

MPC with Casadi/Python

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What's in mpc-tools-casadi?

A Python package: `mpctools`¹

- You should put the `mpctools` folder somewhere on your Python path.
- In Python, use `import sys; print sys.path` to see what folders are on your path.

A cheatsheet (in the `doc` folder).

- Should get you started writing your own code.
- Compares plain CasADi vs. CasADi + `mpctools`.

A bunch of example files, e.g.,

- `nmpcexample.py`: Example of linear vs. nonlinear MPC.
- `cstr_startup.py`: startup and a setpoint change (with no disturbances) for the CSTR system from Example 1.11.

¹See `install.pdf` for detailed installation instructions.

Why did we write this code?

- We plan to solve nonlinear MPC problems.
- CasADi is more robust than our `mpc-tools`
- However, setting up an MPC problem in CasADi takes a fair bit of code
- Everyone copy/pasting their own code is bad.
- A simpler interface means we (and others) can save a lot of time.

From official CasADi Examples

```
# For all collocation points: eq 10.4 or 10.17 in Biegler's book
# Construct Lagrange polynomials to get the polynomial basis at
# the collocation point
for j in range(deg+1):
    L = 1
    for j2 in range(deg+1):
        if j2 != j:
            L *= (tau-tau_root[j2])/(tau_root[j]-tau_root[j2])

    lfcn = SXFunction([tau],[L])
    lfcn.init()
    # Evaluate the polynomial at the final time to get the
    # coefficients of the continuity equation
    lfcn.setInput(1.0)
    lfcn.evaluate()
    D[j] = lfcn.getOutput()

# Evaluate the time derivative of the polynomial at all
# collocation points to get the coefficients of the
# continuity equation
tfcn = lfcn.tangent()
tfcn.init()
for j2 in range(deg+1):
    tfcn.setInput(tau_root[j2])
    tfcn.evaluate()
    C[j][j2] = tfcn.getOutput()
```

We don't want
everyone writing
this themselves!

Python Basics

For our purposes Python+Numpy isn't that much different from Octave/MATLAB.

Octave/MATLAB	Python+Numpy
<code>A = zeros(3,2);</code>	<code>A = np.zeros((3,2))</code>
<code>x = ones(2,1); %2D.</code>	<code>x = np.ones((2,)) #1D.</code>
<code>y = A*x;</code>	<code>y = A.dot(x)</code>
<code>z = y.^2; % Elementwise.</code>	<code>z = y**2 # Elementwise.</code>
<code>A(1,1) = 2; % One-based.</code>	<code>A[0,0] = 2 # Zero-based.</code>
<code>A(2,:) = [3,4];</code>	<code>A[1,:] = np.array([3,4])</code>
<code>s = struct('field',1);</code>	<code>s = {"field" : 1}</code>
<code>disp(s.field);</code>	<code>print s["field"]</code>

- Remember that indexing is 0-based.
- Use NumPy's `array` instead of `matrix`.
 - Despite what Octave/MATLAB says, everything is *not* a matrix of doubles
 - Have to use `A.dot(x)` instead of `A*x`
 - However, indexing is MUCH easier with arrays
- Use `bmat([[A,B],[C,D]])` to assemble matrices from blocks.
 - Equivalent to `[A, B; C, D]` in Octave/MATLAB
 - Trailing `.A` casts back to array type from `matrix`
- Use `scipy.linalg.solve(A,b)` to compute $A^{-1}b$.

