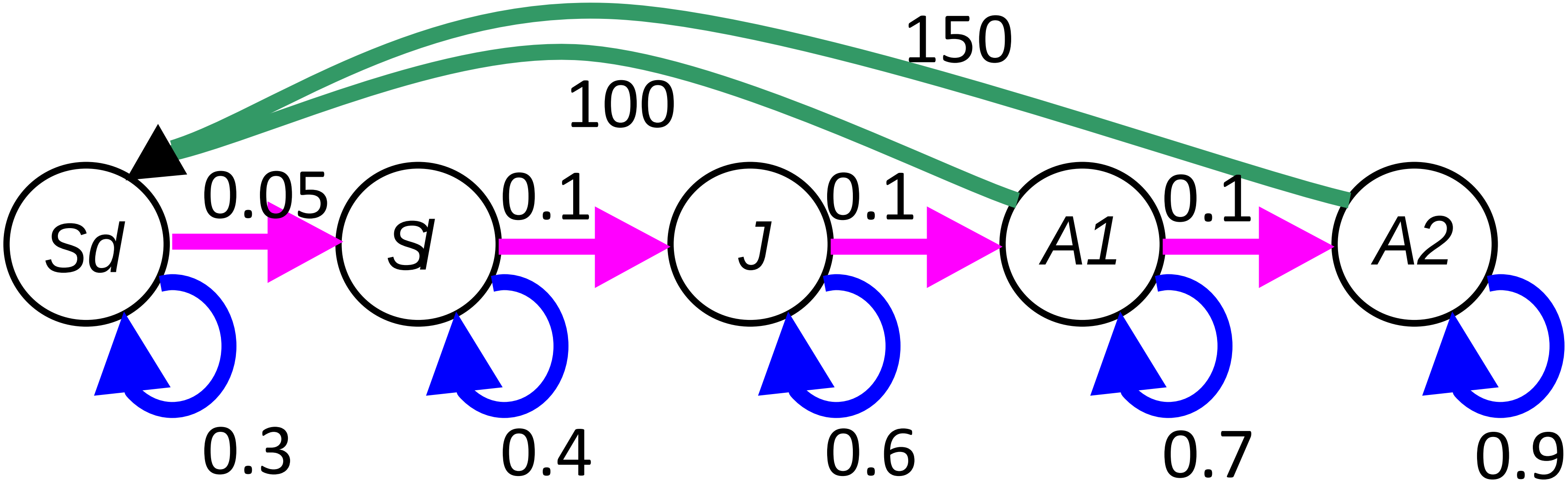


Making size categories

Sd *Sl* *J* *A1* *A2*

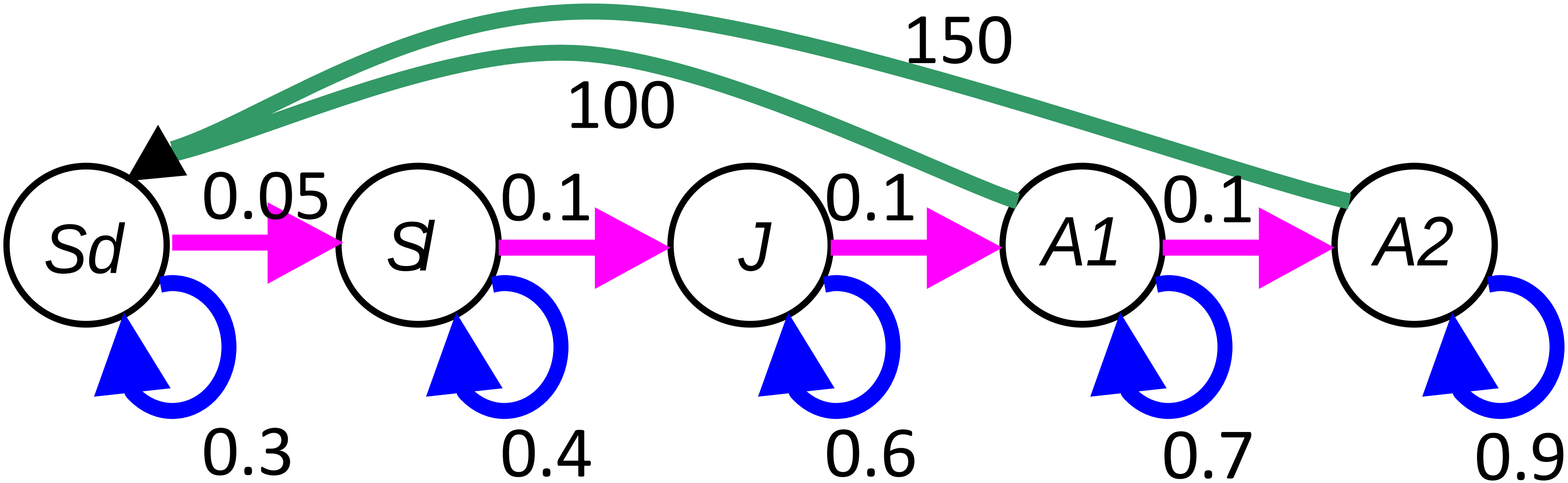
Life cycle





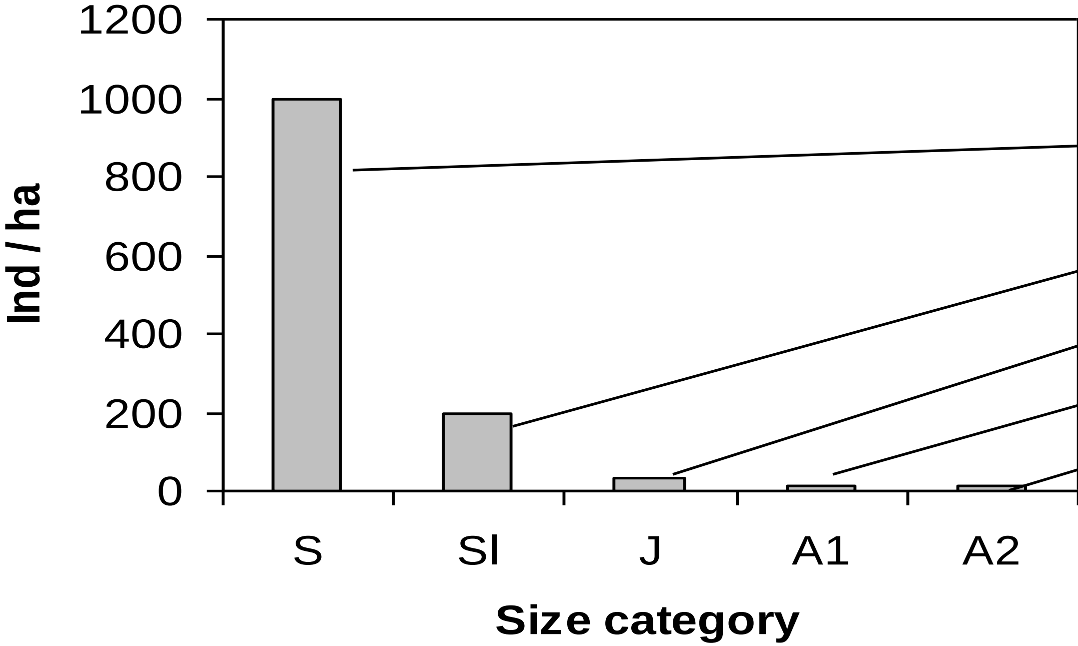
At time = t

At time = t+1	*Sd*	*Sl*	*J*	*A1*	*A2*
Sd					
Sl					
J					
A1					
A2					



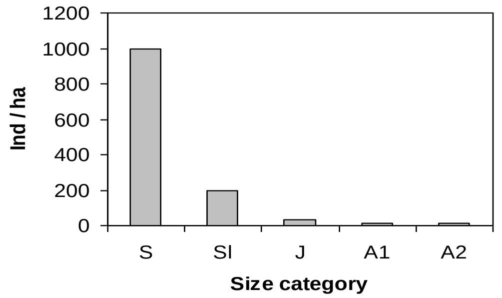
$$
\begin{array}{r c}
 & \textit{At time} = t \\
 & \begin{array}{ccccc} \quad Sd \quad & \quad Sl \quad & \quad J \quad & \quad A1 \quad & \quad A2 \quad \end{array} \\
\textit{At time} = t+1 \quad \begin{array}{r} Sd \\ Sl \\ J \\ A1 \\ A2 \end{array} & \begin{pmatrix} 0.3 & 0 & 0 & 100 & 150 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0 & 0.1 & 0.6 & 0 & 0 \\ 0 & 0 & 0.1 & 0.7 & 0 \\ 0 & 0 & 0 & 0.1 & 0.9 \end{pmatrix}
\end{array}
$$

