Simulation Analysis of a Periodically Structured Dielectric Material

by

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A dissertation submitted to the Department of Physics, University of Surrey, in partial fulfillment of the degree of Master of Science in Physics/Medical Physics/Radiation and Environmental Protection/Nuclear Sciences and Applications

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September 2022

Abstract

In this dissertation I explore various aspects related to the lasing action in structured photonic materials. Building upon recent advances in lasing technology and design, I employed a computational formalism to study photonic crystals, periodically structured dielectric materials, focusing on identifying the photonic modes which are providing most promising characteristic for low-threshold, small footprint lasing. The photonic crystal considered consists of pockets of air within the dielectric slab, which gives rise to a photonic bandstructure displaying bandgap in which light propagation is forbidden and complete reflection takes place. In order for a mode to provide most promising characteristics, lasing with a minimalistic spectrum width will need to occur and to review the transmission rate that correlates with the mode. After completing the simulation, the data shows that as the pumping power is increased the population within the levels changed as a function of time. This change in population allowed for transmission to occur resulting in a lasing wavelength. This result shows that in this field of research a laser is able to be generated using this photonic crystal and enough pumping power.

Acknowledgement

I would like to thank Dr. Marian Florescu, my research project supervisor, for his expertise on the subject matter. Dr. Florescu has been a wonderful guidance in furthering my understanding of the subject matter discussed in this paper. He was there to help me understand concepts I was unable to grasp by either being prompt with his responses via email or in person. Dr. Florescu kept me on track to complete my research project by helping set up a reasonable and attainable timeline on what needs to be accomplished in order to fully flush out everything from the beginning of the concept to modern research.

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Introduction

Albert Einstein was one of the most influential scientists of the 20th Century having been the one to postulate groundbreaking theories such as Mass-Energy Equivalence, Bose-Einstein Condensate, Gravitational Waves, General Relativity and many others [1]. In 1916, Albert Einstein proposed that excess energy will be released as light if an atom is under the correct conditions. The released energy could either be stimulated by a light source or could be released spontaneously [2]. Although Einstein was the one who proposed this phenomenon it would take another Physicist to observe this phenomenon. Having another scientist observe the proposed phenomenon is not abnormal in the science world. On this occasion, Rudolf Walther Ladenburg, who was the head of the physics division at the Institute in Berlin at the time, first observed stimulated emission in 1928 [3,2]. Ladenburg would not see how this phenomenon could have any practical use. However, Charles H. Townes (American Physicist) who was at Columbia at the time (1953), was able to demonstrate a working device that emitted at microwave frequencies. Townes' demonstration would lead to an explosion of research [4].

In quantum mechanics, an atom's electron takes on different energy states. These energy states are labelled as E_n where n (n being an integer) is the level starting at the ground state or level 1. The lower-level energy states have a higher level of stability than the higher energy levels. Higher energy level electrons decay giving off electromagnetic radiation into lower energy levels, this is named Spontaneous Radiation [5]. The energy released has a relationship of:

$$E_i - E_f = h\nu = \frac{hc}{\lambda}, \tag{1}$$

where E_i is the initial energy level, E_f is the final energy level, h is the Plank's constant, v is the frequency of the electromagnetic radiation, c is the speed of light and λ is the wavelength. Electrons transferring from higher energy levels to lower energy levels may also release energy through kinetic or internal energy when a collision occurs [5].

1.1 Lasers

The 1960's-1970's saw the invention of optical-fiber communication. With these inventions telecommunication was able to have far-reaching effects which would lead to optic-fiber communication networks [6]. In order for an optical-fiber system to be high performance they need to have a few features. A standard device, Distributed Feedback lasers, have the desired characteristic as they are able to be stable, single frequency output, and manufacturable [6]. With all of these qualities within the Distributed Feedback laser, they have been a topic of intrigue for research.

1.2 Early Development of Lasers

It is important to know where the term "Laser" comes from. While this term has been commonly used throughout science, scientific fiction and literature, many may be unaware that the term is actually an acronym that stands for "*L*ight *A*mplification by the *S*timulated *E*mission of *R*adiation" [7].

While the He-Ne gas laser was the first commercial laser, Fabry-Pérot laser were the earliest form of diode lasers. They are small cross section general purpose lasers.

The propagation coefficient (β) of the laser is:

$$\beta = \frac{2\pi}{\lambda_m}, \tag{2}$$

with λ_m representing the material wavelength [6]. Using these equations, a team of research scientists could keep making advancements within the field of semiconductor lasers. This could be strengthening the effectiveness of Fabry-Pérot lasers or by creating new devices that are more powerful.

With several nanometers being covered by the spontaneous emission the phase condition can be defined by:

$$\beta L = N\pi, \tag{3}$$

with N being integers. Since the phase condition is satisfied by a number of modes or frequencies. The gain condition lets scientist know that spontaneous emission amplifies significantly only when the modes allow the following to be true:

$$\left|\frac{1}{1 - G_{round}}\right| \gg 1 \tag{4}$$

where G_{round} is the magnitude of gain in a round trip. With this condition met, we are nearing/reached λ_{peak}, which will provide the spontaneous emission that is desired for lasing [6].

1.3 Computational Approaches

Today, research of any laser can be completed in a laboratory or with the aid of a computer simulation. Simulations can and have been used in many areas of research. Such areas of research include but are not limited to predicting the growth pattern of plants based on multiple environmental factors or simulating how two binary stars will interact with each other or predict an election.

There are a few types of definitions for what a computer simulation is. The first, is the narrow definition which is a simulation that models a real-world system by approximating the behavior using a step-by-step computer model. This method can be utilized when a continuous differential equation within the model of interest cannot be solved using analytical methods. The narrowing definition would imply that the implementation of the algorithm needs to be done on a specific computer, in a specified language, and with a particular compiler to compile the simulation. If these conditions are not met then there could be a variation within the results. The second definition is known as a broad definition which incorporates the entire research process from the model selection to the way the results will be visualized. The third and final definition is the alternative point of view. This definition is defined as any system that has a dynamic behavior which can be studied and learned from [8].

When contemplating using a computer simulation one must determine what kind of outcome, they want to receive in order to determine the type of simulation they will use. The two types of simulation that are the most common are the equation-based simulations and agent-based simulations. Equation-based simulations are used when a theory governs the mathematical models within the simulation. Due to this, equation-based simulations are the most common simulations within physical sciences. This type of simulation can be field-based, particle-based or a set number of equations which will evolve as time elapses due to a continuous medium or field. Agent-based simulations are commonly used in research regarding many individuals interacting with each other. The difference between agent-based and equation-based simulations is that agent-based simulations do not have governing differential equations, they use the behavior of the individual to dictate the results of the simulations [8].

Simulations are used in place of or in conjunction with experiments to either compare the simulated results versus what was found during a laboratory experiment or to replace the laboratory experiment with the simulations entirely. The use of simulations, like anything, have their positive and negative consequences when in use. Simulations and experiments both allow the researchers to generate new knowledge [9]. As both methods can be used to generate new knowledge, how can researchers decide which one to use? Well, when exploring the difference between simulations and experiments and their desired outcome, scientists are able to determine the best technique.

On the exploration front both methods of research allow for scientific explorations. With the changing of inputs, conditions and control parameters the researchers are able to study the systems reactions to the changing factors in order to answer the specific question that was raised. However, the two methods of research differ in how they go about the scientific explorations. The simulation explores theoretical implications based on mathematics. The experimental method uses observations and measurements to explore the phenomena being studied [9]. Simulations can be used to predict data or they can be used to understand data that has already been collected. Due to this they are able to predict the future or they can infer about the past based on evidence [8]. By understanding what the researchers need to explore, they are able to generate the new knowledge through the experiment or simulation that previously would have never been discovered.

