深蓝学院
shenlanxueyuan.com

Motion Planning with Model Predictive Control

主讲人 赖叔朋

新加坡国立大学 Research Fellow
主要研究方向为 Motion Planning 以及
Nonlinear Model Predictive Control

目录

Introduction: Model Predictive Control

- Model
  - System model
  - Problem model

$$\min(p - p_d)$$

$$F = ma$$

$$\dot{p} = v$$

$$\dot{v} = a$$

- Prediction
  - State space
  - Input space
  - Parameter space

- Control
  - The process of choosing the best policy

Introduction: Model

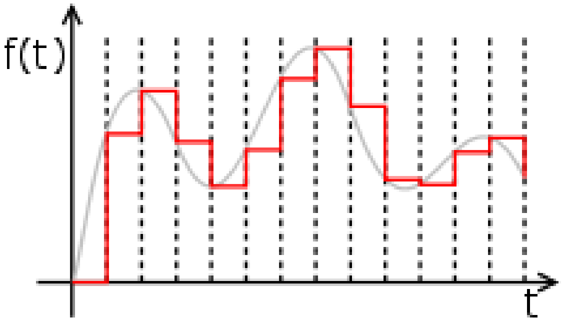
$$\min C_F\left(x(t_f)\right) + \int_{t=t_0}^{t_f} C_R(x,u)\,dt$$

$$\dot{x} = f(x,u)$$
$$g(x,u) < 0$$
$$h(x,u) = 0$$
$$x \notin Obstacle$$

Introduction: Parameter space

$$\min_{u} C_F\left(x(t_f)\right) + \int_{t=t_0}^{t_f} C_R(x,u)\,dt$$

- Zero order hold (direct discretization)
- Polynomial
- B-spline
- Numerical mapping
  - Jerk limited trajectory
  - Neural network based method



$$u(t) = at^3 + bt^2 + ct + d$$

Introduction: Optimization

Searching:

- Graph search
- Random sampling based search

Convex optimization:

- Quadratic programming

Nonconvex optimization:

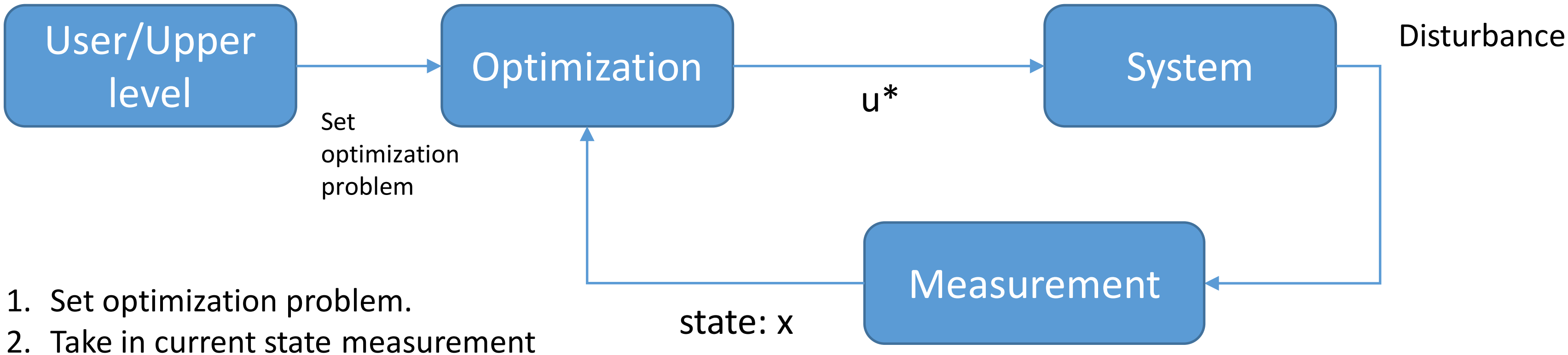
- Sequential quadratic programming
- Particle swarm optimization

Nonconvex, nonlinear, discontinuous

Introduction: Control

$$\min_u C_F\left(x(t_f)\right) + \int_{t=t_0}^{t_f} C_R(x,u)\,dt$$
$$\dot{x} = f(x,u)$$
$$g(x,u) < 0$$
$$h(x,u) = 0$$
$$x \notin Obstacle$$

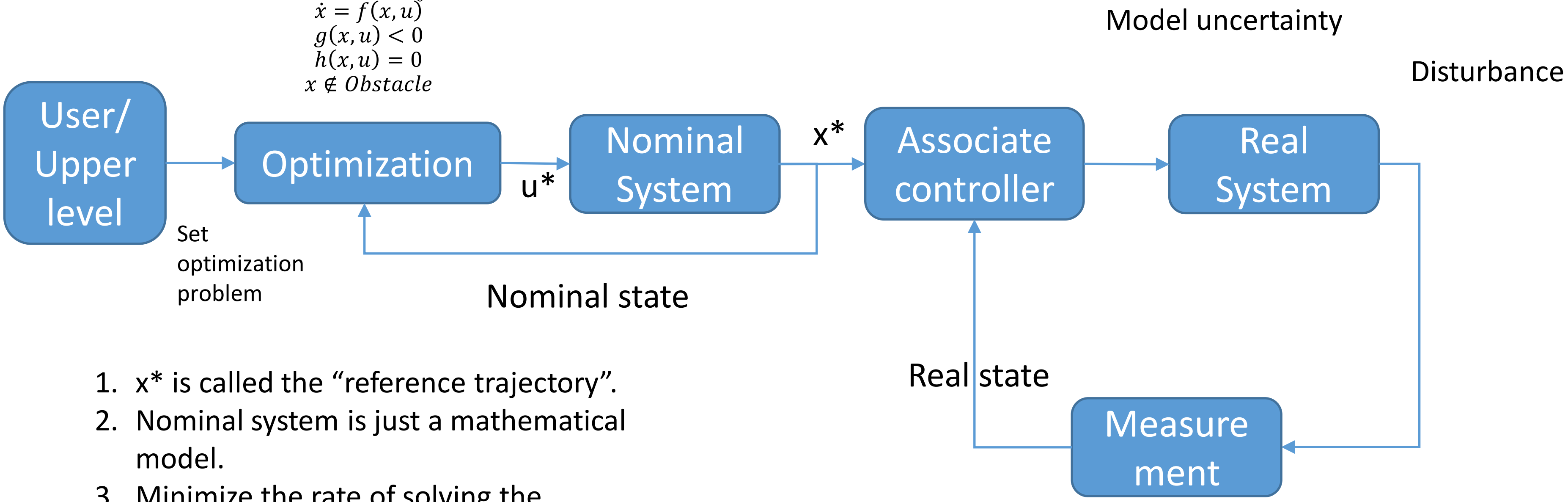
Model uncertainty



1. Set optimization problem.
2. Take in current state measurement and finalize the optimization problem.
3. Solve the optimization and get u*.
4. Apply u* for a short period of time.

Introduction: Tube based MPC

$$\min_{u} C_F\left(x(t_f)\right) + \int_{t=t_0}^{t_f} C_R(x,u)\,dt$$
$$\dot{x} = f(x,u)$$
$$g(x,u) < 0$$
$$h(x,u) = 0$$
$$x \notin Obstacle$$



1. x* is called the "reference trajectory".
2. Nominal system is just a mathematical model.
3. Minimize the rate of solving the optimization problem.
4. Leave the robust tracking problem to the associate controller.

"Tube-Based MPC: a Contraction Theory Approach"

Introduction: Convenient sources

- Matlab MPC toolbox: https://www.mathworks.com/products/mpc.html
- µAO-MPC: http://ifatwww.et.uni-magdeburg.de/syst/muAO-MPC/
- Acado toolkit: https://acado.github.io/
- YANE: http://www.nonlinearmpc.com/
- Multi-Parametric Toolbox 3: https://www.mpt3.org/

Linear MPC: basics

Continuous model

$$\dot{p} = v$$
$$\dot{v} = a$$
$$\dot{a} = j$$

$$p_i = p(i \cdot dt)$$
$$v_i = v(i \cdot dt)$$
$$a_i = a(i \cdot dt)$$

$\longrightarrow$

Time discretization

Discrete model

$$p_{i+1} = p_i + v_i dt + \frac{1}{2} a_i dt^2 + \frac{1}{6} j_i dt^3$$
$$v_{i+1} = v_i + a_i dt + \frac{1}{2} j_i dt^2$$
$$a_{i+1} = a_i + j_i dt$$

$$i = [0,2, \dots, 19]$$

$$dt = 0.2$$

Linear MPC: basics

Discrete model

$$p_{i+1} = p_i + v_i dt + \frac{1}{2} a_i dt^2 + \frac{1}{6} j_i dt^3$$

$$v_{i+1} = v_i + a_i dt + \frac{1}{2} j_i dt^2$$

$$a_{i+1} = a_i + j_i dt$$

$$i = [0,2, \dots, 19]$$

$$dt = 0.2$$

Linear Matrix form

Prediction model

$$P = T_p J + B_p$$

$$V = T_v J + B_v$$

$$A = T_a J + B_a$$

$$P = [p_1, p_2, p_3, \dots, p_{20}]^T$$

$$V = [v_1, v_2, v_3, \dots, v_{20}]^T$$

$$A = [a_1, a_2, a_3, \dots, a_{20}]^T$$

$$J = [j_0, j_1, j_2, \dots, j_{19}]^T$$

Linear MPC: basics

Prediction model

$$P = T_p J + B_p$$

$$V = T_v J + B_v$$

$$A = T_a J + B_a$$

```
function [Tp, Tv, Ta, Bp, Bv, Ba] = getPredictionMatrix(K,dt,p_0,v_0,a_0)
Ta=zeros(K);
Tv=zeros(K);
Tp=zeros(K);

for i = 1:K
    Ta(i,1:i) = ones(1,i)*dt;
end

for i = 1:K
    for j = 1:i
        Tv(i,j) = (i-j+0.5)*dt^2;
    end
end

for i = 1:K
    for j = 1:i
        Tp(i,j) = ((i-j+1)*(i-j)/2+1/6)*dt^3;
    end
end

Ba = ones(K,1)*a_0;
Bv = ones(K,1)*v_0;
Bp = ones(K,1)*p_0;

for i=1:K
    Bv(i) = Bv(i) + i*dt*a_0;
    Bp(i) = Bp(i) + i*dt*v_0 + i^2/2*a_0*dt^2;
end
```

Linear MPC: basics

Problem model:

Target 1: Zero position, zero velocity and zero acceleration.
Target 2: Smooth trajectory.

Optimization target 1: $\min_{J} w_1 P^T P + w_2 V^\top V + w_3 A^\top A$

Optimization target 2: $\min_{J} w_4 J^\top J$

Linear MPC: basics

Optimization:

The overall optimization target:

$$\min_J w_1 P^\top P + w_2 V^\top V + w_3 A^\top A + w_4 J^\top J$$

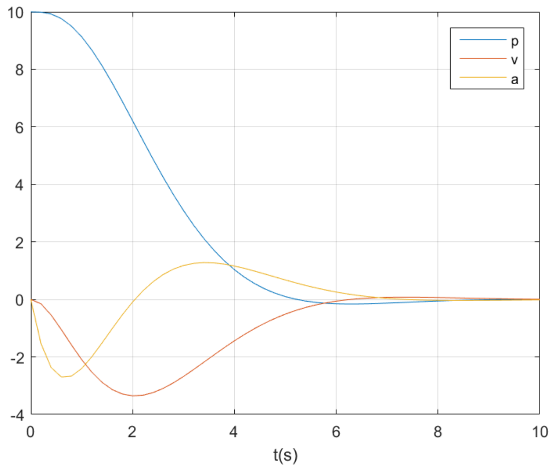
Combine with

$$P = T_p J + B_p \qquad V = T_v J + B_v \qquad A = T_a J + B_a$$

We have

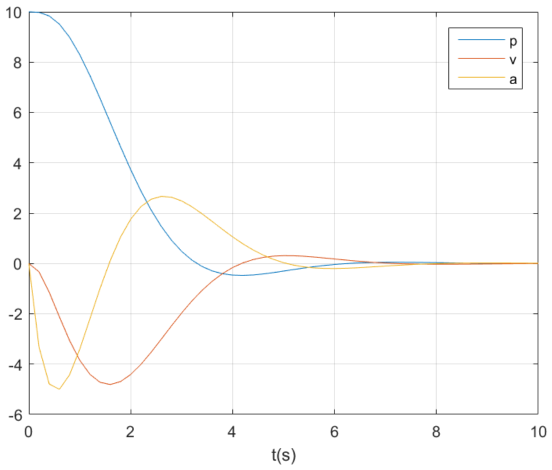
$$\min_J J^\top \left(w_1 T_p^\top T_p + w_2 T_v^\top T_v + w_3 T_a^\top T_a + w_4 I \right) J +$$
$$2\left(w_1 B_p^\top T_p + w_2 B_v^\top T_v + w_3 B_a^\top T_a \right) J + \text{constant}$$

