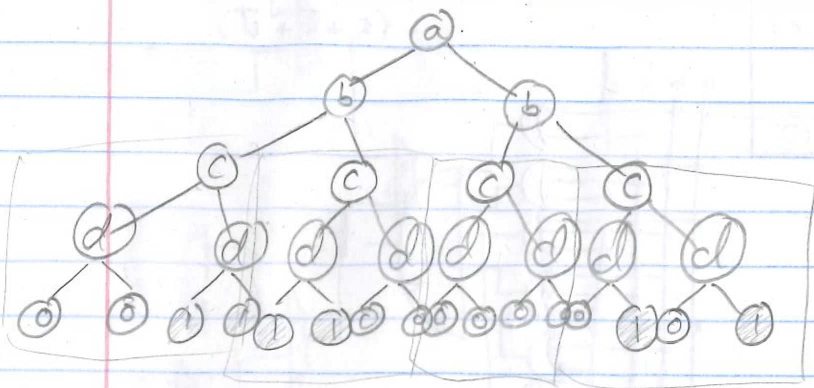
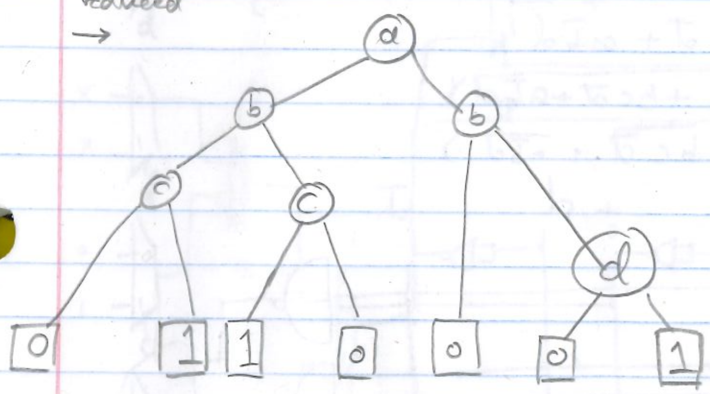
Edward Hong

HW2

4. $f(a,b,c,d) = abd + \bar{a} b \bar{c} + \bar{a} \bar{b} c$



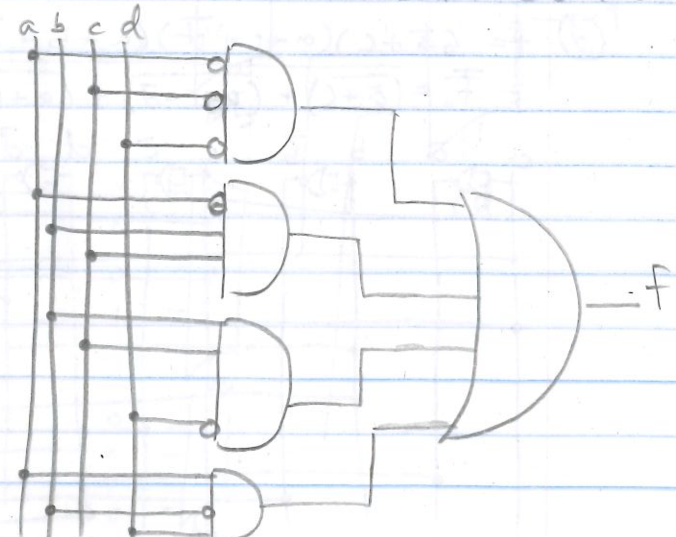
reduced →



5. (a) $f(a,b,c,d) = \Pi M(0, 4, 7, 11, 14) \cdot d(6, 8, 9, 13)$

cd \ ab	00	01	11	10
00	1	1		X
01			X	X
11		1		1
10		X	1	

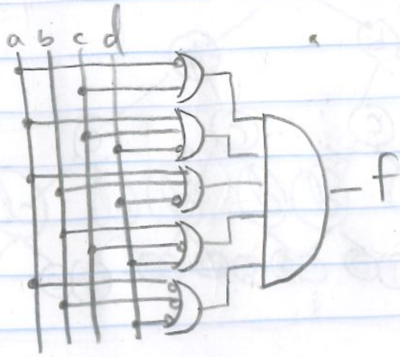
$f = \bar{a}\bar{c}\bar{d} + \bar{a} b c + b c \bar{d} + a \bar{b} d$



