

# SISMID 2023 Spatial Statistics Waller Point Process 1: Monte Carlo Test of CSR

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- What we have

- Event locations for a strand of 327 myrtle trees in a rectangular plot  $170.5 \times 213.0$  meters.
  - 221 healthy trees.
  - 106 diseased trees.
  - Research question: Is the spatial pattern of diseased trees the same as the spatial pattern of healthy trees?
- 

- Reading in the data, basic R commands

```
# Set my working directory (Lance's here for example)
#setwd("~/OneDrive - Emory University/meetings/SISMID.2021/SISMID.2021.Waller.Rcode")
myrtles.healthy = scan(here("data", "myrtles.healthy.d"),list(x=0,y=0))
```

```
#####
# Let's see what we have. Typing the
# name "myrtles.healthy" and hitting return
# prints out the values.
#####
```

```
# Commented out so it doesn't list everything in the handout
#myrtles.healthy
```

```
#####
# The "names" command just give the
# names of the variables inside the data frame.
#####
```

```
names(myrtles.healthy)
```

```
## [1] "x" "y"
```

```
#####
# To access the value within a data frame
# type the name of the frame, a dollar sign,
# then the name of the variable.
#####
```

```
#Commented out so it doesn't list every value in the handout
#myrtles.healthy$x
```

```
#####
# To find out how many observations are
# in myrtles.healthy$x, use the "length"
# command.
#####
```

```
length(myrtles.healthy$x)
```

```
## [1] 221
```

---

- **Plotting the data**

- Read in data on diseased trees.
- Plot patterns
- Take care to make *square* plots, covering the *same* area. \*\*\*