1.4 Research Project

This dissertation’s goal is to understand the fundamentals of photonic crystals used in lasers as well as to explore the physics utilized in lasing. Employing the knowledge of photonic crystal lasing, an exploration can be done into the mechanism that drive lasing within a periodically structure material. This exploration will be completed with the help of a computer simulation. The use of a simulation allows the exploration of the benefits (lasing threshold and footprint) a periodic dielectric material device offers us without the need to create a physical device or be in a laboratory. After the execution of the simulation used in this dissertation, various graphs will be able to show us what benefits the periodic dielectric material offers us. These graphs will show us where the bandgap of the device is as well as show us at which pumping power will lasing occur. Understanding the dielectric properties, we can identify at which pumping power will we start to see transmission resulting in a lasing frequency. The power in versus power out of the pumping can illustrate a trend which can be used to identify the modes of intrigue from a researcher’s point of view.

Background

The scientific principles of laser devices are rooted within the field of electromagnetism. A wave of photons (photons being the basic unit) contain the two properties, amplitude and wavelength. These two properties inform us of the amount of energy (amplitude) and what type of electromagnetic wave is produced [10]. The following **Figure 1** [11] shows the electromagnetic spectrum.

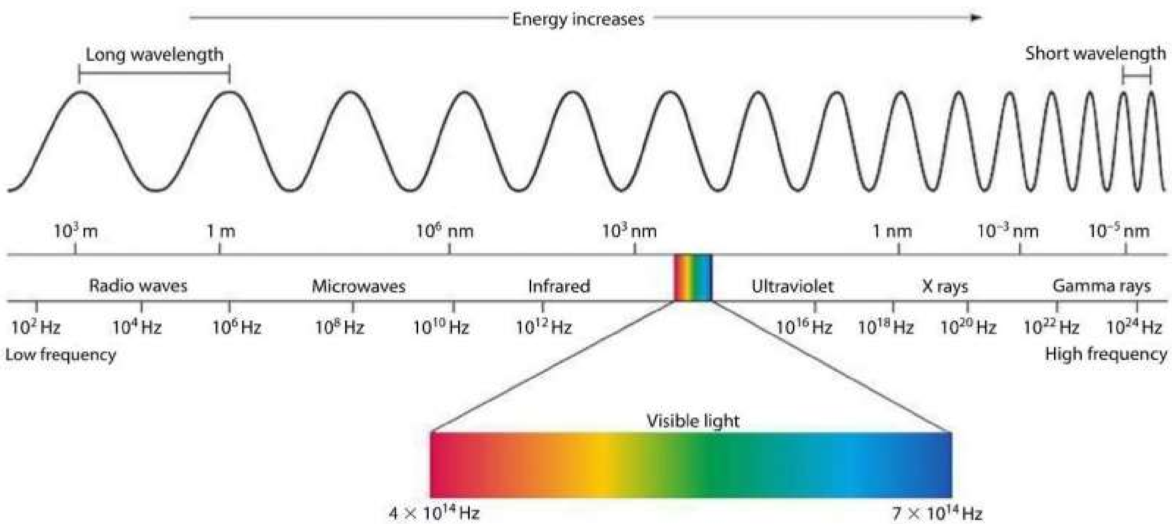

Figure 1: *This figure represents the electromagnetic spectrum which ranges from the longest wavelengths (Radio waves) to the shortest wavelengths (Gamma Rays).*

The three forms of emissions are due to absorption, spontaneous emission and stimulated emission. The figure, **Figure 2** [7], demonstrates the difference between all three forms.

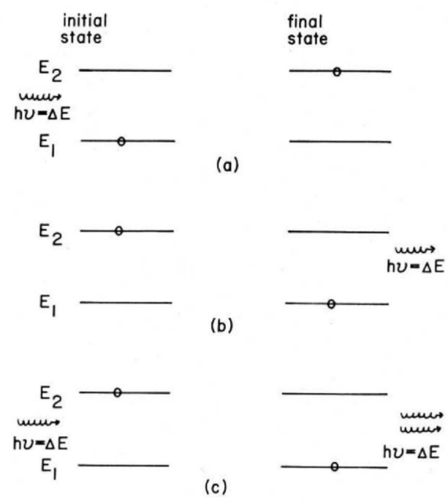

Figure 2: *This figure shows the three types of emission,* ***a*** *shows absorption,* ***b*** *shows spontaneous emission, and* ***c*** *shows stimulated emission.*

In **Figure 2a** we see that a system in a low-level state prior to absorption of a resonant photon raising the state to a higher-level state. In **Figure 2b** we see spontaneous emission which allows the emission of a resonant photon causing the system to go from a higher-level to a lower-level state. **Figure 2c** shows a stimulated emission which describes when a system already in an excited higher-level state interacts with a resonant photon causes two resonant photons to be released [7].

2.1 Fabry-Pérot Laser

A Fabry-Pérot laser, named after Charles Fabry and Alfred Pérot who were French scientists, consist of two optical amplifiers (mirrors) at the edge of the device enclosing a cavity within the laser device [12,13]. Essentially, the Fabry-Pérot laser can be seen as an LED with two end mirrors [14]. For the device to ‘turn on’, the pumping of the amplifier through a pumping mechanism must begin. Lasing begins when the specific laser’s minimum pump rate (also known as threshold condition) is established. In order for the threshold to be satisfied, two conditions will be required to be met. The two required conditions are an inversion in the gain medium and stimulated emission within the cavity producing a light beam [13]. The resonant cavity is created by the two mirrors separated by an integral multiple of half wavelength or they will undergo destructive interference [14]. With these two conditions met the device can be considered ‘turned on’. If the threshold conditions are mathematically written the intensity is shown by:

$$I_{after} = R_1^2 e^{\gamma L} R_2^2 e^{\gamma L} I_{before}, \tag{5}$$

where R_1 and R_2 are the optical amplifiers, and L is the length of the amplifier length. Since the threshold has to be maintained by stimulated emissions, we see I_{after} is equal to I_{before} which means the threshold gain must be:

$$R_1^2 R_2^2 e^{2\gamma L} = 1, \tag{6}$$

or

$$\gamma = -\frac{1}{L} ln(R_1 R_2)\,. \tag{7}$$

These two equations allow scientists to see that if there is an increase in L or R_1, R_2 then the threshold gain per length is reduced [13].

If there is a large number of resonant wavelengths that are close together, we lose the filtering characteristics of a Fabry-Pérot cavity. A Fabry-Pérot laser’s wavelength must be large in relation to the cavity. The following formula shows how the refractive index of the medium inside the Fabry-Pérot laser creates a shorter wavelength due to the speed of propagation being much lower than the speed of light:

$$L = \frac{\lambda x}{2n}, \tag{8}$$

where λ is the wavelength, x is an arbitrary integer and n represents the refractive index of the medium [14]. In the above equation n would be a number greater than 1 since 1 represents the refractive index within a vacuum.

2.2 Advancing the design of Fabry-Pérot lasers

The advancement of technology is nothing new. Very little, if any, inventions are perfected when they are released. Due to this, research scientists have been able to create better Fabry-

Pérot devices. Required features such as high output power, narrow spectrum and small emitting area may be needed for specific applications. An example of advancing the Fabry-Pérot laser is by changing the output itself. A Fabry-Pérot laser has light coming out of both facets equally, but a higher efficiency Fabry-Pérot laser only allows a small amount of light to come from the other side. This small amount of light is used as monitoring [6]. This is done by using a high reflective coating that is placed near the mirror in the device and a reduced reflective coating at the front of the laser. The product produced by the two reflectivity's is approximate to that within an uncoated laser which means that the gain for oscillation and the net feedback within this higher efficient laser are equal to that of the uncoated laser. We can see the ratio of the facet power by the following equation:

$$\frac{P_L}{P_R} = \frac{\left(\frac{\rho_L}{\rho_R}\right)(1-\rho_L^2)}{(1-\rho_R^2)}, \tag{9}$$

with ρ being the reflectivity of the coating [6].