(b)

cd \ ab	00	01	11	10
00			0	X
01	0	0	X	X
11	0		0	
10	0	X		0

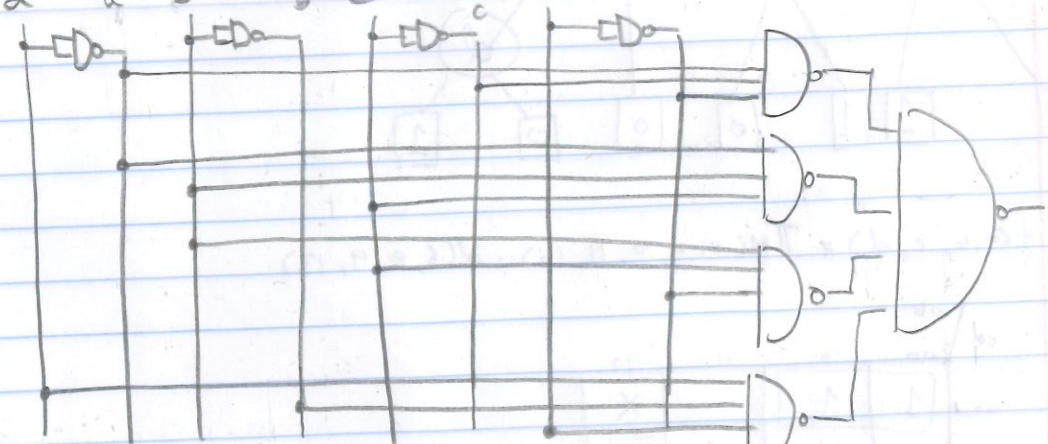
$f = (\bar{a}+c)(a+c+\bar{d})(a+b+\bar{d})(b+\bar{c}+d) \cdot (\bar{a}+\bar{b}+\bar{d})$



(c) $f = \bar{a}\bar{c}\bar{d} + \bar{a}bc + bc\bar{d} + a\bar{b}d$

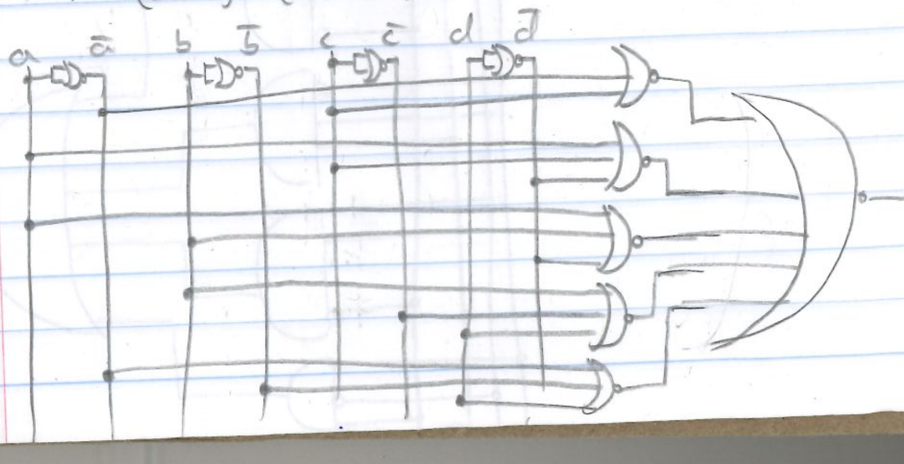
$f = \overline{\overline{F}} = \overline{\overline{(\bar{a}\bar{c}\bar{d} + \bar{a}bc + bc\bar{d} + a\bar{b}d)}}$

$= \overline{(\overline{\bar{a}\bar{c}\bar{d}} \cdot \overline{\bar{a}bc} \cdot \overline{bc\bar{d}} \cdot \overline{a\bar{b}d})}$