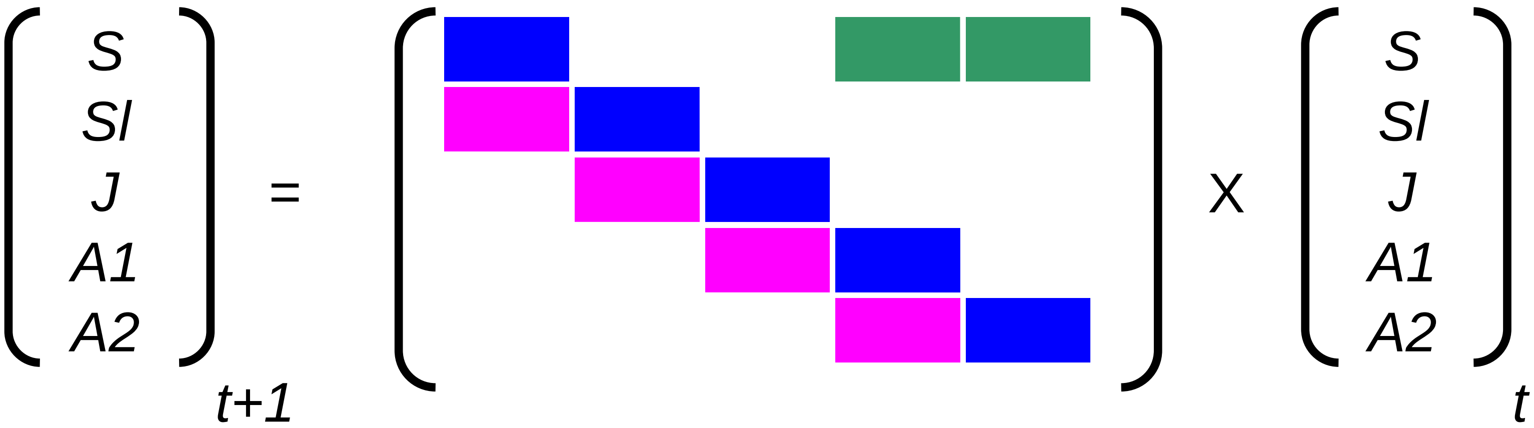
Population vector

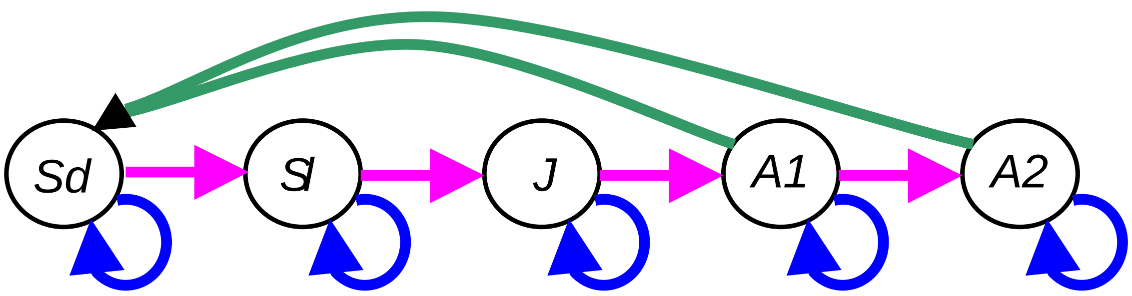


$$\begin{pmatrix} S \\ Sl \\ J \\ A1 \\ A2 \end{pmatrix}_t = \boldsymbol{n}$$

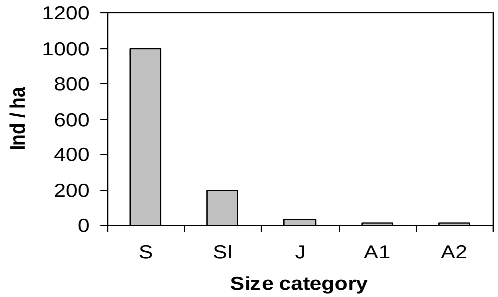
Projections

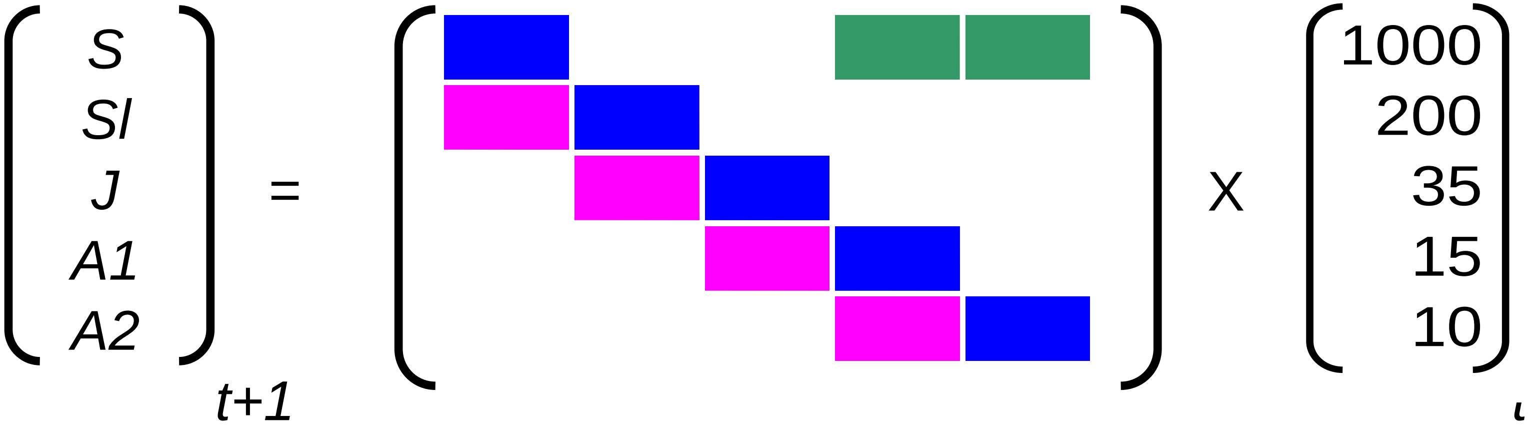


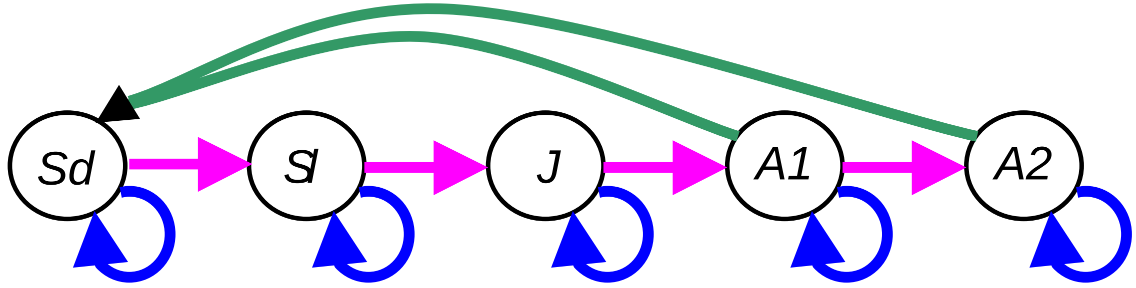




Projections

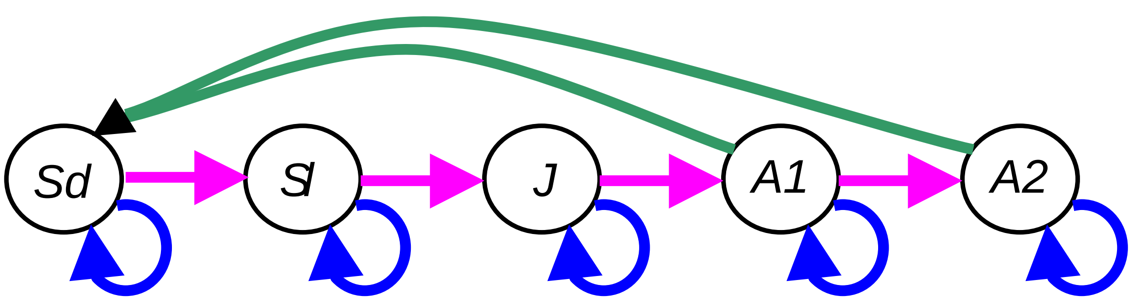






Projections

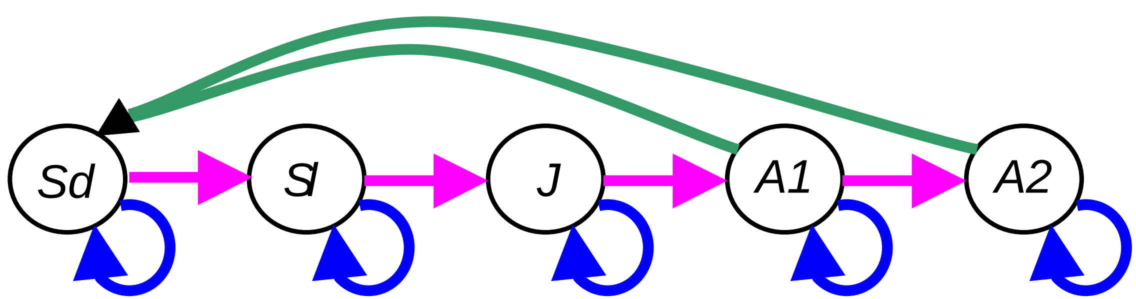
$$\begin{bmatrix} \\ \\ \\ \\ \\ \end{bmatrix} = \begin{pmatrix} \boxed{0.3 \quad 0 \quad 0 \quad 100 \quad 150} \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0 & 0.1 & 0.6 & 0 & 0 \\ 0 & 0 & 0.1 & 0.7 & 0 \\ 0 & 0 & 0 & 0.1 & 0.9 \end{pmatrix} \text{ X } \begin{bmatrix} 1000 \\ 200 \\ 35 \\ 15 \\ 10 \end{bmatrix}$$