Advancing the Fabry-Pérot laser even further will get us to the extended cavity laser. The requirements are that the collimating lens, and diffraction grating all are located rigidly with one another. Another requirement is that the angle between the lens and grating are precisely adjustable [15]. An optimized narrow-line continuous output can be produced by varying the dimensions of the external cavity as well as varying the reflectivity of the laser. The extended cavity laser requires the phase condition to be satisfied within the external resonator and the laser simultaneously. This phase condition can be adjusted to only meet once by changing the length of the external resonator. A change in the length of the external resonator can allow researchers to produce their desired output. However, researchers need to be careful as a change in temperature can disrupt this narrow line output, which is why there will need to be a readjustment within the device [6]. In 1997, scientists at the University of Sussex demonstrated an extended cavity laser using a design that was built on the Newport Ultima U100-P mirror mount [15].

With taking the advancement a step further we go from the extended cavity laser to the

External grating device. An external grating device is a variant on the external cavity which has grown in popularity for many optical measurements. The change within this device is a diffraction grating replacing the external mirror. A single frequency is ensured by the grating selecting a wavelength through the feedback [6]. An issue with the extended cavity lasers is the use of them within commercial communication systems because the assembly is far too expensive and delicate [6].

2.3 Distributed Feedback Laser

A Distributed Feedback laser contains a Bragg grating in the cavity of a Fabry-Pérot laser but does not use the two mirrors at the edge of the laser like the Fabry-Pérot does [16,12]. The grating within the laser can have a variety of designs to allow the Distributed Feedback Lasers to perform a desired task. At each corrugation (refractive index change) there will be small reflections due to the presence of the grating. Constructive interference will occur if the corrugation is at a multiple of the wavelength. The wide spectral widths that a Fabry-Pérot laser creates will be narrowed down due to non-wanted wavelengths being subjected to destructive interference allowing them to not be reflected [16]. The different types of Bragg gratings are, but not limited to, a Simply Grating, Chirped Grating, a Grating with a Phase Slip and Apodized grating. **Figure 3** [17]:

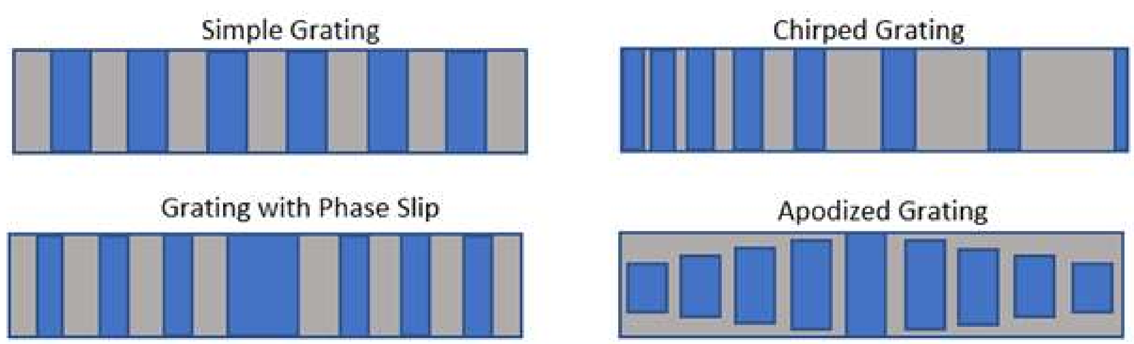

Figure 3: *This figure shows the four basic types of Bragg grating.*

As the above figure shows, a Simple Grating resonates at one wavelength in a periodic structure. The wavelength of a Simple grating resonates at the Bragg Wavelength. The grating reflects a narrow spectrum around the wavelength and all other wavelengths are transparent. An Apodized Grating uses the adjustment in the refractive index amplitude, however, keeping the uniform periodicity within the grating. A Phase slip grating carries a large center slip with a uniform periodicity and a chirped grating uses a varying periodicity of the Bragg grating [17,18]. Each of the different types of grating has their own use within fiber optic applications.

2.4 Application of Fabry-Pérot and Distributed Feedback Lasers

Fabry-Pérot lasers are the most common lasers used in communication fiber optic cables such as local optical networks, optical transmission and data communication [12]. When it comes to extended distance, coherent reception or wavelength multiplexing, the Fabry-Pérot laser is not a suitable device for this application. This result is due to the device producing a wide spectral width [14]. If we would like to use lasers for long distance communication, the Fabry-Pérot laser will just not suffice. This is because those types of lasers have a few significant problems. These problems include mode hopping and chromatic dispersion [16]. With these problems the communication industry wanted to find a fix to these issues. For them one potential answer to this issue is the Distributed Feedback laser. This is why the communication industry uses Distributed Feedback lasers for clean single-mode operation and high data rate long-distance transmission because of its high output power and stability [12]. These lasers also provide tunable control wavelengths which allows them to be used in fiber optic sensors, Metrology and different spectroscopy applications [12].

2.5 Photonic Crystals

Photonic crystals are materials that inherits a large number of its properties from the structure of the material instead of what the material is made of [19]. The application of photonic crystals can be split into two categories, these categories are the ones that rely on a complete bandgap or the ones that rely on the dispersion of the bandgaps. The photonic crystals that rely on a complete bandgap will see a complete suppression from emission in the bandgap [19]. The photonic bandgap can lead into the development of “efficient low-lose dielectric reflectors” [20]. The low-lose reflectors can use their light wavelength comparable size to confine radiation. Since light can propagate unimpeded within free space, confining light is a difficult task [20].

The photonic crystal was conceptualized by Yablonovitch in 1987 when he proposed that a lattice was composed of high and low reflective index materials. He also proposed that these materials would lead to forbidden and allowed propagation in the similar fashion to a lattice of atoms and their electron energies [21]. From ultra violet to infrared a photonic crystal can provide a single mode of operation, while a single mode fiber is unable to be used as effectively. The typical photonic crystal structure will be a two-dimensional cross section which will have a pure silica core which is surrounded by air holes [22]. This allows the photonic crystal to be a cavity in a Fabry-Pérot laser or the photonic crystal can be used employed in a distributed feedback laser due to the periodic modulation effect.

Theory

3.1 Pumping

During the early days of development of laser devices, a laser was considered to be an intense, sharp beam source. It was quickly comprehended that the most fundamental attribute of a laser was not the high intensity but rather the coherence of photons. A first and second order correlation function specifically describes the coherence of a laser [23]. A pumped pulsing square wave will excite a laser during the time short interval of $(0 < t < t_p)$ in which the laser will go from below to above the threshold. If certain conditions are met a phenomenon can be seen in which a laser may emit a pulse after the pump's time interval has concluded [24]. A L-I curve (Light-current) such as **Figure 4** [17] shows how much current will be needed to be applied in order for a desired amount of power to be produced from the laser [25].