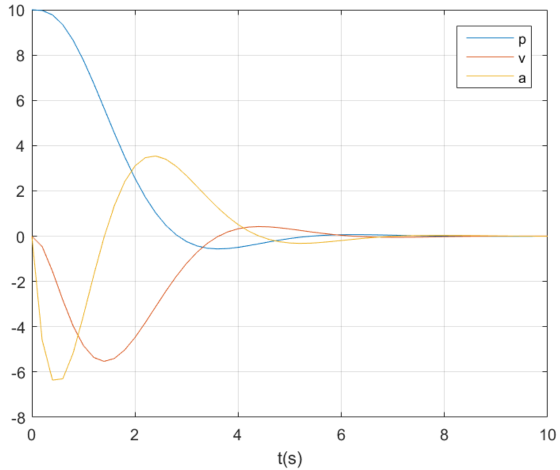
```
p_0 = 10;
v_0 = 0;
a_0 = 0;
K=20;
dt=0.2;
log=[0 p_0 v_0 a_0];
w1 = 1;
w2 = 1;
w3 = 1;
w4 = 1;
for t=0.2:0.2:10
    %% Construct the prediction matrix
    [Tp, Tv, Ta, Bp, Bv, Ba] = getPredictionMatrix(K,dt,p_0,v_0,a_0);

    %% Construct the optimization problem
    H = w4*eye(K)+w1*(Tp'*Tp)+w2*(Tv'*Tv)+w3*(Ta'*Ta);
    F = w1*Bp'*Tp+w2*Bv'*Tv+w3*Ba'*Ta;

    %% Solve the optimization problem
    J = quadprog(H,F,[],[]);

    %% Apply the control
    j = J(1);
    p_0 = p_0 + v_0*dt + 0.5*a_0*dt^2 + 1/6*j*dt^3;
    v_0 = v_0 + a_0*dt + 0.5*j*dt^2;
    a_0 = a_0 + j*dt;

    %% Log the states
    log = [log; t p_0 v_0 a_0];
end
```



w1=1
w2=1
w3=1
w4=1

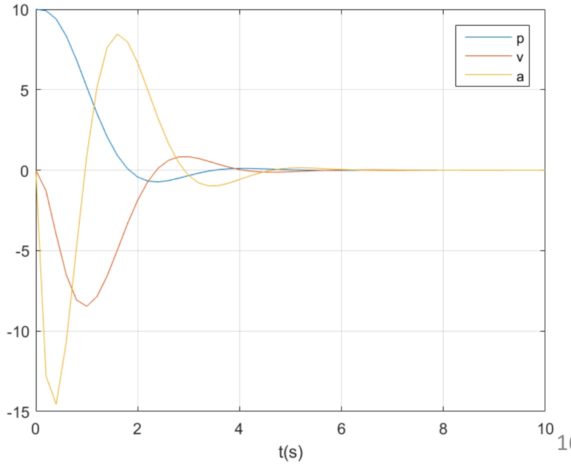
w1=5
w2=1
w3=1
w4=1





w1=10
w2=1
w3=1
w4=1

w1=100
w2=1
w3=1
w4=1



Linear MPC: basics

Optimization problem

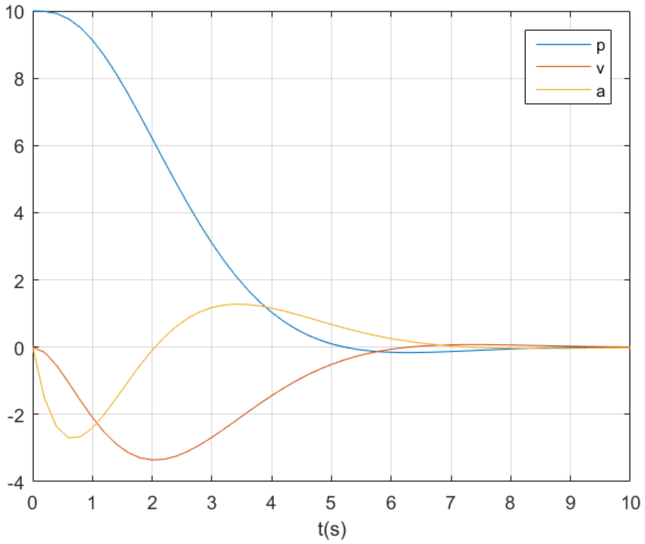
$$\min_J J^\top\left(w_1 T_p^\top T_p + w_2 T_v^\top T_v + w_3 T_a^\top T_a + w_4 I\right)J +$$
$$2\left(w_1 B_p^\top T_p + w_2 B_v^\top T_v + w_3 B_a^\top T_a\right)J + \text{constant}$$

$$\text{Let } H = \left(w_1 T_p^\top T_p + w_2 T_v^\top T_v + w_3 T_a^\top T_a + w_4 I\right)$$
$$F = \left(w_1 B_p^\top T_p + w_2 B_v^\top T_v + w_3 B_a^\top T_a\right)$$

$$\min_J J^\top H J + 2FJ$$

$$\Downarrow$$

$$J = -H^{-1}F$$




```
%% Solve the optimization problem
J = -H\F';
```

Linear MPC: hard constraints

Constraints

$$-1 \leq v_i \leq 1, \forall i \in \{1, 2, 3 \cdots, 20\}$$

$$-1 \leq a_i \leq 1, \forall i \in \{1, 2, 3, \ldots, 20\}$$

Matrix form:

$$-1_{20\times1} \leq V \leq 1_{20\times1}$$

$$-1_{20\times1} \leq A \leq 1_{20\times1}$$

$$V = T_v J + B_v$$

$$A = T_a J + B_a$$

Final form:

$$-1_{20\times1} \leq T_v J + B_v \leq 1_{20\times1}$$

$$-1_{20\times1} - B_v \leq T_v J \leq 1_{20\times1} - B_v$$

$$-1_{20\times1} - B_a \leq T_a J \leq 1_{20\times1} - B_a$$

Less equal form:

$$T_v J \leq 1_{20\times1} - B_v$$
$$-T_v J \leq 1_{20\times1} + B_v$$
$$T_a J \leq 1_{20\times1} - B_a$$
$$-T_a J \leqslant 1_{20\times1} + B_a$$

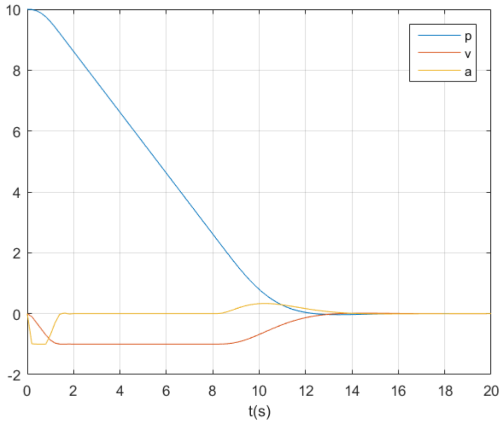
Linear MPC: hard constraints

$$T_v J \leq 1_{20\times1} - B_v$$
$$-T_v J \leq 1_{20\times1} + B_v$$
$$T_a J \leq 1_{20\times1} - B_a$$
$$-T_a J \leqslant 1_{20\times1} + B_a$$

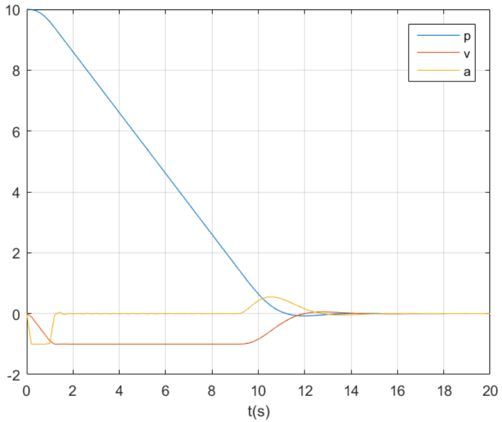
```
A = [Tv;-Tv;Ta;-Ta];
b = [ones(20,1)-Bv;ones(20,1)+Bv;ones(20,1)-Ba;ones(20,1)+Ba];
```

```
J = quadprog(H,F,A,b);
```

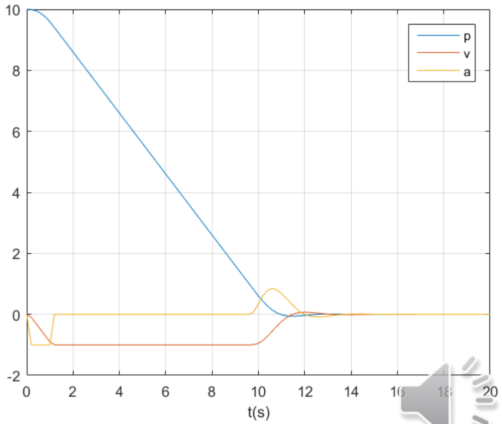
w1=1, w2=1, w3=1, w4=1



w1=10, w2=1, w3=1, w4=1



w1=100, w2=1, w3=1, w4=1



Linear MPC: soft constraints

- What if the velocity and acceleration constraints are inevitably violated?

$$v_0 = 3m/s$$

$$-1 \leq v_i \leq 1, \forall i \in \{1, 2, 3 \cdots, 20\}$$

$$-1 \leq a_i \leq 1, \forall i \in \{1, 2, 3, \ldots, 20\}$$

The solver will report no solution!!

The controller won't know what to do.

Linear MPC: soft constraints

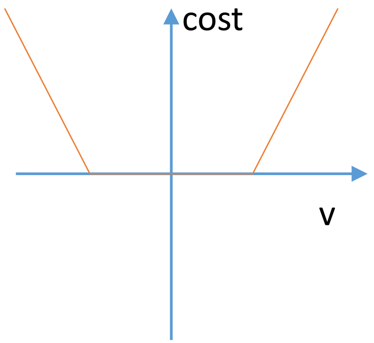
Add a penalty function to the optimization target

$$\min_{J} w_1 P^\top P + w_2 V^\top V + w_3 A^\top A + w_4 J^\top J + S(V)$$

$$S(V) = \sum_{i=1}^{20} s(v_i)$$

$$s(v_i) = \begin{cases} 0 & \text{if} \quad \|v_i\| \leqslant 1 \\ M \cdot (\|v_i\| - 1) & \quad \text{else} \end{cases}$$

M is just a large positive number



Linear MPC: soft constraints

Original constraints

$$
\begin{aligned}
T_v J &\leq 1_{20\times1} - B_v \\
-T_v J &\leq 1_{20\times1} + B_v \\
T_a J &\leq 1_{20\times1} - B_a \\
-T_a J &\leqslant 1_{20\times1} + B_a
\end{aligned}
$$

Add the slack variable L

$$
\begin{aligned}
-T_v J &\leqslant 1_{20\times1} + B_v + L \\
-L &\leqslant 0
\end{aligned}
$$

$$L = [l, l_2, l_3, \cdots, l_{20}]^\top$$

New optimization target

$$\min_{J,L} w_1 P^T P + w_2 V^\top V + w_3 A^\top A + w_4 J^\top J + w_5 L^\top L$$

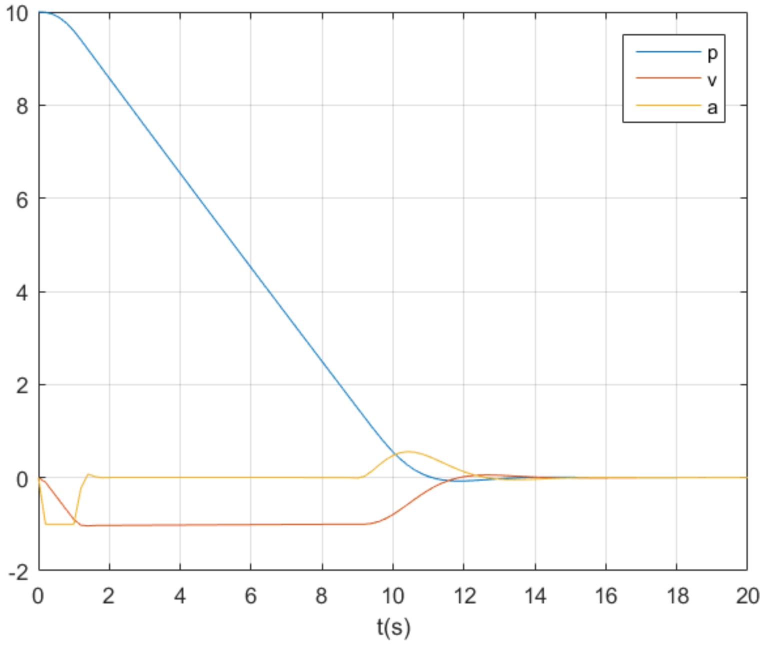
Set new programming variable as $\bar{J} = \begin{bmatrix} J \\ L \end{bmatrix}$

Then all the H, F, A, b matrix shall be adjusted accordingly

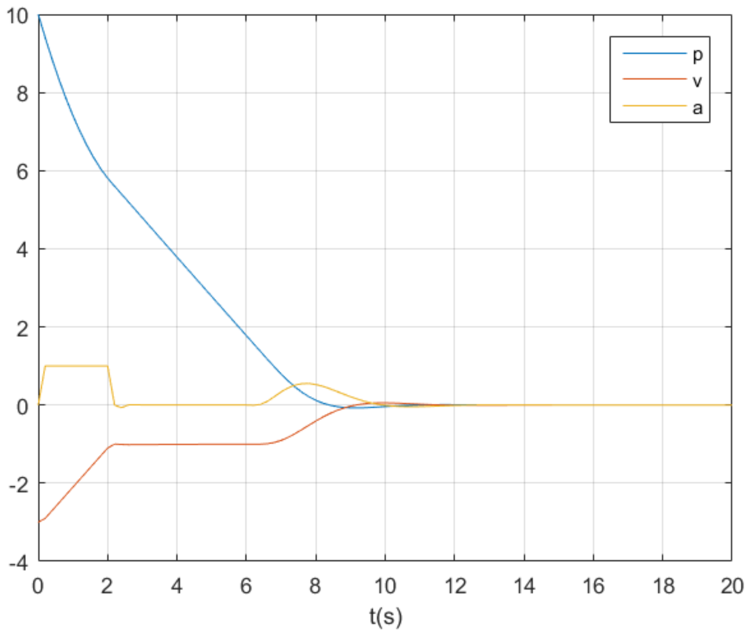
```
%% Construct the prediction matrix
[Tp, Tv, Ta, Bp, Bv, Ba] = getPredictionMatrix(K,dt,p_0,v_0,a_0);

%% Construct the optimization problem
H = blkdiag(w4*eye(K)+w1*(Tp'*Tp)+w2*(Tv'*Tv)+w3*(Ta'*Ta),w5*eye(K));
F = [w1*Bp'*Tp+w2*Bv'*Tv+w3*Ba'*Ta zeros(1,K)];

A = [Tv zeros(K);-Tv -eye(K);Ta zeros(K); -Ta zeros(K); zeros(size(Ta)) -eye(K)];
b = [ones(20,1)-Bv;ones(20,1)+Bv;ones(20,1)-Ba;ones(20,1)+Ba; zeros(K,1)];
%% Solve the optimization problem
J = quadprog(H,F,A,b);
```