Projections

$$
\begin{pmatrix} 3300 \\ \\ \\ \\ \\ \end{pmatrix} = \begin{pmatrix} \boxed{0.3 \quad 0 \quad 0 \quad 100 \quad 150} \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0 & 0.1 & 0.6 & 0 & 0 \\ 0 & 0 & 0.1 & 0.7 & 0 \\ 0 & 0 & 0 & 0.1 & 0.9 \end{pmatrix} \times \begin{pmatrix} 1000 \\ 200 \\ 35 \\ 15 \\ 10 \end{pmatrix}
$$

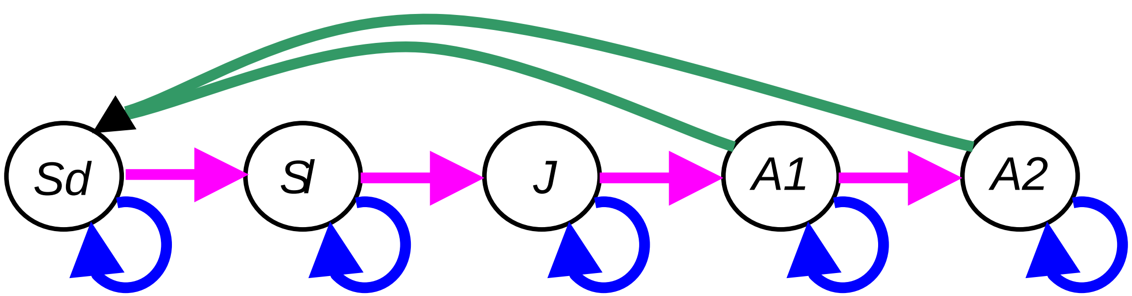
$Sd_{t+1} = 0.3*1000 + 0*200 + 0*35 + 100*15 + 150*10 = 3300$



Projections

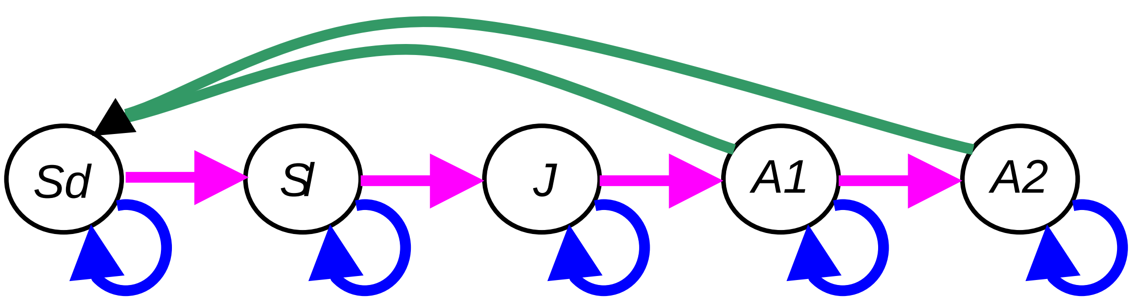
$$\begin{pmatrix} \\ \\ \\ \\ 10.5 \end{pmatrix} = \begin{pmatrix} 0.3 & 0 & 0 & 100 & 150 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0 & 0.1 & 0.6 & 0 & 0 \\ 0 & 0 & 0.1 & 0.7 & 0 \\ \boxed{0} & \boxed{0} & \boxed{0} & \boxed{0.1} & \boxed{0.9} \end{pmatrix} X \begin{pmatrix} 1000 \\ 200 \\ 35 \\ 15 \\ 10 \end{pmatrix}$$

$$A2_{t+1} = 0*1000 + 0*200 + 0*35 + 0.1*15 + 0.9*10 = 10.5$$

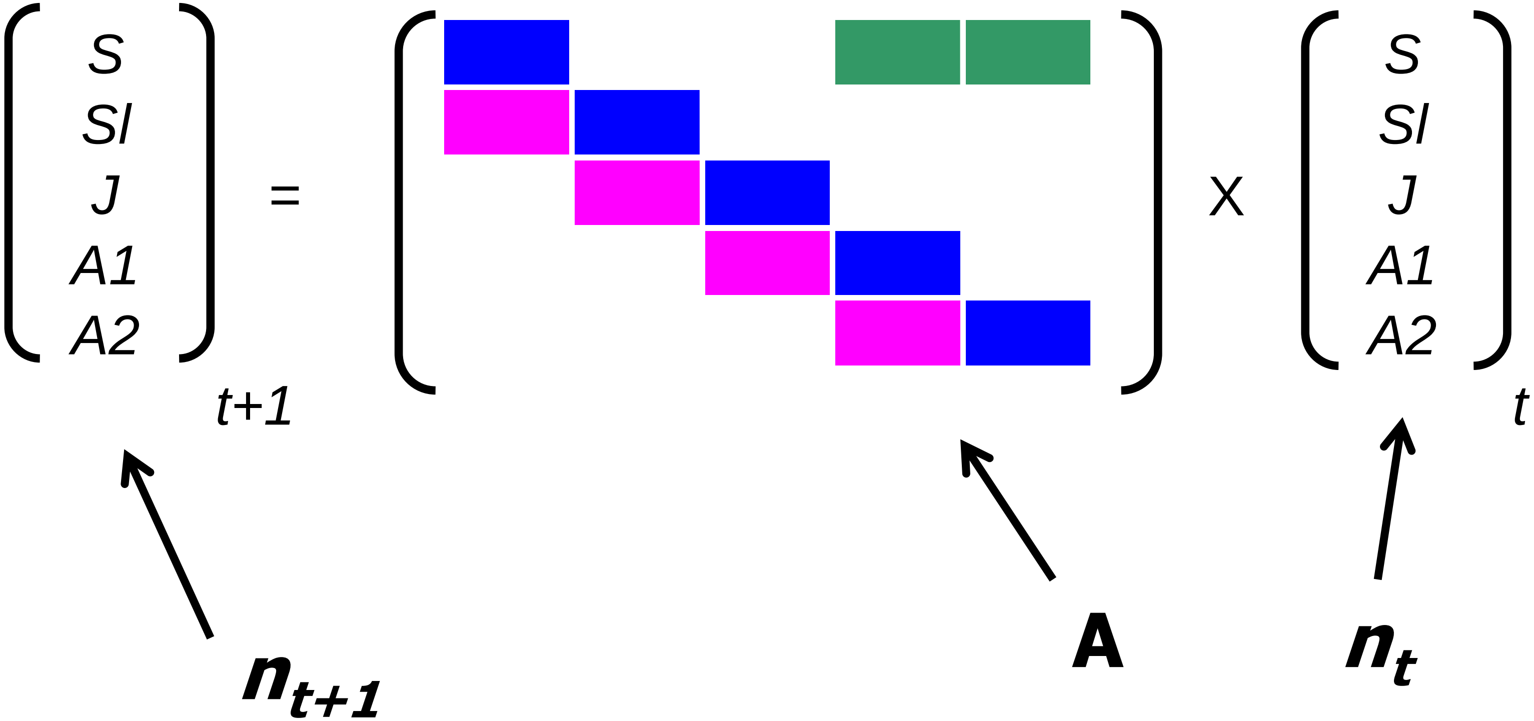


Projections

$$\begin{pmatrix} 3300 \\ 130 \\ 41 \\ 14 \\ 10.5 \end{pmatrix} = \begin{pmatrix} 0.3 & 0 & 0 & 100 & 150 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0 & 0.1 & 0.6 & 0 & 0 \\ 0 & 0 & 0.1 & 0.7 & 0 \\ 0 & 0 & 0 & 0.1 & 0.9 \end{pmatrix} \times \begin{pmatrix} 1000 \\ 200 \\ 35 \\ 15 \\ 10 \end{pmatrix}$$