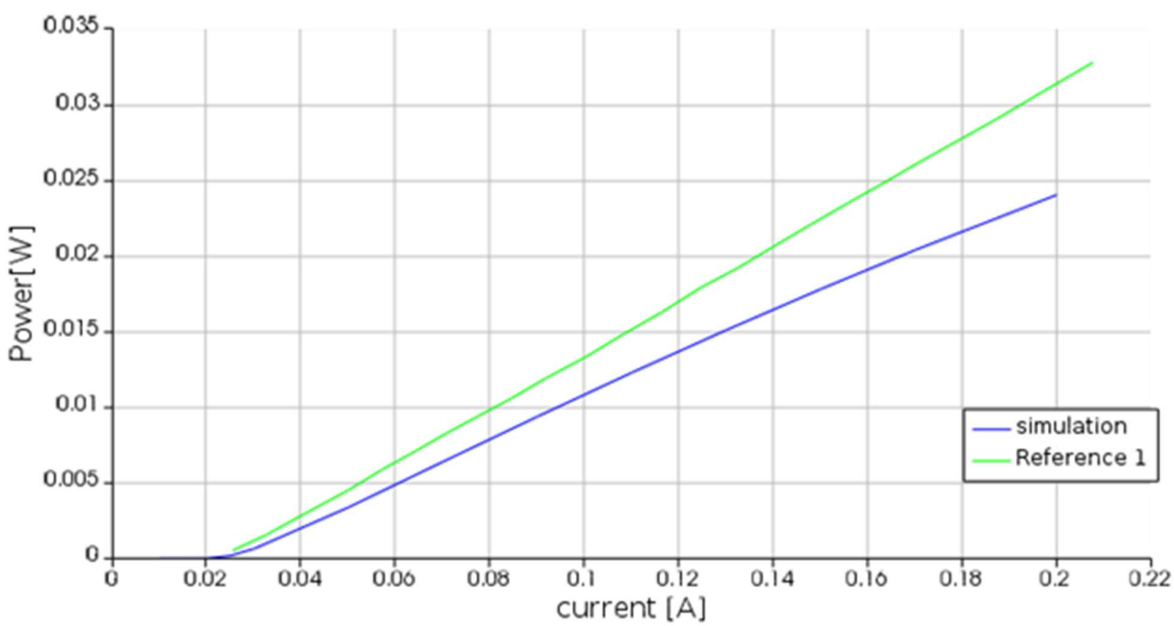

Figure 4: *A L-I curve such as this figure shows how until a certain current is obtained lasing will not begin.*

As the graph illustrates the laser device will not start lasering until the threshold is met.

If the laser has a current bias, then we can take the current to be:

$$I(t) = I + \Delta I(t), \tag{10}$$

making the output power equation to be:

$$P(t) = P + \Delta P(t). \tag{11}$$

In a communication system the transfer of information uses the current modulation to transfer electrical domain to optical domain [26].

3.2 Gain Medium

A gain medium is a material that needs to have quantum properties so that the material amplifies a laser beam by the way of stimulated emission. The material within the laser is the source of gain that allows the transition from an excited state due to pumping to a lower energy state [27].

Gain media comes in various types that can have a wide or linear spectra. Types of gain media include: gas laser, certain glasses or crystals, laser dyes and semiconductors. The gas lasers are pumped with electrical discharge into certain gas mixtures. Active ions are pumped into certain glasses or crystals which include “Nd:YAG, Yb:YAG, Rr:YAG, or Ti:sapphire”. The laser dyes are in liquid form, electrical currents are pumped as quantum wells into semiconductors such as gallium nitride [27]. Some of the key properties associated with a gain media are chemical stability, high quantum efficiency, laser transition takes place in the desired wavelength and high energy pulse amplification [27].

We must have the gain large enough so that the following equation is met in order for lasing to occur,

$$R_1 R_2 e^{(\Gamma_a \tilde{g} - \tilde{\alpha})^2 L} = 1, \tag{12}$$

where g is the threshold gain. If the left-hand side of the equation above is below 1 then the gain is said to be small, this means that all photons that enter the cavity will either leave through the mirrors or will be absorbed by the cavity. When the gain is large enough for the above equation to be correct then a buildup of photons populates the cavity which will cause spontaneous emission leading to lasing [26].

3.3 Distributed Feedback Laser & Quarter-Wave Shifted Distributed Feedback

Laser Theory

Within distributed feedback lasers the threshold gain and the photon's lifetime are related by the equation:

$$\Gamma_a v_g \widetilde{g_{th}} = \frac{1}{\tau_g}, \tag{13}$$

where τ_g is the photon's lifetime. The photon's lifetime is determined by:

$$\frac{1}{\tau_g} = v_g(\tilde{\alpha}_m + \tilde{\alpha}), \tag{14}$$

due to the losses experienced by the mirror or external loss ($\tilde{\alpha}_m$) and the cavity internal loss ($\tilde{\alpha}$) [28].

To create a quarter-wave shift one half of the device's grating will need to be shifted by one-quarter of a wavelength with respect to the other half, creating a crystal defect [28]. From solid state physics we know that a crystal defect may contain energy in the bandgap. In a quarter-wave shifted distributed feedback laser the frequency of the lasing mode falls within the bandgap and we see the intensity of lasing mode near the defect. A unique mode to the quarter-wave shifted distributed feedback laser is the defect mode. The defect mode holds the lowest threshold gain compared to any other mode, and this allows the lasing frequency to not worry about the structural imperfections while it is being designed for high accuracy [28].

The side-mode suppression rate of a distributed feedback laser is a ratio between the main and largest side mode and has a typical value that is greater than 30dB (Decibel - a logarithmic measurement of sound levels [29]) which indicates that most power exist in the main mode. To generate more power in the main mode you will need to use a higher performance distributed feedback laser [30].

Methodology

To conduct the research within this dissertation, the utilization of the software Lumerical by Ansys was utilized as the software works well with high-performance computing allowing us greater speeds for large simulations [31]. Within Lumerical's FDTD Solver we are able to access multiple pre-made simulations that cover a wide range of scientific research. Each simulation has the ability to allow the user to modify any aspect to their specific need, this can be done by changing the properties of an object found within the simulation or by changing the script of the simulation. During this project we called upon the use of the pre-built simulation *DFB laser using travelling wave laser model (TWLM)* [17]. The chosen pre-built simulation allows the user to simulate an index-coupled distributed feedback laser that has undergone a quarter-wave shift [17]. The grating of which the laser initially starts with a Simple grating, however, the Simple grating can be changed to any variation of a Chirped, Phase slip or Apodized grating that the user wants to study.

When the chosen simulation has finished running, the results will consist of a multi-mode spectrum vs current, spectral gain width, a side-mode suppression ratio function due to current and a L-I curve plot. The L-I curve plot will be compared to the results from [32] [17].

