

# A Game-Theoretic Approach to Coalition Formation in Fog Provider Federations

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**Abstract**—In this paper we deal with the problem of making a set of *Fog Infrastructure Providers* (FIPs) increase their profits when allocating their resources to process the data generated by IoT applications that need to meet specific QoS targets in face of time-varying workloads. We show that if FIPs cooperate among them, by mutually sharing their workloads and resources, then each one of them can improve its net profit. By using a game-theoretic framework, we study the problem of forming *stable* coalitions among FIPs. Furthermore, we propose a mathematical optimization model for profit maximization to allocate IoT applications to a set of FIPs, in order to reduce costs and, at the same time, to meet the corresponding QoS targets. Based on this, we propose an algorithm, based on cooperative game theory, that enables each FIP to decide with whom to cooperate in order to increase its profits. The effectiveness of the proposed algorithm is demonstrated through an experimental evaluation considering various workload intensities. The results we obtain from these experiments show the ability of our algorithm to form coalitions of FIPs that are stable and profitable in all the scenarios we consider.

**Index Terms**—Fog computing, Fog federation, Game Theory, Coalition Formation

## 1. Introduction

In Fog Computing, compute, storage, and network resources are provided by *fog nodes* that are placed at the edge of the network, in close proximity to where data are generated. In this way, the data generated by IoT devices can be processed at the edge of the network, thus greatly reducing latency and avoiding to congest the core network, with respect to centralized cloud solutions.

The reduction of latency, however, may be quite costly in terms of resources, since it is proportional to the inverse square root of the number of fog nodes [1]. This cost may be prohibitive for enterprises whose population of users is spread across different geographic areas, as they would need to purchase, operate, and manage a large number of fog nodes distributed over these areas.

It has been anticipated [1] that *Fog Infrastructure Providers* (FIPs), i.e., operators that provide individual enterprises with the computing and networking infrastructure needed to host their fog nodes on a pay-per-use basis, will eliminate these costs. Indeed, on the one hand individual enterprises would not have to purchase, deploy, and manage their own fog infrastructures, while FIPs would amortize these costs by multiplexing the same physical infrastructure among multiple tenants, given that each enterprise would use only a fraction of the physical

capacity of fog nodes because of the variations in volume, variety, and velocity of generated data [2].

Furthermore, it has also been anticipated [1] that in many cases (e.g., small urban centers or rural areas) FIPs will be regional providers that exploit co-location facilities to cut down operational costs, a solution that is growing fast in the small-to-medium-size enterprise arena [3].

In the case where several FIPs share the same co-location facility (and, hence, their resources are indistinguishable from the perspective of latency perceived by a user), their profits can be further increased if they cooperate among them by mutually sharing their users and infrastructures. In particular, each FIP can improve its net profit by either (a) reducing energy costs by switching off (a part of) its infrastructure and offloading its users to resources belonging to other cooperating FIPs, or (b) increasing its earnings by attracting users from other FIPs, or (c) relying on resources of other FIPs to accept more users than what could do by working alone.

Obviously, it is unreasonable to expect that a FIP is willing to unconditionally cooperate with the other ones regardless the benefits it receives. Such a cooperation arises indeed only if suitable benefits result from it, and if the risks of monetary losses (caused by violations in the QoS parameters negotiated with its clients) are kept within acceptable limits.

In this paper, we devise a decision algorithm that provides a set of FIPs with suitable means to decide whether to cooperate with other FIPs, and if so with whom to cooperate. Our algorithm is based on game-theoretic techniques, where the process of establishing cooperation among the FIPs is modeled as a *cooperative game with transferable utility* (in particular, as a *hedonic game*, whereby each FIP bases its decision on its own preferences).

More specifically, we propose a game-theoretic framework to study the problem of forming *stable* coalitions among FIPs, and a mathematical optimization model to allocate IoT services to a set of resources, in order to improve profits and, at the same time, to meet service QoS for FIPs inside the same coalition. We achieve our goal by devising a *coalition formation algorithm* to form stable coalitions that allows each FIP to autonomously and selfishly decide whether to leave the current coalition to join a different one or not on the basis of the net profit it receives for doing so.

In our approach, each FIP pays for the energy consumed to serve each service, whether it belongs to it or to another FIP, but receives a payoff (computed as discussed later) for doing so. We use the game theory framework proposed in [4] to obtain Nash-stable coalitions [5], where no FIP can benefit to leave the current coalition to join

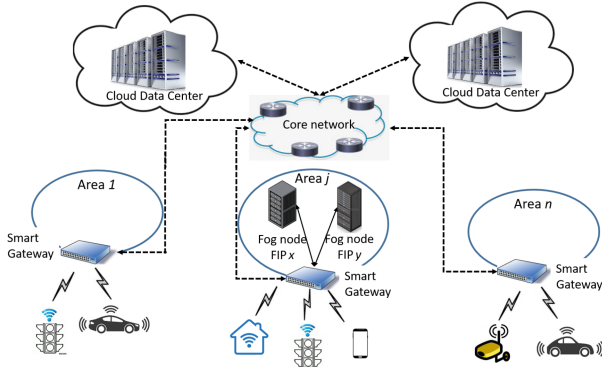


Figure 1. System architecture.

a different one. The solution we propose can be easily implemented in a distributed fashion.

To demonstrate the effectiveness of the algorithm we propose, we carry out a set of experimental evaluations. The results we obtain indicate that our algorithm allows indeed a population of FIPs to significantly improve their profits thanks to the combination of energy reduction and satisfaction of QoS requirements.

The contributions of this paper can be summarized as follows:

- we consider the problem of improving the profit of a set of FIPs that share the same co-location facility;
- we model the problem as a cooperative game with transferable utility;
- we devise a distributed algorithm enabling operators to find the coalition improving their profits under stability concerns.