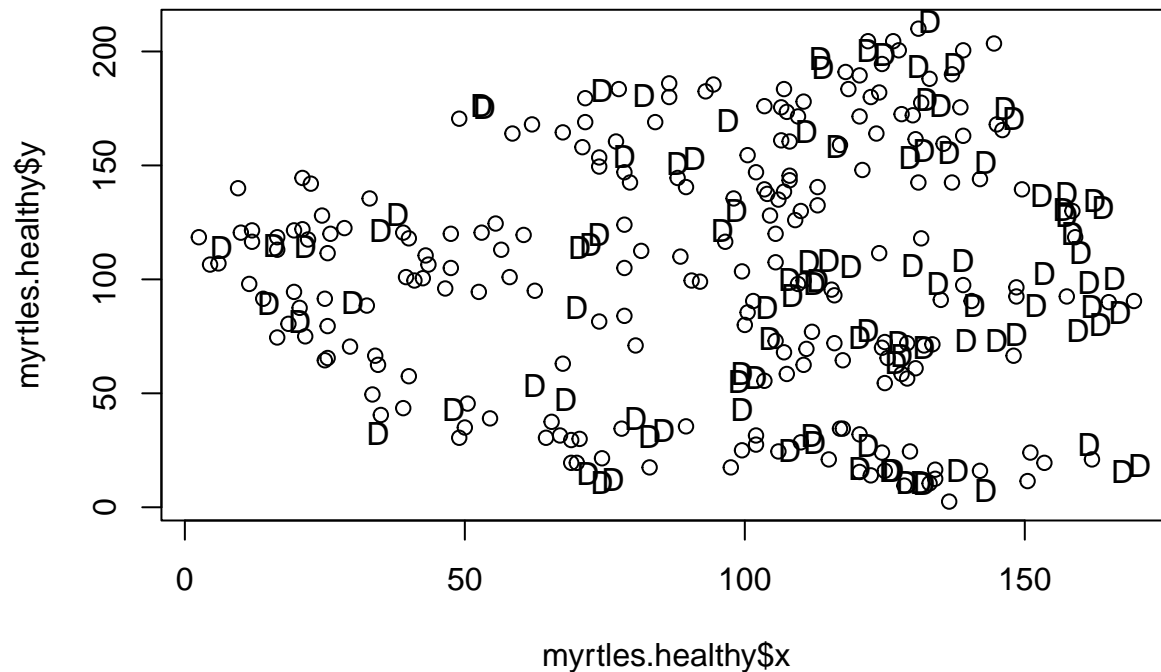
```
myrtles.all = scan(here("data", "myrtles.d"),list(x=0,y=0))
myrtles.diseased = scan(here("data", "myrtles.diseased.d"),list(x=0,y=0))
```

```
#####
# Let's plot the data
#####
```

```
plot(myrtles.healthy$x, myrtles.healthy$y)
```

```
#####
# The "points" command adds points to a plot,
# and the "pch" option changes the "plot character".
# Let's add the diseased myrtle locations and plot
# them as "D"s.
#####
```

```
points(myrtles.diseased$x,myrtles.diseased$y,pch="D")
```



```
#####
# Notice that the plot is sort of square, but
# that the range of values for x is different
# from that for y. Let's set the limits
# so that they are the same. First, we find
# min and max of the x and y coordinates for
# ALL myrtle locations (healthy and diseased).
#####
```

```
min(myrtles.all$x)
```

```
## [1] 2.5
```

```
max(myrtles.all$x)
```

```
## [1] 170.5
```

```
min(myrtles.all$y)
```

```
## [1] 2.5
```

```
max(myrtles.all$y)
```

```
## [1] 213
```

```
#####
# We can also use the "range" command
# to do this.
#####
```

```
range(myrtles.all$x)
```

```
## [1] 2.5 170.5
```

```
range(myrtles.all$y)
```

```
## [1] 2.5 213.0

#####
# Looks like if we set the plot boundaries
# for (0,215) for x and y, we'll catch all
# of the points. We use the "xlim" and
# "ylim" parameters in the plot command.
# NOTE: we can continue a command onto the next
# line if we end with a comma and don't include
# a closing paranthesis until we are ready.
# ALSO NOTE: "c(0,215)" concatenates the values
# 0 and 215 into a vector.
#####

plot(myrtles.healthy$x,myrtles.healthy$y,xlim=c(0,215),
     ylim=c(0,215))

#####
# Finally, to make sure R draws the plotting area
# as a square, we introduce the "par" command.
# "par" sets plotting parameters and is a very,
# very, very, very, very, very important
# command with lots of uses. You have to set
# "par" before plotting, but the settings stay until
# the next "par" command resets them.
# "pty" = "plot type" and "pty=s" means "set plot type
# to square".
#####

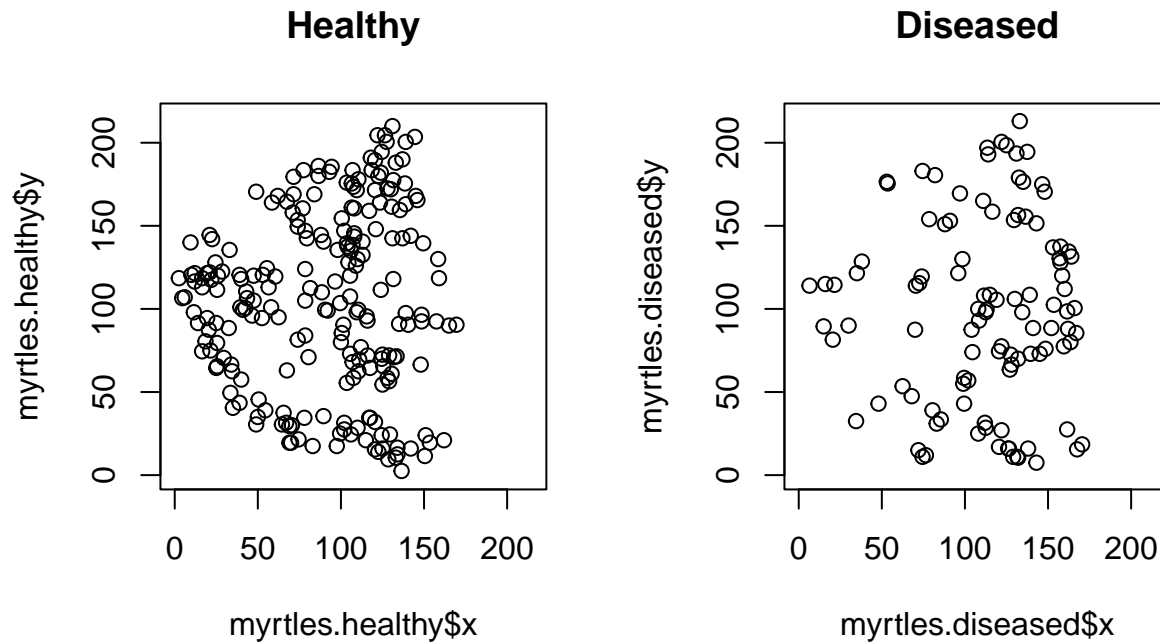
par(pty="s")
plot(myrtles.healthy$x,myrtles.healthy$y,xlim=c(0,215),
     ylim=c(0,215))

#####
# We can also use "par" to put put multiple plots
# in the same window.
# "mfrow" means "multiple figures by row".
# "mfrow=c(1,2)" means "multiple figures, one row
# containing two figures". Let's try it.
#####

par(pty="s",mfrow=c(1,2))

plot(myrtles.healthy$x,myrtles.healthy$y,xlim=c(0,215),
     ylim=c(0,215))
title("Healthy")

plot(myrtles.diseased$x,myrtles.diseased$y,xlim=c(0,215),
     ylim=c(0,215))
title("Diseased")
```