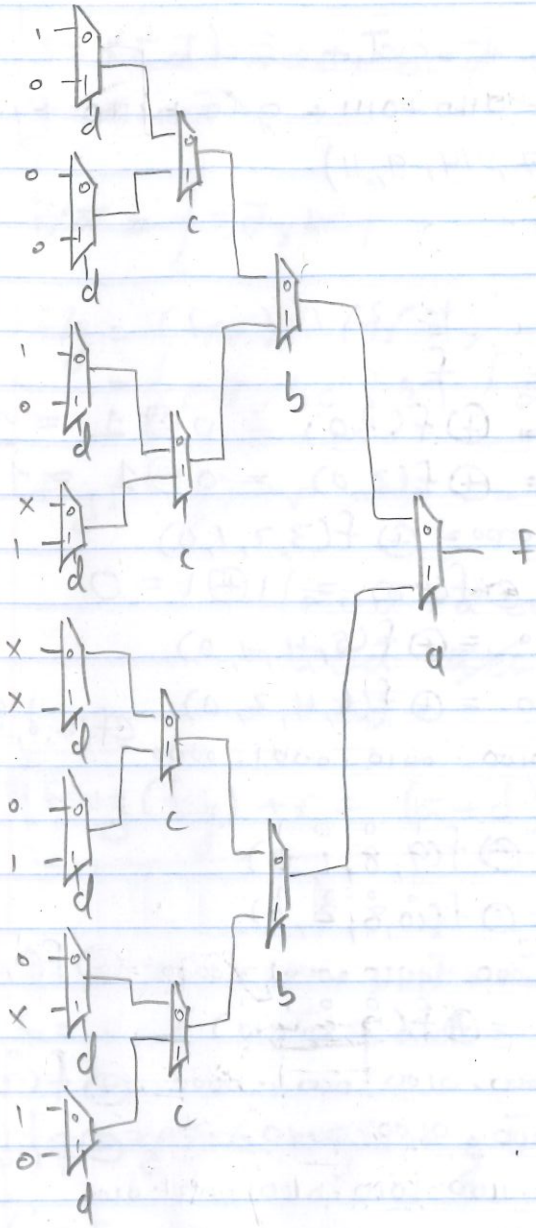


(d) $f = (\bar{a}+c)(a+c+\bar{d})(a+b+\bar{d})(b+\bar{c}+d)(\bar{a}+\bar{b}+d)$

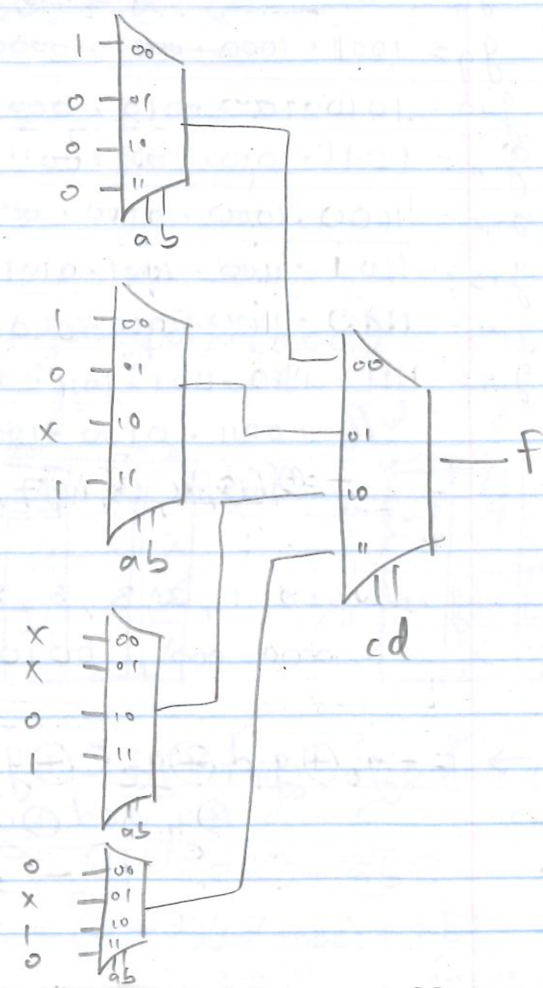
$= \overline{F} = \overline{(\bar{a}+c)} + \overline{(a+c+\bar{d})} + \overline{(a+b+\bar{d})} + \overline{(b+\bar{c}+d)} + \overline{(\bar{a}+\bar{b}+d)}$