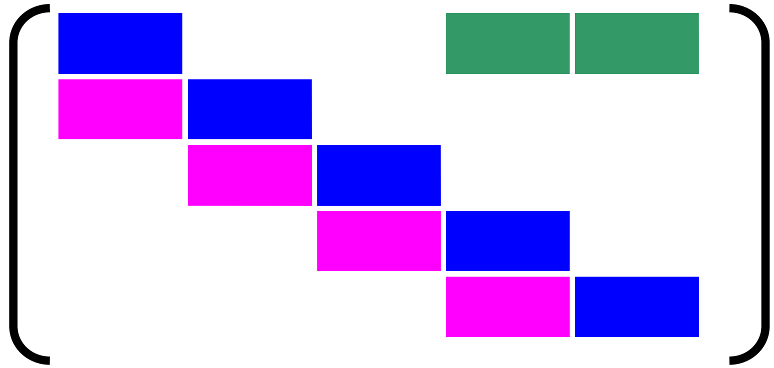


Transient population growth



Transient population growth

$$\begin{pmatrix} S \\ SI \\ J \\ A1 \\ A2 \end{pmatrix}_{t+1} =$$



$$\times \begin{pmatrix} S \\ SI \\ J \\ A1 \\ A2 \end{pmatrix}_{t}$$

$$n_{t+1} = \mathbf{A} \times n_t$$

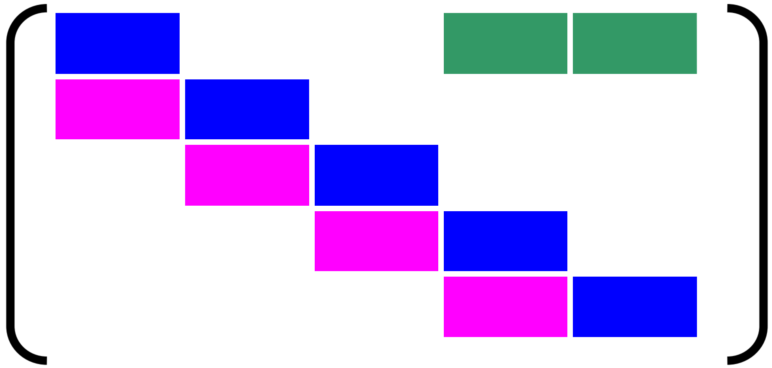
$$n_{t+2} = \mathbf{A} \times n_{t+1}$$

$$n_{t+3} = \mathbf{A} \times n_{t+2}$$

etc ...

Transient population growth

$$\begin{pmatrix} S \\ SI \\ J \\ A1 \\ A2 \end{pmatrix}_{t+1} = \mathbf{A} \times \begin{pmatrix} S \\ SI \\ J \\ A1 \\ A2 \end{pmatrix}_{t}$$



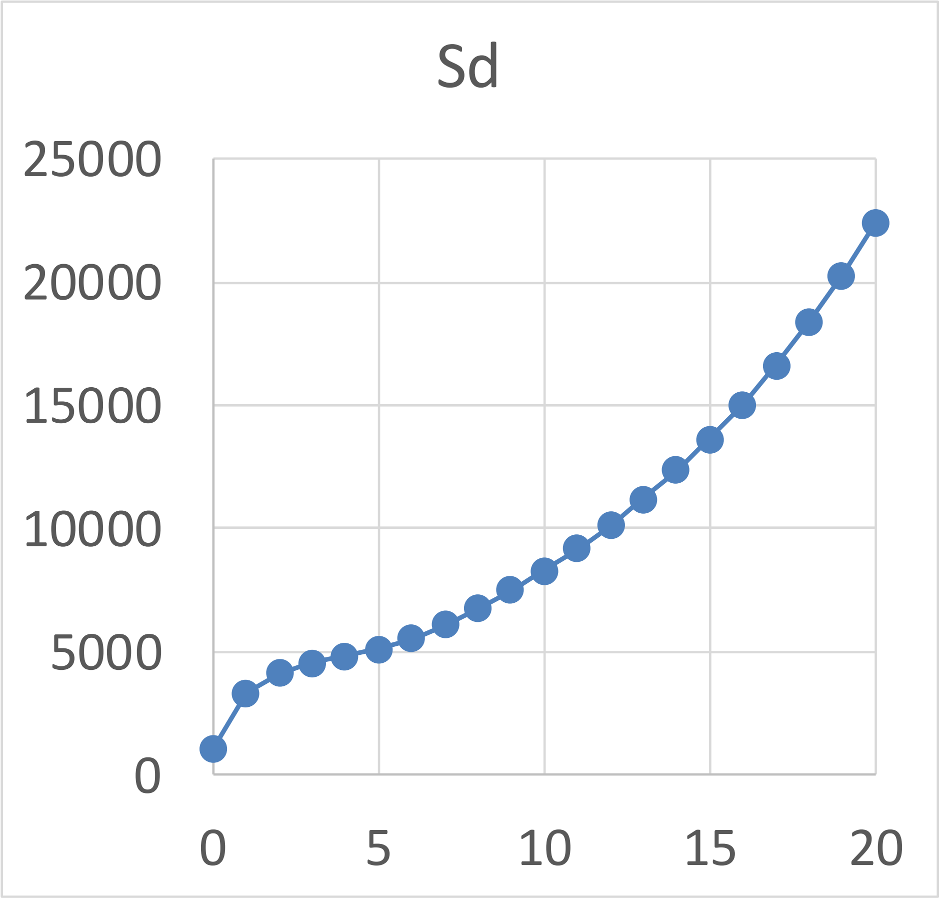
$$n_{t+1} = \mathbf{A} \times n_t$$

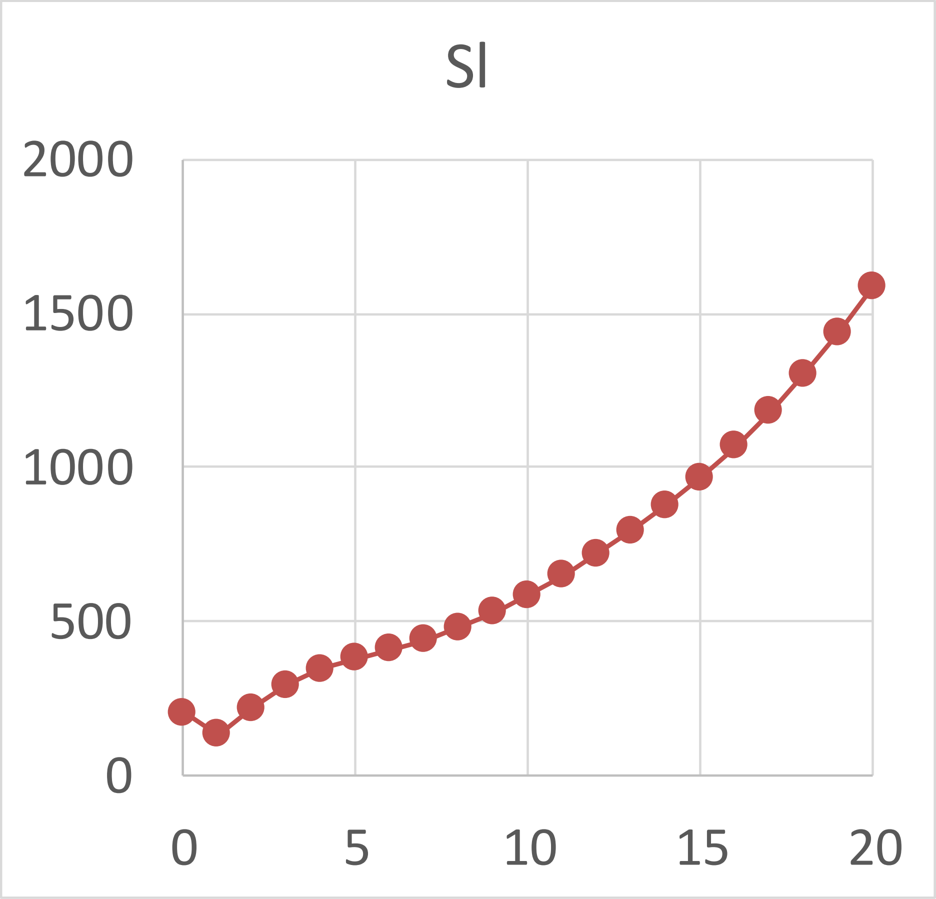
$$n_{t+2} = \mathbf{A}^2 \times n_t$$

$$n_{t+3} = \mathbf{A}^3 \times n_t$$

etc ...

Projections per category





What are the assumptions?