Linear MPC: soft constraints

w1=10, w2=1, w3=1, w4=1, w5 = 1e4



w1=10, w2=1, w3=1, w4=1, w5 = 1e4



Linear MPC: soft constraints

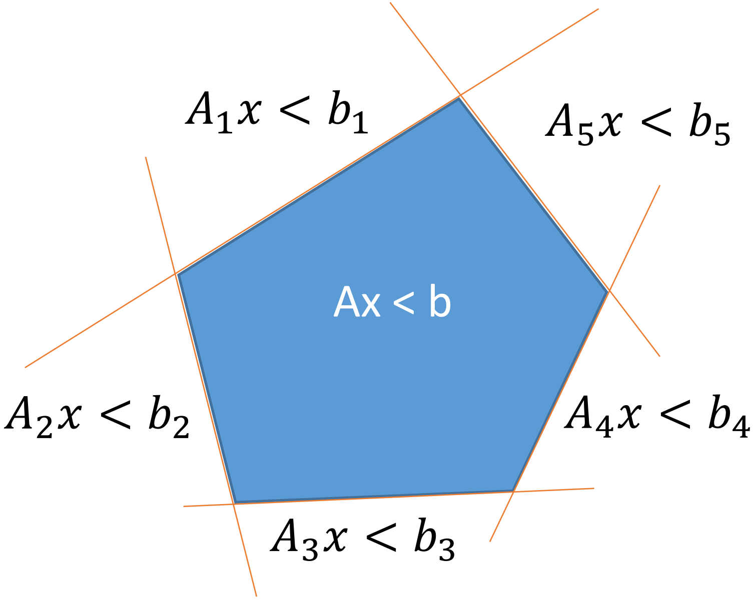
- State constraints -> use soft constraints
  - They are effected by measurement noise and disturbances

- Input constraints -> use hard constraints
  - They can be varied arbitrarily, and their violation might harm the physical system.

Linear MPC: limitation

- It usually requires a linear model, or the model can be reasonably linearized (adaptive MPC).

- Obstacle constraints are usually non-convex by nature.



Parameter space: Boundary constrained motion primitives (BSCP)

- Polynomial parameter space

Initial state

$$\text{Parameter: a b c d} \xrightarrow{u(t) = at^3 + bt^2 + ct + d} \text{Input: u} \xrightarrow{\text{Simulate}} \text{State trajectory}$$

- Zero order hold parameter space

Initial state

$$\text{Parameter: J} \xrightarrow{zero\ order\ hold} \text{Input: u} \xrightarrow{\text{Simulate}} \text{State trajectory}$$

- BSCP

Parameter:

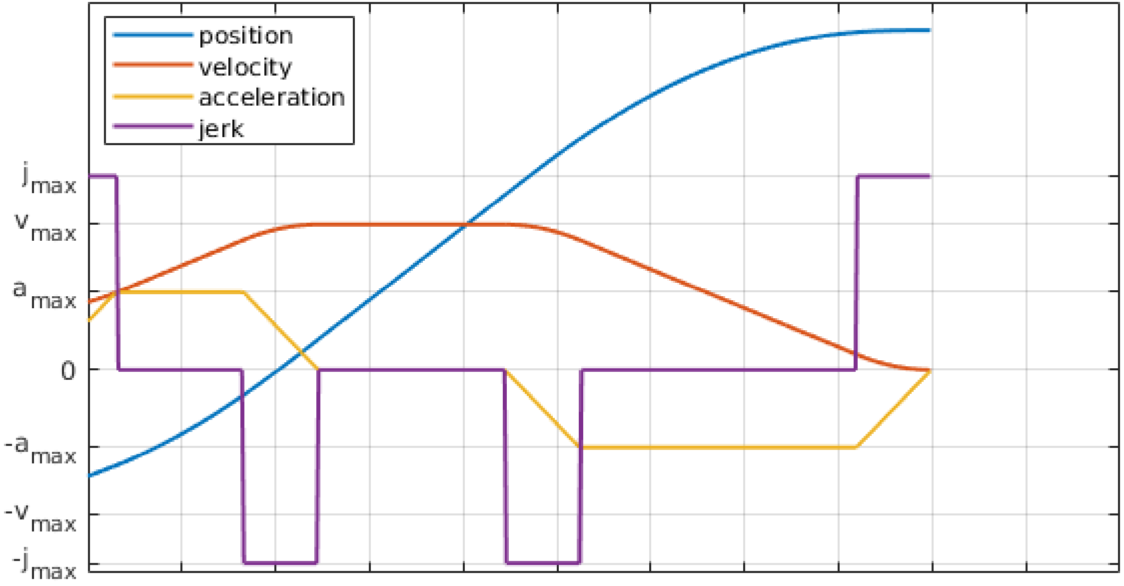
Initial state

Desired final state

$$\xrightarrow{\text{Solve boundary value problem (BVP)}} \text{Input: u} \xrightarrow{\text{Simulate}} \text{State trajectory}$$

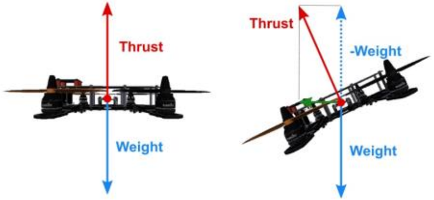
Parameter space: Jerk limited trajectory

- Not only limited jerk, but also limited acceleration and velocity.
- We will refer it as JLT later.



The jerk limited trajectory for a single axis

Parameter space: Jerk limited trajectory



<u>Inner loop constraints:</u>

The limited *body rate* $\omega_{\max}$ and *total thrust* $f_{\max}$

Sufficient condition

<u>Outer loop constraints:</u>

The limited acceleration and jerk

$$\sqrt{\dddot{x}^2 + \dddot{y}^2 + \dddot{z}^2} \leq (\ddot{z}_{\min} + g)\omega_{\max}$$

$$\sqrt{\ddot{x}^2 + \ddot{y}^2 + (\ddot{z} + g)^2} \leq \frac{f_{\max}}{m_{\mathrm{v}}}$$

$$\ddot{z} \geq \ddot{z}_{\min} \geq \frac{f_{\min}}{m_{\mathrm{v}}} - g$$

Decoupling

Sufficient condition

<u>Single axis constraints:</u>

The limited acceleration and jerk

$$-a_{\max} \leqslant a(t) \leqslant a_{\max},$$

$$-u_{\mathrm{j}\max} \leqslant u_{\mathrm{j}}(t) \leqslant u_{\mathrm{j}\max},$$

Limit the velocity for safety

<u>Final constraints:</u>

The limited acceleration and jerk

$$-v_{\max} \leqslant v(t) \leqslant v_{\max},$$

$$-a_{\max} \leqslant a(t) \leqslant a_{\max},$$

$$-u_{\mathrm{j}\max} \leqslant u_{\mathrm{j}}(t) \leqslant u_{\mathrm{j}\max},$$

Parameter space: Jerk limited trajectory

The jerk limited trajectory problem considers a system characterized by

$$\begin{pmatrix} \dot{p} \\ \dot{v} \\ \dot{a} \end{pmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} p \\ v \\ a \end{pmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_{\mathrm{j}}$$

with $-v_{\max} \leqslant v(t) \leqslant v_{\max}, \quad -a_{\max} \leqslant a(t) \leqslant a_{\max}, \quad -u_{\mathrm{j}\max} \leqslant u_{\mathrm{j}}(t) \leqslant u_{\mathrm{j}\max}$

The traditional Time Optimal Control (TOC) or bang-bang control problem considers

$$\dot{x}(t) = Ax(t) + Bu(t)$$

with

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ a \end{bmatrix}, \quad x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} y \\ v \end{pmatrix} = \begin{pmatrix} y \\ \dot{y} \end{pmatrix} \qquad (4.6)$$

Note that $v = \dot{y}$ is the velocity of the system. Let the control input be constrained as follows:

$$| u(t) | \leq u_{\max}$$

Parameter space: Jerk limited trajectory

Problem:

From arbitrary state to a desired velocity.
Subject to:

$$v(0) = v_0, \quad v(T) = v_{\mathrm{ref}}$$

$$a(0) = a_0, \quad a(T) = 0$$

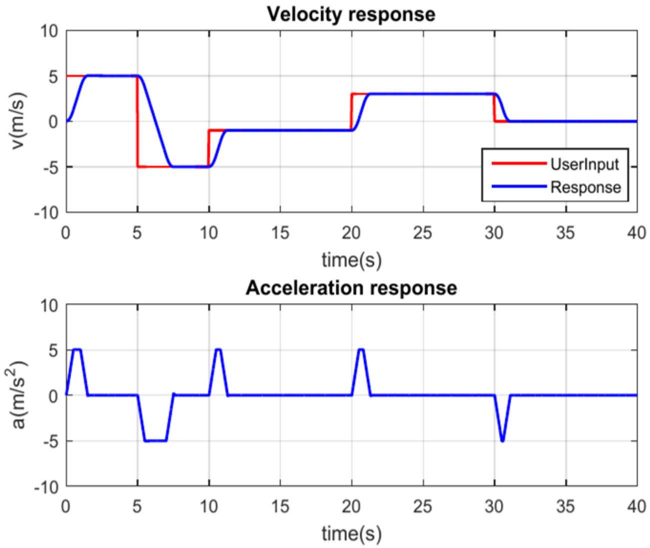
$$\dot{v}(t) = a(t)$$

$$\dot{a}(t) = u_{\mathrm{j}}(t)$$

$$-a_{\max} \leqslant a(t) \leqslant a_{\max}, \qquad \forall t \in [0, T]$$

$$-u_{\mathrm{j}\max} \leqslant u_{\mathrm{j}}(t) \leqslant u_{\mathrm{j}\max}, \quad \forall t \in [0, T]$$

An Example:

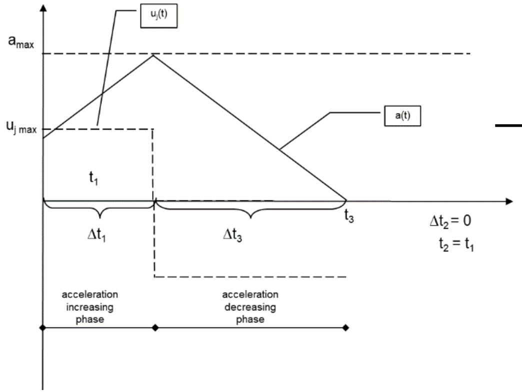


It is shown the trajectory consists of at most three segments with u = ± u_jmax and 0.

On-Line Trajectory Generation in Robotic Systems

Parameter space: Jerk limited trajectory

Intuition on the solution: Covered area (acc) = Δ Velocity



Smaller area

Larger area

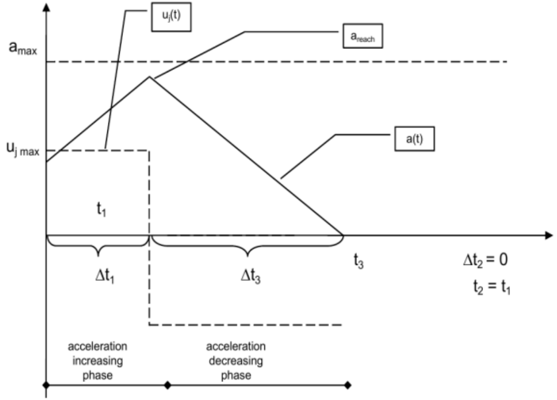


Figure 4.13: Wedge acceleration profile

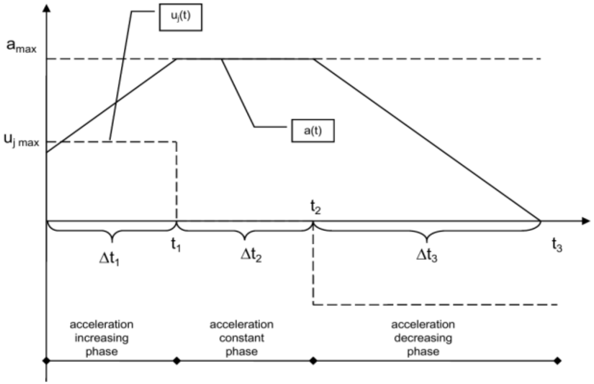


Figure 4.12: Trapezoidal acceleration profile

Parameter space: Jerk limited trajectory

The Problem:

