

Octave vs. Python for Example 1.11

Below, we present an example file to show that, for our purposes, Python isn't that much different from Octave/MATLAB. On the left side, we show the script written using Octave (MATLAB compatibility requires breaking out the subfunctions), while on the right, we show the script rewritten to use Python+Casadi (with a bit of `mpc-tools-casadi` as well).

```
% cstr.m applies offset free linear MPC
% to the linearized and
% nonlinear CSTR.
% See Pannocchia and Rawlings, AIChE J, 2002.
%
% Edits by Michael Risbeck (April 2015).

global F F0 r E k0 DeltaH rhoCp T0 c0 U Tc hs

% parameters and sizes for the nonlinear system
delta = 1;
n = 3;
m = 2;
nmeas = n;
np = 1;
small = 1e-5; % Small number.

F0 = 0.1; % m^3/min
T0 = 350; % K
c0 = 1; % kmol/m^3
r = 0.219; % m
k0 = 7.2e10; % min^-1
E = 8750; % K
U = 54.94; % kJ/(min m^2 K)
rho = 1e3; % kg/m^3
Cp = 0.239; % kJ/(kg K)
DeltaH = -5e4; % kJ/kmol
Tcs = 300; % K
hs = 0.659; % m
ps = 0.1*F0; % m^3/min

rhoCp = rho*Cp;

function df = partial(x)
    global F F0 r E k0 DeltaH rhoCp T0 c0 U Tc
    c = x(1);
    T = x(2);
    h = x(3);
    k = k0*exp(-E/T);
    kprime = k*E/(T^2);
    Fprime = F0/(pi*r^2*h);
    df = [
        -Fprime - k, -kprime*c, ...
        -Fprime*(c0-c)/h, ...
        0, 0, (c0-c)/(pi*r^2*h);
        %
        -k*DeltaH/rhoCp, -Fprime ...
        - kprime*c*DeltaH/(rhoCp) ...
        - 2*U/(r*rhoCp), ...
        -Fprime*(T0-T)/h, ...
        2*U/(r*rhoCp), 0, (T0-T)/(pi*r^2*h);
        %
        0, 0, 0, 0, -1/(pi*r^2), 1/(pi*r^2)
```

```
# Example 1.11 from Rawlings and Mayne.
import mpctools as mpc
import numpy as np
from scipy import linalg
import matplotlib.pyplot as plt
from matplotlib import gridspec

# Define some parameters and then the CSTR model.
Nx = 3
Nu = 2
Nd = 1
Ny = Nx
Delta = 1
eps = 1e-6 # Use this as a small number.

T0 = 350
c0 = 1
r = .219
k0 = 7.2e10
E = 8750
U = 54.94
rho = 1000
Cp = .239
dH = -5e4

def ode(x,u,d):
    # Grab the states, controls, and disturbance.
    [c, T, h] = x[0:Nx]
    [Tc, F] = u[0:Nu]
    [F0] = d[0:Nd]

    # Now create the ODE.
    rate = k0*c*np.exp(-E/T)

    dxdt = [
        F0*(c0 - c)/(np.pi*r**2*h) - rate,
        F0*(T0 - T)/(np.pi*r**2*h)
        - dH/(rho*Cp)*rate
        + 2*U/(r*rho*Cp)*(Tc - T),
        (F0 - F)/(np.pi*r**2)
    ]
    return np.array(dxdt)

# Turn into casadi function and simulator.
ode_casadi = mpc.getCasadiFunc(ode,
```

```

];
endfunction

function rhs = massenbalstst(x)
    global F F0 r E k0 DeltaH rhoCp T0 c0 U Tc hs
    c = x(1);
    T = x(2);
    h = x(3);
    k = k0*exp(-E/T);
    rate = k*c;
    dcdt = F0*(c0-c)/(pi*r^2*h) - rate;
    dTdt = F0*(T0-T)/(pi*r^2*h) - ...
        DeltaH/rhoCp*rate + ...
        2*U/(r*rhoCp)*(Tc-T);
    % fix the reactor height
    dhdt = h - hs;
    rhs = [dcdt; dTdt; dhdt];
endfunction