The rest of this paper is organized as follows. Section 2 describes the details of the system model, then in Section 3 we present the problem statement. Section 4 introduces the coalition formation game and we evaluate the system in Section 5 using simulations. Finally, we end the paper with related work (in Section 6) and conclusions (in Section 7).

## 2. System Model

We consider a typical configuration [6] of a fog system, whose architecture is schematically depicted in Figure 1, where a large population of geo-distributed IoT devices, located in a set of  $n$  distinct geographic areas, generates very large amounts of data that require real-time or near-real-time processing. These devices may be either stationary (e.g., smart homes, IP cameras, smart traffic lights, etc.) or mobile (e.g., smartphones, tablets, connected cars, etc.).

Without loss of generality, we assume that each area  $j$  is covered by a single co-location facility, each one hosting a set of fog nodes that run a set of applications in charge of processing the data generated by the devices. These applications are encapsulated into a set of Virtual Machines (VMs), hosted on the fog nodes. In the rest of this section, we describe the characteristics of the three main components of the system, namely the fog nodes, the applications, and the VMs.

### 2.1. The Fog Nodes

We assume that each co-location facility hosts a set of resource-rich fog nodes [7] (e.g., *Cloudlets* or *Micro Data Centers*) belonging to a population of  $m$  FIPs. For instance, the co-location facility of area  $j$  in Figure 1 hosts fog nodes belonging to FIPs  $x$  and  $y$ .

Each fog node  $z$  is characterized by its CPU capacity  $c_z$ , which is measured by means of a suitable benchmark, and by its power consumption  $w_z(u)$ , which is computed as in [8] by the following equation:

$$w_z(u) = W_z^{\min} + u \cdot (W_z^{\max} - W_z^{\min}) \quad (1)$$

where  $u \in [0, 1]$  is the CPU utilization of the fog node, and  $W_z^{\min}$  and  $W_z^{\max}$  denote its power consumption (in Watts) when its CPU is in the idle state and when it is fully utilized, respectively.<sup>1</sup>

We assume all the fog nodes of the same FIP  $i$  located in a given area  $j$  are identical among them, i.e. they are characterized by the same CPU capacity and energy consumption. We also assume that they are provided by suitable dynamic resource allocation mechanisms (e.g. [9], [10], [11]) enabling them to partition their physical capacity across the VMs they run.

### 2.2. The Applications

The data generated by the IoT devices are processed by a set of  $K$  distributed applications  $\mathcal{S} = \{S_1, S_2, \dots, S_K\}$ . Each application  $S_i$  is associated with its *reference FIP*  $Ref(S_i)$ , i.e., the FIP that hosts the VMs running the instances of  $S_i$ , and we assume that a given FIP  $k$  can be the reference FIP of several distinct applications, that we denote as  $App(k)$ .

We assume that each application  $S_i$  is characterized by its QoS target, quantified by the maximum value  $Q_i$  that the average request processing time  $T_i$  can take (in other words, it must be ensured that  $T_i \leq Q_i$ ). To achieve its QoS target, the owner of application  $S_i$  makes an agreement with  $Ref(S_i)$  stating that it will pay to  $Ref(S_i)$  a certain amount of money per each unit of time, computed according to an agreed-upon *revenue rate*  $R_{Ref(S_i),i}$ . In addition,  $Ref(S_i)$  will pay the owner of  $S_i$  an amount of money per each unit of time during which the QoS target of  $S_i$  is not met, computed according to the agreed-upon *penalty rate*  $L_{Ref(S_i),i}$ .

To cover all the various areas of its interest, application  $S_i$  is present with one or more *instances* in the various areas, whereby each instance corresponds to a VM, and its instances located in a given area  $j$  process all the data generated for  $S_i$  by the devices located in that area. We assume that the load of data processing requests submitted to application  $S_i$  in any area  $j$  varies over time, and is described by its *load profile curve*  $l(i, j)$  expressing, as function of time, the rate at which requests are submitted. We also assume that, for each application  $S_i$  and area  $j$ ,  $l(i, j)$  is known in advance. The load profile curve can be indeed accurately built by estimating both the request rates generated by stationary devices and mobile users, and by aggregating them into a single measure.

1. This model, albeit simple, has been shown to provide accurate estimates of power consumption for different host types when running several benchmarks representative of real-world applications [8].

### 2.3. The Virtual Machines

The instances of any application  $S_j$  are embedded into a set of identical VMs, each one hosting a single instance, that are instantiated from a common *master* VM, denoted as  $VM_j$ .

$VM_j$  is characterized by the amount of time  $\tau_j$  it takes to process a single request that, without loss of generality, it is assumed to be determined only by the amount of physical CPU capacity allocated to that VM,<sup>2</sup> and that this amount is the same for all its instances, and remains constant for all their lifetimes.

To ensure that all the instances of  $VM_j$  exhibit the same value of  $\tau_j$ , we assume that each one of them receives, on the fog node  $k$  on which it runs, a suitable amount of CPU capacity  $U_{k,j}$  computed by Eq. 2 [12] (see [13] for more details):

$$U_{k,j} = U_{x,j} \frac{c_x}{c_k} \quad (2)$$

where  $c_k$  and  $c_x$  denote the physical CPU capacity of fog node  $k$  and  $x$ , respectively, that are measured as discussed in Section 2.1.

In order to meet the QoS target of  $S_i$  in area  $j$ , it is necessary to suitably choose the number  $N_{i,j}$  of VMs allocated on fog nodes located in that area so as to ensure that  $T_i \leq Q_i$ . This value, however, depends on the value of the load intensity  $\lambda_{i,j}(t)$ , which is not constant but varies, as already discussed, according to the load profile  $l(i, j)$ .