From arbitrary state to a desired point subject to:

$$p(0) = p_0, \quad p(T) = p_{\text{ref}}$$

$$v(0) = v_0, \quad v(T) = 0$$

$$a(0) = a_0, \quad a(T) = 0$$

$$\dot{p}(t) = v(t)$$

$$\dot{v}(t) = a(t)$$

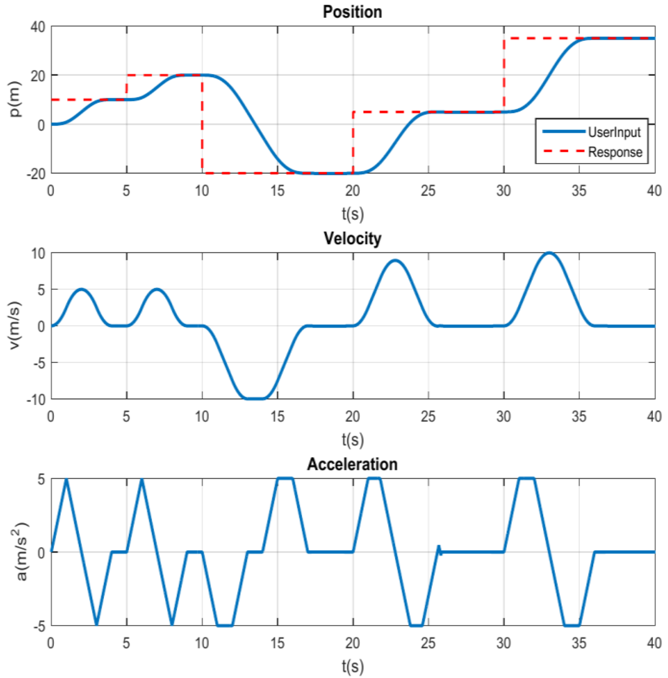
$$\dot{a}(t) = u_j(t)$$

$$-v_{\max} \leqslant v(t) \leqslant v_{\max}, \quad \forall t \in [0, T]$$

$$-a_{\max} \leqslant a(t) \leqslant a_{\max}, \quad \forall t \in [0, T]$$

$$-u_{\mathrm{j}\max} \leqslant u_{\mathrm{j}}(t) \leqslant u_{\mathrm{j}\max}, \quad \forall t \in [0, T]$$

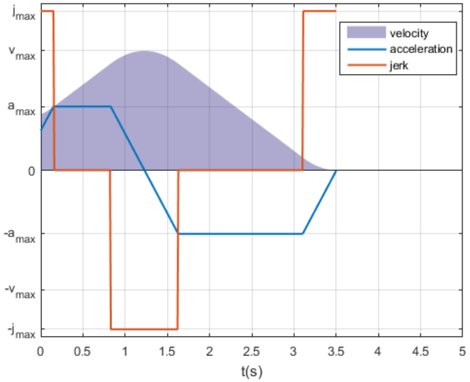
An Example:



Parameter space: Jerk limited trajectory

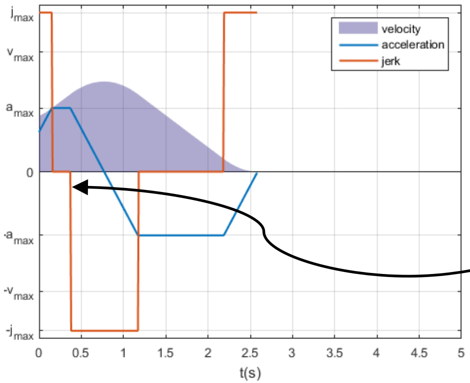
Intuition on the solution:

Covered area (vel) = Δ Position



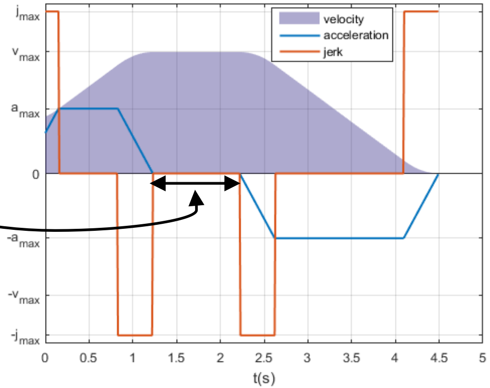
Need smaller area

Need larger area



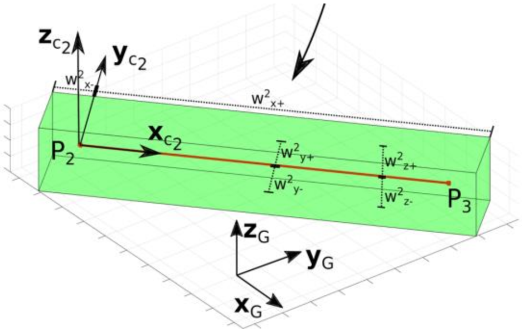
Determine switching moment

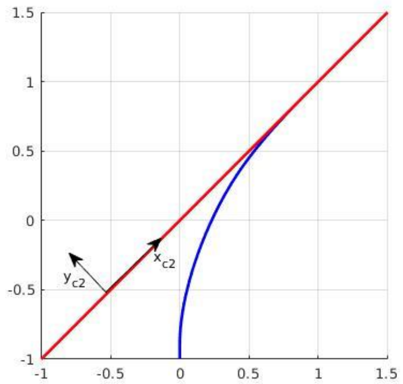
Determine cruise time



Parameter space: Jerk limited trajectory

For the trajectory to "converge" to a given line segment.





On the y and z axes:

- Set the position set-point to zero.

On the x axis:

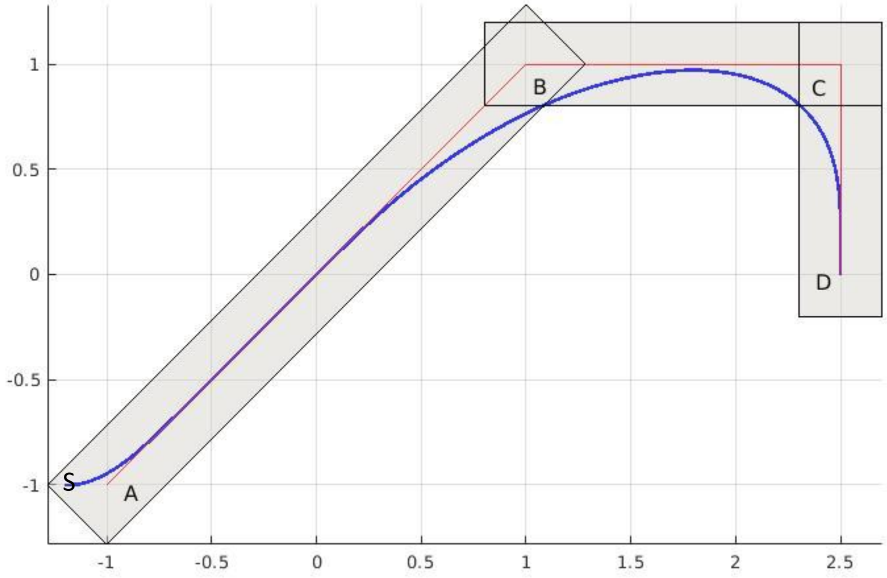
- Set the position set-point to $|P_3 – P_2|$

Result:

- The trajectory converge to the desired line-segment path.

Parameter space: Jerk limited trajectory

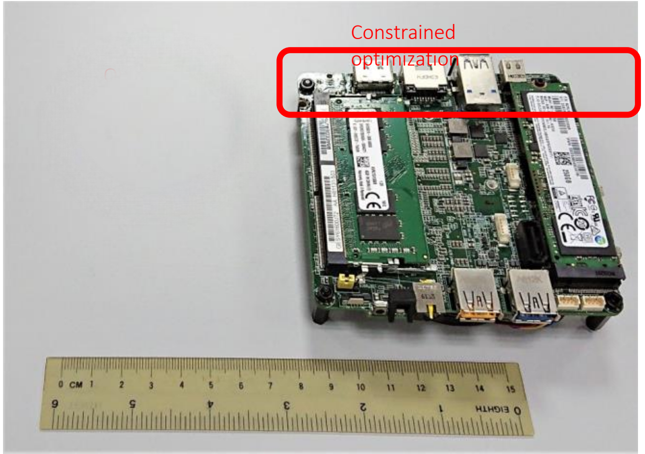
Online & incremental,
sample & throw

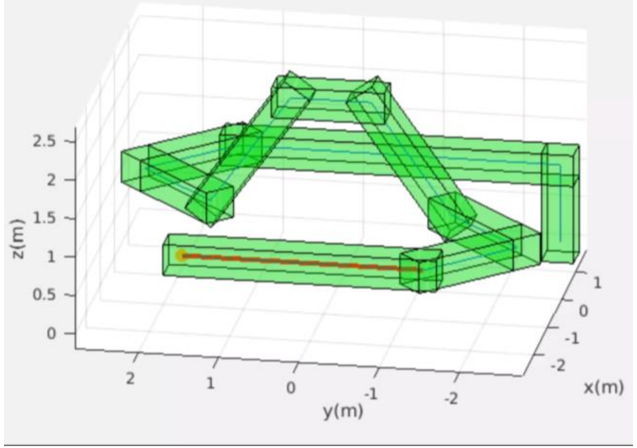


Parameter space: Jerk limited trajectory

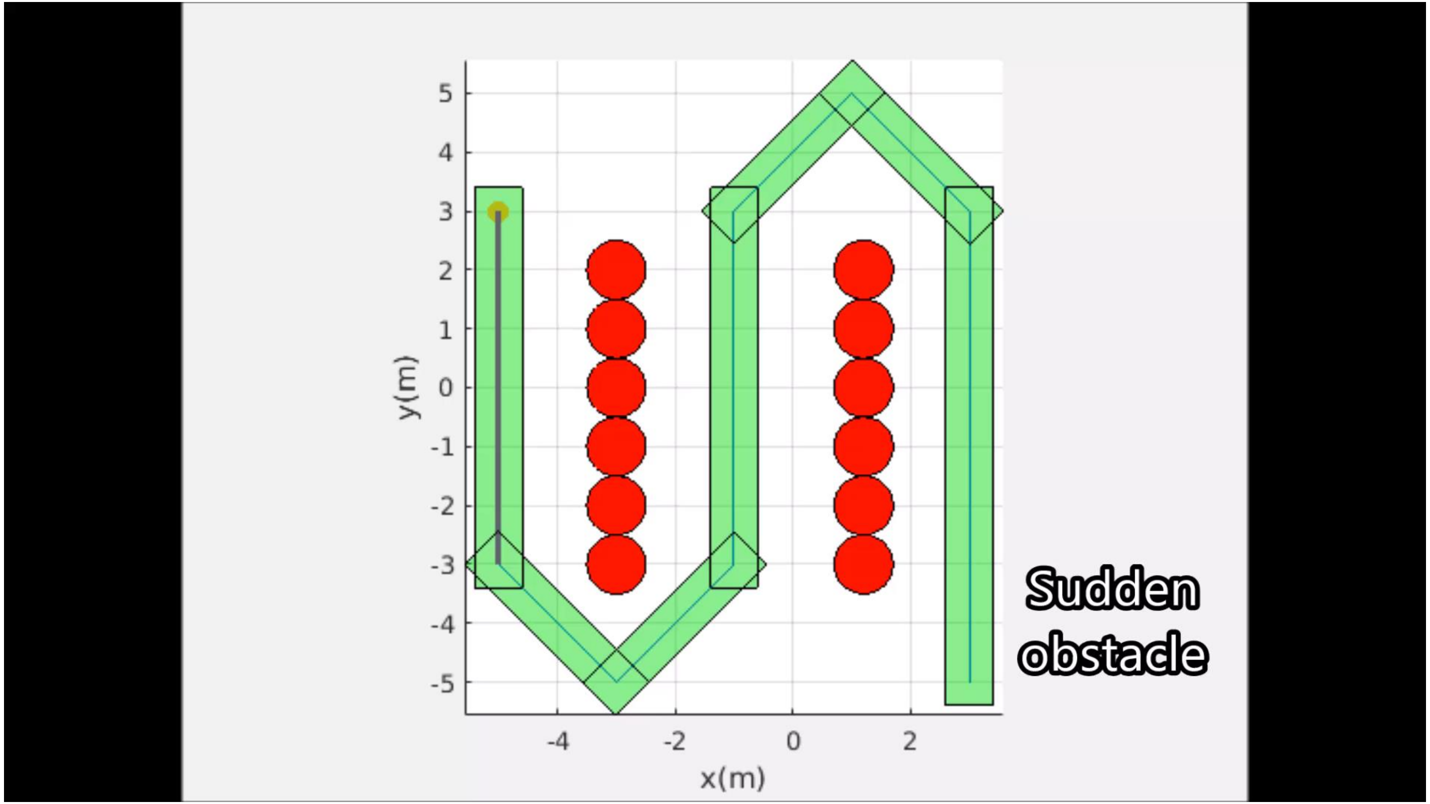
- An ***efficient*** algorithm to generate ***3D*** velocity, acceleration and jerk-limited trajectory.

- Instant reaction to changes (environment/mission)





Parameter space: Jerk limited trajectory



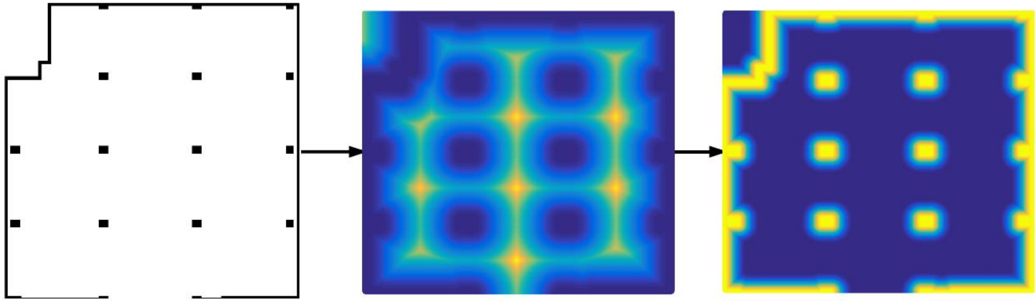
Parameter space: Jerk limited trajectory

JLT: Nonlinear-MPC

Environment perception

Pixelized environment

- **Occupancy map**
  - Whether a pixel is occupied or not
- **Cost map**
  - The cost of stepping into certain pixel
  - The information on distance to nearest obstacle



Occupancy Map: Black means occupied, white means unoccupied

EDT: The darker the color, the closer to an obstalce.