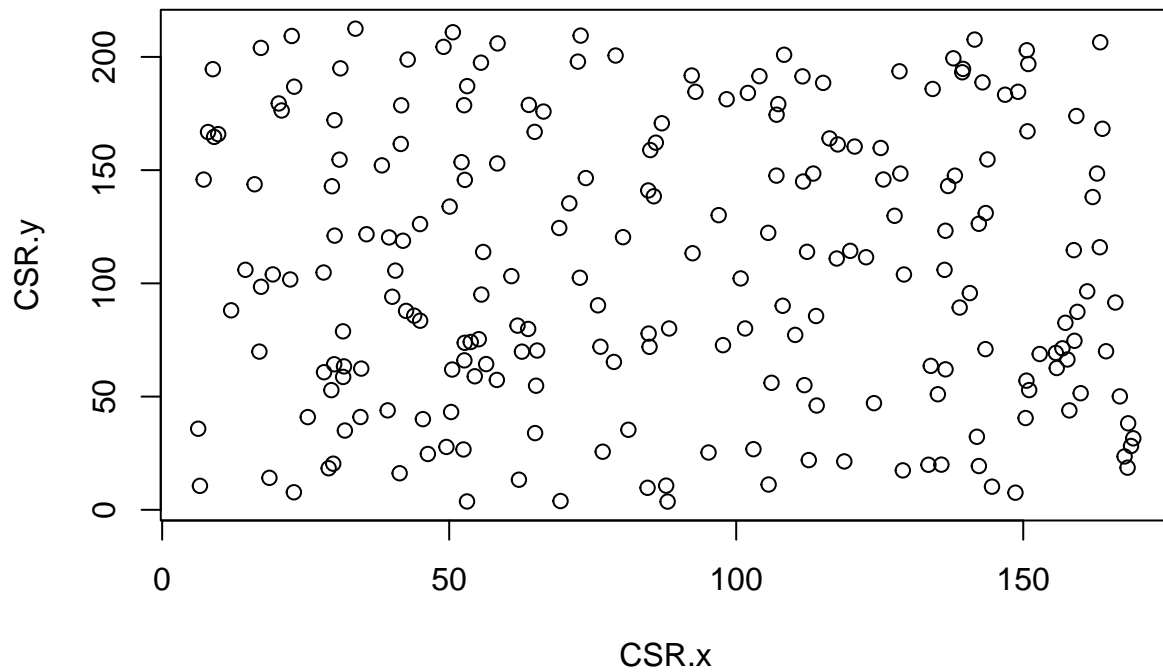
- 
- Let's test for CSR
  - Consider test statistic by Pielou (1959)
  - Test statistic  $P = \pi \lambda \sum X_i^2 / n$
  - $X_i$  = distance from event  $i$  to its nearest neighbor
  - Pielou (1959) suggests  $P \stackrel{a}{\sim} N(1, 1/n)$ .\ [link] (<http://methodsblog.com/2017/03/10/ec-pielou/>)
  - Let's try a Monte Carlo test.
- 

```
#####
# To generate realizations from CSR in the range
# of values of the data we use "runif"
# command that generates uniformly distributed
# random numbers.
# NOTE: We don't want to generate on the interval
# (0,215) since we want to limit the range of values
# to the range of the data. We extended the region to
# get a square plot, but we want to limit simulations
# to the area with data.
#####
# Let's set the number of events to simulate to
# match the observed number of events.

num.events = length(myrtles.healthy$x)

CSR.x <- runif(num.events,min(myrtles.all$x),max(myrtles.all$x))
CSR.y <- runif(num.events,min(myrtles.all$y),max(myrtles.all$y))

plot(CSR.x,CSR.y)
```