To determine  $\lambda_{i,j}(t)$ , we proceed as follows: we discretize  $l(i, j)$  by splitting the time axis into uniform disjoint sub-intervals  $[r, r + \Delta t)$  of length  $\Delta t$  time units. Then, the value  $\lambda_{i,j}(s)$  for any sub-interval  $s$  is approximated as a constant value set to the peak load in that sub-interval. Once the values of  $\lambda_{i,j}(s)$  have been computed as above, they are fed as input into a queuing model representing the set of VMs of  $S_i$  allocated in area  $j$ . The solution of this model yields the minimum number  $N_{i,j}(s)$  of VMs in sub-interval  $s$  that results in the satisfaction of the inequality  $T_i \leq Q_i$ .

In particular, the set of VMs associated to application  $S_i$  in area  $j$  is modeled as an M/M/c-FCFS queuing station [14] with  $c = N_{i,j}(s)$ , given that the service times of all the instances of  $VM_i$  are identical and the incoming stream of processing requests is fairly distributed among them.

For these queuing systems, it can be shown that in any time interval – and in particular in each sub-interval  $s$  – the average response time  $T_i$  is given by Eq. (3) (where, for readability purposes, we drop the dependence on  $s$ ):

$$T_i = \frac{\bar{n}}{\lambda_{i,j}} = \frac{C}{\mu N_{i,j} - \lambda_{i,j}} + \frac{1}{\mu} \quad (3)$$

where:

- $\mu = \frac{1}{\tau_i}$  is the request service rate;
- $\bar{n} = \frac{\rho}{1-\rho} C + \rho N_{i,j}$  is the average number of requests in the station, both in the queue and

receiving service, and  $\rho = \frac{\lambda_{i,j}}{\mu N_{i,j}}$  is the offered load to the station;

- $C = [1 + (1 - \rho) \left( \frac{N_{i,j}!}{(\rho N_{i,j})^{N_{i,j}}} \right) \sum_{k=0}^{N_{i,j}-1} \frac{(N_{i,j} \rho)^k}{k!}]^{-1}$  is the probability of a request to be enqueued before being served.

From Eq. (3) it follows that, in order to have  $T_i \leq Q_i$ ,  $N_{i,j}$  must satisfy the following inequality:

$$N_{i,j} \geq \frac{C}{Q_i - \frac{1}{\mu}} + \frac{\lambda_{i,j}}{\mu} \quad (4)$$

Furthermore, to ensure the stability of the system, we must have that:

$$N_{i,j} > \frac{\lambda_{i,j}}{\mu} \quad (5)$$

### 3. Problem Statement

Let us now describe the problem that is faced by a FIP  $i$  that has to allocate, for each application  $S_k \in \text{App}(i)$ , the corresponding set of VMs on its fog nodes located in area  $j$  (denoted as  $FN(i, j)$ ). Without loss of generality, we focus on a single area, given that FIPs allocate their respective VMs submitted in a given area independently from those submitted to other areas. The extension to other areas is thus straightforward.

FIP  $i$  aims at getting a *net profit* (i.e., the difference between its revenues and costs) as high as possible, given the set of requests for the allocation of all the applications  $S_k \in \text{App}(i)$ .

The net profit rate  $P_{i,j}$  (i.e., the profit FIP  $i$  makes per unit of time) is computed as the following difference:

$$P_{i,j} = \sum_{S_k \in \text{App}(i)} R_{i,k} n_{k,j} - \left( \sum_{f \in FN(i,j)} w_f(u_f) E_{i,j} + \sum_{S_k \in \text{App}(i)} \mathbf{1}_{[0, N_{k,j}]}(n_{k,j}) L_{i,k} \right) \quad (6)$$

where  $n_{k,j} \leq N_{k,j}$  is the number of VMs for  $S_k$  that are actually allocated (see below),  $E_{i,j}$  is the energy cost for FIP  $i$  in area  $j$  (expressed as a cost rate per unit of time),  $u_f$  is the overall physical capacity of fog node  $FN(i, j)$  allocated to the VMs it hosts, and  $\mathbf{1}_\Omega(x)$  is the *indicator function* which has value 1 if  $x \in \Omega$  and 0 otherwise.

Eq.(6) has the following meaning:

- the first term of the difference is the sum of the revenue rates  $R_{i,k}$  that FIP  $i$  charges (per unit of time) to each application  $S_k$  for hosting  $n_{k,j}$  of its VMs;
- the second term of the difference represents the costs that FIP  $i$  incurs (per unit of time) to run the above VMs. This cost, in turn, is given by the sum of two costs, namely (a) the *energy cost rates* resulting from the execution of overall CPU capacity allocated to all the VMs it hosts (see Eq. 1), and (b) the possible *monetary penalty rates*  $L_{i,k}$  that FIP  $i$  incurs when the QoS of some application  $S_k \in \text{App}(i)$  is not met (i.e., when  $n_{k,j} < N_{k,j}$ ).

Maximizing the net profit rate is a challenging task which involves solving an optimization problem that takes

2. The extension to multiple types of physical resources (e.g., RAM and storage) and to multiple classes of VMs, each one with different physical resource requirements, is straightforward (e.g., see [12]).

into account the current workload, the electricity price and the application penalties. In Section 4.2, we provide more details about the solution to this optimization problem.