System Model

Start by defining the system model as a Python function.

```
def ode(x,u,d):
    # Grab the states, controls, and disturbance.
    [c, T, h] = x[:Nx]
    [Tc, F] = u[:Nu]
    [F0] = d[:Nd]

    # Now create the right-hand side function of the ODE.
    rate = k0*c*np.exp(-E/T)

    dxdt = [
        F0*(c0 - c)/(np.pi*r**2*h) - rate,
        F0*(T0 - T)/(np.pi*r**2*h)
        - dH/(rho*Cp)*rate
        + 2*U/(r*rho*Cp)*(Tc - T),
        (F0 - F)/(np.pi*r**2)
    ]

    return np.array(dxdt)
```

The nonlinear system can be simulated using CasADi integrator objects with a convenient wrapper.

```
# Turn into casadi function and simulator.
ode_casadi = mpc.getCasadiFunc(ode,
    [Nx,Nu,Nd],["x","u","d"],funcname="ode")
cstr = mpc.DiscreteSimulator(ode, Delta, [Nx,Nu,Nd], ["x","u","d"])

# Simulate with nonlinear model.
x[n+1,:] = cstr.sim(x[n,:] + xs, u[n,:] + us, d[n,:] + ds) - xs
```


Calls to LQR and LQE

The functions `dlqr` and `dlqe` are also provided in `mpc-tools-casadi`.

Octave/MATLAB

```
% Get LQR.  
[K, Pi] = dlqr(A, B, Q, R);  
  
% Get Kalman filter.  
[L, M, P] = dlqe(Aaug, ...  
    eye(naug), Caug, Qw, Rv);  
Lx = L(1:n,:);  
Ld = L(n+1:end,:);
```

CasADi/Python

```
# Get LQR.  
[K, Pi] = mpc.util.dlqr(A,B,Q,R)  
  
# Get Kalman filter.  
[L, P] = mpc.util.dlqe(Aaug,  
    Caug, Qw, Rv)  
Lx = L[:Nx,:]  
Ld = L[Nx:,:]
```

Controller Simulation

Octave/MATLAB

```
for i = 1:ntimes
    % Take plant measurement.
    y(:,i) = C*x(:,i) + v(:,i);

    % Update state estimate with measurement.
    ey = y(:,i) - C*xhat(:,i) - Cd*dhat(:,i);
    xhat(:,i) = xhat(:,i) + Lx*ey;
    dhat(:,i) = dhat(:,i) + Ld*ey;

    % Steady-state target.
    H = [1 0 0; 0 0 1];
    G = [eye(n)-A, -B; H*C, zeros(size(H,1), m)];
    qs = G\[Bd*dhat(:,i); ...
            H*(target.yset-Cd*dhat(:,i))];
    xss = qs(1:n);
    uss = qs(n+1:end);

    % Regulator.
    u(:,i) = K*(xhat(:,i) - xss) + uss;
    if (i == ntimes) break; end

    % Simulate with nonlinear model.
    t = [time(i); mean(time(i:i+1)); time(i+1)];
    z0 = x(:,i) + zs; F0 = p(:,i) + Fs;
    Tc = u(1,i) + Tcs; F = u(2,i) + Fs;
    [tout, z] = ode15s(@massenbal, t, z0, opts);
    x(:,i+1) = z(end,:) - zs;

    % Advance state estimate.
    xhat(:,i+1) = A*xhat(:,i) ...
        + Bd*dhat(:,i) + B*u(:,i);
    dhat(:,i+1) = dhat(:,i);
end
```