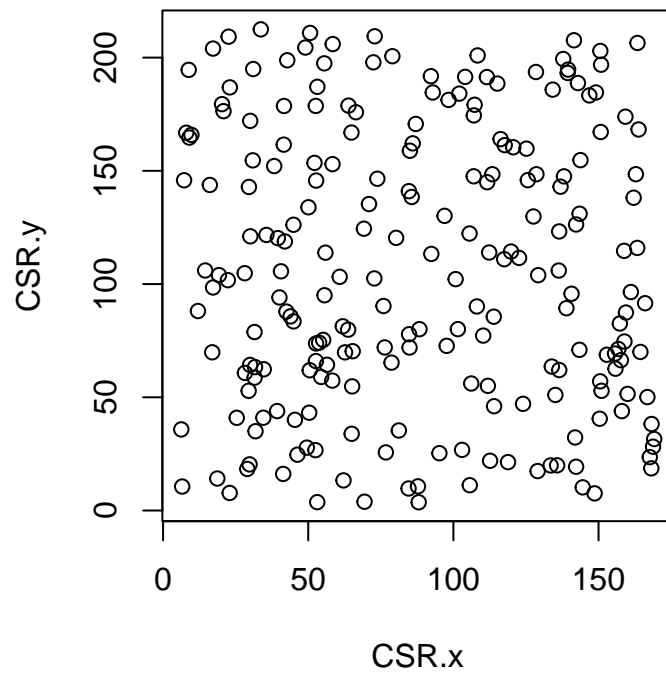
\*\*\*

- Need to reset plot to be in a square

---

```
#####
# Ops need to reset "mfrow" and a square plot using "par"
#####

par(mfrow=c(1,1),pty="s")
plot(CSR.x,CSR.y)
```



For a Monte Carlo test, we will want to make a loop of CSR simulations and calculate the test statistic for each simulated realization.

Let's calculate a clustering statistic due to Pielou (1959). This statistic requires us to find the distance from each event to its nearest neighbor. (This also gives us a chance to try out some other R functions.)

---

First, for each event, calculate the distance to all other events.

We access individual x or y values by brackets with the index, i.e., `x[1]` is the first element of `x`.

We can find the distance between `(x[1],y[1])` and all other observations by...

```
dist1 = sqrt( (myrtles.all$x[1] - myrtles.all$x)^2 + (myrtles.all$y[1] - myrtles.all$y)^2 )

#####
# This is a little tricky since
# "myrtle.all$x[1] - myrtle.all$x" is a number (x[1])
# minus a vector (x). In R this results in subtracting the number from
# all elements of the vector.
#####
```