Intuitively, when the number of VMs to allocate on a fog node  $FN(i, j)$  is so small that the resulting net profit is negative, FIP  $i$  must decide whether it is more profitable to not allocate any VM on  $FN(i, j)$  (thus opting to pay the monetary penalties for violating the QoS of the related applications), or it is instead better to allocate the VMs anyways to avoid paying high application penalties. Also, when the number of VMs is so large that it needs more than one fog node to allocate them, FIP  $i$  must decide whether it is more profitable to allocate all of them, or it is instead better to allocate only the ones that leads to a positive profit (thus paying the monetary penalties for those applications whose QoS is not met).

In this paper, we show that a way to improve the net profit of a FIP is through cooperation, meaning that two or more FIPs in the same area of interest join to form a coalition where they share their workloads and their fog nodes to serve them.

Specifically, with cooperation, FIP  $i$  can try to reduce its energy consumption costs by allocating (some of) its VMs to the fog nodes of other FIPs, so that its fog nodes can be turned off. Also, FIP  $i$  can try to increase its revenues either by hosting VMs from other FIPs (so that it can better amortize its energy consumption costs) or by relying on fog nodes of other FIPs to allocate VMs that, if working alone, it could not host (thus incurring into monetary penalties for violating the QoS of the related applications).

Clearly, FIPs are willing to cooperate with each other only if they receive suitable incentives to do so that make cooperation at least as profitable as working alone. The lack of these suitable incentives leads to the so called unstable coalitions, that is to coalitions where a participating FIP prefers either to leave its current coalition to move to a more profitable one or to work alone.

#### 4. The coalition formation game

We assume that the agents (also called players) may join/leave a coalition without any permission requirements, that is, a player is always accepted by a coalition to which it is willing to join, and it can leave a coalition without any permission. One way to describe such a process is to model it as a *coalition formation game*. In particular, in this paper we use a type of coalition formation games, known as *hedonic coalition formation games* (also called hedonic games) [5].

An hedonic game is a game where: *i*) the gain of any player depends solely on the members of the coalition to which the player belongs, and *ii*) the coalitions arise as a result of the preferences of the players over their possible set of coalitions. The hedonic games are usually analyzed in terms of the *stability* of coalition structures: the focus lies on finding the conditions for the existence of stable outcomes (i.e., a coalition or a set of coalitions). An outcome (e.g., a set of coalitions) is said to be *stable* if no player (or possibly no coalition of players) can deviate from the outcome so as to reach a subjectively better outcome. Several notions of stability have been defined in literature some of which allow to guarantee

stability against single player moves, i.e. Nash stability, while others also allow group movements, i.e. the core (see [5] for details).

A coalition  $C \subseteq \mathcal{N}$  represents an agreement among the players in  $C$  to act as a single entity (i.e., they must agree to share their own resources and users among them). At any given time, the set of players is partitioned into a *coalition partition*  $\Pi$ , that we define as the set  $\Pi = \{C_1, C_2, \dots, C_l\}$ . That is, for  $k = 1, \dots, l$ , each  $C_k \subset \mathcal{N}$  is a disjoint coalition such that  $\bigcup_{k=1}^l C_k = \mathcal{N}$  and  $C_j \cap C_k = \emptyset$  for  $j \neq k$ . Given a coalition partition  $\Pi$ , for any player  $i \in \mathcal{N}$ , we denote by  $C_\Pi(i)$  the coalition  $C_k \in \Pi$  such that  $i \in C_k$ .

An hedonic coalition formation game is a pair  $(\mathcal{N}, \succeq)$ , where  $\mathcal{N}$  is the set of players, and  $\succeq_i$  a preference profile that specifies, for every player  $i \in \mathcal{N}$ , a reflexive, complete, and transitive binary relation  $\succeq_i$  on  $\mathcal{N}_i$  (set of all coalitions that include player  $i$ ). The binary relation  $\succeq_i$  is called *preference relation*.

In its partition form, a coalition game is defined on the set  $\mathcal{N}$  by associating a utility value  $u(C|\Pi)$  to each subset of any partition  $\Pi$  of  $\mathcal{N}$ . For hedonic games the utility value of a coalition  $C$  is independent of the other coalitions, that is, and therefore,  $u(C|\Pi) = u(C)$ . In particular, we define the coalition value  $u(C)$  as the net profit rate of coalition  $C$  that we compute as the solution of the profit maximization problem that is presented in Section 4.2.

To set up the coalition formation process, we need to specify the preference relation so that each player can order and compare all the possible coalitions it belongs to, and hence it can build preferences over them. This can be done by using a preference function  $\pi_i(C)$  that describes the preference of player  $i$  (with  $i \in \mathcal{N}$ ) for any coalition  $C \in 2^{\mathcal{N}}$ . Formally the preference function for player  $i$  can be defined as  $\pi_i : 2^{\mathcal{N}} \rightarrow \mathbb{R}$ . In this manner we can say that a player  $i$  prefers the coalition  $C$  to  $\mathcal{T}$  iff,

$$\pi_i(C) \geq \pi_i(\mathcal{T}) \iff C \succeq_i \mathcal{T}. \quad (7)$$

We assume that the preference relation is chosen to be equal to the utility allocated to the player in a coalition. In other words, we have that  $\pi_i(C) = \phi_i(C)$ , where  $\phi_i(C)$  is the utility received by player  $i$  in coalition  $C$ .

An allocation is said to be *efficient* if for any coalition  $C \in \Pi$ , the sum of the individual utilities allocated to the coalition participants is equal to the coalition utility, that is

$$\sum_{i \in C} \phi_i(C) = u(C). \quad (8)$$

A partition  $\Pi$  is said to be Nash-stable if no player can benefit from moving from his coalition  $C_\Pi(i)$  to another existing coalition  $C_k$ , that is,

$$\forall i, k : i \in C_\Pi(i) \succeq_i C_k \cup \{i\}, \text{ where } C_k \in \Pi \cup \{\emptyset\}. \quad (9)$$

Note that, as pointed out in [5], the Nash-stability is a noncooperative notion of stability in the sense that players do not need permission to leave/join a coalition.

