OBVP作业推导

- 针对末状态部分自由的线性双积分系统求解最优控制量及状态轨线
- 由于原题中机器人可独立分析三个轴向的运动,此处仅分析单个轴向

优化模型

$$egin{aligned} minJ &= rac{1}{T} \int_0^T j(t)^2 dt \ s.t.rac{ds}{dt} &= As + Bu \ A &= egin{bmatrix} 0 & 1 & 0 \ 0 & 0 & 1 \ 0 & 0 & 0 \end{bmatrix}, B &= egin{bmatrix} 0 \ 0 \ 1 \end{bmatrix} \ s &= (p,v,a)^T, s(0) = (p_0,v_0,a_0)^T, s_1(T) = p_T \end{aligned}$$

推导

构造Hamilton方程:

$$H(s,u,\lambda) = rac{1}{T}j^2 + \lambda^Trac{ds}{dt} = rac{1}{T}j^2 + \lambda_1 v + \lambda_2 a + \lambda_3 j$$

由正则方程:

$$\dot{\lambda} = -rac{\partial H}{\partial s} = (0, -\lambda_1, -\lambda_2)^T \ \dot{s} = f(s, u) = As + Bu$$

解得

$$\lambda(t) = rac{1}{T} egin{bmatrix} -2lpha \ 2lpha t + 2eta \ -lpha t^2 - 2eta - 2\gamma \end{bmatrix}$$

由横截条件:

$$s_1(T)=p_T$$
 $\lambda_2(T)=\lambda_3(T)=0$

得约束条件

$$2lpha t + 2eta = 0$$
 $-lpha t^2 - 2eta - 2\gamma = 0$

由庞特里亚金极小值原理

$$u^*(t) = rgmax_{j(t)} H(s,u,\lambda) = rac{1}{2} lpha t^2 + eta t + \gamma$$

代入状态方程

$$s^*(t) = egin{bmatrix} rac{lpha}{120} t^5 + rac{eta}{24} t^4 + rac{\gamma}{6} t^3 + rac{a_0}{2} t^2 + v_0 t + p_0 \ rac{lpha}{24} t^4 + rac{eta}{6} t^3 + rac{\gamma}{2} t^2 + a_0 t + v_0 \ rac{lpha}{6} t^3 + rac{eta}{2} t^2 + \gamma t + a_0 \end{bmatrix}$$

结合末端位置约束条件

$$egin{bmatrix} rac{1}{120}T^5 & rac{1}{24}T^4 & rac{1}{6}T^3 \ 2T & 2 & 0 \ T^2 & 2T & 2 \end{bmatrix} egin{bmatrix} lpha \ eta \ \gamma \end{bmatrix} = egin{bmatrix} \Delta p \ 0 \ 0 \end{bmatrix}$$
 $\Delta p = p_f - p_0 - v_0 T - rac{1}{2}a_0 T^2$

解得

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} \frac{20\Delta p}{T^5} \\ -\frac{20\Delta p}{T^4} \\ \frac{10\Delta p}{T^3} \end{bmatrix}$$

则最优控制输入为

$$u^*(t) = -rac{20arDelta p}{T^4}t + rac{10arDelta p}{T^3}$$

最优状态轨线为

$$s^*(t) = egin{bmatrix} rac{\Delta p}{6T^5} t^5 - rac{5\Delta p}{6T^4} t^4 + rac{5\Delta p}{3T^3} t^3 + rac{a_0}{2} t^2 + v_0 t + p_0 \ rac{5\Delta p}{6T^5} t^4 - rac{10\Delta p}{3T^4} t^3 + rac{5\Delta p}{T^3} t^2 + a_0 t + v_0 \ rac{10\Delta p}{3T^5} t^3 - rac{10\Delta p}{T^4} t^2 + rac{10\Delta p}{T^3} t + a_0 \end{bmatrix}$$