Transitions do not change over time

- No stochastic variation included
- No exploitation or disturbance
- No gradual change (e.g, climate change)

Transitions independent on population size

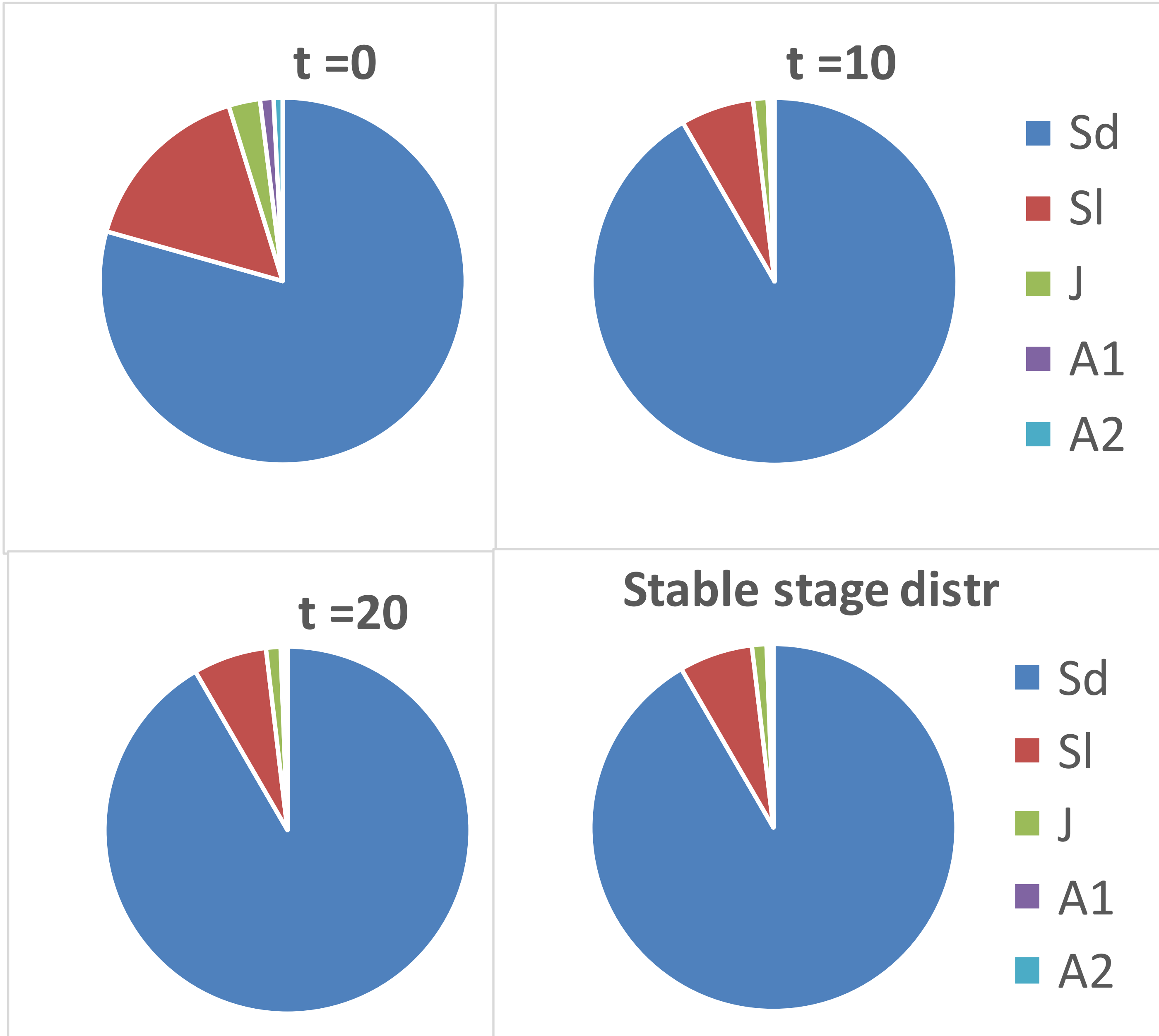
- No ‘density-dependence’
- Exponential population growth (or decline)

So: projections, not predictions

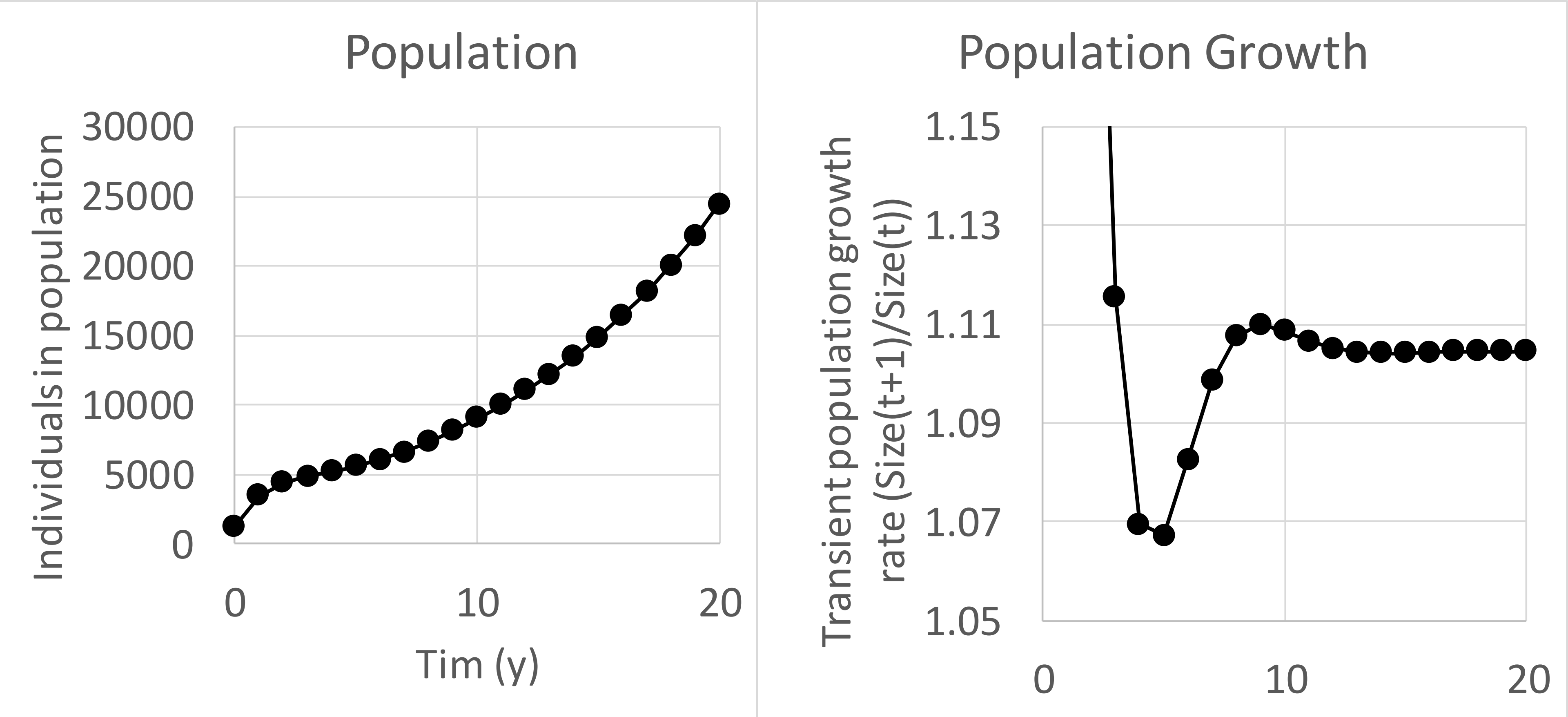
Transient population growth

- Transient population dynamics = projecting the population for some time into the future
- Transient population growth = change in population size during this period

Population structures over time



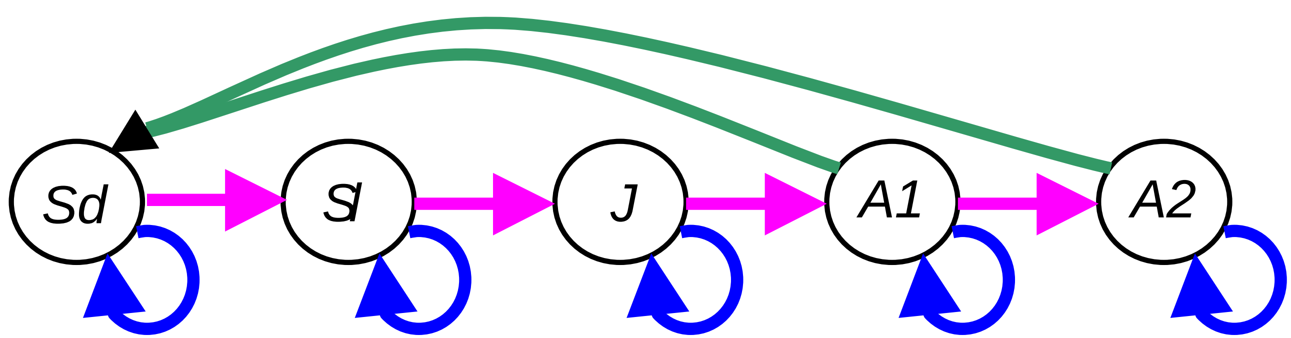
Population growth (rate) over time



$$\lambda = 1.104$$

Lecture: Intro matrix models

- Applications
- Basics
- Asymptotic growth rate
- Elasticity analysis
- Introduction to example species



Stable stage (or age) distribution

If t becomes large (formally if $t \to \infty$)

Then:

- Proportions in population vector are stable
- This is the stable (st)age distribution
- = <u>independent</u> of starting conditions
- = mathematical property of the <u>transition matrix</u>, the ‘right eigenvector’

Stable stage (or age) distribution

- The distribution of projected populations gradually ‘approaches’ the stable stage (or age) distribution
- The stable stage or age distribution is a <u>relative</u> distribution
- It is a mathematical property of the transition matrix.