Now we want to find the minimum element of “dist1” that is NOT 0. We can use a nifty feature of R namely we can put logical expressions inside brackets and get only the elements where that expression is true. For instance,

```
# Commented out so it doesn't list output
# dist1[dist1!=0]
```

gives the elements of `dist1` that are not equal to zero. So

```
min(dist1[dist1!=0])
```

```
## [1] 4.472136
```

gives the nearest neighbor distance! Now we just need it for all values. Let's use a loop to get this.

```
#####
# Set a vector of zeros with length equal to the number of locations in myrtles.all (length(myrtles.all))

mindist.all <- 0*(1:length(myrtles.all$x))

for (i in 1:length(myrtles.all$x)) {
  dist = sqrt( (myrtles.all$x[i] - myrtles.all$x)^2 + (myrtles.all$y[i] - myrtles.all$y)^2 )
  mindist.all[i] = min(dist[dist!=0])
}
```

---

Now we need to calculate Pielou's statistic. \*\*\*

```
n.all = length(myrtles.all$x)
area.all = (max(myrtles.all$x) - min(myrtles.all$x)) * (max(myrtles.all$y) - min(myrtles.all$y))
lambda.all = n.all/area.all
pielou.all = pi*lambda.all*sum(mindist.all^2)/n.all

#####
# Print out the value using 'paste' to put text before it. (sep="" to have no
# separator between the text and the value)
```

```
print(paste("Pielou's statistic value = ",pielou.all,sep=""))
```

```
## [1] "Pielou's statistic value = 0.65680109594231"
```

---

Let's round the statistic to make the print output a little cleaner.

```
print(paste("Pielou's statistic value = ",round(pielou.all,4),sep=""))
```

```
## [1] "Pielou's statistic value = 0.6568"
```

---

Now to set up the Monte Carlo test\*

To get a Monte Carlo p-value we need to do these same calculations to data generated under CSR. First define the number of simulations.

```
num.sim = 99
```

then define a vector to hold the values of Pielou's statistic for each simulated data set.

```
pielou.all.sim = 0*(1:num.sim)
```

---

Now set up the simulation loop

```
for (sim in 1:num.sim) {  
  
  # define CSR data  
  CSR.x = runif(num.events,min(myrtles.all$x),max(myrtles.all$x))  
  CSR.y = runif(num.events,min(myrtles.all$y),max(myrtles.all$y))  
  
  #define vector of min NN distances  
  mindist.sim = 0*(1:length(myrtles.all$x))  
  
  # find min distances (a loop within the simulation loop)  
  
  for (i in 1:length(CSR.x)) {  
    dist = sqrt( (CSR.x[i] - CSR.x)^2 + (CSR.y[i] - CSR.y)^2 )  
    mindist.sim[i] = min(dist[dist!=0])  
  }  
  
  # calculate pielou.all.sim[sim] (Pielou's statistic for the "sim-th" CSR data set).  
  pielou.all.sim[sim] = pi*lambda.all*sum(mindist.sim^2)/n.all  
  
}
```