Python+Numpy

```
for n in range(Nsim + 1):
    # Take plant measurement.
    y[n,:] = C.dot(x[n,:]) + v[n,:]

    # Update state estimate with measurement.
    err[n,:] = (y[n,:] - C.dot(xhat[n,:])
                - Cd.dot(dhat[n,:]))
    xhat[n,:] = xhat[n,:] + Lx.dot(err[n,:])
    dhat[n,:] = dhat[n,:] + Ld.dot(err[n,:])

    # Make sure we aren't at the last timestep.
    if n == Nsim: break

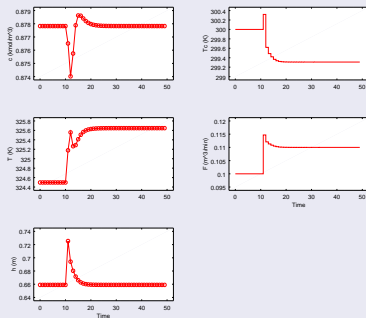
    # Steady-state target.
    rhs = np.concatenate((Bd.dot(dhat[n,:]),
                           H.dot(ysp[n,:] - Cd.dot(dhat[n,:]))))
    qsp = linalg.solve(G,rhs) # i.e. G\rhs.
    xsp = qsp[:Nx]
    usp = qsp[Nx:]

    # Regulator.
    u[n,:] = K.dot(xhat[n,:] - xsp) + usp

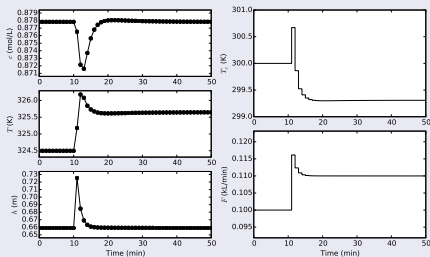
    # Simulate with nonlinear model.
    x[n+1,:] = cstr.sim(x[n,:] + xs,
                       u[n,:] + us, d[n,:] + ds) - xs

    # Advance state estimate.
    xhat[n+1,:] = (A.dot(xhat[n,:])
                  + Bd.dot(dhat[n,:]) + B.dot(u[n,:]))
    dhat[n+1,:] = dhat[n,:]
```

Octave/MATLAB



Python+Numpy



What can we do with `mpc-tools-casadi`?

- Discrete-time linear MPC
- Discrete-time nonlinear MPC
 - Explicit models
 - Runge-Kutta discretization
 - Collocation
- Discrete-time nonlinear MHE
 - Explicit models
 - Runge-Kutta discretization
 - Collocation
- Basic plotting function
- Example scripts
 - Linear
 - Solution of linear as nonlinear
 - Periodic linear
 - Example 2-8
 - Simple collocation
 - Example 1-11

Example Script I

Example script for a simple nonlinear MPC problem.

```
# Control of the Van der Pol oscillator.
import mpctools as mpc
import numpy as np

# Define model and get simulator.
Delta = .5
Nsim = 20
Nx = 2
Nu = 1
def ode(x,u):
    dxdt = [(1 - x[1]*x[1])*x[0] - x[1] + u, x[0]]
    return np.array(dxdt)

# Create a simulator.
vdp = mpc.DiscreteSimulator(ode, Delta, [Nx,Nu], ["x","u"])

# Then get nonlinear casadi functions and a linearization.
ode_casadi = mpc.getCasadiFunc(ode, [Nx,Nu], ["x","u"], funcname="f")
lin = mpc.util.getLinearization(ode_casadi,[0,0],[0],Delta=Delta)

# Also discretize using RK4.
ode_rk4_casadi = mpc.getCasadiFunc(ode_rk4, [Nx,Nu], ["x","u"], funcname="F",
                                   rk4=True, Delta=Delta, M=1)
```

Example Script II

```
# Define stage cost and terminal weight.
def lfunc(x,u): return mpc.mtimes(x.T,x) + mpc.mtimes(u.T,u)
l = mpc.getCasadiFunc(lfunc, [Nx,Nu], ["x","u"], funcname="l")

def Pffunc(x): return 10*mpc.mtimes(x.T,x)
Pf = mpc.getCasadiFunc(Pffunc, [Nx], ["x"], funcname="Pf")

# Create linear discrete-time model for comparison.
def Ffunc(x,u): return (mpc.mtimes(mpc.util.DMatrix(lin["A"]),x) +
    mpc.mtimes(mpc.util.DMatrix(lin["B"]),u))
F = mpc.getCasadiFunc(Ffunc, [Nx,Nu], ["x","u"], funcname="F")

# Make optimizers.
x0 = np.array([0,1])
Nt = 20
commonargs = dict(
    N={"x":Nx, "u":Nu, "t":Nt},
    verbosity=0,
    l=l,
    x0=x0,
    Pf=Pf,
    lb={"u" : -.75*np.ones((Nsim,Nu))},
    ub={"u" : np.ones((Nsim,Nu))},
    runOptimization=False,
)
solvers = {}
```