(e)



(f)



(9) $f(a,b,c,d) = \bar{a}\bar{c}\bar{d} + \bar{a}bc + bc\bar{d} + a\bar{b}d$

$= 0000 + 0100 + 0110 + 0111 + \cancel{0110} + 1110 + 1001 + 1011$

$= \sum m(0, 4, 6, 7, 14, 9, 11)$

$f_{1,2,3,5,8,10,12,13,15} = 0$

$f_{0,4,6,7,9,11,14} = 1$

1	$g_0 = 0000$	$= f_0$
1	$g_1 = 0001 \ldots 0000$	$= \oplus f(1,0) = 0 \oplus 1 = 1$
1	$g_2 = 0010 \ldots 0000$	$= \oplus f(2,0) = 0 \oplus 1 = 1$
1	$g_3 = 0011 \ldots 0010 \ldots 0001 \ldots 0000$	$= \oplus f(3,2,1,0)$
0	$g_4 = 0100 \ldots 0000$	$= \oplus f(4,0) = 1 \oplus 1 = 0$
0	$g_5 = 0101 \ldots 0100 \ldots 0001 \ldots 0000$	$= \oplus f(5,4,1,0)$
1	$g_6 = 0110 \ldots 0100 \ldots 0010 \ldots 0000$	$= \oplus f(6,4,2,0)$
0	$g_7 = 0111 \cdot 0110 \cdot 0101 \cdot 0011 \cdot 0100 \cdot 0010 \cdot 0001 \cdot 0000$	$= \oplus f(\overset{1}{7}, \overset{1}{6}, \overset{0}{5}, \overset{1}{4}, \overset{0}{3}, \overset{0}{2}, \overset{0}{1}, \overset{1}{0})$
1	$g_8 = 1000 \cdot 000$	$= \oplus f(8,0)$
0	$g_9 = 1001 \cdot 1000 \cdot 0001 \cdot 0000$	$= \oplus f(\overset{1}{9}, \overset{0}{8}, \overset{0}{1}, \overset{1}{0})$
1	$g_{10} = 1010 \cdot 1000\ 0010 \cdot 0000$	$= \oplus f(\overset{0}{10}, \overset{0}{8}, \overset{0}{2}, \overset{1}{0})$
1	$g_{11} = 1011 \cdot 1010 \cdot 1001 \cdot 0011 \cdot 1000 \cdot 0010 \cdot 0001 \cdot 0000$	$= \oplus f(\overset{1}{11}, \overset{0}{10}, \overset{1}{9}, \overset{0}{8}, \overset{0}{3}, \overset{0}{2}, \overset{0}{1}, \overset{1}{0})$
0	$g_{12} = 1100 \cdot 1000 \cdot 0100 \cdot 0000$	$= \oplus f(\overset{0}{12}, \overset{0}{8}, \overset{1}{4}, \overset{1}{0})$
1	$g_{13} = 1101 \cdot 1100 \cdot 1001 \cdot 0101 \cdot 1000 \cdot 0100 \cdot 0001 \cdot 0000$	$= \oplus f(\overset{0}{13}, \overset{0}{12}, \overset{1}{9}, \overset{0}{8}, \overset{0}{5}, \overset{1}{4}, \overset{0}{1}, \overset{1}{0})$
0	$g_{14} = 1110 \cdot 1100 \cdot 1010 \cdot 0110 \cdot 1000 \cdot 0100 \cdot 0010 \cdot 0000$	$= \oplus f(\overset{1}{14}, \overset{0}{12}, \overset{0}{10}, \overset{0}{8}, \underset{1}{6}, \underset{1}{4}, \underset{0}{2}, \underset{1}{0})$
1	$g_{15} = 1111 \cdot 1110 \cdot 1101 \cdot 1011 \cdot 0111 \cdot 1100 \cdot 1010 \cdot 0110 \cdot 1001 \cdot 0101 \cdot 0011 \cdot 0100 \cdot 1000 \cdot 0010 \cdot \cancel{0010} \cdot 0001 \cdot 0000$	$= \oplus f(15, 14, 13, 11, 7, 12, 10, 6, 9, 5, 3, 4, 8, 2, 1, 0)$

