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1.) Define an appropriate language and formalize the following sentences using FOL formulas.

A. All the children are younger than their parents.

Variables: x, y. Quantifiers Used: ∀

Functions & Predicates: Child(), Parent(), Younger()

For all children and parents, the child is younger than the parent.

For all x that is a child and y that is a parent, x is younger than y

For all x and y. If x is a child and y is a parent, then x is younger than y.

FOL: $\forall x, y. (Child(x) \wedge Parent(y) \rightarrow Younger(x, y))$

B. Every student takes exactly one course.

Variables: x, y, z. Quantifiers: $\forall x, y,\ \exists ! z \in y$

Functions & Predicates: Student(), Course, Takes

All students x take one course y.

For all students x and a course y. A student can take only that course y.

For all x: If x is a student, There exist only one course, where x is taking y.

For all x and y. If x is a student and y is a course. Then there exist one z of y where x is taking z.

FOL: $\forall x, y\ (Student(x) \wedge Course(y) \rightarrow \exists z(Takes(x, z) \leftrightarrow z = y)$

C. All insects are lighter than some mammal.

Variables: x, y Quantifiers: $\forall x,\ \exists y$

Functions & Predicates: Lighter(), Insect(), Mammal()

All insects x are lighter than a mammal y.

For all insects x, there exist a mammal y where x is lighter than y.

For all x and a Y: If x is an insect and y is a mammal, then x is lighter than y.

FOL: $\forall x \exists y. (Insect(x) \wedge Mammal(y) \rightarrow Lighter(x, y))$

D. Some connected people at Facebook live in the same city.

Variable: a, b, x Quantifiers: $\exists a, b, x$

Functions & Predicates: Connected(), People(), Facebook(), LiveInCity()

Some people x who are at facebook and are connected. Also live in the same city.

For some a. a is a person, and is a part of facebook, and is connected;

Also live in the same city.

For some a, x, and y. a is a person and x is a city, and y is a facebook. x is connected, and live in y.

There exist a, b, x: If a and b are people at facebook, and x is a city, then a and b are connected, and a and b live in x.

FOL: $\exists a, b, x. (People(a) \wedge People(b) \wedge Facebook(a) \wedge Facebook(b) \wedge City(x)$
$\rightarrow LiveInCity(a, x) \wedge LiveInCity(b, x))$

E. Not all prime numbers are odd.

Variables: x, y Quantifiers: $\forall x \ \exists y$

Functions & Predicates: Odd(), Prime(), P()

There exist y in the set of prime numbers that is not odd.

For all prime numbers, there exists an element x, where x is not odd.

For all x there exist y. If x is all the prime numbers, and y is an element of x. Then y is not odd.

For all x. If x is prime, there exist y. Where y is not odd, and y is x.

FOL: $\forall x \exists y. \ (Prime(x) \wedge \neg Odd(y) \wedge P(x = y))$

F. Every box contains at most one coin.

Variables: x, y, z Quantifiers: $\forall x, y \exists z$

Functions & Predicates: Box(), Coin(), Contains()

Every box x contains at most one coin y.

For all boxes x, there exist at most one coin inside.

For all x. If x is a box then there exist one y that is a coin, and the box contains at most one.

For all x there exist y. If x is a box and y is a coin, then x contains y if and only if there is one y.

For all x and y. If x is a box and y is a coin, then there exist z, where x contains z and z = y.

FOL: $\forall x, y. \ (\ Box(x) \wedge Coin(y) \rightarrow \ \exists z.(Contains(x, z) \leftrightarrow z = y) \ )$

G. All red objects are to the left of green objects.

Variables: x, y Quantifiers: $\forall x, y$

Functions & Predicates: RedObject(), GreenObject, Left()

All red objects x are to the left of green object y.

For all red objects x, and green objects y, red objects are to the left of green objects.

For all x and y. If x is red object and y is green object, then x is to the left of y.

FOL: $\forall x, y. \ (RedObject(x) \wedge GreenObject(y) \rightarrow Left(x, \ y))$

2.)

(a.1)

$F = F1 \wedge F2 \wedge F3$

$F1 = \forall x \forall y \forall z((P(x,y) \wedge P(y,z)) \rightarrow P(x,z))$

$F2 = \forall x \forall y((P(x,y) \wedge P(y,x)) \rightarrow x = y)$

$F3 = \exists x \exists y(P(a,y) \rightarrow P(x,b))$

$P^A = \{(m,n) \in U \mid m <= n\}$

A(F) = x = 1, y = 2, z = 3, a = 1, and b = 1.