---

Make a histogram of the CSR values

---



```

par(pty="m") # makes plot type "maximum" (rectangular in window).

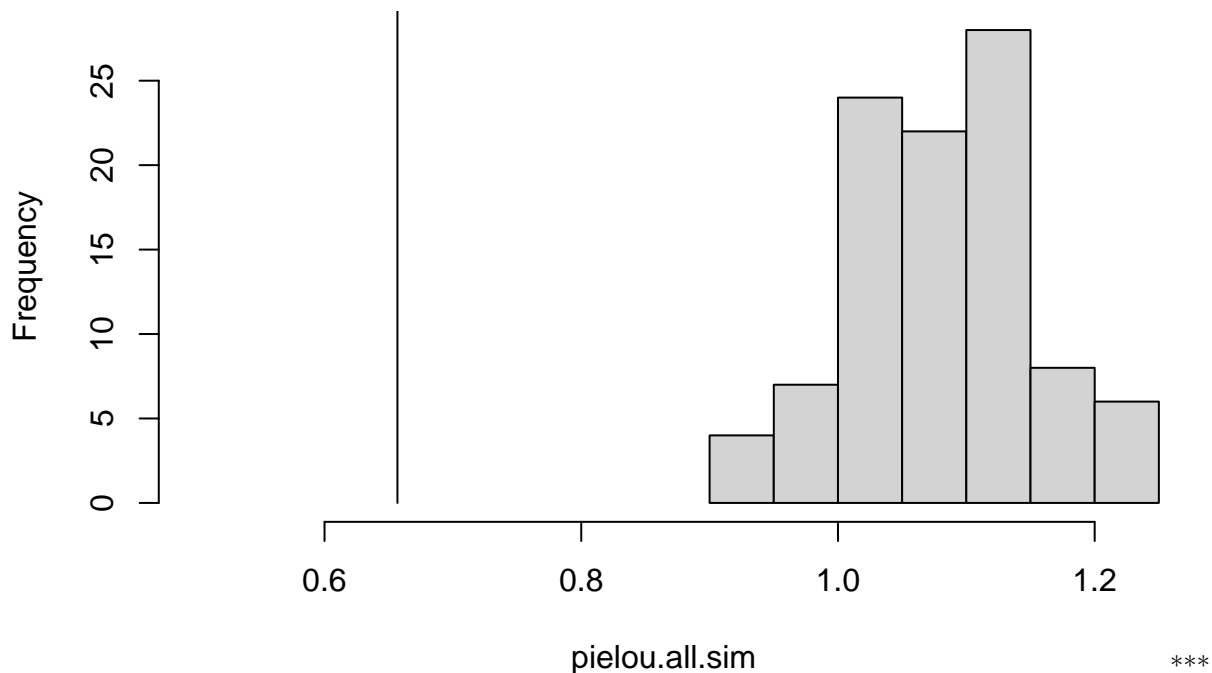
hist(pielou.all.sim,xlim=c(0.5,max(pielou.all.sim)))

# add a vertical line showing Pielou's statistic from the observed data.
# ("segments(x1,y1,x2,y2)" draws a line segments between (x1,y1) and (x2,y2).

segments(pielou.all,0,pielou.all,100)

```

## Histogram of pielou.all.sim



Now we can calculate Monte Carlo p-value (the number of statistics from simulated data that exceed the statistic from the observed data divided by the number of simulations + 1).

```

p.val = length(pielou.all.sim[pielou.all.sim>pielou.all])/(num.sim+1)

print(paste("Peilou's statistic: ",round(pielou.all,4)," p-val = ",round(p.val,4),sep=""))

## [1] "Peilou's statistic: 0.6568 p-val = 0.99"

```

\*\* Now to do this for the healthy and diseased subsets

```

mindist.healthy = 0*(1:length(myrtles.healthy$x))

for (i in 1:length(myrtles.healthy$x)) {
  dist = sqrt( (myrtles.healthy$x[i] - myrtles.healthy$x)^2 + (myrtles.healthy$y[i] - myrtles.healthy$y)^2 )
  mindist.healthy[i] = min(dist[dist!=0])
}

```

```

n.healthy = length(myrtles.healthy$x)
# NOTE: We still use area.all to cover the entire study area.
lambda.healthy = n.healthy/area.all
pielou.healthy = pi*lambda.healthy*sum(mindist.healthy^2)/n.healthy
print(paste("Peilou's statistic, healthy myrtles:",pielou.healthy))

## [1] "Peilou's statistic, healthy myrtles: 0.657911544652011"

```

---

Now for the diseased trees

---

```

mindist.diseased = 0*(1:length(myrtles.diseased$x))

for (i in 1:length(myrtles.diseased$x)) {
  dist = sqrt( (myrtles.diseased$x[i] - myrtles.diseased$x)^2 + (myrtles.diseased$y[i] - myrtles.diseased$y)^2 )
  mindist.diseased[i] = min(dist[dist!=0])
}

n.diseased = length(myrtles.diseased$x)
# NOTE: We still use area.all to cover the entire study area.
lambda.diseased = n.diseased/area.all
pielou.diseased = pi*lambda.diseased*sum(mindist.diseased^2)/n.diseased
print(paste("Peilou's statistic, diseased myrtles:",pielou.diseased))

## [1] "Peilou's statistic, diseased myrtles: 0.507452851359244"

```

---

*What we have:* Separate tests for healthy trees and for diseased trees.

*What we don't have:* A comparison between healthy and diseased trees.