g values: 0, 1, 2, 3, 6, 8, 10, 11, 13, 15

= 0000, 0001, 0010, 0011, 0110, 1000, 1010, 1011, 1101, 1111

$\rightarrow$ $\boxed{F = g_0 \oplus g_1 d \oplus g_2 c \oplus g_3 cd \oplus g_6 bc \oplus g_8 a \oplus g_{10} ac \oplus g_{11} acd \oplus g_{13} abd \oplus g_{15} abcd}$

$\rightarrow$

(h) $f = \bar{a}\bar{c}\bar{d} + \bar{a}bc + bc\bar{d} + a\bar{b}d$

$g = \bar{a} + \bar{d}$

$f/\bar{a} = \{\bar{c}\bar{d}, bc\}$ $\qquad$ $f/\bar{d} = \{\bar{a}\bar{c}, bc\}$

$f/g = (f/\bar{a}) \cap (f/\bar{d})$

$= \{\bar{c}\bar{d}, bc\} \cap \{\bar{a}\bar{c}, bc\} = bc$

$f = g(f/g) + r \Longrightarrow r = f - g(f/g)$

$= (\bar{a}\bar{c}\bar{d} + \bar{a}bc + bc\bar{d}) - (\bar{a} + \bar{d})(bc)$

$= \bar{a}\bar{c}\bar{d} + \cancel{\bar{a}bc} + \cancel{bc\bar{d}} - \cancel{\bar{a}bc} - \cancel{bc\bar{d}}$

$= \bar{a}\bar{c}\bar{d}$

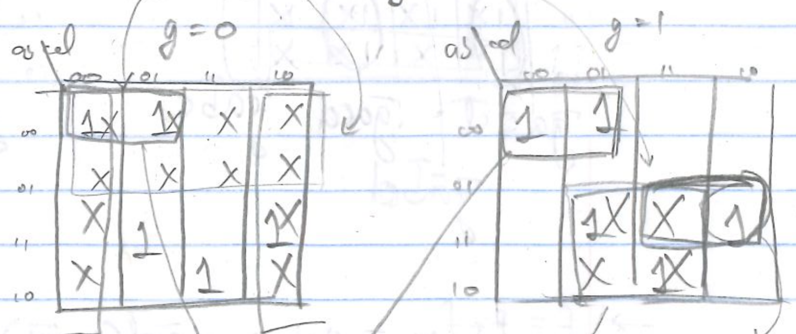
$\rightarrow \boxed{f = g(f/g) + r = (\bar{a} + d)(bc) + \bar{a}\bar{c}\bar{d}}$

(i) $f = \bar{a}\bar{c}\bar{d} + \bar{a}bc + bc\bar{d} + a\bar{b}d$

$G = \bar{a} + \bar{d}$, $\bar{G} = ad$

$f_{DC} = g \oplus G = \bar{g}G + g\bar{G} = (\bar{g}\bar{a} + \bar{g}\bar{d}) + (gad)$

$\tilde{f} = f + f_{DC}$



$\tilde{f} = f + f_{DC} = \bar{a}\bar{b}\bar{c}(\bar{g} + g) + gad + gabc$

$= g(ad + abc) + \bar{a}\bar{b}\bar{c}$

$f = (\bar{a} + \bar{d})\underbrace{(ad + abc)}_{\text{quotient}} + \underbrace{\bar{a}\bar{b}\bar{c}}_{\text{remainder}}$ $\rightarrow$

6. (a) $F = ab + acd + a\bar{c}\bar{d} + \bar{a}\bar{b}c + \bar{a}\bar{b}d$

$G = a + \bar{b}$

$f/a = \{b, cd, \bar{c}\bar{d}\}$ $\quad f/\bar{b} = \{\bar{a}c, \bar{a}d\}$

$f/g = (f/a) \cap (f/\bar{b}) = \{b, cd, \bar{c}\bar{d}\} \cap \{\bar{a}c, \bar{a}d\} = \{\} = \phi$