In this paper we use the approach to the Nash-stability proposed in [4]. Due to the lack of space in this paper we present a summary of the main steps of this approach; all the details can be found in [13].

In [4] the set of efficient allocations of a Nash-stable coalition partition has been called *Nash-stable core*, and the Nash-stability has been rephrased to an optimization problem aiming at deriving the functions  $v(\cdot, \cdot)$  (for any player  $i \in \mathcal{N}$ ) satisfying the property defined by the following equation

$$C_1 \succeq_i C_2 \iff \sum_{j \in C_1} v(i, j) \geq \sum_{j \in C_2} v(i, j), \quad (10)$$

with  $v(i, i) = 0$ . The method presented in [4] derives the functions  $v(\cdot, \cdot)$  satisfying the property defined by Equation (10), and the preference relations  $\succeq_i$  that satisfy the Nash-stability condition.

According to the results presented in [4], a feasible solution of the following linear program guarantees the Nash-stability condition

$$\begin{aligned} \max \quad & \sum_{\forall C \in 2^{\mathcal{C}}} \sum_{i, j \in C: j > i} v(i, j) \quad \text{subject to} \\ & \sum_{i, j \in C: j > i} v(i, j) \leq \frac{1}{2} \Delta(C), \quad \forall C \in 2^{\mathcal{C}}, \end{aligned} \quad (11)$$

where  $\Delta(C) = u(C) - \sum_{i \in C} u(\{i\})$  is the *profit* due to coalition  $C$ .

**A three players example.** We illustrate the previous concepts by means of a simple three players game, where

$$\begin{aligned} u(\{1\}) &= u(\{2\}) = u(\{3\}) = -0.002, \\ u(\{1, 2\}) &= -0.001, u(\{1, 3\}) = -0.005, \\ u(\{2, 3\}) &= -0.003, u(\{1, 2, 3\}) = -0.0035. \end{aligned}$$

A utility allocation method based on the relaxed efficiency can be found by solving the linear program defined in (11). In particular we have that

$$\begin{aligned} \max \quad & \{v(1, 2) + v(1, 3) + v(2, 3)\} \text{ subject to} \\ v(1, 2) &\leq \frac{0.003}{2}, v(1, 3) \leq \frac{-0.001}{2}, v(2, 3) \leq \frac{0.001}{2}, \\ v(1, 2) + v(1, 3) + v(2, 3) &\leq \frac{0.0025}{2}, \end{aligned}$$

which produces the following values:

$$v(1, 2) = 0.00125; \quad v(1, 3) = -0.0005; \quad v(2, 3) = 0.0005.$$

From this we can derive the following preference profile

$$\begin{aligned} \{1, 2\} &\succ_1 \{1, 2, 3\} \succ_1 \{1\} \succ_1 \{1, 3\} \\ \{1, 2, 3\} &\succ_2 \{1, 2\} \succ_2 \{2, 3\} \succ_2 \{2\} \\ \{2, 3\} &\succ_3 \{3\} \succeq_3 \{1, 2, 3\} \succ_3 \{1, 3\}. \end{aligned}$$

By assigning  $\phi_i(C) = u(\{i\}) + \sum_{j \in C} v(i, j)$  we have that

$$\begin{aligned} \phi_1(\{1, 2\}) &= \phi_2(\{1, 2\}) = -0.00075; \\ \phi_1(\{1, 3\}) &= \phi_3(\{1, 3\}) = -0.0025; \\ \phi_2(\{2, 3\}) &= \phi_3(\{2, 3\}) = -0.0015; \\ \phi_1(\{1, 2, 3\}) &= -0.00125; \phi_2(\{1, 2, 3\}) = -0.00025; \\ \phi_3(\{1, 2, 3\}) &= -0.002. \end{aligned}$$

TABLE 1. A POSSIBLE EVOLUTION OF THE DECENTRALIZED ALGORITHM

Round / Schedule	Step	Player	Strategy tuple	Partition
0			$\{s_1, s_2, s_3\}$	$\{(1), (2), (3)\}$
1 - (3,1,2)	1	3	$\{s_1, s_2, s_{2,3}\}$	$\{(1), (2, 3)\}$
	2	1	$\{s_{1,2,3}, s_2, s_{2,3}\}$	$\{(1, 2, 3)\}$
	3	2	$\{s_{1,2,3}, s_{1,2,3}, s_{2,3}\}$	$\{(1, 2, 3)\}$
2 - (2,3,1)	1	2	$\{s_{1,2,3}, s_{1,2,3}, s_{2,3}\}$	$\{(1, 2, 3)\}$
	2	3	$\{s_{1,2,3}, s_{1,2,3}, s_{1,2,3}\}$	$\{(1, 2, 3)\}$
	3	1	$\{s_{1,2,3}, s_{1,2,3}, s_{1,2,3}\}$	$\{(1, 2, 3)\}$

#### 4.1. The Coalition Formation Algorithm

In this section, we succinctly describe a decentralized algorithm to reach a Nash-stable partition inspired to the proposal of [4]. All the details of this algorithm can be found in [13], and for the theoretical foundations in [4]. We assume that the game evolves in turns and players are allowed to choose their strategies once in each turn.