Cost Map: The lighter the color, the higher the cost.

JLT: Nonlinear-MPC

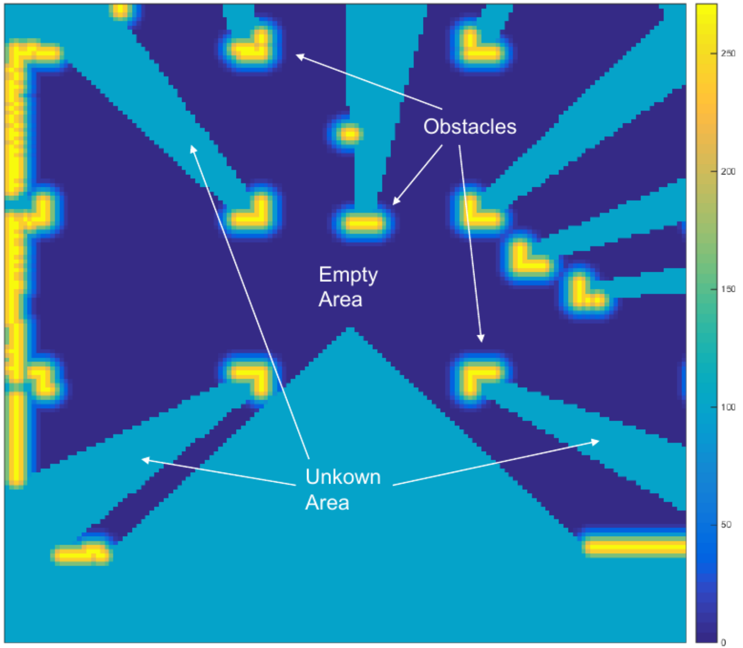
Environment perception

Sensor

- **Laser scanner**

Mapping procedure

1. **Read sensor data**
2. **Update the occupancy map**
   a. Add in new obstacle
   b. Clean LOS area
3. **Perform EDT**
4. **Perform Cost assigning**



JLT: Nonlinear-MPC

Two level guidance

Cut the problem into a series of TPBVP.

- **Global planner**

  Provides a series of connected line segments

- **Trajectory planner**

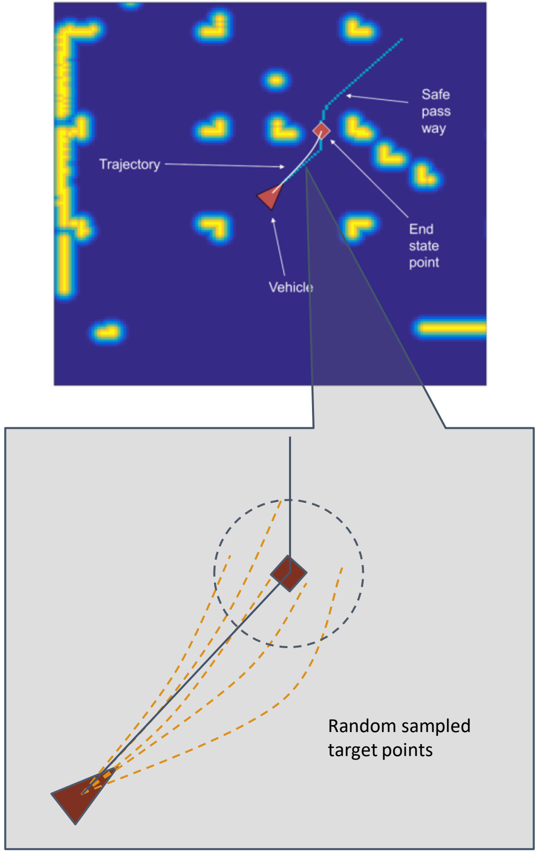
  Solve TPBVP for jerk limited trajectory.

  The trajectory leads the vehicle towards the first sharp turning point on line segment path.

  It propose multiple trajectories for evaluator.

- **Evaluator**

  Evaluate the quality of trajectories.



JLT: Nonlinear-MPC

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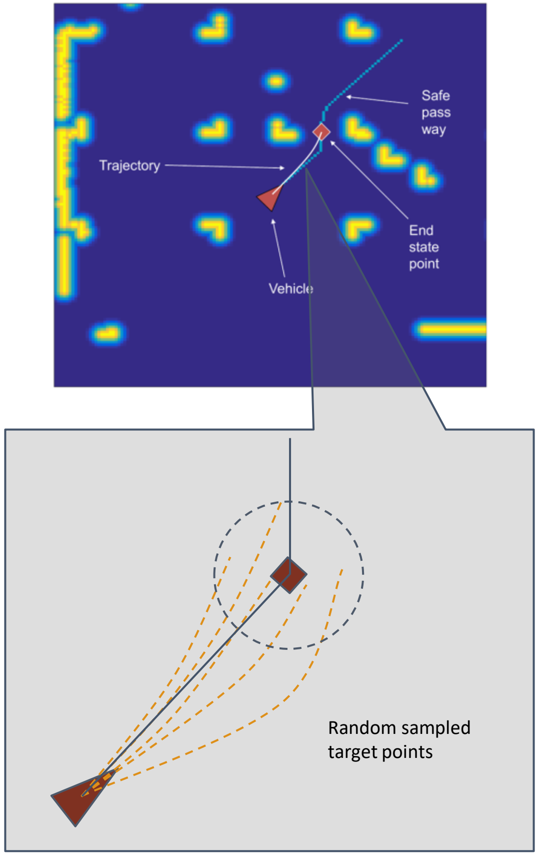
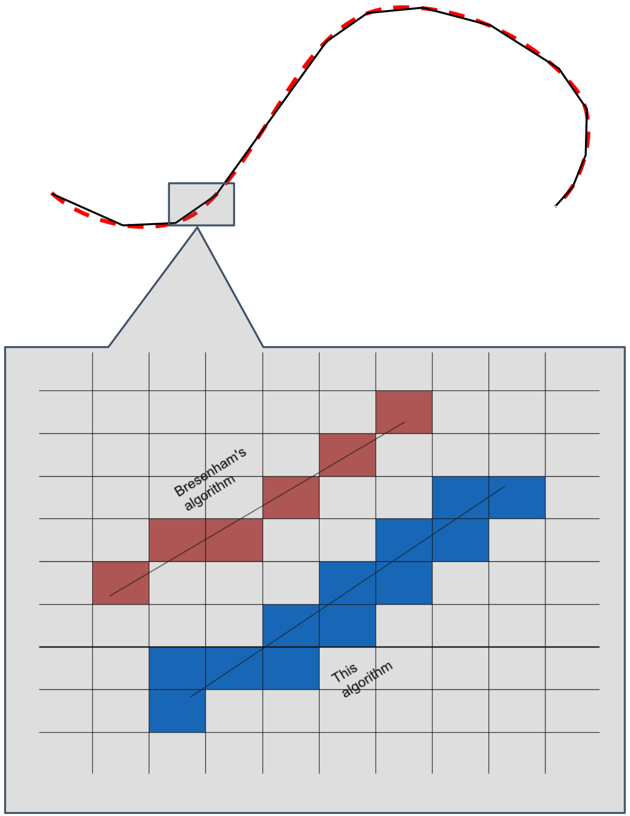


JLT: Nonlinear-MPC

Evaluator

Check the quality of each trajectory

- Represent original trajectory using connected line segments.
- Perform DDA marching to exam each pixel covered by trajectory.
- Find the pixel with highest cost value.
- Evaluate the trajectory based on its time duration and clearness.



JLT: Nonlinear-MPC

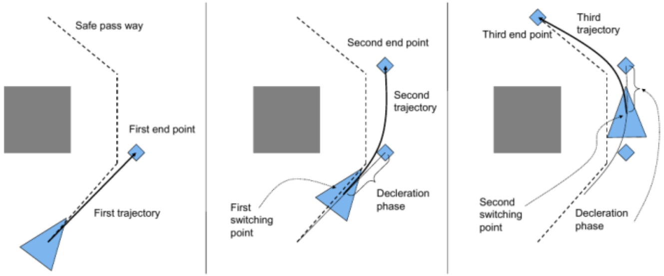
Event Manager

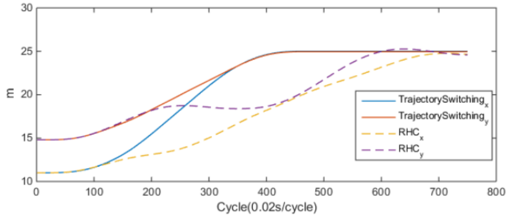
Determine when to use new trajectory

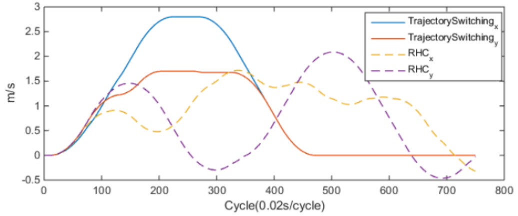
- **Trajectory governor**
  - Constantly monitoring whether the current trajectory is safe.

- **Event handler**
  - Fire a replanning event if the current trajectory is about to end
  - Fire emergency replanning event if the current trajectory is unsafe

- **Saving computational power and smoother flight**



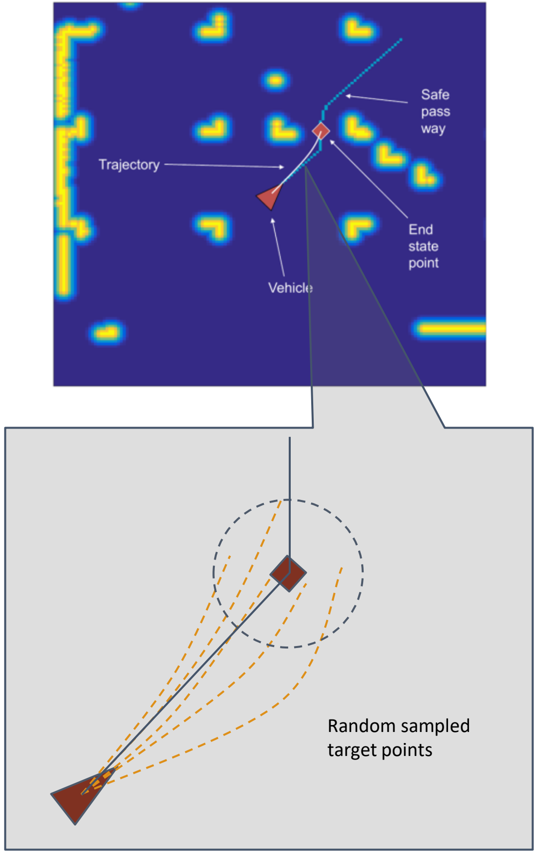




JLT: Nonlinear-MPC

$$\min C_F\left(x(t_f)\right) + \int_{t=t_0}^{t_f} C_R(x,u)\, dt$$

$$\begin{gathered}\dot{x} = f(x,u)\\ g(x,u) < 0\\ h(x,u) = 0\\ x \notin Obstacle\end{gathered}$$

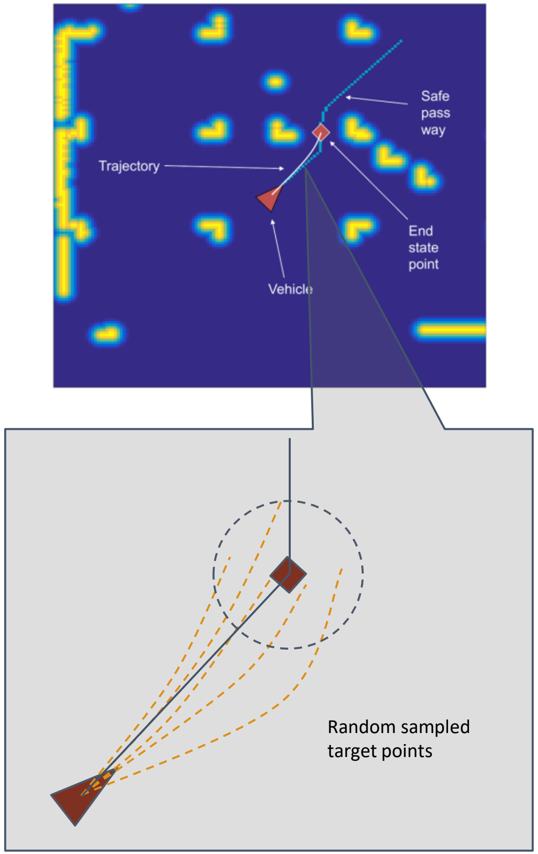


JLT: Nonlinear-MPC

$$\min C_F\left(x(t_f)\right) + \int_{t=t_0}^{t_f} C_R(x,u)\,dt$$

Hard constraint: $x \notin Obstacle$

Soft constraint: $s(x) = \begin{cases} 0, if x \notin Obstacle \\ M, otherwise \end{cases}$