```

```

function rhs = massenbal(t, x)
    global F F0 r E k0 DeltaH rhoCp T0 c0 U Tc
    c = x(1);
    T = x(2);
    h = x(3);
    k = k0*exp(-E/T);
    rate = k*c;
    dcdt = F0*(c0-c)/(pi*r^2*h) - rate;
    dTdt = F0*(T0-T)/(pi*r^2*h) - ...
        DeltaH/rhoCp*rate + ...
        2*U/(r*rhoCp)*(Tc-T);
    dhdt = (F0 - F)/(pi*r^2);
    rhs = [dcdt; dTdt; dhdt];
endfunction

```

```

%% find the steady-state
Fs = F0; F = F0;
Tc = Tcs;
z0 = [c0; Tc; hs];
[z, fval, info] = fsolve(@massenbalstst, z0);
if ( info ~= 1 )
    warning('failure to find steady state!')
endif
cs = z(1);
Ts = z(2);
hs = z(3);
zs = [cs; Ts; hs];

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%check the linear model
G = partial(zs);
Ac = G(:, 1:n);
Bc = G(:, n+1:n+2);
Bpc = G(:, n+3:end);
C = eye(n);
sys = ss(Ac, [Bc, Bpc], C, ...

```

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[Nx,Nu,Nd], ["x","u","d"], funcname="ode")
cstr = mpc.DiscreteSimulator(ode, Delta,
    [Nx,Nu,Nd], ["x","u","d"])

```

```

# We don't need to take any derivatives by hand
# because Casadi can do that.

```

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# Steady-state values.
cs = .878
Ts = 324.5
hs = .659
Fs = .1
Tcs = 300
F0s = .1

# Update the steady-state values a few times to make
# sure they don't move.
for i in range(10):
    [cs,Ts,hs] = cstr.sim([cs,Ts,hs],[Tcs,Fs],
        [F0s]).tolist()
xs = np.array([cs,Ts,hs])
us = np.array([Tcs,Fs])
ds = np.array([F0s])

# Now get a linearization at this steady state.
ss = mpc.util.linearizeModel(ode_casadi,
    [xs,us,ds], ["A","B","Bp"], Delta)
A = ss["A"]
B = ss["B"]
Bp = ss["Bp"]
C = np.eye(Nx)

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        zeros(size(C,1),size([Bc, Bpc],2)));
dsys = c2d(sys, delta);
A = dsys.a;
B = dsys.b(:,1:m);
Bp = dsys.b(:,m+1:end);

% set up linear controller
Q = diag(1./zs.^2);
R = diag(1./[Tcs; Fs].^2);
[K, P] = dlqr(A, B, Q, R);
K = -K;

% Pick whether to use good disturbance model.
useGoodDisturbanceModel = true();

if useGoodDisturbanceModel
    % disturbance model 6; no offset
    nd = 3;
    Bd = zeros(n, nd);
    Bd(:,3) = B(:,2);
    Cd = [1 0 0; 0 0 0; 0 1 0];
else
    % disturbance model with offset
    nd = 2;
    Bd = zeros(n, nd);
    Cd = [1 0; 0 0; 0 1];
end

Aaug = [A, Bd; zeros(nd, n), eye(nd)];
Baug = [B; zeros(nd, m)];
Caug = [C, Cd];
naug = size(Aaug,1);

% detectability test of disturbance model
detec = rank([eye(n+nd) - Aaug; Caug]);
if (detec < (n+nd))
    warning('augmented system is not detectable\n')
endif

% set up state estimator; use KF
Qw = zeros(naug);
Qw(1:n,1:n) = small*eye(n);
Qw(n+1:end,n+1:end) = small*eye(nd); Qw(end,end)=1.0;
Rv = diag((5e-4*zs).^2);
[L, M, P] = dlqe(Aaug, eye(naug), Caug, Qw, Rv);
Lx = L(1:n,:);
Ld = L(n+1:end,:);

ntimes = 50;
x0 = zeros(n, 1);
x = zeros(n, ntimes);
x(:, 1) = x0;
y = zeros(nmeas, ntimes);

```

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# Weighting matrices for controller.
Q = np.diag(xs**-2)
R = np.diag(us**-2)

[K, Pi] = mpc.util.dlqr(A,B,Q,R)

# Define disturbance model.
useGoodDisturbanceModel = True

# Bad disturbance model with offset.
if useGoodDisturbanceModel:
    Nid = Ny # Number of integrating disturbances.
else:
    Nid = Nu

Bd = np.zeros((Nx,Nid))
Cd = np.zeros((Ny,Nid))

if useGoodDisturbanceModel:
    Cd[0,0] = 1
    Cd[2,1] = 1
    Bd[:,2] = B[:,1] # or Bp[:,0]
else:
    Cd[0,0] = 1
    Cd[2,1] = 1