To describe the decentralized coalition formation algorithm we denote a scheduler by  $\Sigma = \{s(1), s(2), \dots, s(N)\}$ . A scheduler is a random permutation of players' indices and, in particular, the scheduler of  $l$ -th round denoted by  $s(l)$  is a tuple  $s(l) = \{s_1(l), s_2(l), \dots, s_N(l)\}$ , where  $s_i(l)$  identifies the  $i$ -th player selected to play in the round  $l$ . Hence, each round includes  $N$  steps that correspond to single players' decisions. It is important to notice that in this game, the set of actions available to each player is equal to the number of coalitions in the game in that moment. For example, in the case of all players apart and considering singletons as coalitions a player may join any other agent or not to move.

A strategy tuple in step  $s$  is denoted as  $\sigma^{(s)} = \{\sigma_1^{(s)}, \sigma_2^{(s)}, \dots, \sigma_N^{(s)}\}$ , where  $\sigma_i^{(s)}$  is the strategy of player  $i$  in step  $s$ . The strategy tuples  $\sigma^{(s)}$  and  $\sigma^{(s-1)}$  differ in at most one position. That is, the position corresponding to the player that takes its turn in step  $s$ . On the other hand  $\sigma^{(s)}$  and  $\sigma^{(s-1)}$  are identical if the player that takes its turn in step  $s$  does not change its previous strategy. We denote by  $\Pi_l^{(s)}$  the partition in step  $s$  of the  $l$ -th round, and by  $\Pi_l$  the partition obtained at the end of the  $l$ -th round.

Furthermore, we denote by  $C_i^{(s)} = \{j : \sigma_j^{(s)} = \sigma_i^{(s)}, \forall j \in \mathcal{N}\}$  the set of players that share the same strategy with player  $i$ . In this manner we have that preference relation of player  $i$ , denoted by  $\pi_i(C)$ , verifies the following relation

$$\pi_i(C_i^{(s)}) > \pi_i(C_i^{(s-1)}) \iff C_i^{(s)} \succ_i C_i^{(s-1)}.$$

It is easy to verify that a Nash-stable partition is reached, at a given  $l+1$ -th round, when  $\Pi_l = \Pi_{l+1}$ .

**A three players example (cont'd).** The algorithm starts from the  $\Pi_0 = \{(1), (2), (3)\}$ , that is the partition where each player is alone. This corresponds to the strategy tuple  $\{s_1, s_2, s_3\}$  (each player chooses a different strategy). Table 1 shows a possible evolution of the example in the previous section.

In the first example, the game begins with all players apart and the extracted scheduler is (3, 1, 2). Player 3 begins and coalesces with Player 2. Player 1 is the next to play, and the only move available to him better than playing alone is to join the grand coalition. Therefore, at this point of the game, the formed coalition gathers

all the players. For Player 2, the coalition  $\{1, 2, 3\}$  is the most preferable and it is happy to be part of it. After Player 2 moves the round is over and the second round begins with scheduling  $(2, 3, 1)$ . Now is Player 1 turn and it faces the same choice as before; since at the previous step Player 1 chose the best it could and now the situation is unchanged, it will choose again to stay in the grand coalition. Eventually is Player 3 turn which does not get any benefit from leaving the grand coalition so it will remain there. Given that there is no any player who benefits from unilaterally deviate from the strategy profile  $\{s_{1,2,3}, s_{1,2,3}, s_{1,2,3}\}$  a Nash-stable equilibrium is reached.

## 4.2. Computation of the Optimal Coalition Allocation Profit

The coalition formation process described in the above sections requires the computation of the coalition net profit rate  $u(\mathcal{C})$  for any coalition  $\mathcal{C}$  of FIPs that may form.

As discussed in Section 3, this computation involves solving an optimization problem that, given a geographic area  $h$  and a coalition  $\mathcal{C}$  of FIPs located in this area, seeks to find the optimal allocation of the set  $\mathcal{V}$  of VMs where to run instances of the applications  $\mathcal{A} = \bigcup_{i \in \mathcal{C}} \text{App}(i)$  to host in this area onto the set  $\mathcal{F} = \bigcup_{i \in \mathcal{C}} \text{FN}(i, h)$  of fog nodes, so as to maximize the overall net profit rate of coalition  $\mathcal{C}$ . The set  $\mathcal{V}$  is given by the union of the  $N_{j,h}(t)$  VMs required by each application  $j \in \mathcal{A}$  to meet its target QoS in the discretization interval  $t$ .

To this purpose, we define a *Mixed Integer Linear Program* (MILP) to model the problem of allocating a set  $\mathcal{V}$  of VMs onto a set  $\mathcal{F}$  of fog nodes so that the overall net profit rate of the coalition  $\mathcal{C}$  is maximized.

The resulting optimization model is shown in Figure 2, where we use the same notation defined in Section 2 and where we denote as  $p(\cdot)$  the function  $p : \mathcal{F} \rightarrow \mathcal{C}$  which maps, in the given area of interest  $h$ , a fog node to its FIP, and as  $s(\cdot)$  the function  $s : \mathcal{V} \rightarrow \mathcal{A}$  which maps a VM to the (instance of) application it runs. Also, to ease readability, we simplify the model by dropping from it the dependence from the discretization interval  $t$  (e.g., we denote as  $N_{j,h}$ , instead of  $N_{j,h}(t)$ , the required number of VMs to allocate in order to meet the QoS of application  $j$ ).

In the optimization model, we define the following decision variables:

- $x_i$ : a binary decision variable which is equal to 1 if fog node  $i$  is powered on, and 0 otherwise;
- $y_{i,j}$ : a binary decision variable which is equal to 1 if VM  $j$  is allocated on fog node  $i$ , and 0 otherwise;
- $u_i$ : a non-negative real decision variable which represents the total fraction of CPU capacity of fog node  $i$  that has been allocated to the VMs it hosts;
- $n_k$ : a non-negative integer decision variable which denotes the number of VMs allocated to run instances of application  $k$ .