4.1 Understanding Lumerical

The first stage of completing the research of this project is to acquire a working understanding of the Lumerical software. This involves exploring a number of examples included in the software, understanding the general setting of a simulation and performing a detailed analysis of the results. As an example, I have focused on the script *spectrum.lsf* which is associated with testing which wavelength, given the parameters, will there be a peak. A run of the script with the parameters of FWHM = 1000nm and 28mA for the bias current we find the results shown in **Figure 5** [17]:

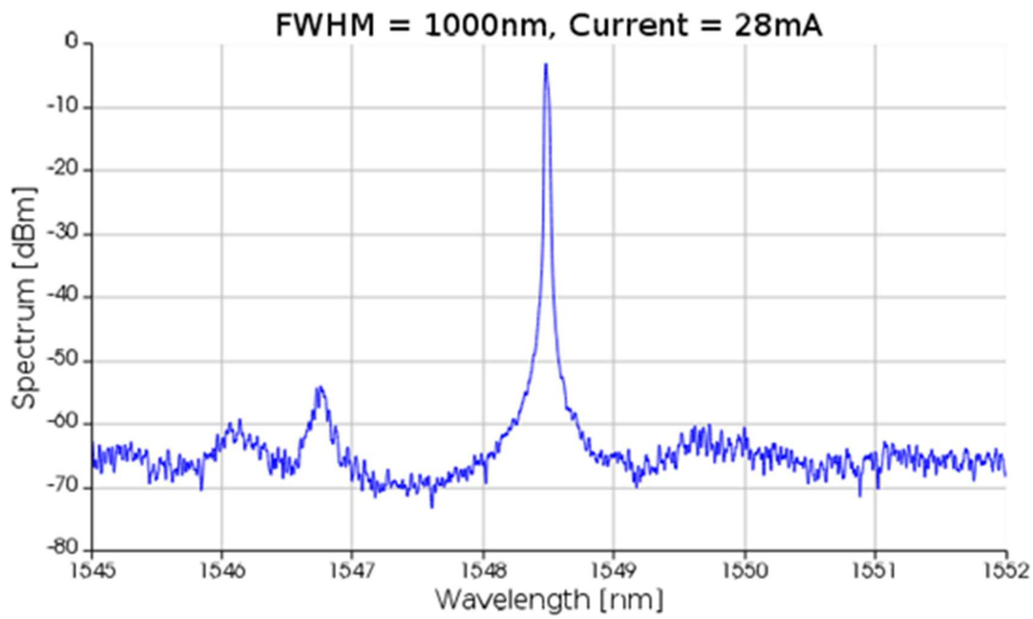

Figure 5: *This is the given result for the chosen Lumerical simulation. The graph shows at which wavelength a FWHM = 1000nm and Current = 28mA would produce a peak*

If we were to run the script with these parameters in place than we have the printed plot **Figure 6**:

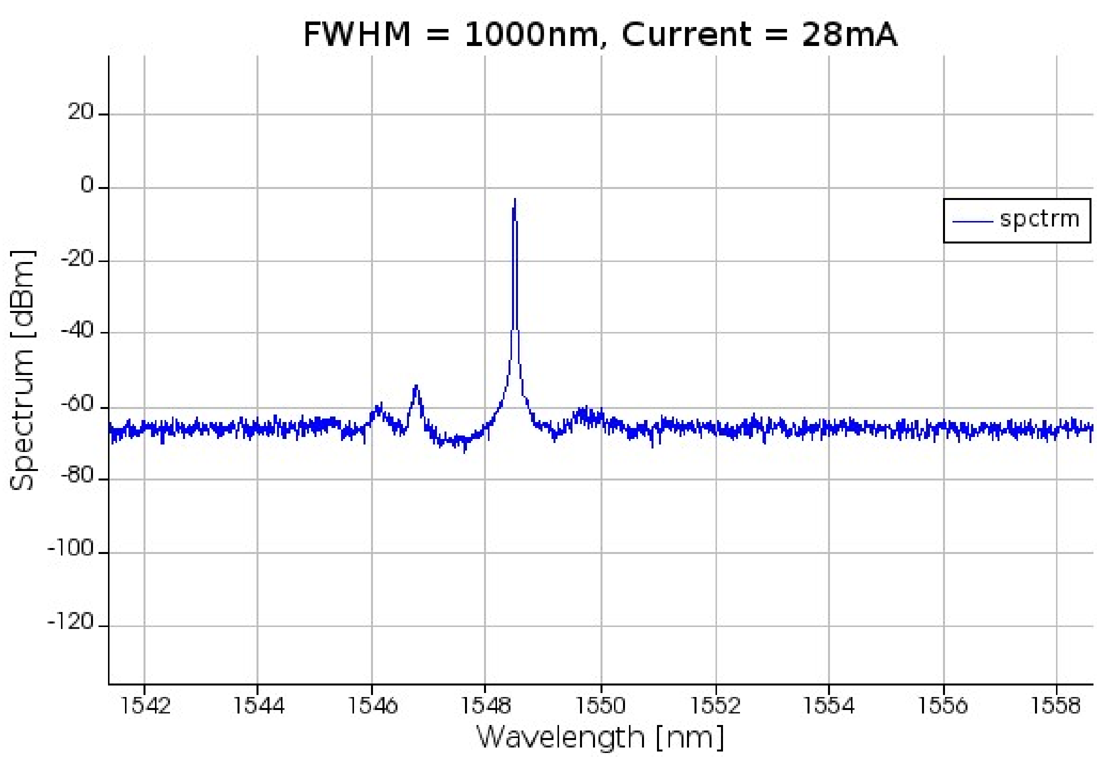

Figure 6: *This is the result when the chosen Lumerical simulation was run using my device with the same parameters (FWHM = 1000nm, Current = 28mA) as* ***Figure 5****.*

Although looking at first glance one may think the graphs are different, a look closer at the graph shows that the results are nearly identical. Both graphs show a peak at roughly 1548.5nm and the spectrum dB comes close to reaching zero at the peak. The reasons the graphs look different can be due to the fact that the wavelength range used in [17] could be different than the one that we used in our test run of the program.

After studying to confirm that the predicted graph and the test graph are nearly identical for this file, we went on to complete the rest of the “Run and results” section from [17] to confirm their accuracy as well, these tests are omitted from this paper as they do not apply to the project’s goal. This stage of research was to gain valuable insight in how the Lumerical simulations software works.

4.2 Maxwell’s Equations

Using the plane wave expansion method allows the determination of the mode dispersion. The plane wave expansion method expresses the field as a sum of periodic boundary condition plane waves while having converted Maxwell’s equations into linear eigenproblems [33]. In order to understand that the solutions of Maxwell’s equations will be a set of harmonic modes in which an eigenvalue equation can be constructed, we need to first know the math behind these solutions. To start we can first write the Maxwell equations: [33]

$$\nabla \cdot \mathrm{B} = 0, \tag{15}$$

$$\nabla \cdot \mathrm{D} = \rho, \tag{16}$$

$$\nabla \times \mathrm{E} + \frac{\partial \mathrm{B}}{\partial \mathrm{t}} = 0, \tag{17}$$

$$\nabla \times \mathrm{H} - \frac{\partial \mathrm{D}}{\partial \mathrm{t}} = \mathrm{J}. \tag{18}$$

In Maxwell’s equations the variables stand for as follows: *B* represents the Magnetic Flux Density, *D* stands for the Electric Displacement, *ρ* is the Resistivity, *E* is the Electric Field, *H* is the Magnetic Field and *J* is the Current Density [34,35]. To arrive at Maxwell’s equations in a simplified form we will need to make a few assumptions. These assumptions allow us to say that if we have a transparent media, we know that the permittivity is real, or that the displacement field and the electric field are related linearly or if there is no movement with the material, then there is no time dependence of the electric permittivity [33]. After applying these assumptions, we arrive at:

$$\nabla \cdot \mathrm{H}(\mathrm{r},\mathrm{t}) = 0, \tag{19}$$

$$\nabla \cdot [\epsilon(\mathrm{r})\mathrm{E}(\mathrm{r},\mathrm{t})] = 0, \tag{20}$$

$$\nabla \times \mathrm{E}(\mathrm{r},\mathrm{t}) + \mu_0 \frac{\partial \mathrm{H}(\mathrm{r},\mathrm{t})}{\partial \mathrm{t}} = 0, \tag{21}$$

$$\nabla \times \mathrm{H}(\mathrm{r},\mathrm{t}) - \epsilon_0 \frac{\partial \mathrm{E}(\mathrm{r},\mathrm{t})}{\partial \mathrm{t}} = 0. \tag{22}$$

We can further express this by using the complex forms of both H(r,t) and E(r,t):

$$\mathrm{E}(\mathrm{r},\mathrm{t}) = \mathrm{E}(\mathrm{r}) \cdot \mathrm{e}^{-\mathrm{i}\omega\mathrm{t}}, \tag{23}$$

$$\mathrm{H}(\mathrm{r},\mathrm{t}) = \mathrm{H}(\mathrm{r}) \cdot \mathrm{e}^{-\mathrm{i}\omega\mathrm{t}}, \tag{24}$$

with ω being the angular frequency [33].