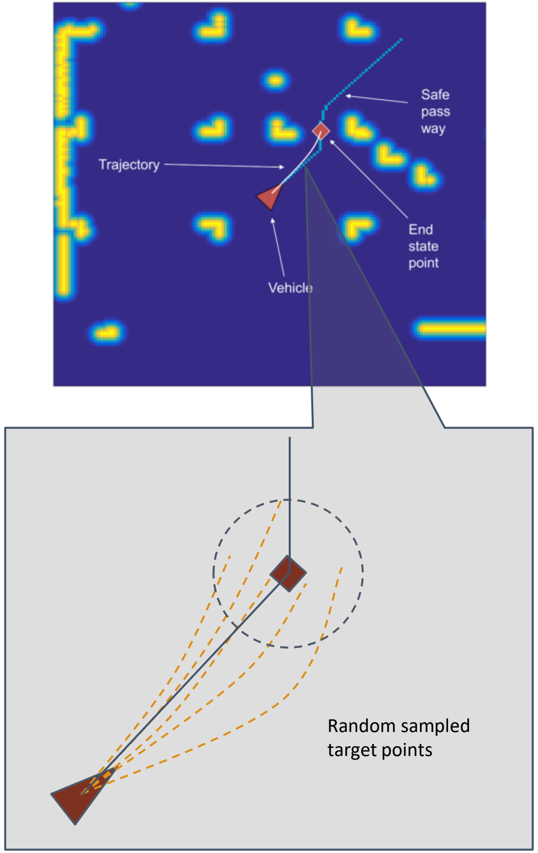
JLT: Nonlinear-MPC

$$\min C_F\left(x(t_f)\right) + \int_{t=t_0}^{t_f} C_R(x,u)\,dt$$

$$C_R(x,u) = (x - x_d)^T Q (x - x_d) + u^T R u + s(x)$$

$$C_F(x) = (x - x_d)^T W (x - x_d) + s(x)$$

$$s(x) = \begin{cases} 0, if\, x \notin Obstacle \\ M, otherwise \end{cases}$$



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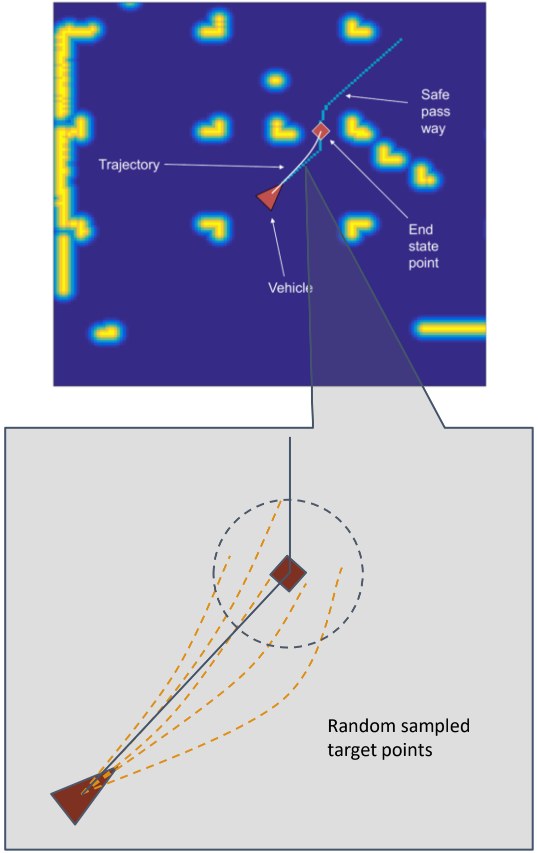
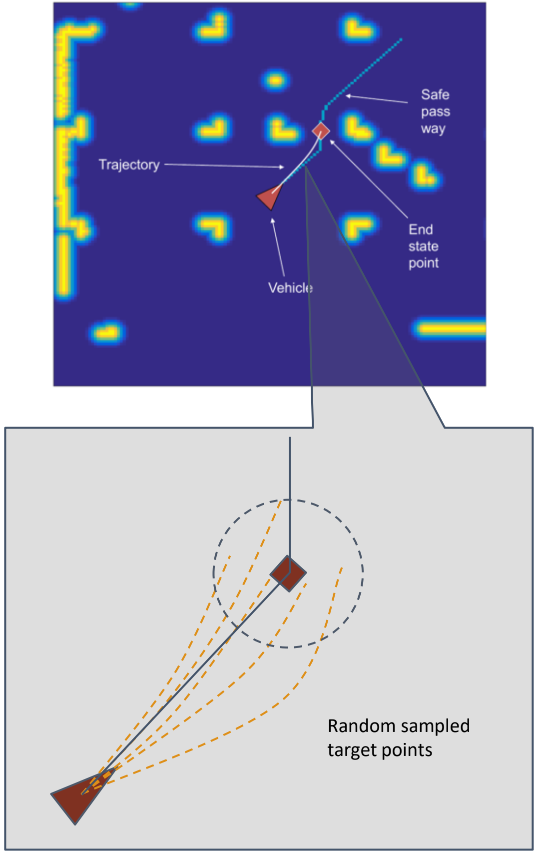


JLT: Nonlinear-MPC

Algorithm 1: Particle Swarm Optimization.

Input: $\mathrm{x}_{\mathrm{ini}}$, $\mathcal{M}$(map)
Output: $\boldsymbol{\theta}^*$ (best end state constraint)

$\Theta \leftarrow$ Particle_Initialization();
$c_i^* \leftarrow \infty,\ \theta_i^* \leftarrow \theta_i,\ \delta_i \leftarrow \mathrm{rand},\ \forall i \in [1, \mathrm{size}(\Theta)]$
for $m = 1$ **to** MAX_ITERS **do**
  for each $\boldsymbol{\theta}_i \in \Theta$ **do**
    $[\mathrm{x}(t), \mathrm{u}(t)] = \mathcal{S}_{\mathrm{NN}}(\mathbf{x}_{\mathrm{ini}}, \boldsymbol{\theta}_i)$
    $c_i = J(\mathrm{x}(t), \mathrm{u}(t), \mathcal{M})$
    if $c_i < c_i^*$ **then**
      $c_i^* = c_i$
      $\boldsymbol{\theta}_i^* = \boldsymbol{\theta}_i$
  $i^* = \underset{i}{\mathrm{argmin}}(c_i^*)$
  $\boldsymbol{\theta}^* = \boldsymbol{\theta}_{i^*}$
  for each $\boldsymbol{\theta}_i \in \Theta$ **do**
    $\boldsymbol{\delta}_i = \boldsymbol{\delta}_i + k_1 \cdot \mathrm{rand} \cdot (\boldsymbol{\theta}_i^* - \boldsymbol{\theta}_i) + k_2 \cdot \mathrm{rand} \cdot (\boldsymbol{\theta}^* - \boldsymbol{\theta}_i)$
    $\boldsymbol{\theta}_i = \boldsymbol{\theta}_i + \boldsymbol{\delta}_i$



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JLT: Nonlinear-MPC

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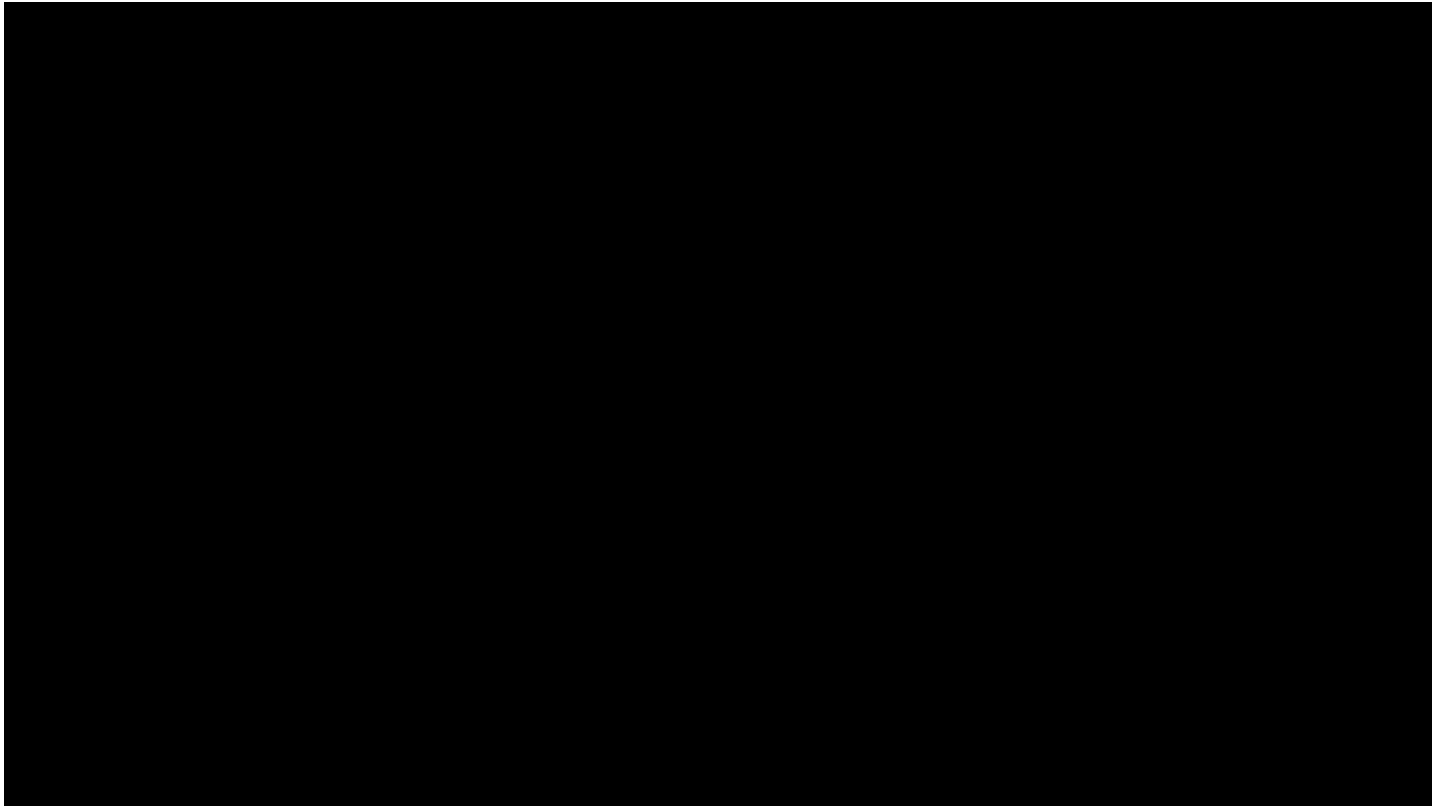

Parameter space: General BSCP

A general BVP problem:

$$\min_{\mathbf{u}(t),\mathbf{x}(t),t_f} \quad G(\mathbf{x}(t), \mathbf{u}(t), t_f)$$

$$g(\mathbf{x}(t_f), \boldsymbol{\theta}) = 0$$

$$h(\mathbf{x}, \mathbf{u}) = 0,\ \tilde{h}(\mathbf{x}, \mathbf{u}) \leq 0$$

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}),$$

Often requires a numerical optimization.

Some times can be solved through a controller:

$$\|g(\mathbf{x}(t_f), \boldsymbol{\theta})\| < \epsilon,\ \forall t \geq t_f,$$

$$\mathcal{S}:\ \langle \mathbf{x}_0, \boldsymbol{\theta} \rangle \rightarrow \left\langle \hat{\mathbf{u}}(t), \hat{\mathbf{x}}(t), \hat{t}_f \right\rangle.$$

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Model Predictive Local Motion Planning With Boundary State Constrained Primitives

Parameter space: General BSCP

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$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}),$$

$$\min C_F\left(x\left(t_f\right)\right) + \int_{t=t_0}^{t_f} C_R(x,u)\,dt$$

$$\mathcal{S}:\ \langle \mathbf{x}_0, \boldsymbol{\theta} \rangle \rightarrow \left\langle \hat{\mathbf{u}}(t), \hat{\mathbf{x}}(t), \hat{t}_f \right\rangle .$$

$$\min_{\theta} L_F(\theta) + \int_{t=t_0}^{t_f} L_R(\theta)\,dt$$

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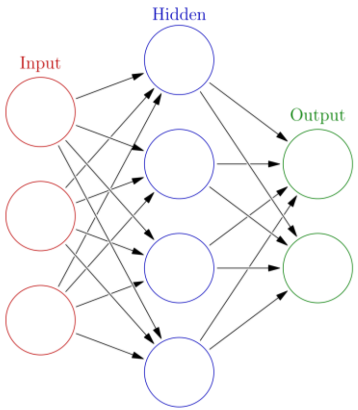


$$\min C_F\left(x(t_f)\right) + \int_{t=t_0}^{t_f} C_R(x,u)\,dt$$

Update x_0.

Optimize $\min_{\theta} L_F(\theta) + \int_{t=t_0}^{t_f} L_R(\theta)\,dt$ and get θ^*.

1) Use the NN approximate input u.
2) Solve the original BVP

$$\min_{\mathbf{u}(t),\mathbf{x}(t),t_f} \quad G(\mathbf{x}(t), \mathbf{u}(t), t_f)$$

$$g(\mathbf{x}(t_f), \boldsymbol{\theta}) = 0 \qquad \dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}),$$

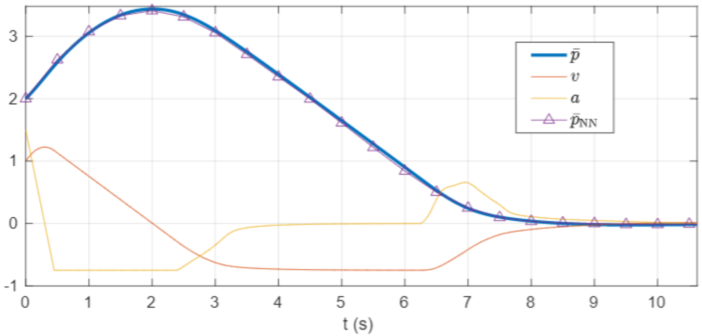
$$h(\mathbf{x}, \mathbf{u}) = 0,\ \tilde{h}(\mathbf{x}, \mathbf{u}) \leq 0$$

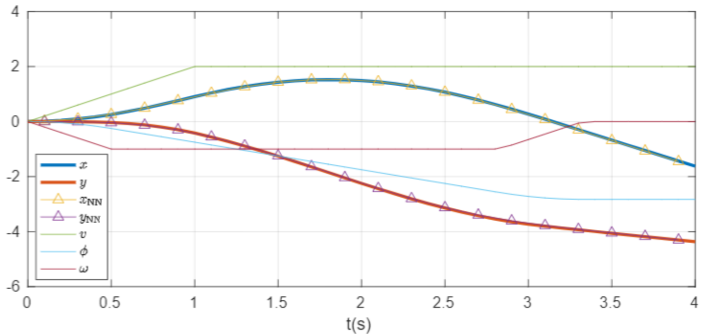
Parameter space: General BSCP

Network size and training details.

