Strathmore University

Strathmore University Examination

Strathmore Institute of Mathematical Sciences

Master of Science in Mathematical Finance & Risk Analytics

Assignment: Computational Methods in Finance - MFI 8302

1. Consider the stochastic volatility process

$$\frac{dS}{S} = \mu dt + \sqrt{v} dZ_S$$
$$dv = \kappa_v(\bar{v} - v)dt + \delta_v \sqrt{v} dZ_v$$

where Z_S and Z_v are correlated Wiener processes so that $\mathbb{E}(dZ_S dZ_v) = \rho dt$. Simulate the dynamics for S and v and obtain the distribution for S at $t = 1$ conditional on the initial stock price S_0.

Take $\mu = 0.15$, $\bar{v} = 0.04$, $\delta_v = 0.2$, $\kappa_v = 1$, and $S_0 = 0.5$. Simulate from $t = 0$ to $t = 1$, using 10,000 simulations. Experiment with the step size until you get a "good" distribution. Take $\rho = -0.5$, 0, and 0.5. Compare the distribution for S you obtain with the log-normal distributions obtained by using the mean and variance of the simulated time series.

In order to see clearly the differences in the tails of the distribution also plot the distributions on a log scale. As a further experiment to gauge the impact of δ_v, simulate the $\rho = 0$ case with $\delta_v = 0.5$, 1, and 1.5 and compare the distributions (on both standard scale and log scale).

2. Consider the geometric Brownian motion model under the risk-neutral measure:

$$\frac{dS_t}{S_t} = r\, dt + \sigma\, dW_t$$

Simulate the dynamics of the asset price S_t over the interval $t \in [0, T]$, and use Monte Carlo methods to estimate the price of a **European-style average strike Asian call option** with maturity $T = 1$, where the payoff is given by:

$$\max(S_T - \bar{S}, 0) \quad \text{with} \quad \bar{S} = \frac{1}{N} \sum_{i=1}^{N} S_{t_i}.$$

Given the parameters: $S_0 = 100$, $r = 0.05$, $\sigma = 0.2$, $T = 1$, Number of monitoring points: $N = 50$, Number of Monte Carlo simulations: $M = 100,000$:

a Simulate the paths of S_t using Euler discretization for geometric Brownian motion. Compute the average strike $\bar{S}$ and terminal price S_T for each path, and estimate the option price using:

$$C_{\text{MC}} = e^{-rT}\, \mathbb{E}[\max(S_T - \bar{S}, 0)]$$

Report the estimated option price along with a 95% confidence interval.

b Improve the estimate using the **control variate method**. Use the payoff:

$$\max(S_T - G, 0) \quad \text{where} \quad G = \left(\prod_{i=1}^{N} S_{t_i}\right)^{1/N}$$

as a control variate, where G is the *geometric average* of the asset price. Note that the price of the corresponding *geometric average strike Asian option* is available in closed form under the Black-Scholes framework. Let $\mu_Y = \mathbb{E}[Y]$ be the analytic value of the geometric strike option. Define the adjusted estimator:

$$Z = X + \theta(\mu_Y - Y) \quad \text{where} \quad \theta = \frac{\mathrm{Cov}(X, Y)}{\mathrm{Var}(Y)}$$

with X the arithmetic payoff and Y the geometric payoff. Estimate θ empirically. Report the control variate-based estimate:

$$C_{\mathrm{CV}} = e^{-rT}\,\mathbb{E}[Z]$$

along with a 95% confidence interval.

c Compare the two estimates in terms of accuracy and variance. Report the **percentage reduction in variance** achieved using the control variate technique.

d As an extension, explore how the estimate improves as you vary:

- The number of time steps N
- The number of simulations M
- The volatility σ

Plot how the confidence interval narrows as a function of M, and discuss the efficiency of the control variate method.