The goal is to use the frequency-domain and frequency time-domain method to solve these equations numerically with the adoption of various initial conditions.

4.3 Frequency-Domain and Frequency Time-Domain Method

The Frequency-Domain and Time-Domain methods are the two most common computational approaches to photonic crystal study. With each of the two methods providing their own advantages and disadvantages we can see how they can each find their own place within research [36].

The frequency-domain methods have an advantage when it comes to the calculation of eigenstates and band structures. A disadvantage to this method is that all lower eigenstates (even if they are not important) will need to be computed until the desired one. The Time-Domain method’s advantage is with computing evolution of fields. Another advantage is that through the use of the Fourier transform of the system we can determine all peak frequencies of a single field. However, the Time-Domain does have several disadvantages. These disadvantages include a lack of confidence if all states are found as some weak states may have been coupled, and in the Fourier Transform the resolution is inversely related to the simulation time. A few more disadvantages are that the spatial-grid size and the time-step must be proportional in order to have numerical stability and only the frequency of the states will be determined, not their eigenstates [36]. The Massachusetts Institute of Technology (MIT) Photonic Band software represents a frequency-domain method due to its ability to use planewave basis computation of the eigenstate and eigenvalue [36].

4.4 MPB (MIT Photonic Bands)

The next stage of the research project was to use a freely available MIT developed software package. The MPB software is compatible with 1-3D systems to compute both the band structure and mode profile using a periodic boundary condition in dielectric structures (Richard Thesis). This software's main application is in the photonic crystal field of study; however, it can be used to compute eigenstates and other optical dispersion [36].

4.5 Plane Wave Expansion Method

Using the same assumptions as we did in the Maxwell's equations the eigenvalue equation obtained is:

$$\nabla \times \left(\frac{1}{\epsilon(r)} \nabla \times \mathrm{H(r)} \right) = \left(\frac{\omega}{c} \right)^2 \mathrm{H(r)}, \tag{25}$$

which allows the use of the plane wave expansion method to be applied to periodic dielectric functions [33].

4.6 Design of Lattice Structure

The in-plane structure is shown in **Figure 7**, this structure allows us to see that we have created a structure which has circular holes (Lumerical allows the circular figures to represent air by setting certain conditions of the material (the material chosen is a real dielectric material which has a refractive index of n = 3.6) to that of air, but will not create a visual representation of holes) within its lattice. The height of the slab used is 0.7a (a is the unit height used in MPB ~ 0.23971 μm).

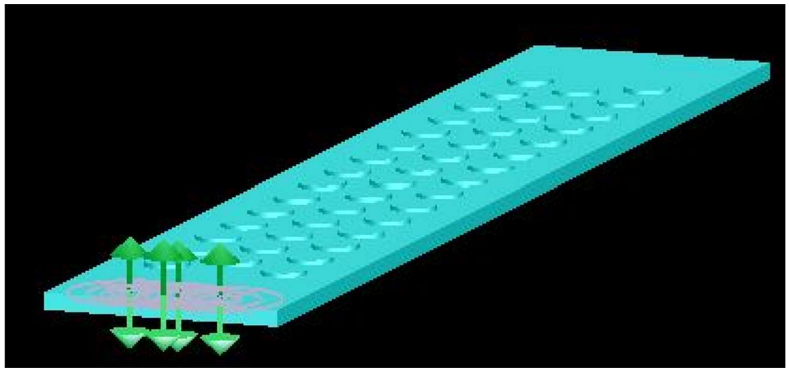

Figure 7: *This figure represents the used dielectric slab within the computational aspect of the research project. The slab is made of three parts, the diodeIn/diodeOut and the dielectric material which consist of uniformed air pockets.*

The slab of material also has an in and an out rectangle attached to the lattice which will be where the pumping of the material occurs. The green arrows are coming from the rectangle which houses dipoles creating the electromagnetic pumping in this system.

Results

5.1 Photonic Bandgaps

An important concept in solid state physics is the bandstructure. Two quantum numbers characterize the electronic levels in a photonic crystal structure [37]. The frequency of the propagating photonic modes as a function of the wave number. The resulting bandstructures of transverse electric modes for the represented slab gives us the following result in **Figure 8**.

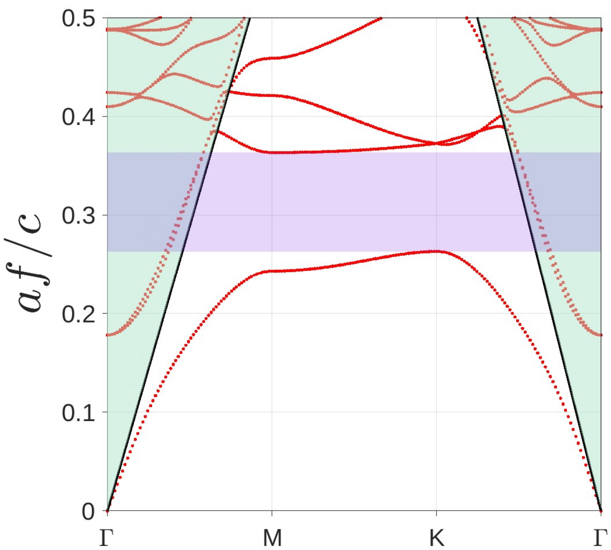

Figure 8: *After running the simulation using the dielectric slab in **Figure 7**, this is the resulting bandstructure produced.*

The green area is associated with modes in the slab which couple to modes in the continuum and eventually decay as a function of time. The modes in the white area are modes in the slab which do not couple to the modes in the continuum, they are called guided modes and are confined to the extent of the slab. A bandgap describes the region in which the light being propagated in the axis of the periodicity is reflected at 100% efficiency making propagation unobtainable in the specific spectral range [33]. Optimizing the bandgap size is studied extensively due to the increase in control of propagation direction in the bandgap region even though there is no spontaneous emission [38].

When the next stage of research was conducted, we keep the same parameters as before as we want to see the transmission rate throughout the bandstructure which we see in **Figure 9**.