P(x, y) = True
P(y, z) = False
P(x, z) = True
P(y, x) = False
P(a, y) = True
P(x, b) = True

Under this assignment, F1 is true. F2 is false, and F3 is true.
Since F1, F2, and F3 must be true, F is false. Thus A(F) is false.

(a.2)

$F = F1 \wedge F2 \wedge F3$

$F1 = \exists x \exists y \exists z((P(x,y) \wedge P(y,z) \wedge \neg(x = y) \wedge \neg(y = z)) \rightarrow P(x,z))$

$F2 = \forall x \forall y((P(x,y) \wedge P(y,x)) \rightarrow x = y)$

$F3 = \forall x \forall y(P(a,y) \rightarrow P(x,b))$

$P^A = \{(m,n) \in U \mid m < n\}$

A(F) = x = 1, y = 1, z = 3, a = 1, b = 3

P(x, y) = True
P(y, z) = True
P(x, z) = True
P(y, x) = True
P(a, y) = True
P(x, b) = True

Under this assignment F1 is True, F2 is true, and F3 is true. Since F1, F2, and F3 are all true, F is true and A(F) = True.

2.)

b.1) $\neg((\forall x)P(x) \rightarrow (\forall x)(\exists y)(\exists z)Q(x,y,z))$

Prenex:

1. $\neg(\neg(\forall x)P(x) \vee (\forall x)(\exists y)(\exists z)Q(x,y,z))$ 2. $\neg\neg(\forall x)P(x) \wedge \neg(\forall x)(\exists y)(\exists z)Q(x,y,z)$ 3. $(\forall x)P(x) \wedge \neg(\forall x)(\exists y)(\exists z)Q(x,y,z)$ 4. $(\forall x)P(x) \wedge (\exists x)(\forall y)(\forall z)\neg Q(x,y,z)$ 5. $(\forall x)P(x) \wedge (\exists u)(\forall y)(\forall z)\neg Q(u,y,z)$ 6. $(\forall x)(\exists u)(\forall y)(\forall z)(P(x) \wedge \neg Q(u,y,z))$	1. Implication 2. De Morgan 3. Double Implication 4. Distribute Negation down to preciates 5. Turn q's x to u. 6. Move quantifiers out, and CNF.

Skolem:

1. $(\forall x)(\forall y)(\forall z)(P(x) \wedge \neg Q(f(x),y,z))$	1. Remove u existential quantifier and put f(x) instead.

b.2) $\exists z(\exists xQ(x,z) \vee \exists xP(x)) \rightarrow \neg(\neg\exists xP(x) \wedge \forall x\exists zQ(z,x))$

Prenex:

1. $\neg\exists z(\exists xQ(x,z) \vee \exists xP(x)) \vee \neg(\neg\exists xP(x) \wedge \forall x\exists zQ(z,x))$ 2. $\forall z(\forall x\neg Q(x,z) \wedge \forall x\neg P(x)) \vee (\exists xP(x) \vee \exists x\forall z\neg Q(z,x))$ 3. $\forall z\forall x(\neg Q(x,z) \wedge \neg P(x)) \vee \exists x(P(x) \vee \forall z\neg Q(z,x))$ 4. $\forall z\forall x(\neg Q(x,z) \wedge \neg P(x)) \vee \exists u(P(u) \vee \forall v\neg Q(v,u))$ 5. $\forall z\forall x(\neg Q(x,z) \wedge \neg P(x)) \vee \exists u\forall v(P(u) \vee \neg Q(v,u))$ 6. $\forall z\forall x\exists u\forall v((\neg Q(x,z) \wedge \neg P(x)) \vee (P(u) \vee \neg Q(v,u)))$	1. Implication 2. De Morgan 3.Take $\forall x$ and $\exists x$ to left. 4. Rename P's x -> u, and Q's z -> v. 5.Move all on the right side of the first or over to left. 6.Move quantifiers completely out.