	NN structure	Training set size	Testing set size	Batch size	Epoch	Average MSE	Maximum MSE
Quadrotor horizontal	64-128-128-41 MLP	800,000	200,000	20	12	4.72×10^{-4}	0.071
Quadrotor vertical	64-128-128-41 MLP	800,000	200,000	10	15	1.2×10^{-3}	0.088
2nd order unicycle	80-256-128-81 MLP	860,000	130,000	10	20	3.4×10^{-3}	0.094

* MSE: Mean Squared Error. MLP: Multi-Layer Perceptron.





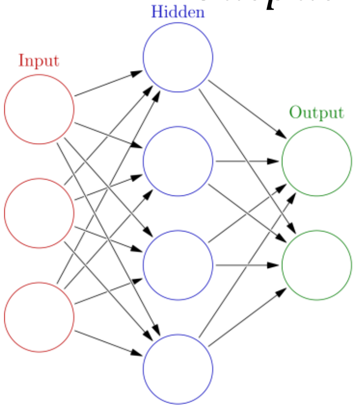
Parameter space: General BSCP

Approximated gradient

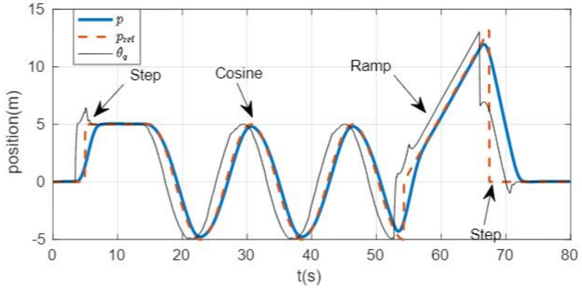
$$input = \theta, x_0$$

$$output = x(t), u(t)$$

$$\frac{dx}{d\theta}, \frac{du}{d\theta}$$



$$\frac{dJ}{dx}, \frac{dJ}{du}$$

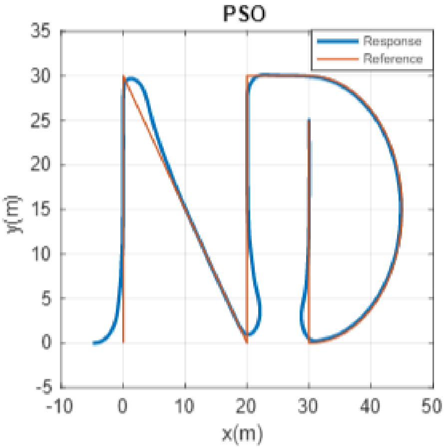


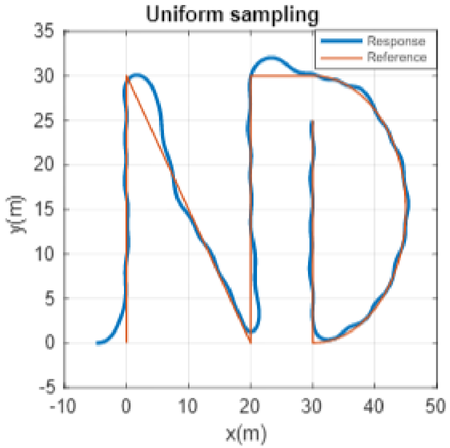
$$J = C_F\left(x(t_f)\right) + \int_{t=t_0}^{t_f} C_R(x,u)\, dt$$

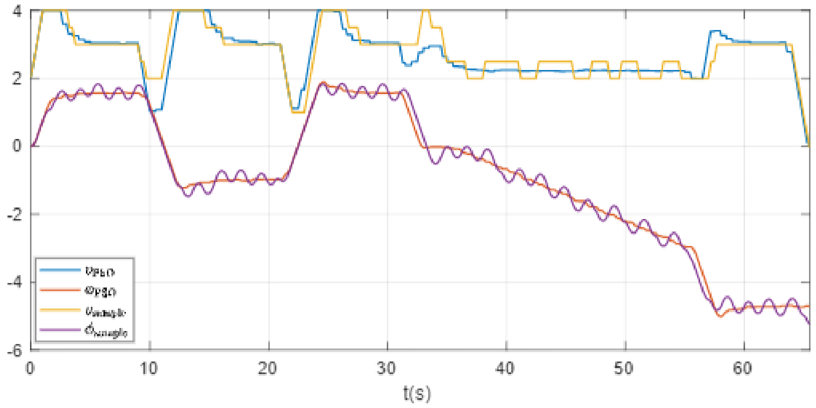
$$\frac{dJ}{d\theta}$$

Parameter space: General BSCP

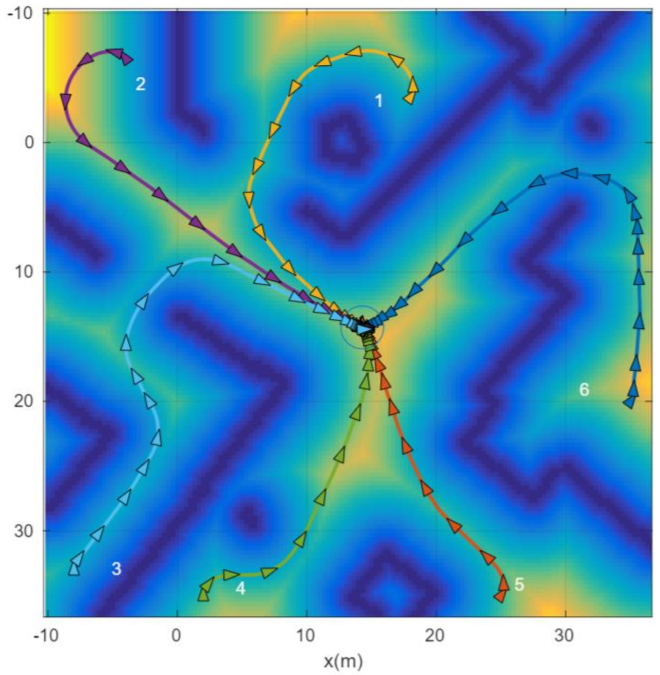
PSO: 10 particle, 10 iterations. VS Uniform sampling: 100 particles 1 iteration.







Parameter space: General BSCP



$$\min C_F\left(x(t_f)\right) + \int_{t=t_0}^{t_f} C_R(x,u)\,dt$$

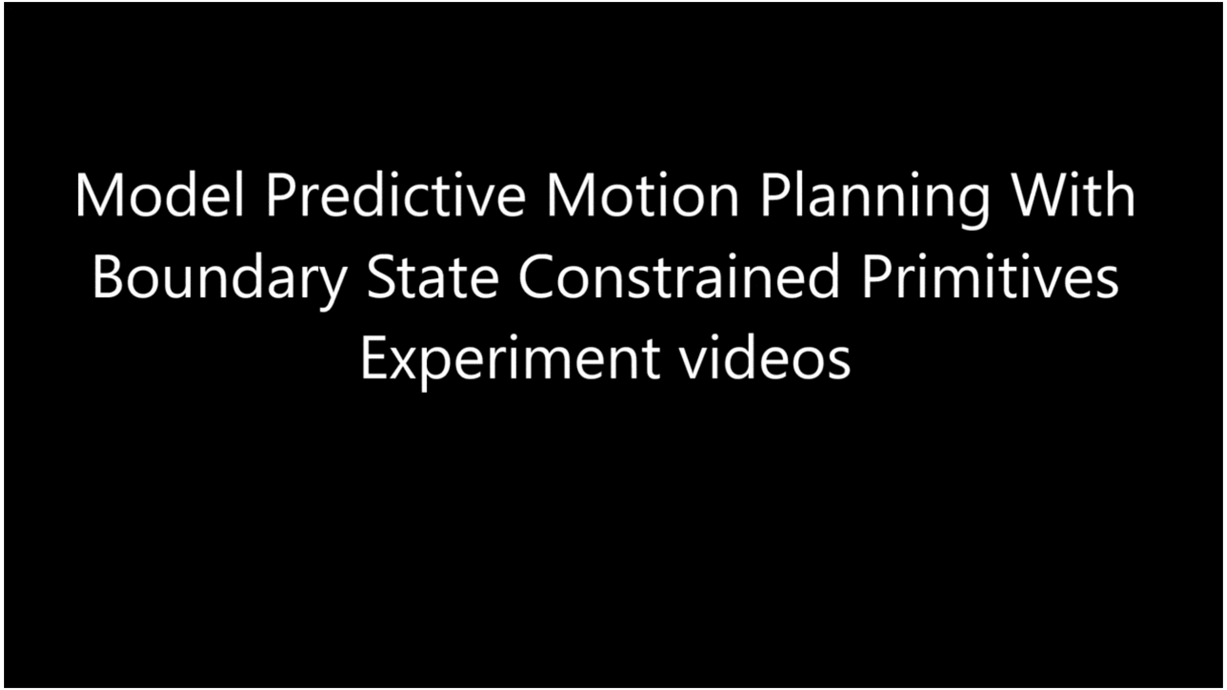
$$C_R(x,u) = (x - x_d)^T Q(x - x_d) + u^T R u + s(x)$$

$$C_F(x) = (x - x_d)^T W(x - x_d) + s(x)$$

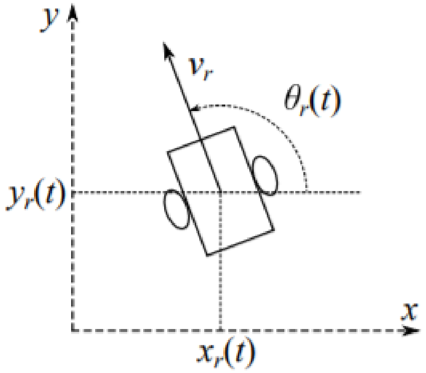
$$s(x) = \begin{cases} 0, if x \notin Obstacle \\ M, otherwise \end{cases}$$

cost_to_goal(x), through method like Dijkstra.

Parameter space: General BSCP



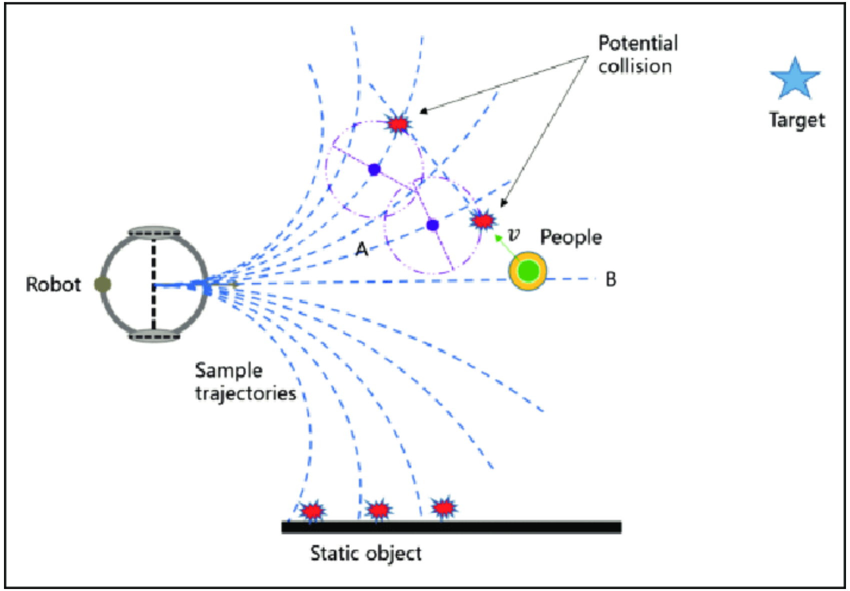
Unicycle model



$$\begin{cases} \dot{x}_r(t) = v_r \cos\theta_r(t) \\ \dot{y}_r(t) = v_r \sin\theta_r(t) \\ \dot{\theta}_r(t) = u(t) \in [-u_M, u_M] \end{cases}, \quad \begin{matrix} x_r(0) = x_0 \\ y_r(0) = y_0 \\ \theta_r(0) = \theta_0 \end{matrix}$$

Input to this model is the linear speed (v_r) and the angular speed (u).
This model is suitable for low speed and small unicycle robot.
For larger vehicle, linear and angular acceleration is usually limited.

Dynamic window approach



Given a pair of expected linear and angular speed.

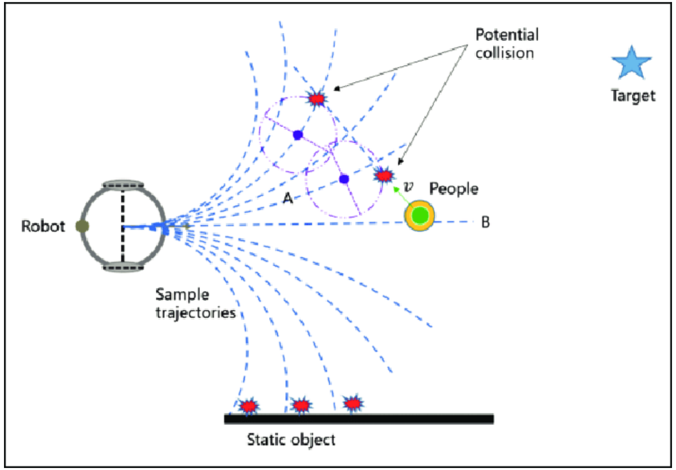
The predicted trajectory is an arch.

