Strathmore University Examination

Strathmore Institute of Mathematical Sciences

Master of Science in Mathematical Finance & Risk Analytics

Assignment 2: Computational Methods in Finance - MFI 8302

1. Consider the expression for the price of a European call option, namely

$$C(S,t) = e^{-r(T-t)}\,\widetilde{\mathbb{E}}_t\big[C(S_T,T)\big],$$

where $\widetilde{\mathbb{E}}_t$ is generated according to the process

$$dS = rS\,dt + \sigma S\,d\widetilde{z}(t).$$

   (a) By simulating M paths for S, approximate the expectation with

$$\frac{1}{M}\sum_{i=0}^{M} C\big(S_T^{(i)},T\big),$$

   where i indicates the ith path. Take the following parameters:

$$r = 5\%\text{ p.a.},\quad \sigma = 20\%\text{ p.a.},\quad S = 100,\quad E = 100,\quad T = 6\text{ months}.$$

   (b) Compare graphically the simulated values for various M with the true Black–Scholes value.

   (c) Instead of using discretisation to simulate paths for S, use instead the explicit solution for the GBM.

2. Consider a stochastic process such that the underlying security S follows the model:

$$dS_t = \mu S_t dt + \sigma_t S_t dZ_t,$$

where Z is a standard Brownian motion. Suppose the variance $v(t) = \sigma_t^2$ also follows a stochastic process given by

$$dv_t = \kappa(\theta - v(t))dt + \gamma\sqrt{v_t}dW_t,$$

where W is a standard Brownian motion. If the correlation coefficient between W and Z is denoted by ρ,

$$\mathrm{Cov}(dZ_t, dW_t) = \rho\,dt,$$

show that the generalized Black–Scholes equation is

$$\frac{1}{2}vS^2\frac{\partial^2 U}{\partial S^2} + \rho\gamma v S\frac{\partial^2 U}{\partial v\,\partial S} + \frac{1}{2}\gamma^2 v\frac{\partial^2 U}{\partial v^2} + rS\frac{\partial U}{\partial S} + [\kappa(\theta - v) - \lambda v]\frac{\partial U}{\partial v} - rU + \frac{\partial U}{\partial t} = 0.$$