Example Script III

```
solvers["lmpc"] = mpc.nmpc(f=F,**commonargs)
solvers["nmpc"] = mpc.nmpc(f=ode_rk4_casadi,**commonargs)

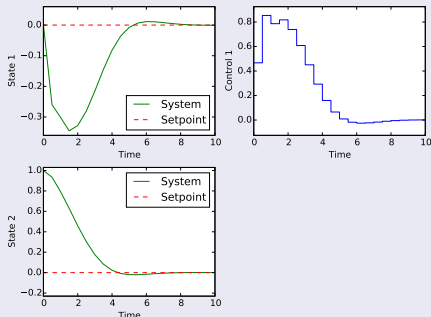
# Now simulate.
times = Delta*Nsim*np.linspace(0,1,Nsim+1)
x = {}
u = {}
for method in solvers.keys():
    x[method] = np.zeros((Nsim+1,Nx))
    x[method][0,:] = x0
    u[method] = np.zeros((Nsim,Nu))
    for t in range(Nsim):
        solvers[method].fixvar("x",0,x[method][t,:])
        solvers[method].solve()
        print "%5s %d: %s" % (method,t,solvers[method].stats["status"])
        u[method][t,:] = solvers[method].var["u",0,:]
        x[method][t+1,:] = vdp.sim(x[method][t:],u[method][t,:])
fig = mpc.plots.mpcplot(x[method],u[method],times,title=method)
fig.savefig("vdposcillator_%s.pdf" % (method,))
```

Output

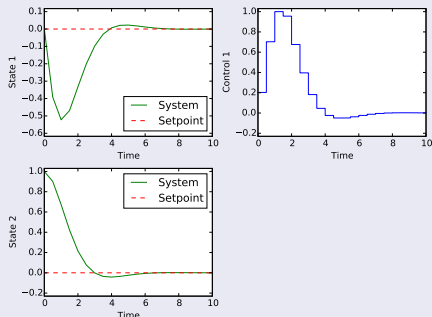
For this problem, nonlinear MPC performs slightly better.

- The computation isn't much more time-consuming because of the power of Casadi.
- The problem isn't difficult to set up because of `mpc-tools-casadi`.

Linear MPC



Nonlinear MPC

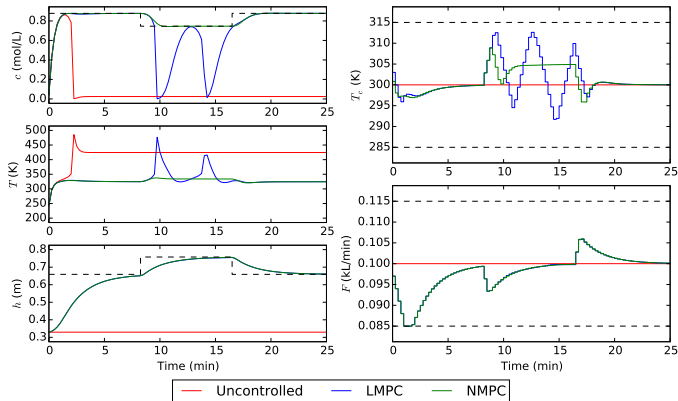


More Complicated Example

Using `mpc-tools-casadi`, we can replace the LQR and KF from Example 1.11 with nonlinear MPC and MHE.

- `cstr_startup.py` shows basic structure and a setpoint change.
- `cstr_nmpc_nmhe.py` shows steady-state target finding and NMHE.
- See the cheatsheet for important functions and syntax.

Here, nonlinear MPC knows to be less aggressive.



What can't we do yet?

- True continuous-time formulation
 - Continuous-time models with explicit time dependence are not supported
 - Quadrature for continuous-time objective function is available via collocation
 - DAE systems are possible in principle
- Quality guess generation
 - Solve sequence of smaller problems
 - Use as initial guess for large problem
 - Must do “by hand”