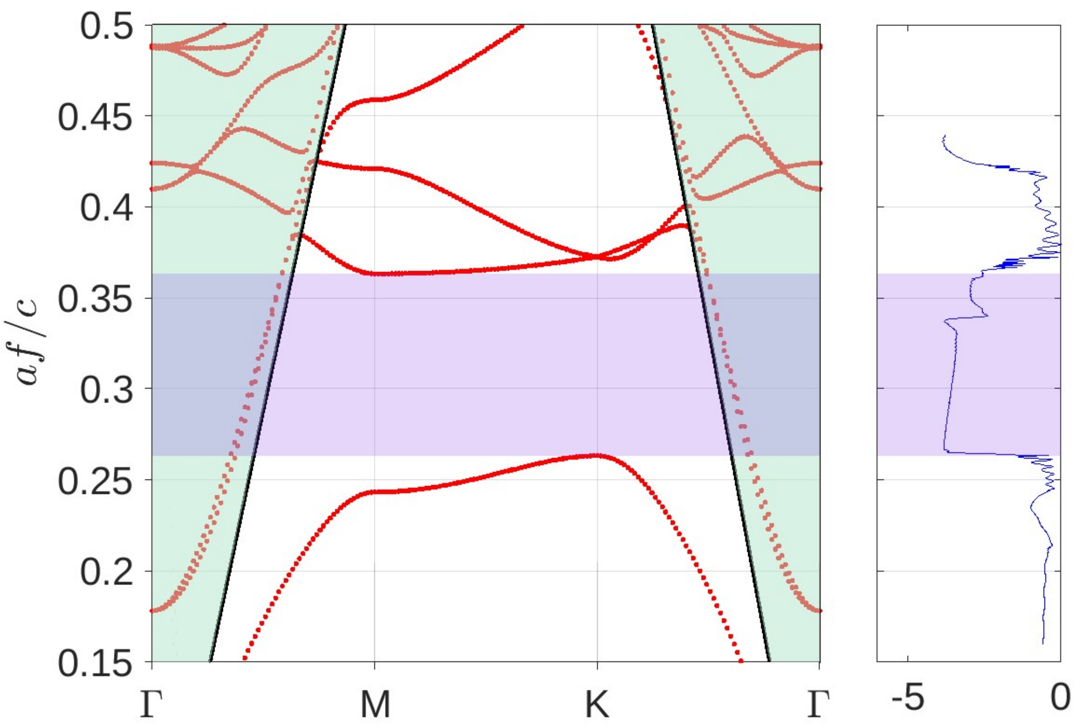

Figure 9: *Using the same bandstructure found in* ***Figure 8****, we can now see the transmission rate on a logarithmic scale.*

Figure 9 is an expansion on **Figure 8**. We are now able to see a transmission spectrum which has a logarithmic scale for the x-axis. We will now need to analyze what the transmission spectrum graph represents.

5.2 Transmission Spectrum

The maximum intensity of a transmission spectrum will occur when the corresponding wavelengths absorption is at the minimum, at this wavelength the most amount of light is transmitted through the sample. In the scope of this experiment, we will measure the amount of radiation that is transmitted through the structure with a detector placed at the end of the structure and a source at the other. Since the laser threshold is derived from the output power and the output power is calculated using the transmission spectrum, it is important that an analysis is done. So, when analyzing the transmission spectrum a few factors will need to be considered. These factors are the reflective loss, propagation loss, localized modes and the photonic band gap. These factors need to be considered are for reflective loss, the source will have reflection at each and every air-dielectric interface. For propagation loss, a leaky mode can make the propagation suffer if it is highly localized at the frequency. The reason a localized mode will affect the propagation loss is that light propagated through full structure length modes easier than that of a non-full-size mode and as for the photonic band gap there is a relationship between the density of states and the transmission [33].

Looking at the transmission spectrum from **Figure 9** we can view this by turning the graph horizontally instead of vertically to get **Figure 10**.

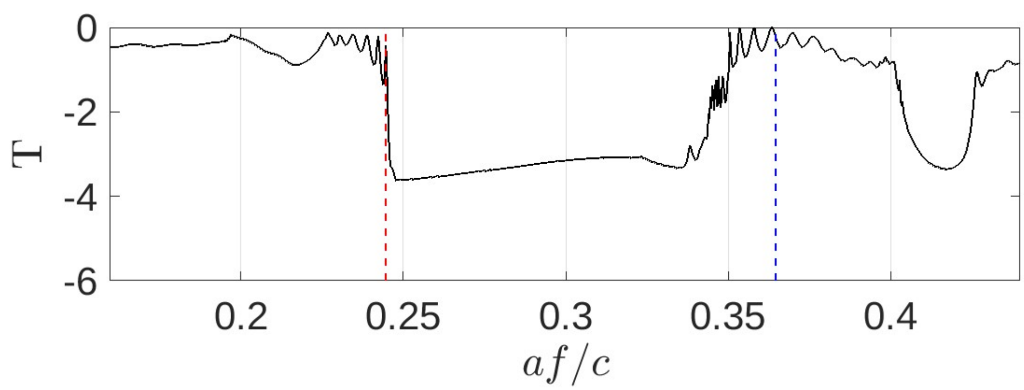

Figure 10: *A stand along look at the transmission rate using a logarithmic scale vs af/c. On the graph is two marked regions which although both show higher transmission, one falls into the bandgap while the other does not.*

Looking at the transmission spectrum for **Figure 10** a dip in transmission is seen in the same af/c as we see the bandgap. The reason for this is that in the bandgap when there are no spontaneous emissions there will be no transmission between the bandstructures. Now describing the two lines on the spectrum graph. The line (lasing frequency) sitting around 0.25 af/c falls into the bandgap whereas the line (pump frequency) above 0.35 af/c will have a large transmission. We are able to determine the optimal frequency by taking the cross-section (this step will not be completed in this research project) which is the product of the pump, transmission and gain spectra [33].

5.3 Mode Lasing Relation

In order to optimize the laser, the engineering of the laser can be tinkered with in order to do this. By constructing a laser with a mode where a high transmission occurs due to the pump frequency. If we simulate the slab through different modes, increasing modes from 1 to 42, we can see how the power in (P_{in}) versus power out (P_{out}) are related. We see based on **Figure 11 a-d** that as we increase the P_{in} and the number of modes then we start to see transmission within the population graph. We also see a peak emerge from lasering.