# Augmented system. Trailing .A casts to array type.
Aaug = np.bmat([[A,Bd],
                 [np.zeros((Nid,Nx)),np.eye(Nid)]]).A
Baug = np.vstack((B,np.zeros((Nid,Nu))))
Caug = np.hstack((C,Cd))

# Check rank condition for augmented system.
svds = linalg.svdvals(np.bmat([[np.eye(Nx) - A,
                                -Bd],[C,Cd]]))

rank = sum(svds > 1e-10)
if rank < Nx + Nid:
    print "*Warning: system not detectable!"

# Build augmented penalty matrices for KF.
Qw = eps*np.eye(Nx + Nid)
Qw[-1,-1] = 1
Rv = eps*np.diag(xs**2)

# Get Kalman filter.
[L, P] = mpc.util.dlqe(Aaug, Caug, Qw, Rv)
Lx = L[:Nx,:];
Ld = L[Nx:,:];

# Now simulate things.
Nsim = 50
t = np.arange(Nsim+1)*Delta
x = np.zeros((Nsim+1,Nx))
u = np.zeros((Nsim,Nu))

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u = zeros(m, ntimes);
randn('seed', 0);
v = zeros(nmeas, ntimes);
xhat_ = zeros(n, ntimes);
dhat_ = zeros(nd, ntimes);
xhat = xhat_;
dhat = dhat_;
time = (0:ntimes-1)*delta;
xs = zeros(n, ntimes);
us = zeros(m, ntimes);
ys = zeros(nmeas, ntimes);
etas = zeros(nmeas, ntimes);
options = [];

% Disturbance and setpoint.
p = [zeros(np, 10), ps*ones(np, ntimes-10)];
yset = zeros(nmeas, 1);

% Steady-state target matrices.
H = [1 0 0; 0 0 1];
Ginv = inv([eye(n)-A, -B; ...
            H*C, zeros(size(H,1), m)]);

for i = 1: ntimes
    %% measurement
    y(:,i) = C*x(:,i) + v(:,i);

    %% state estimate
    ey = y(:,i) - C*xhat_(:,i) - Cd*dhat_(:,i);
    xhat(:,i) = xhat_(:,i) + Lx*ey;
    dhat(:,i) = dhat_(:,i) + Ld*ey;

    % Stop if at last time.
    if (i == ntimes) break; endif

    %% target selector
    tmodel.p = dhat(:,i);

    qs = Ginv*[Bd*dhat(:,i); H*(yset-Cd*dhat(:,i))];
    xss = qs(1:n);
    uss = qs(n+1:end);
    xs(:,i) = xss;
    us(:,i) = uss;
    ys(:,i) = C*xss + Cd*dhat(:,i);

    %% control law
    x0 = xhat(:,i) - xs(:,i);
    u(:,i) = K*x0 + us(:,i);

    %% plant evolution
    t = [time(i); mean(time(i:i+1)); time(i+1)];
    z0 = x(:,i) + zs;
    Tc = u(1,i) + Tcs;
    F = u(2,i) + Fs;
    F0 = p(:,i) + Fs;

```

```

y = np.zeros((Nsim+1,Ny))
err = y.copy()
v = y.copy()
xhat = x.copy() # State estimate after measurement.
xhatm = xhat.copy() # ... before measurement.
dhat = np.zeros((Nsim+1,Nid))
dhatm = dhat.copy()

# Pick disturbance and setpoint.
d = np.zeros((Nsim,Nd))
d[:,0] = (t[:-1] >= 10)*1*F0s
ysp = np.zeros(y.shape)
contVars = [0,2] # Concentration and height.

# Steady-state target selector matrices.
H = np.zeros((Nu,Ny))
H[range(len(contVars)),contVars] = 1
Ginv = np.array(np.bmat([
    [np.eye(Nx) - A, -B],
    [H.dot(C), np.zeros((H.shape[0], Nu))]]).I) # Take inverse.

for n in range(Nsim + 1):
    # Take plant measurement.
    y[n,:] = C.dot(x[n,:]) + v[n,:]

    # Update state estimate with measurement.
    err[n,:] = (y[n,:] - C.dot(xhatm[n,:])
                - Cd.dot(dhatm[n,:]))
    xhat[n,:] = xhatm[n,:] + Lx.dot(err[n,:])
    dhat[n,:] = dhatm[n,:] + Ld.dot(err[n,:])