Skolem:

$\forall z\forall x\forall v((\neg Q(x,z) \wedge \neg P(x)) \vee (P(f(x)) \vee \neg Q(u,f(x))))$	1. Change $\exists u$ to f(x)

b.3) $\exists x\forall yP(x,y) \vee \neg\exists y(Q(y) \rightarrow \forall zR(z))$

Prenex:

1. $\exists x\forall yP(x,y) \vee \neg\exists y(-Q(y) \vee \forall zR(z))$ 2. $\exists x\forall yP(x,y) \vee \forall y\neg(\neg Q(y) \vee \forall zR(z))$ 3. $\exists x\forall yP(x,y) \vee \forall y(Q(y) \wedge \neg\forall zR(z))$ 4. $\exists x\forall yP(x,y) \vee \forall y(Q(y) \wedge \exists z\neg R(z))$ 5. $\exists x\forall yP(x,y) \vee \forall y\exists z(Q(y) \wedge \neg R(z))$ 6. $\exists x\forall yP(x,y) \vee \forall u\exists z(Q(u) \wedge \neg R(z))$ 7. $\exists x\forall y\forall u\exists z(P(x,y) \vee (Q(u) \wedge \neg R(z)))$	1.Implication 2. Negation 3. De Morgan 4.Move negation to R predicate 5. Move $\exists z$ out. 6. Rename Q's y to u. 7. Move quantifiers out.

Skolem:

1. $\forall y\forall u\exists z(P(a,y) \vee (Q(u) \wedge \neg R(z)))$ 2. $\forall y\forall u(P(a,y) \vee (Q(u) \wedge \neg R(f(y))))$ 3. $\forall y\forall u((P(a,y) \vee Q(u)) \wedge (P(a,y) \vee \neg R(f(y))))$	1. Make x a constant a. 2.Make z a f(y). 3. CNF

3.) Find Most General Unifier (if exists) for:
a){P(f(x), z), P(y, a)};
[f(x)/y, z/a], {P(y, a)}

b) {P(f(x), a), P(y, f(w))};
[f(x)/y, a/f(w)] Not possible.

c) {P(a, x, f(g(y))), P(b, h(z,w), f(w))};
[a/b, x/h(z,w), g(y)/w], {P(b, h(z,w), f(w)}

d) {S(x, y, z), S(u, g(v, v), v)};
[x/u, y/g(v, v), z/v], {S(u, g(v, v), v)}

e) {P(x, x), P(y, f(y))};
[x/y] Not Possible.

f) {Q(f(x), y),Q(z, g(w))};
[f(x)/z, y/g(w)], {Q(z, g(w)}

4.) $F = \exists x(\neg P(x) \wedge \neg P(f(v)) \wedge \exists z Q(z)) \vee \exists w(\neg P(g(w,x)) \wedge \neg Q(x)) \vee \exists y P(y)$

Negation: $\neg(\exists x(\neg P(x) \wedge \neg P(f(v)) \wedge \exists z Q(z)) \vee \exists w(\neg P(g(w,x)) \wedge \neg Q(x)) \vee \exists y P(y))$

Prenex and Skolem:

1. $\neg \exists x(\neg P(x) \wedge \neg P(f(v)) \wedge \exists z Q(z)) \wedge \neg \exists w(\neg P(g(w,x)) \wedge \neg Q(x)) \wedge \neg \exists y P(y)$ 2. $\forall x \neg(\neg P(x) \wedge \neg P(f(v)) \wedge \exists z Q(z)) \wedge \forall w \neg(\neg P(g(w,x)) \wedge \neg Q(x)) \wedge \forall y \neg P(y)$ 3. $\forall x(P(x) \vee P(f(v)) \vee \neg \exists z Q(z)) \wedge \forall w(P(g(w,x)) \vee Q(x)) \wedge \forall y \neg P(y)$ 4. $\forall x(P(x) \vee P(f(v)) \vee \forall z \neg Q(z)) \wedge \forall w(P(g(w,x)) \vee Q(x)) \wedge \forall y \neg P(y)$ 5. $\forall x \forall z(P(x) \vee P(f(v)) \vee \neg Q(z)) \wedge \forall w(P(g(w,x)) \vee Q(x)) \wedge \forall y \neg P(y)$ 6. $\forall x \forall z \forall w \forall y((P(x) \vee P(f(v)) \vee \neg Q(z)) \wedge (P(g(w,x)) \vee Q(x)) \wedge \neg P(y))$	1.De Morgan 2. Negations 3. De Morgan's 4.Negation 5. Scope out $\forall z$ 6.Scope out universals and CNF.