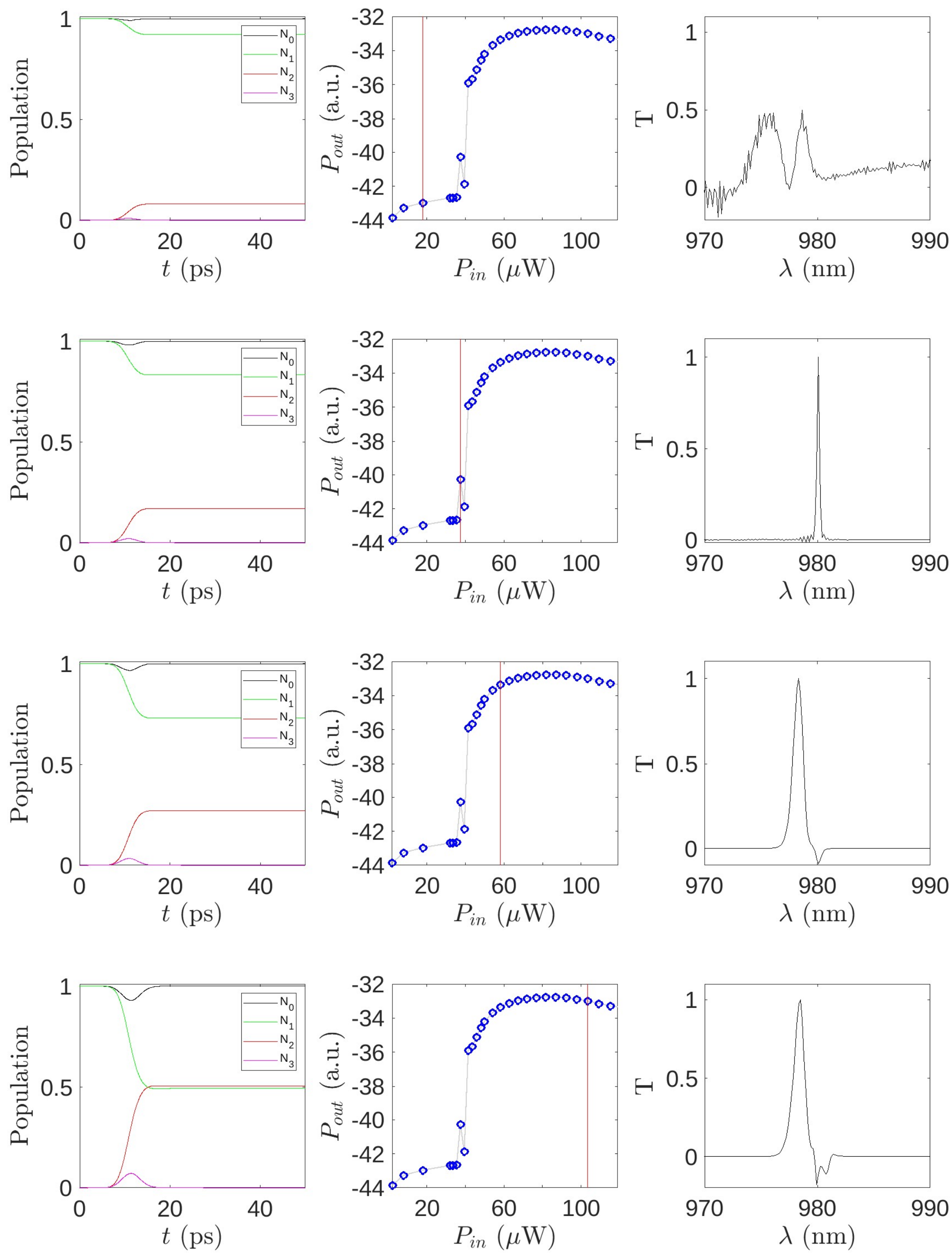

Figure 11 a-d: *In these four figures we see how the Population at the ground and excited state start to absorb and emit at larger quantities when the mode is increased.*

The figure, **Figure 11 a-d**, the left column shows the evolution of the population of the four levels as a function of time based on the four different pumping powers. The middle column depicts at which power in versus power out point the pumping power is representing. The vertical lines correlates to the mode chosen for the pumping power. The right column shows the transmission versus wavelength produced by the various pumping powers.

If we investigate further, we know that the incoming pumping frequency will undergo a few losses as well as scattering prior to reaching lasing. This will create a time delay, to put the time delay in a spectrogram we will determine **Figure 12 a-d**.

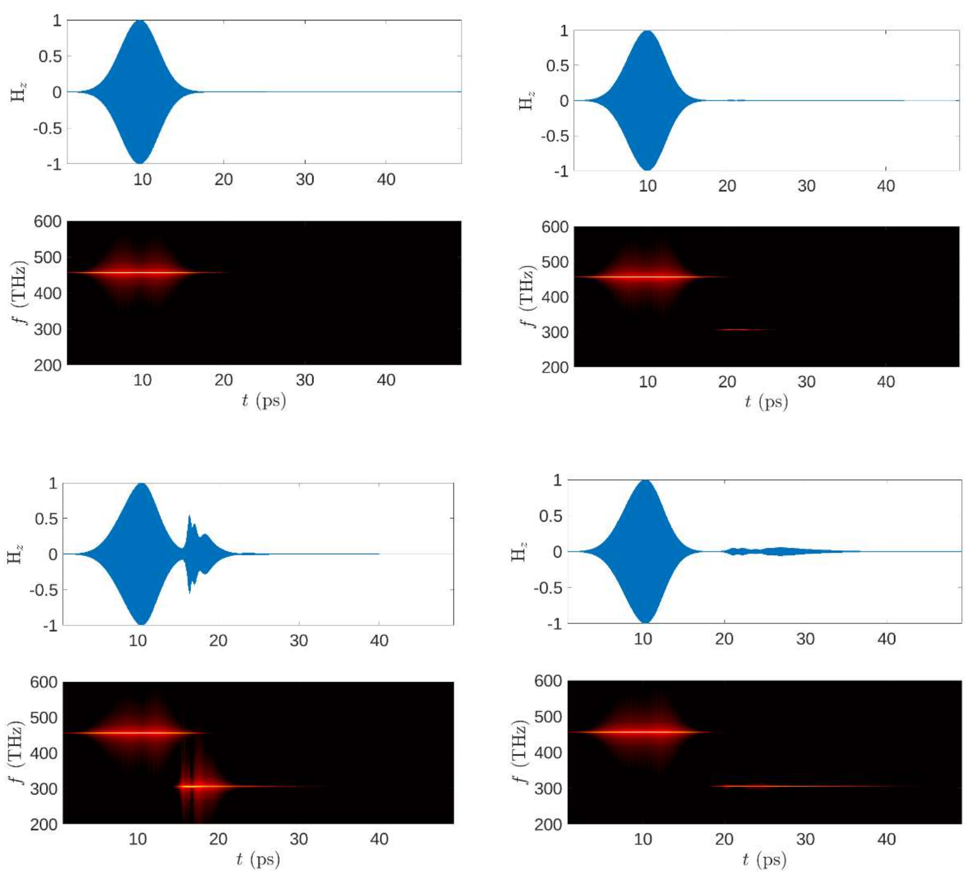

Figure 12 a-d: *These four graphs show that as the modes increase a resulting lasing frequency is produced. However, a longer delay of lasing is a result of increasing the number of modes as well.*

As the **Figure 12 a-d** illustrates, the pumping frequency is seen in each of the graphs. When lasing starts to occur at higher modes, we start to see a time delay before producing lasing within the slab. The lasing from time 1-20ps that the lasing frequency has a spectral width that does not rest at one wavelength however, as the modes increase, a singular wavelength is produced having a small spectral width. The results also suggest that for greater pump power we have shorter time delay between the lasing output and pump pulse [33].

Discussion and Future Works

6.1 Discussion

This dissertation explored the relationship between the structure of photonic materials and the lasing action. Using a computational approach, I studied which photonic mode in a periodically structured dielectric material would show the most promising desired characteristics for low-threshold lasing. In **Figure 11 a-d** one can notice that with the increase in mode count, an increase in transmission increases as well. In **Figure 12 a-d** we can see how there exists a delay in the laser emitting a pulse at the desired frequency. The delay time grows larger as the mode number increases. Seeing the two sets of figures together, we see that the delayed emission starts to make its presence as a negative transmission rate.

6.2 Future Work

In 1965 an American engineer, Gordon Moore, predicted that a silicon chip's transistors would double every year [39]. This is known as Moore's Law, although this law originated from the field of computer science, the theory has been used by other fields such as semiconductors. If we see how well this theory has stood up over time, we can say that for over a 50-year period this prediction has held up perfectly [40]. At the current time traditional semiconductor lasers are difficult to use in metal cutting due to their beam quality. Fiber laser's beam quality surpasses that of semiconductors in direct material processing but when it comes to thin plate welding and cutting applications semiconductors are a suitable option. Therefore, many important applications can be made possible from the development of high-power semiconductor lasers [40]. Future research will be needed to create these high-power semiconductor lasers using the ground works of optimized modes.

Metals are the most studied efficient electromagnetic wave expeller and reflector that is naturally known. They are ideal candidates for confining electromagnetic radiation as they exhibit high reflectivity while having low absorption loss. In the microwave regime, $\lambda \sim 1$cm, aluminium experiences ~0.01% absorption loss, however, when in the visible regime, λ~ 400nm-800nm, or the infrared, λ~ 800nm-20μm, aluminium has 10-20% absorption loss [20]. Photonic crystals have many applications in which further research can yield exciting results. Within a microelectronic device such as solar cells the suppression of spontaneous emission can be used to help make advancements in the field. Microwave application could be made to use substrates to have emission in only the desired direction. New applications can even spring entirely from the new physics photonic bandgaps have brought [19].

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