Asymptotic population growth

If relative size structure of simulated population equals the stable stage distribution, then:

- Population grows at constant rate
- So, instead of writing

$$n_{t+1} \quad = \quad \mathbf{A} \quad \times \quad n_t$$

- We can write

$$n_{t+1} \quad = \quad \lambda \quad \times \quad n_t$$

Asymptotic population growth

Lambda (λ) = asymptotic population growth rate:

- >1: growth; <1: shrinkage; =1: stable
- Mathematical property of transition matrix **A**, the dominant eigenvalue
- Independent of starting population vector

	Stable stage distribution	Asymptotic population growth rate (λ)
Definition	• Relative population structure • Right eigenvector of transition matrix	• Population growth rate • When SSD is reached • Dominant eigenvalue of matrix

- Reached when t is large
- Mathematical properties transition matrix

Asymptotic population growth

Two important assumptions:

1. Transitions remain unchanged
    1. independent of population size
    2. Independent of 'environmental changes’ or disturbances or harvesting!!
2. Population structure is equal to (or very similar to) stable (st)age distribution

Transient vs. asymptotic dynamics

Transient

- Matrix projection
- Pop development
- For years or decades
- Starting pop structure important

Asymptotic

- After many decades/centuries
- Stable stage distribution
- Lambda
- Sensitivity/elasticity

Lecture: Intro matrix models

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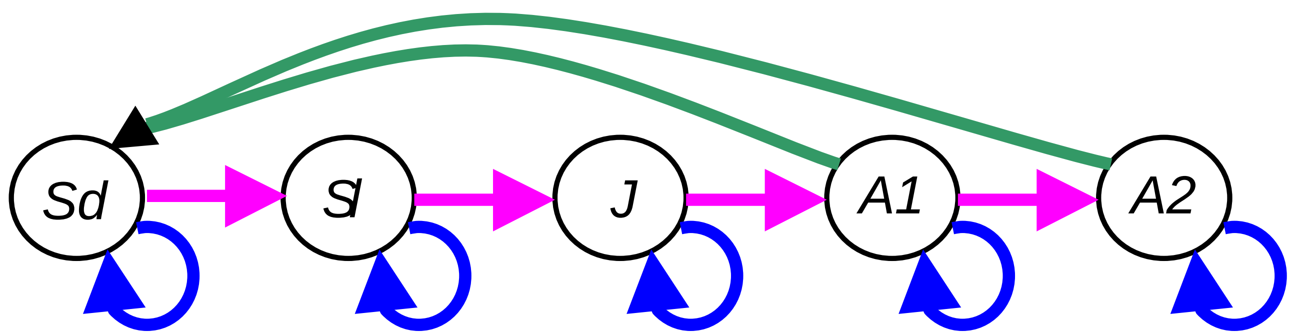


Sensitivity and elasticity

What transitions in transition matrix **A** are most important for λ? Or what stages are most important?

To find out, change the question:

- What would happen with the population growth rate (λ) if values of a transition changes?
- Or, how sensitive is λ to changes in values of **A**?

=> Sensitivity analysis & Elasticity analysis

Elasticity: numerically

- Add a very small proportional change to a transition
- Calculate the change in lambda
- Elasticity = (proportional change in lambda)/(proportional change in transition)
- So, proportional change in lambda due to a proportional change in a transition

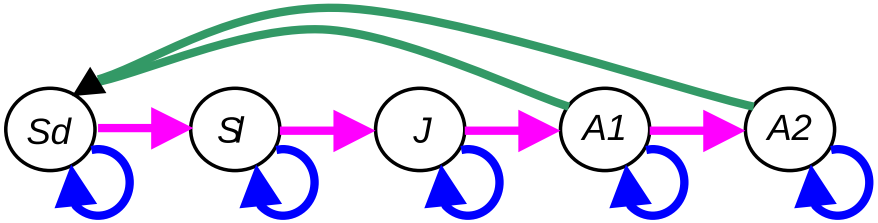
Elasticity: analytically

$$e_{ij} = \frac{\partial(\log \lambda)}{\partial(\log a_{ij})} = \frac{a_{ij}}{\lambda}\frac{\partial\lambda}{\partial a_{ij}}$$

$$s_{ij} = \frac{\partial\lambda}{\partial a_{ij}} = \frac{v_i w_j}{\langle \mathbf{w}, \mathbf{v} \rangle}$$

- v = Right eigenvector = stable stage distribution
- w = Left eigenvector = reproductive values
- <**w,v**> = scalar product of vectors
- Elasticity also depends on transition matrix

Elasticity: numerically



$$\begin{bmatrix} 0.3 & 0 & 0 & 100 & 150 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0 & 0.1 & 0.6 & 0 & 0 \\ 0 & 0 & 0.1 & 0.7 & 0 \\ 0 & 0 & 0 & 0.1 & 0.9 \end{bmatrix}$$

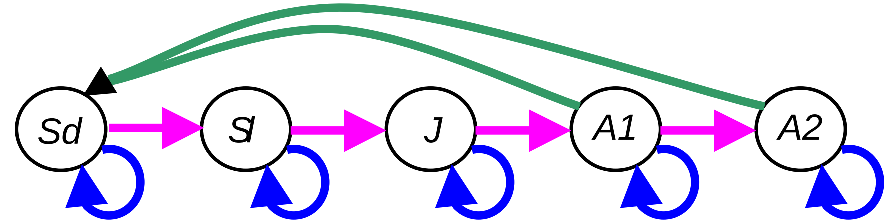
$\lambda_1 =$ **1.075**

a_{ij}

$$\begin{bmatrix} 0.3 & 0 & 0 & 110 & 150 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0 & 0.1 & 0.6 & 0 & 0 \\ 0 & 0 & 0.1 & 0.7 & 0 \\ 0 & 0 & 0 & 0.1 & 0.9 \end{bmatrix}$$

$\lambda_2 =$ **1.08** **e =** $(\delta\lambda/\lambda) / (\delta a/a)$

Elasticity: numerically



$$\begin{bmatrix} 0.3 & 0 & 0 & 100 & 150 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0 & 0.1 & 0.6 & 0 & 0 \\ 0 & 0 & 0.1 & 0.7 & 0 \\ 0 & 0 & 0 & 0.1 & 0.9 \end{bmatrix}$$

$\lambda_1 =$ **1.075**

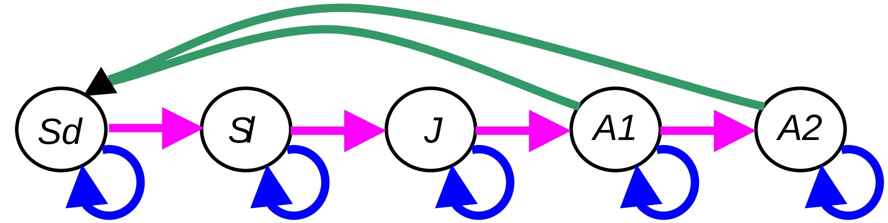
a_{ij}

$$\begin{bmatrix} 0.3 & 0 & 0 & 110 & 150 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0 & 0.1 & 0.6 & 0 & 0 \\ 0 & 0 & 0.1 & 0.7 & 0 \\ 0 & 0 & 0 & 0.1 & 0.9 \end{bmatrix}$$

$\lambda_2 =$ **1.08**

$$\begin{aligned} e &= (\delta\lambda/\lambda) \,/\, (\delta a/a) \\ &= (0.005/1.075)/(10/100) \\ &= 0.05 \end{aligned}$$