Clause form: F = { {P(x), P(f(v)), ¬ Q(z))}, {P(g(w, x)), Q(x)}, {¬ P(y)} }

C1 = { P(x), P(f(v)), ¬ Q(z) }
C2 = { P(g(w, x)), Q(x) }
C3 = { ¬ P(y) }

U1 = C1 & C3, [x/y] = { P(y), P(f(v)), ¬ Q(z), ¬ P(y) } = { P(f(v)), ¬ Q(z) }
U2 = C2 & U1, [x/z, f(v)/g(w,x)] = { P(g(w, z)), Q(z), P(g(w, z)), ¬ Q(z) } = { P(g(w, z)) }

¬ F is satisfiable, F is not Valid

5.) Prove by resolution refutation that B(a) is a logical consequence of the conjunction of (6.1), (6.2), (6.3), (6.4), and (6.5):

$\forall x(R(x) \lor G(x) \lor B(x))$; (6.1)

$\forall x \forall y(L(x,y) \rightarrow \neg RR(x,y))$; (6.2)

$\forall x \forall y((R(x) \land G(y)) \rightarrow L(x,y))$; (6.3)

$RR(a,b)$; (6.4)

$G(b) \land \neg G(a)$; (6.5)

F =

$$\forall x(R(x) \lor G(x) \lor B(x)) \land \forall x \forall y(L(x,y) \rightarrow \neg RR(x,y)) \land \forall x \forall y((R(x) \land G(y)) \rightarrow L(x,y)) \land RR(a,b) \land G(b) \land \neg G(a)$$

$F \mid= B(a)$

$F \land \neg B(a)$

Prenex & Skolem:

1. Implication

$\forall x(R(x) \lor G(x) \lor B(x))$

$\land \forall x \forall y(\neg L(x,y) \lor \neg RR(x,y))$

$\land \forall x \forall y(\neg(R(x) \land \neg G(y)) \lor L(x,y))$

$\land RR(a,b) \land G(b) \land \neg G(a) \land \neg B(a)$

3. De Morgan's

$\forall x(R(x) \lor G(x) \lor B(x))$

$\land \forall x \forall y(\neg L(u,y) \lor \neg RR(u,y))$

$\land \forall x \forall y(\neg R(x) \lor \neg G(y) \lor L(x,y))$

$\land RR(a,b) \land G(b) \land \neg G(a) \land \neg B(a)$

2.Standardize Variables

$\forall x(R(x) \lor G(x) \lor B(x))$

$\land \forall u \forall y(\neg L(u,y) \lor \neg RR(u,y))$

$\land \forall v \forall w(\neg R(v) \land \neg G(w)) \lor L(v,w)$

$\land RR(a,b) \land G(b) \land \neg G(a) \land \neg B(a)$

3. Scope Out Quantifiers

$\forall x \forall u \forall y \forall v \forall w((R(x) \lor G(x) \lor B(x)) \land (\neg L(u,y) \lor \neg RR(u,y))$

$\land (\neg R(v) \lor \neg G(w) \lor L(v,w) \land RR(a,b) \land G(b) \land \neg G(a) \land \neg B(a))$

4. Associative & Remove universals.

$(R(x) \lor G(x) \lor B(x)) \land (\neg L(u,y) \lor \neg RR(u,y))$

$\land (\neg R(v) \lor \neg G(w) \lor L(v,w))$

$\land RR(a,b) \land G(b) \land \neg G(a) \land \neg B(a)$

Clause Form:

{ { R(x), G(x), B(x) }, { ¬L(u, y), ¬RR(u, y) }, { ¬R(v), ¬G(w), L(v, w) }, { RR(a, b) }, { G(b) }, { ¬G(a) }, { ¬B(a) } }

C1 = { R(x), G(x), B(x) }
C2 = { ¬L(u, y), ¬RR(u, y) }
C3 = { ¬R(v), ¬G(w), L(v, w) }
C4 = { RR(a, b) }
C5 = { G(b) }
C6 = { ¬G(a) }
C7 = { ¬B(a) } }

U1 = C2 & C4, [a/u, b/y] = { ¬L(u, y) }
U2 = C1 & C6, [a/x] = { R(x), B(x) }
U3 = C3 & C5, [b/w] = { ¬R(v), L(v, w) }
U4 = U1 & U3, [v/x] = { ¬R(v) }
U5 = U2 & U4, [x/v] = { B(x) }

R = C7 & U5, [x/a] = □

The conjunction of the formula and not B(a) is not valid, as such B(a) is a logical consequence of the formula.