    # Make sure we aren't at the last timestep.
    if n == Nsim: break

    # Steady-state target.
    rhs = np.concatenate((Bd.dot(dhat[n,:]),
                           H.dot(ysp[n,:] - Cd.dot(dhat[n,:]))))
    qsp = Ginv.dot(rhs)
    xsp = qsp[:Nx]
    usp = qsp[Nx:]

    # Regulator.
    u[n,:] = K.dot(xhat[n,:] - xsp) + usp

    # Simulate with nonlinear model.
    x[n+1,:] = cstr.sim(x[n,:] + xs, u[n,:] + us,
                        d[n,:] + ds) - xs

```

```

[tout, z] = ode15s(@massenbal, t, z0, options);
if sum(tout ~= t)
    warning('integrator failed!')
end
x(:,i+1) = z(end,:) - zs;

%% advance state estimates
xhat(:,i+1) = A*xhat(:,i) + ...
    Bd*dhat(:,i) + B*u(:,i);
dhat(:,i+1) = dhat(:,i);

end
u(:,end) = u(:,end-1); % Repeat for stair plot.

% dimensional units
yd = y + kron(ones(1, ntimes), zs);
ud = u + kron(ones(1, ntimes), [Tcs; Fs]);

% *** Plots ***
figure()
axmul = eye(4) + .05*[1,-1,0,0; ...
    -1,1,0,0;0,0,1,-1;0,0,-1,1];
subplot(3,2,1)
plot(time, yd(1,:), '-or')
ylabel('cl(kmol/m3)')
axis(axis()*axmul)
subplot(3,2,3)
plot(time, yd(2,:), '-or')
ylabel('Tl(K)')
axis(axis()*axmul)
subplot(3,2,5)
plot(time, yd(3,:), '-or')
ylabel('hl(m)')
xlabel('Time')
axis(axis()*axmul)
subplot(3,2,2)
stairs(time, ud(1,:), '-r')
ylabel('Tcl(K)')
axis(axis()*axmul)
subplot(3,2,4)
stairs(time, ud(2,:), '-r')
ylabel('Fl(m3/min)')
xlabel('Time')
axis(axis()*axmul)
print('cstr_octave.pdf', '-dpdf', '-S720,600')

# Advance state estimate.
xhatm[n+1,:] = (A.dot(xhat[n,:])
    + Bd.dot(dhat[n,:]) + B.dot(u[n,:]))
dhatm[n+1,:] = dhat[n,:]

# Define plotting function.
def cstrplot(x,u,ysp=None,contVars=[],title=None):
    u = np.concatenate((u,u[-1:,:]))
    t = np.arange(0,x.shape[0])*Delta
    ylabelsx = ["$c$ (mol/L)", "$T$ (K)", "$h$ (m)"]
    ylabelsu = ["$T_c$ (K)", "$F$ (kL/min)"]

    gs = gridspec.GridSpec(Nx*Nu,2)

    fig = plt.figure(figsize=(10,6))
    for i in range(Nx):
        ax = fig.add_subplot(gs[i*Nu:(i+1)*Nu,0])
        ax.plot(t,x[:,i] + xs[i],'-ok')
        if i in contVars:
            ax.step(t,ysp[:,i] + xs[i],'-r',
                where="post")
            ax.set_ylabel(ylabelsx[i])
            mpc.plots.zoomaxis(ax,yscale=1.1)
            mpc.plots.prettyaxesbox(ax)
            mpc.plots.prettyaxesbox(ax,
                facecolor="white",front=False)
        ax.set_xlabel("Time (min)")
    for i in range(Nu):
        ax = fig.add_subplot(gs[i*Nx:(i+1)*Nx,1])
        ax.step(t,u[:,i] + us[i],'-k',where="post")
        ax.set_ylabel(ylabelsu[i])
        mpc.plots.zoomaxis(ax,yscale=1.25)
        mpc.plots.prettyaxesbox(ax)
        mpc.plots.prettyaxesbox(ax,
            facecolor="white",front=False)
        ax.set_xlabel("Time (min)")
    fig.tight_layout(pad=.5)
    if title is not None:
        fig.canvas.set_window_title(title)
    return fig

fig = cstrplot(x,u,ysp=None,contVars=[],title=None)
fig.savefig("cstr_python.pdf",facecolor="none")

```