Elasticity: numerically



$$\begin{pmatrix} 0.3 & 0 & 0 & 100 & 150 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0 & 0.1 & 0.6 & 0 & 0 \\ 0 & 0 & 0.1 & 0.7 & 0 \\ 0 & 0 & 0 & 0.1 & 0.9 \end{pmatrix} \quad \lambda_1 = \mathbf{1.075}$$

a_{ij}

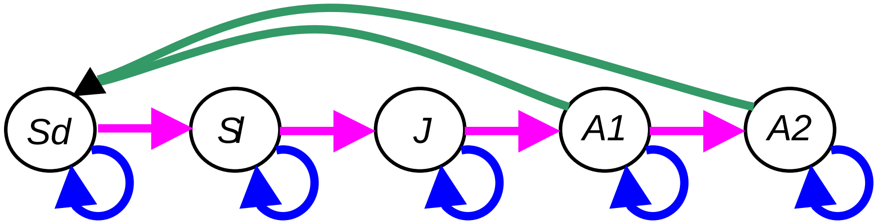
$$\begin{pmatrix} 0.3 & 0 & 0 & 110 & 150 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0 & 0.1 & 0.6 & 0 & 0 \\ 0 & 0 & 0.1 & 0.7 & 0 \\ 0 & 0 & 0 & 0.1 & 0.9 \end{pmatrix} \quad \lambda_2 = \mathbf{1.08}$$

$$\begin{aligned} e &= (\delta\lambda/\lambda) / (\delta a/a) \\ &= (0.005/1.075)/(10/100) \\ &= 0.05 \end{aligned}$$

$$\begin{pmatrix} 0.3 & 0 & 0 & 100 & 150 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0 & 0.1 & 0.6 & 0 & 0 \\ 0 & 0 & 0.1 & 0.7 & 0 \\ 0 & 0 & 0 & 0.1 & 0.99 \end{pmatrix} \quad \lambda_3 = \mathbf{1.104}$$

$$\begin{aligned} e &= (\delta\lambda/\lambda) / (\delta a/a) \\ &= (0.03/1.075)/(0.09/0.9) \\ &= 0.28 \end{aligned}$$

Elasticity: numerically



a_{ij}

Transition matrix

0.3	0	0	100	150
0.05	0.4	0	0	0
0	0.1	0.6	0	0
0	0	0.1	0.7	0
0	0	0	0.1	0.9

e_{ij}

Elasticity matrix

0.04	0	0	0.05	0.04
0.09	0.05	0	0	0
0	0.09	0.12	0	0
0	0	0.09	0.17	0
0	0	0	0.04	0.22

Elasticity analysis

- What processes or classes are most important for population growth?
- Based on asymptotic dynamics, so population at stable stage structure
- Calculated from mathematical properties of matrix

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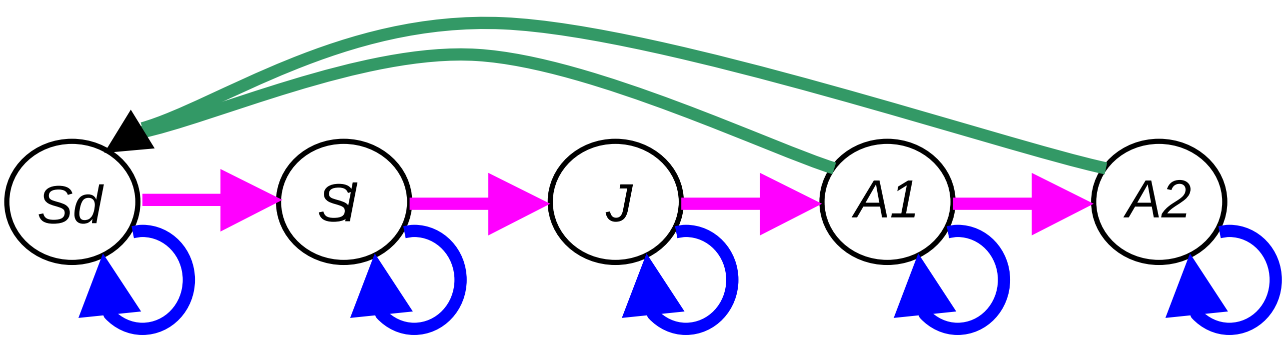
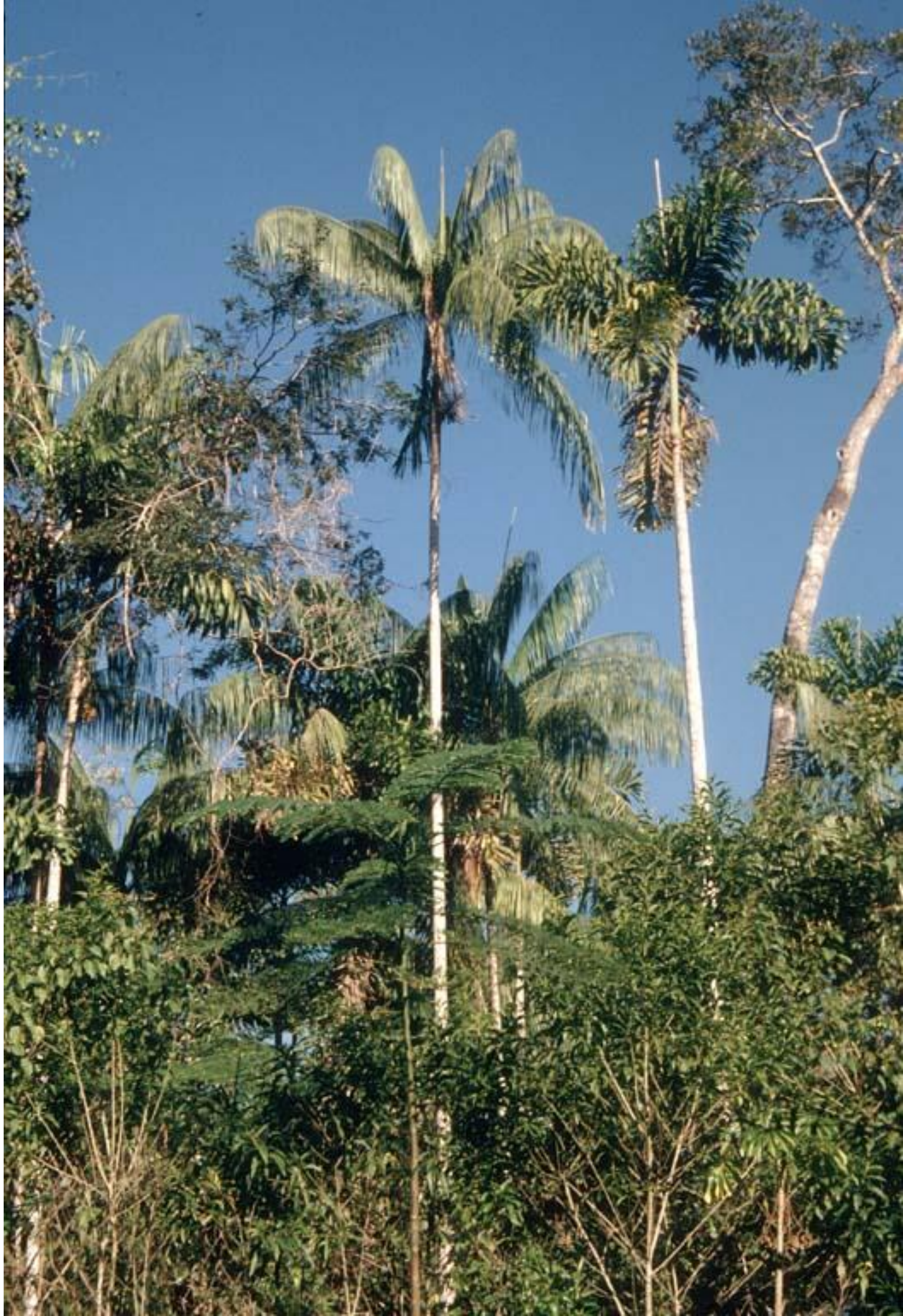