The objective function of the optimization model represents the overall net profit rate earned by the coalition

$$\begin{aligned} \text{maximize } u(\mathcal{C}) = & \sum_{k \in \mathcal{A}} R_{\text{Ref}(k),k} n_{k,h} \\ & - \left[ \sum_{i \in \mathcal{F}} (x_i W_i^{\min} + (W_i^{\max} - W_i^{\min}) u_i) E_{p(i),h} \right. \\ & \left. + \sum_{k \in \mathcal{A}} (n_{k,h} < N_{k,h}) L_{\text{Ref}(k),k} \right] \end{aligned} \quad (12a)$$

subject to

$$\sum_{j \in \mathcal{V}} y_{i,j} \leq |\mathcal{V}| x_i, \quad \forall i \in \mathcal{F}, \quad (12b)$$

$$\sum_{i \in \mathcal{F}} y_{i,j} \leq 1, \quad \forall j \in \mathcal{V}, \quad (12c)$$

$$u_i = \sum_{j \in \mathcal{V}} y_{i,j} U_{i,j}, \quad \forall i \in \mathcal{F}, \quad (12d)$$

$$u_i \leq x_i, \quad \forall i \in \mathcal{F}, \quad (12e)$$

$$n_k = \sum_{i \in \mathcal{F}} \sum_{\substack{j \in \mathcal{V}, \\ s(j)=k}} y_{i,j}, \quad \forall k \in \mathcal{S}, \quad (12f)$$

$$n_k \leq N_{k,h}, \quad \forall k \in \mathcal{A}, \quad (12g)$$

$$x_i \in \{0, 1\}, \quad \forall i \in \mathcal{F}, \quad (12h)$$

$$y_{i,j} \in \{0, 1\}, \quad \forall i \in \mathcal{F}, j \in \mathcal{V}, \quad (12i)$$

$$u_i \in \mathbb{R}^*, \quad \forall i \in \mathcal{F}, \quad (12j)$$

$$n_k \in \mathbb{N}, \quad \forall k \in \mathcal{A}. \quad (12k)$$

Figure 2. The optimization model for the maximization of the coalition net profit rate.

$\mathcal{C}$  of FIPs, which is defined as the difference between the revenues obtained by the allocation of VMs, and the costs due both to the electricity power absorbed by the powered-on fog nodes and to QoS violations (if any).

The maximization of this objective function is bound to the following constraints:

- Eq. (12b) assures that no VM is allocated on a fog node that will be powered off;
- Eq. (12c) states that each VM is hosted by no more than one fog node;
- Eq. (12d) defines the value of the variable  $u_i$  as the sum of the CPU capacity requirements of the VMs allocated on fog node  $i$ ;
- Eq. (12e) ensures that the allocated CPU capacity of a powered-on fog node is not exceeded;
- Eq. (12f) defines the value of the variable  $n_k$  as the number of allocated VMs where to run instances of application  $k$ ;
- Eq. (12g) ensures that for each application no more VMs are allocated than needed;
- Eqs. 12h–12k define the domain of decision variables  $x_i$ ,  $y_{i,j}$ ,  $u_i$  and  $n_k$ , respectively.

## 5. Experimental Evaluation

To assess the effectiveness of the proposed coalition formation algorithm, we perform an experimental evaluation in which we run our algorithm for various scenarios, whose parameters are summarised in Table 2. In these scenarios, we vary the workload of each application and we assess the impact of it on the performance of the proposed algorithm.

In a given geographic area  $h$ , we consider  $m = 3$  identical FIPs, each of which is in charge of running

TABLE 2. PARAMETERS USED IN THE EXPERIMENTAL SCENARIOS.  
SUBSCRIPTS  $i$  AND  $j$  TAKE VALUES ON THE SET  $\{1, 2, 3\}$ .

Parameter	Value
$ App(i) $	Number of applications associated to FIP $i$
$E_{i,j}$	Electricity price for FIP $i$ in area $j$
$ FN(i, j) $	Number of fog nodes for FIP $i$ in area $j$
$L_{i,j}$	Penalty rate for FIP $i$ and application $j$
$m$	Number of FIPs
$n$	Number of applications
$Q_i$	Max request processing time for application $i$
$R_{i,j}$	Revenue rate for FIP $i$ and application $j$
$U_{i,j}$	CPU demand for any VM $j$ and fog node $i$
$W_j^{\max}$	Max power consumption of fog node $j$
$W_j^{\min}$	Idle power consumption of fog node $j$
$\tau_j$	Request processing time of any VM $j$

instances of a different application (i.e.,  $App(i) = \{S_i\}$ , for  $i = 1, \dots, 3$ ). The physical infrastructure of each FIP consists of 3 identical fog nodes whose idle and maximum power consumptions  $W_j^{\min}$  and  $W_j^{\max}$  are set to 100 W and 200 W, respectively (for  $j = 1, \dots, 3$ ). For each application  $j$ , its master VM  $VM_j$  is characterized by a request processing time  $\tau_j$  of 0.5 sec and, to achieve this value, it requires a physical CPU capacity  $U_{i,j}$  of 0.05 for every fog node  $i$  (i.e., each VM of application  $j$  consumes 5% of the physical CPU capacity of each fog node  $i$ ).

We assume that the electricity price  $E_{i,h}$  charged hourly to each FIP  $i$  is the same for all FIPs and we set it to 0.0001 \$/Wh. Also, we set the revenue rate  $R_{i,j}$  that each FIP  $i$  earns for hosting a VM for application  $j$  to 0.0022 \$/hour.