F is not divisible by G, so $r = F$

$\Rightarrow F = g(f/g) + r$

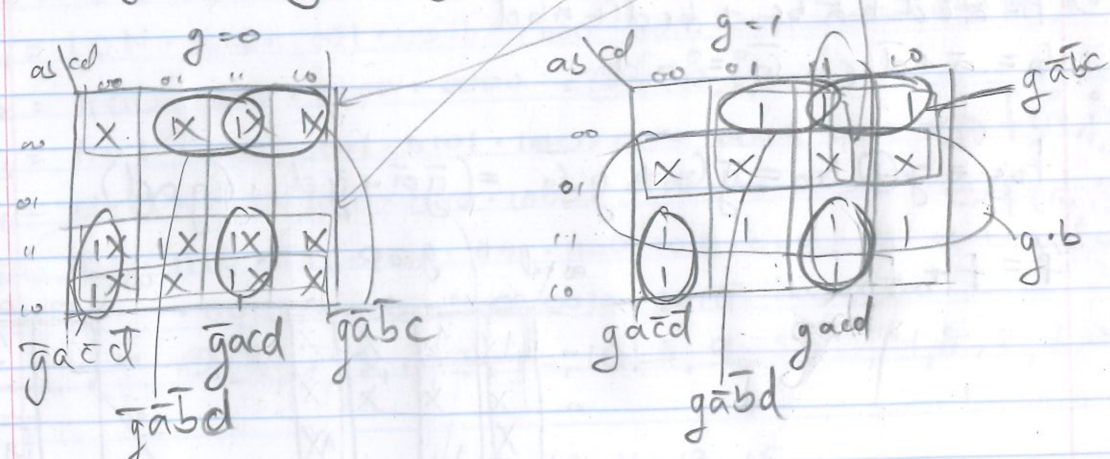
$F = \cancel{(a+\bar{b})(\phi)} + r$, $r = ab + acd + a\bar{c}\bar{d} + \bar{a}\bar{b}c + \bar{a}\bar{b}d$

ϕ

(b) $f = ab + acd + a\bar{c}\bar{d} + \bar{a}\bar{b}c + \bar{a}\bar{b}d$

$G = a + \bar{b}$, $\bar{G} = \bar{a}b$

$f_{DC} = g \oplus G = \bar{g}G + g\bar{G} = \bar{g}a + \bar{g}\bar{b} + g\bar{a}b$



$\rightarrow \tilde{F} = f + f_{DC} = g \cdot b + a\bar{c}\bar{d}(g+\bar{g}) + \bar{a}\bar{b}d(g+\bar{g}) + acd(g+\bar{g}) + \bar{a}\bar{b}c(g+\bar{g})$

$= (a+\bar{b})\underset{\text{quotient}}{b} + \underbrace{a\bar{c}\bar{d} + \bar{a}\bar{b}d + acd + \bar{a}\bar{b}c}_{\text{remander}}$

(c) In cases where weak division can not be used to divide F by G, we can use strong division.

7. $f(w, x, y, z) = \Sigma m(1, 3, 6, 11, 14) + d(9, 15)$

0001, 0011, 0110, 1011, 1110, 1001, 1111

group 1:	1 0001 ✓	(1,3) 00-1 ✓	(1,3,9,11) -0-1 ★
		(1,9) -001 ✓	
group 2:	3 0011 ✓	(3,11) -011 ✓	
	6 0110 ✓	(6,14) -110 ★	
	9 1001 ✓	(9,11) 10-1 ✓	
group 3:	11 1011 ✓	(11,15) 1-11 ★	
	14 1110 ✓	(14,15) 111- ★	
group 4:	15 1111 ✓		

	m_1	m_3	m_6	m_{11}	m_{14}	
✓ (6,14)			X		X	-110
(11,15)				X		
(14,15)					X	
✓ (1,3,9,11)	X	X		X		-0-1

$$\boxed{f = \bar{b}d + bc\bar{d}}$$