Acai / palmito
Euterpe pracatoria

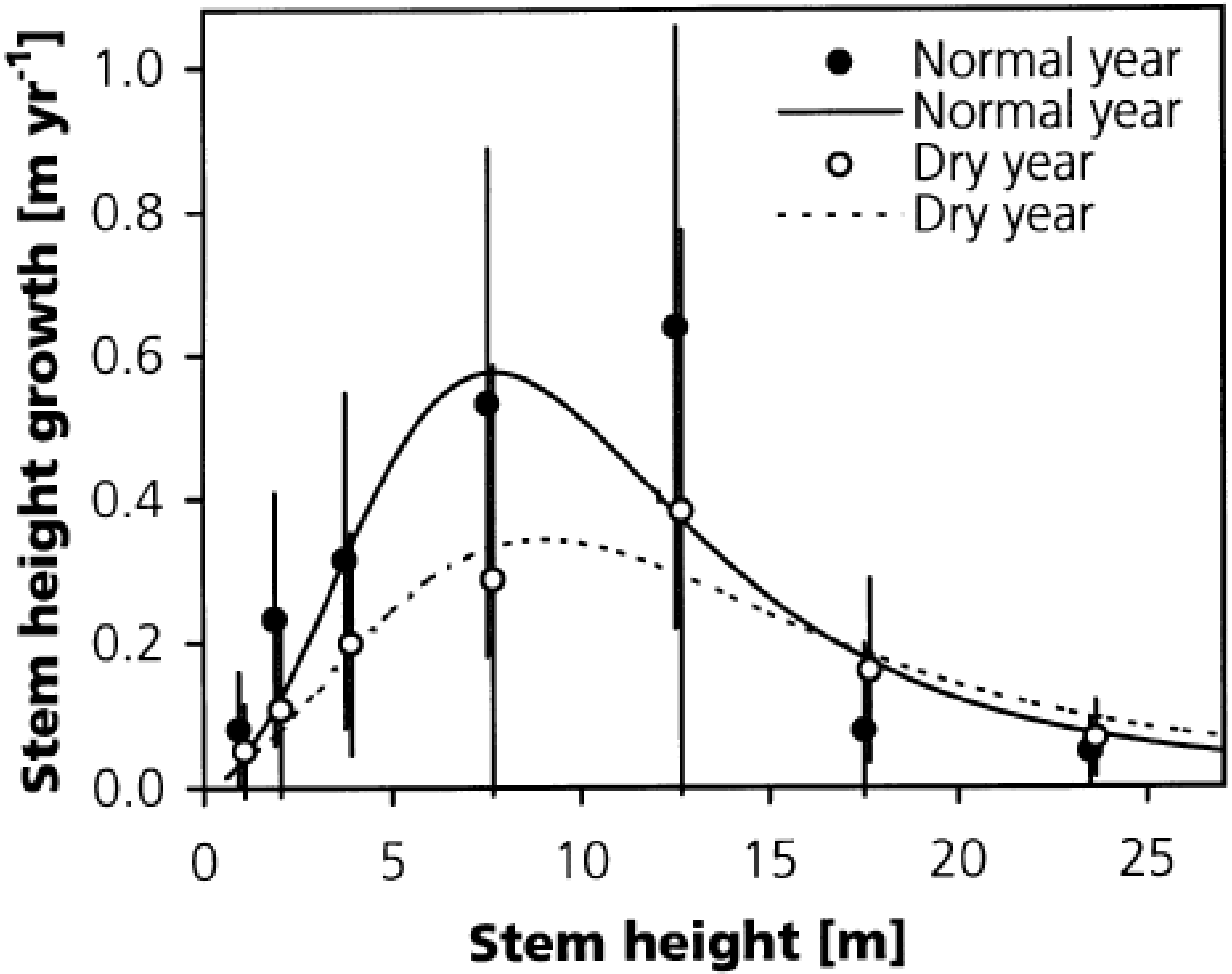




Acai *Euterpe precatoria,* Bolivia

- 6 ha, seedlings in subplots
- 1997-1999
- ~800 individuals
- Height to base last leaf
- No seedlings included to keep things simple
- Survival, growth, recruitment, reproductive status

Acai *Euterpe precatoria,* Bolivia



Mahogany *Swietenia macrophylla*

Journal of Applied Ecology

British Ecological Society

Journal of Applied Ecology 2008 doi: 10.1111/j.1365-2664.2008.01564.x

Silviculture enhances the recovery of overexploited mahogany *Swietenia macrophylla*

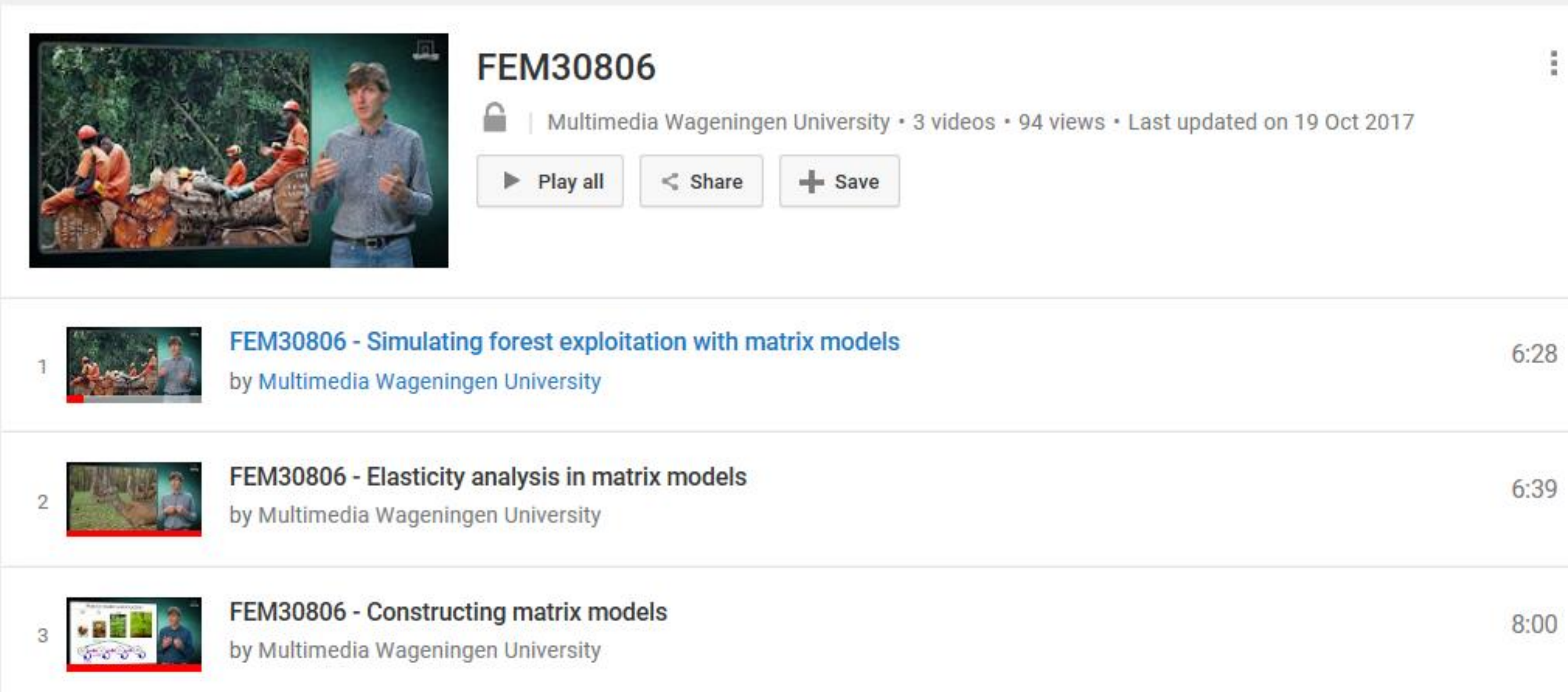
Caspar Verwer[1,2], Marielos Peña-Claros[1,2]*, Daniël van der Staak[1,2], Kristen Ohlson-Kiehn[1] and Frank J. Sterck[2]

[1]*Instituto Boliviano de Investigación Forestal, PO Box 6204, Santa Cruz de la Sierra, Bolivia; and* [2]*Forest Ecology and Forest Management Group, Centre for Ecosystem Studies, Wageningen University, PO Box 47, 6700 AA Wageningen, The Netherlands*

Mahogany *Swietenia macrophylla*

- Silvicultural experiments in Bolivia
- C: control; N: normal; LS: light silvicultural treatments, HS: high silvicultural treatmens
- *Swietenia macrophylla*: mahogany, overexploited
- Can it recover? And does silviculture help?
- So, population models
- Each logging cycle: harvest matrix, then treatment matrix for x year, then control matrix

Knowledge clips



https://www.youtube.com/results?search_query=fem30806

Terms

- Transition matrix = table with transitions between age or size categories
- Matrix model = in fact the transition matrix
- Transitions = probability of individuals moving from one to another category or staying in the same category; or the amount of new individuals produced by an individual
- Vital rates = rates of growth, survival, reproduction, germination
  - are used to calculate transitions
- Population structure = the distribution of individuals in the population over age or size classes

Terms

- Population projection = calculations to simulate the distribution and size of the population in the future
- Transient dynamics = the simulated development of the population when it has not yet reached the stable size distribution
- Stable size/stage (or age) distribution = the relative distribution of individuals over size (or age) classes to which the population will develop when you perform a projection
- Asymptotic population growth rate, lambda, λ = the growth rate of the population once it has reached a relative distribution that is equal to the stable stage/size or age distribution