Finally, for each application  $j$ , we set its QoS parameter  $Q_j$  to 0.7 sec and the related monetary penalty rate  $L_{i,j}$  to 0.022 \$/hour for each FIP  $i$ , which is 10 times the revenue rate  $R_{i,j}$ . We choose this value for the penalty rate so that an FIP always prefer allocating VMs for the applications (s)he hosts than refuse them for reducing energy consumption costs.

Given the above settings, we study the impact of the workload on the coalition formation process by varying, in a controlled way (i.e., by considering each discretization interval of the traffic load curve separately), the workload intensity of each application so that the induced load  $\alpha_i$ , experienced by each FIP  $i$  (when (s)he works alone) on the fog node where the  $N_{h,i}$  VMs have been allocated, ranges from 0.1 (i.e., only 10% of the CPU capacity of the fog node is allocated to VMs) to 0.9 (i.e., the total allocated CPU capacity on the fog node is 90%), with increments of 10% (see [13] for more details). To run our experiments, we develop an ad hoc simulator in C++ where we use the IBM ILOG CPLEX solver 12.7.1 for solving the optimization problem discussed in Section 4.2.

Figure 3 summarizes the behaviour of the coalition formation algorithm for different values of the load (or of the request arrival rates). In particular, we compare the *social welfare*<sup>3</sup> values in case that the providers work independently (symbol \* in the figure) vs the case where the providers set up coalition among them if this increases their profits (symbol □ in the figure). Despite the discrete granularity we can identify three regions in the figure

3. The social welfare, i.e., the sum of the utilities of all the coalitions, is one of the measures that can be defined for coalition formation games (see for instance [15]). In particular, it provides a simple and intuitive measure to compare different coalition partitions (in this paper the partitions originated by our proposal vs a strategy that does not use any coalition strategy).

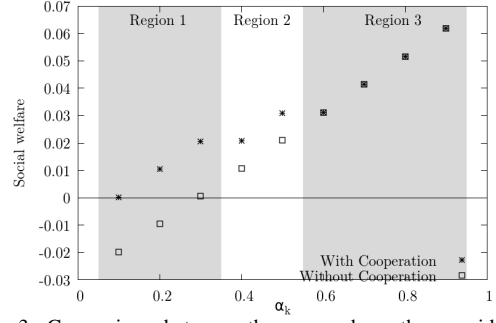


Figure 3. Comparison between the case where the providers work independently with the coalition formation algorithm.

that correspond to different coalition formation algorithm outcomes. Region 1 characterizes the low load values. In these cases the coalition formation algorithm yields the grand coalition, and the coalition always increases social welfare. On the other hand, Region 3 typecasts the high (or relatively high) load values that saturate the nodes. In this case the cooperation does not bring any advantage. In the figure we can note that the social welfare values in this region overlap. Note that in this very simple numerical set of experiments we do not account for the coalition formation algorithm costs and hence the social welfare values of the two strategies (coalitions vs no-coalition) coincide.

This set of experiments allows to point out that the cooperation brings greater benefits in case of low loads. On the other hands, in case of high loads the advantages decrease as load increases. This behavior is due to the reduction of waste of resources that occurs in case of under utilized FIPs.

Despite of its simplicity, the set of experiments summarized by Figure 3, illustrates how to use the distributed coalition formation algorithm. The fog providers, based on its own load profile estimates, can agree the timing and the activation frequency of the distributed algorithm. The goal should be an appropriate trade-off between the benefits due to the algorithm (e.g., obtaining coalitions that allow the reduction of costs) and the overhead derived from too frequent and not very effective activations.

## 6. Related Works

The game theoretical approach to coalition formation has been used in many other scenarios. In the field of cloud computing there is a large body of research, see, for instance [12], [16], [17], [18]. Similar approaches have been used to study cooperative behavior in cellular networks [19], [20]. Coalition formation frameworks for femtocell networks have been used for different purposes such as mitigating the interference, and resource and power allocation; examples of such proposals can be found in [21], [22], [23].

Compared to these works, in our contribution we consider a much different system architecture, which is characterized by different properties and, hence, requires a different solution.

Approaches based on game theory have also been used in the field of fog computing (or edge computing/femto-cloud). In these scenarios the emphasis is on optimizing resources [24], [25], optimization of latency and/or the energy consumption [26], [27], offloading strategies [28],



[29], and approaches tailored for macro/micro cellular network scenarios [30], [31].

In this paper we target a goal which is different from those addressed by the above papers. In particular, we focus on the problem of increasing the profit of different FIPs in the presence of applications characterized by different QoS targets and time-varying workloads.

## 7. Conclusions

In this paper we have dealt with the problem of making a set of FIPs increase their profits when allocating their resources to process the data generated by IoT applications that need to meet specific QoS targets in face of time-varying workloads.

To this end, we proposed a cooperative game-theoretic framework to study the federation formation problem, and a mathematical optimization model to allocate IoT applications of the resources of a FIP in order to increase its net profit.

In the proposed scheme, we model the cooperation among FIPs as a coalition game with transferable utility and we devise a distributed algorithm for coalition formation. With the proposed algorithm, each FIP individually decides whether to leave the current coalition to join a different one according to his preference, meanwhile improving the perceived net profit. Furthermore, we show that the proposed algorithm converges to a Nash-stable partition which determines the resulting coalition structure. Numerical results exhibit the effectiveness of our approach.

The future developments of this research is following several directions. In particular, we would like to enhance the coalition value function in order to account for possible request losses due to lack of physical resources. Furthermore, we want to improve the game-theoretic and optimization models in order to include costs in terms of loss of revenues as well as other aspects like the ones related to trustworthiness among FIPs. Finally, we want to implement and validate the proposed algorithm on a real testbed.

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