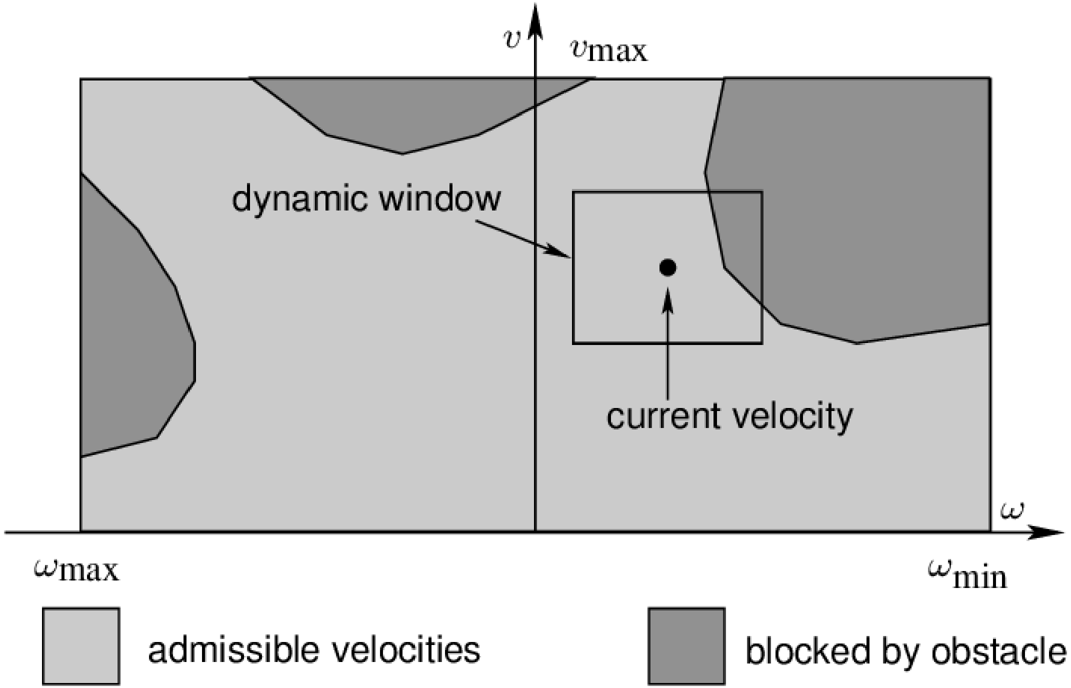
Evaluate the trajectory based on its distance to obstacle, its collision free, its relative distance to the target, etc.

Give each trajectory a score.

Select the trajectory with the highest score.

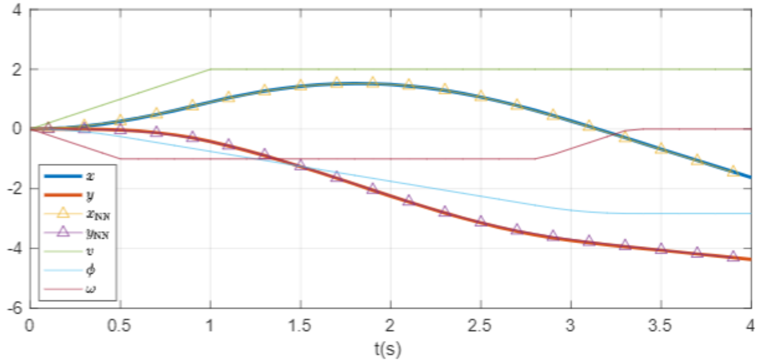
Dynamic window approach





Second order unicycle

$$\dot{x} = vcos(\phi), \quad \dot{\phi} = \omega$$
$$\dot{y} = vsin(\phi), \quad \dot{\omega} = \beta$$
$$\dot{v} = a$$



For larger vehicles, we could consider the second order unicycle model.

Now there will be additional constraints on linear and angular accelerations.

Given a pair of expected linear and angular speed, the future trajectory is non-longer an arch.

An underlying controller is used to regulate the linear/angular velocity to the desired value.

Compare DWA with our approach

- DWA

Optimization params: desired linear and angular velocity

Trajectory prediction: an arch

Evaluate the trajectory: with a user designed cost function

Optimization methods: sample multiple pairs of linear and angular velocity, exam each one of them, select the best trajectory.

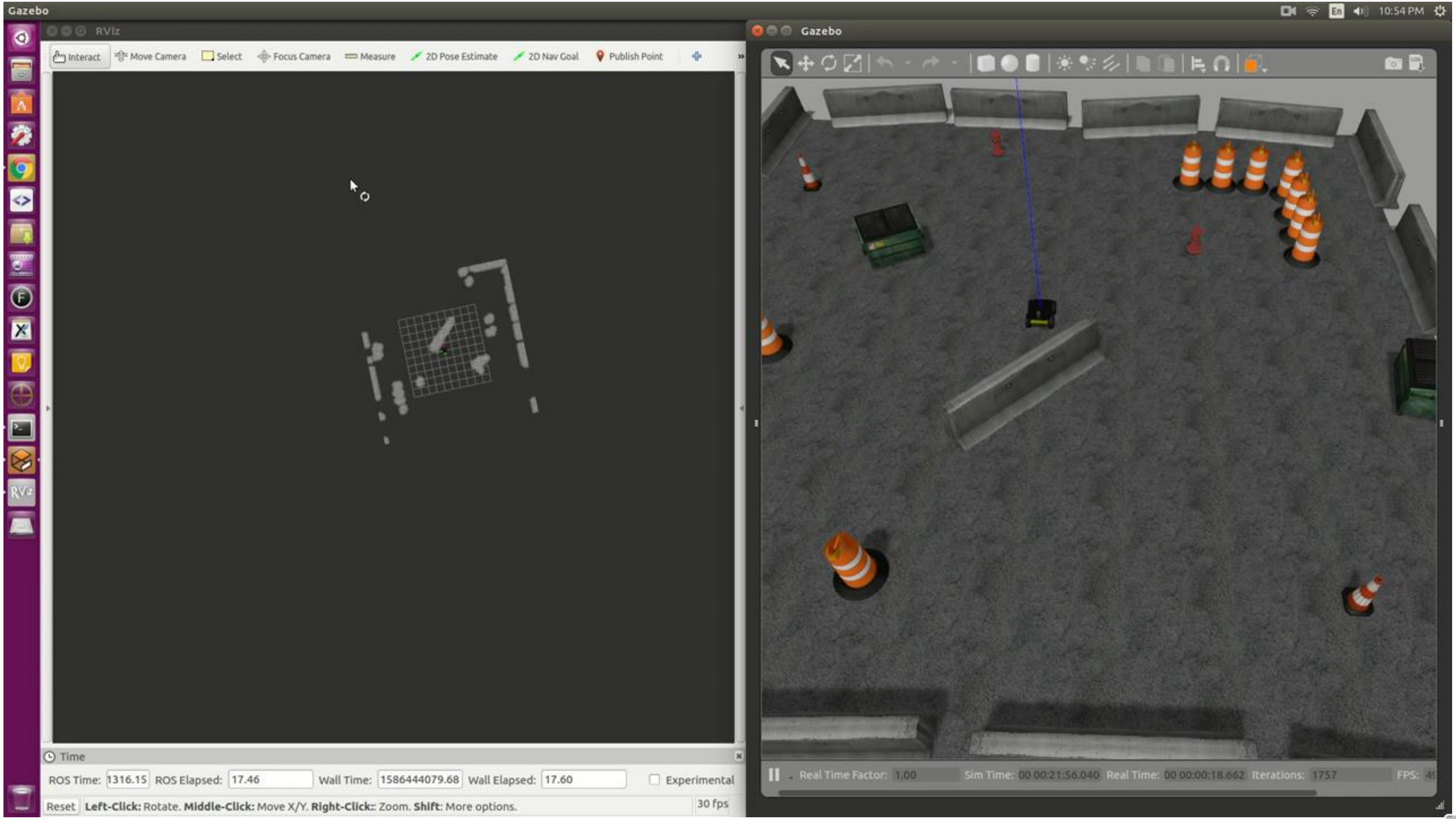
- Our approach

Optimization params: desired linear and angular velocity

Trajectory prediction: forward simulation or using neural networks

Evaluate the trajectory: with a user designed cost function

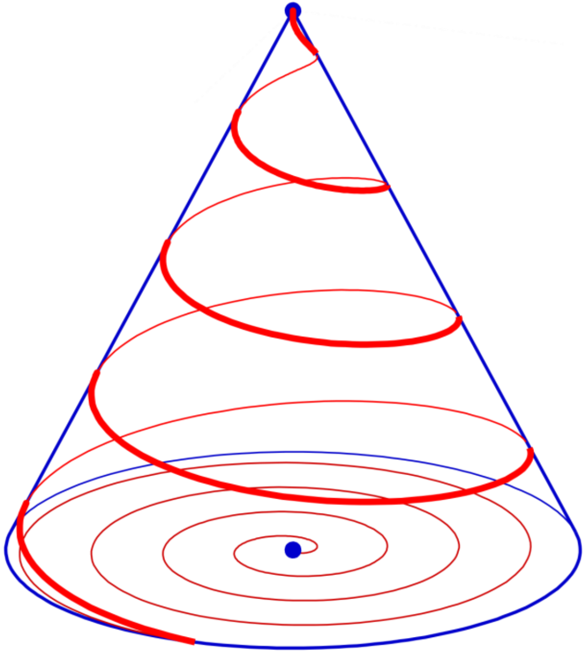
Optimization methods: PSO


Gazebo
10:54 PM
RViz
Interact
Move Camera
Select
Focus Camera
Measure
2D Pose Estimate
2D Nav Goal
Publish Point
Time
ROS Time: 1316.15
ROS Elapsed: 17.46
Wall Time: 1586444079.68
Wall Elapsed: 17.60
Experimental
Reset
Left-Click: Rotate. Middle-Click: Move X/Y. Right-Click:: Zoom. Shift: More options.
30 fps
Gazebo
Real Time Factor: 1.00
Sim Time: 00 00:21:56.040
Real Time: 00 00:00:18.662
Iterations: 1757
FPS:

Homework

Previously, we have discussed how to design a quadratic programming based MPC to allow a single-axis triple integrator to travel from an arbitrary state to the centre of the state space, a.k.a with zero position, velocity and acceleration.

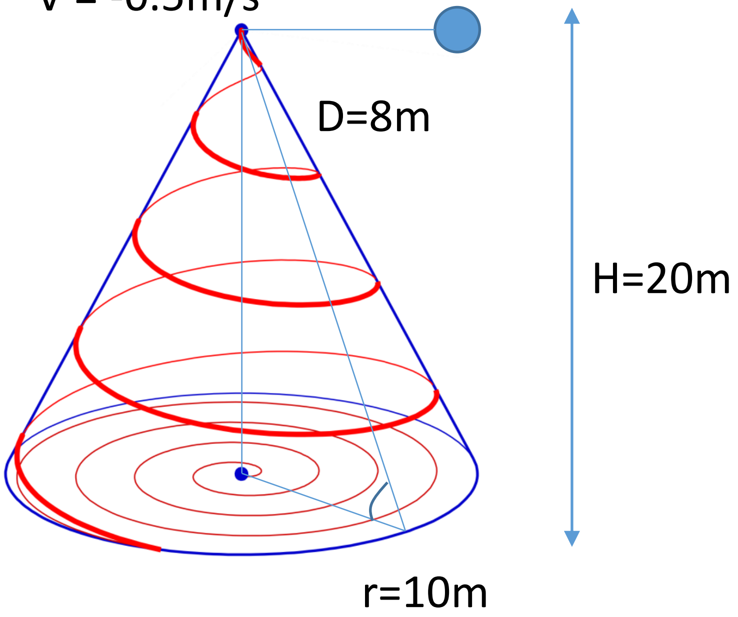
Now please design a quadratic programming based MPC to track conical spiral for a 3-axis triple integrator (basically a quadrotor model).



Homework

W = 0.08 rad /s

V = -0.5m/s

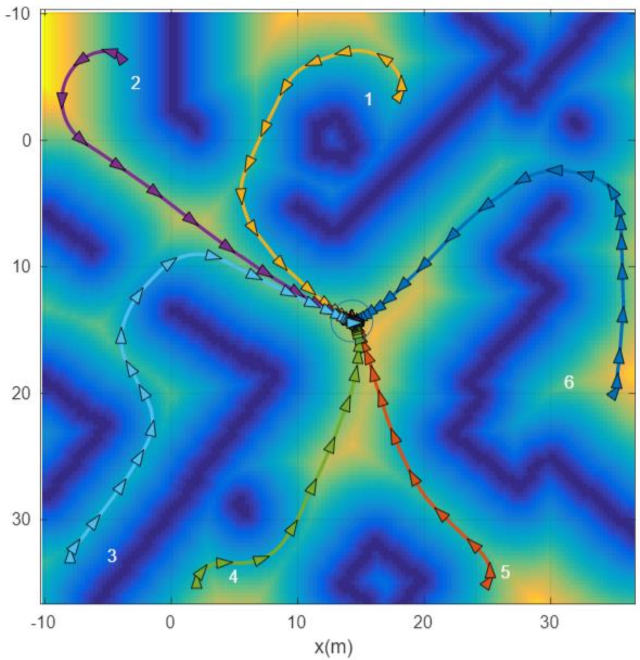


$-6 \leq v_{x,y} \leq 6$ $\quad$ $-1 \leq v_z \leq 6$

$-3 \leq a_{x,y} \leq 3$ $\quad$ $-1 \leq a_z \leq 3$

$-3 \leq j_{x,y} \leq 3$ $\quad$ $-2 \leq j_z \leq 2$

Homework



- Second order unicycle
- The model and trajectory evaluation function is already given
- The task is to go to the map centre from an arbitrary initial state.
- There are constraints on the vehicle linear/angular velocity and acceleration.

感谢各位聆听!

Thanks for Listening