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Quantentransport in Spindichtesystemen mit dem Memory-Matrix-Formalismus

Masterthesis

von

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(**Martin Lietz**)

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Contents

1 Motivation

2 Spin-Fermion-Model

3 Memory-Matrix-Formalsim

4 Calculation

In the last chapter the memory-matrix-formalism was introduced, which give us an exact formula to calculate correlation functions. Now this formalism is used to determine the static conductivity of the spin-fermion-model, introduced in chapter (), pertubate umklapp-scattering.

make link to chapter spin-fermion-model

4.1 Infinite conductivity in systems with unbroken translation symmetry

After Drude published his theory about the electrical transport in metals [Dru00] in the beginning of the last century it is well known that a broken translation symmetry is needed to get a finite static conductivity. Because of Neother's theorem it is also well known that a unbroken symmetry always implies a conserved quantity. In the case of translation symmetry this quantity is the momentum. Phenomenas breaking the translation symmetry are for example impurity scattering, electron-electron scattering and umklapp scattering. Let us firstly assume the standard spin-fermion-model without a translation symmetry breaking pertubation. In chapter () it is showed that in the used model the momentum is conserved but the currunt isn't conserved. This property is needed to calculate the static conductivity.

link to chapter spin-fermion-model

In general the static conductivity is given by taking the small frequencie limit of the conductivity and the conductivity itself is given by the current-current correlation function (J-J correlation function), which reslut directly from the linear response theory.

$$\sigma_{\mathrm{dc}} = \lim_{z\to 0} \sigma(z) = \lim_{z\to 0} \beta\, \mathcal{C}_{\mathrm{JJ}}(z) \tag{4.1}$$

The memory matrix formalism is used to calculate the J-J correlation function. Before we attend us to the calculation of the correlation function we have to think about the set of operators introduced by defining the projection operator. This set of operators has to be choosen for each calculation seperatly depending of the model and the quantity of interest. In our case we choose only a set of two operators namly the momentum and the current, because we want to figure out the influnece of the momentum on the current. That means the projector $\mathcal{P}$ projects into the two dimensional sub-Hilbertspace, spanned P and J. Lets go back to the correlation function, defined by equation (), where the sum over k und l is implied. Because our interest is focused on the J-J correlation function each index j and l is set to J.

reference to correlation function

$$\mathcal{C}_{\mathrm{JJ}}(z) = \frac{i}{\beta}\left[z\delta_{i\mathrm{J}} + i\beta\big(\dot{\mathrm{A}}_i\big|\hat{Q}\frac{i}{z-\hat{Q}\hat{L}\hat{Q}}\hat{Q}\big|\dot{\mathrm{A}}_k\big)\chi^{-1}_{k\mathrm{J}}\right]^{-1}\chi_{i\mathrm{J}}, \tag{4.2}$$

Now the sum over k is performed explicitly where both contriubtions for J and P vanish. Let us look seperatly on J and P starting with the latter case. In our observed model the momentum is conserved and so the time deriviative of P is zero. No more words are needed to see that the expactation value doesn't contribute. In the case of J the time derivative doesn't vanish so the two dimensional sub-Hilbertspace and the action of J in this Hilbertspace has to be consider. In the investigated system the whole current lives in the J-P Hilbertspace to any time, so no single part of J is transported out of the Hilbertspace. The appering operator $\mathcal{Q}$ is the inverse of $\mathcal{P}$ and therefore projected out of the J-P Hilbertspace. Combining both statements it is clear that $\mathcal{Q}|\dot{\mathrm{J}}) = 0$. The only resulting term choosing $i = J$ is

$$\mathcal{C}_{\mathrm{JJ}}(z) = \frac{i}{\beta} z^{-1} \chi_{\mathrm{JJ}}(\omega = 0) = \frac{i}{z} \mathcal{C}_{\mathrm{JJ}}(t = 0), \tag{4.3}$$

reference to scalar product

where the correlation function at $t = 0$ is given by the scalar product $(\mathrm{J}(0)|\mathrm{J}(0))$ defined in equation (). During the motivation of the previous chapter we explained that each observalbe can be split in one secular and one non-secular part. This is equatable with splitting a vector in a parallel and a perpendicular component, respectivily.

$$\big|\mathrm{J}\big) = \big|\mathrm{J}_{||}\big) + \big|\mathrm{J}_{\perp}\big) \tag{4.4}$$

What does this mean in physical language? In the investigated system the current isn't conserved, but a part of him is it. This part is represented by the secular part and has to be parallel with the momentum. Therfore the projection from J at P yield the parallel component of J.

$$\big|\mathrm{J}_{||}\big) = \mathcal{P}\big|\mathrm{J}\big) = \frac{\big|\mathrm{P}\big)\big(\mathrm{P}\big|}{\big(\mathrm{P}\big|\mathrm{P}\big)}\big|\mathrm{J}\big) = \frac{\chi_{\mathrm{PJ}}}{\chi_{\mathrm{PP}}}\big|\mathrm{P}\big) \tag{4.5}$$

$$\mathcal{C}_{\mathrm{JJ}}(t = 0) = \big(\mathrm{J}(0)\big|\mathrm{J}(0)\big) = \big(\mathrm{J}_{||}\big|\mathrm{J}_{||}\big) + \big(\mathrm{J}_{\perp}\big|\mathrm{J}_{\perp}\big) \tag{4.6}$$

$$\mathcal{C}_{\mathrm{JJ}}(t = 0) = \frac{|\chi_{\mathrm{PJ}}|^2}{|\chi_{\mathrm{PP}}|^2}\mathcal{C}_{\mathrm{PP}}(t = 0) + \big(\mathrm{J}_{\perp}\big|\mathrm{J}_{\perp}\big) \tag{4.7}$$

$$\mathcal{C}_{\mathrm{JJ}}(z) = \frac{i}{z\beta}\frac{|\chi_{\mathrm{PJ}}|^2}{|\chi_{\mathrm{PP}}|} + \frac{i}{z}\big(\mathrm{J}_{\perp}\big|\mathrm{J}_{\perp}\big) \tag{4.8}$$

$$\sigma(z) = \frac{|\chi_{\mathrm{PJ}}|^2}{|\chi_{\mathrm{PP}}|}\frac{i}{z} + \sigma_{\mathrm{reg}}(z) \tag{4.9}$$

where the regular conductivity $\sigma_{\mathrm{reg}}(z) = \frac{i\beta}{z}\big(\mathrm{J}_{\perp}\big|\mathrm{J}_{\perp}\big)$ is introduced. Setting $z = \omega + i\eta$, where the limit $\eta \to 0$ is implied.

$$\sigma_{\mathrm{dc}} = \frac{|\chi_{\mathrm{PJ}}|^2}{|\chi_{\mathrm{PP}}|}\left(\mathcal{P}\frac{i}{\omega} + \pi\delta(\omega)\right) \tag{4.10}$$

where $\mathcal{P}$ sympolizied that the prinzipal value is taken.

5 Conclusion

A Appendix

Todo list

Bibliography

[Dru00] P. Drude. „Zur Elektronentheorie der Metalle“. In: *Annalen der Physik* 306.3 (1900), pp. 566–613. DOI: 10.1002/andp.19003060312.