

## Discussion Document

Edward L. Boone  
Virginia Pasour  
Colin Grudzien  
Shirley Han  
Lea Jenkins

October 23, 2014

---

### Abstract

Discussion on Data Assimilation

*Keywords:* Data Assimilation

---

### 1 Introduction

### 2 Kot Model

Here is the Kot (1992) Model used in Figure 1.

$$\begin{aligned}\frac{dS}{dt} &= D \left[ S_i \left( 1 + \epsilon \sin \frac{2\pi}{T} t \right) - S \right] - \frac{\mu_1}{Y_1} \frac{SH}{K_1 + S} \\ \frac{dH}{dt} &= \mu_1 \frac{SH}{K_1 + S} - DH - \frac{\mu_2}{Y_2} \frac{HP}{K_2 + H} \\ \frac{dP}{dt} &= \mu_2 \frac{HP}{K_2 + H} - DP\end{aligned}$$

### 3 Competition Model

Here is my stab at putting together a competition model with predators.

$$\begin{aligned}
\frac{dx_1}{dt} &= r_1x_1 - r_1x_1 \left( \frac{x_1 + \alpha_{12}x_2 + \alpha_{13}x_3}{K_1} \right) \\
\frac{dx_2}{dt} &= r_2x_2 - r_2x_2 \left( \frac{x_2 + \alpha_{21}x_1 + \alpha_{23}x_3}{K_2} \right) - \beta_{24}x_2x_4 \\
\frac{dx_3}{dt} &= r_3x_3 - r_3x_3 \left( \frac{x_3 + \alpha_{31}x_1 + \alpha_{32}x_2}{K_3} \right) - \beta_{35}x_3x_5 \\
\frac{dx_4}{dt} &= \beta_{24}x_2x_4 - c_4x_4 \\
\frac{dx_5}{dt} &= \beta_{35}x_3x_5 - c_5x_5
\end{aligned}$$

- $x_1$  is coral,  $x_2$  is tall algae,  $x_3$  is short algae,  $x_4$  is fish that eat tall algae and  $x_5$  is the fish that eat short algae.
- $r_1$ ,  $r_2$  and  $r_3$  are the recruitment parameters for coral, tall algae and short algae, respectively.
- $K_1$ ,  $K_2$  and  $K_3$  are the carrying capacity parameters for coral, tall algae and short algae, respectively.
- $\alpha_{ij}$  is the competition of  $x_j$  on  $x_i$ .
- $\beta_{ij}$  is the predation success of  $x_j$  on  $x_i$ .
- $c_i$  is the mortality rate for species  $x_i$ .

## Acknowledgements

We thank the opportunities provided by SAMSI.

## References

- Berntsen, J., Espelid, T.O., and Genz, A. (1991) An Adaptive Algorithm for the Approximate Calculation of Multiple Integrals, *ACM Transactions on Mathematical Software*, **17**, 437–451.
- Kot, M., Saylor, G.S. and Schultz, T.W. (1992) Complex Dynamics in a Model Microbial System, *Bulletin of Mathematical Biology*, **54**, 619–648.

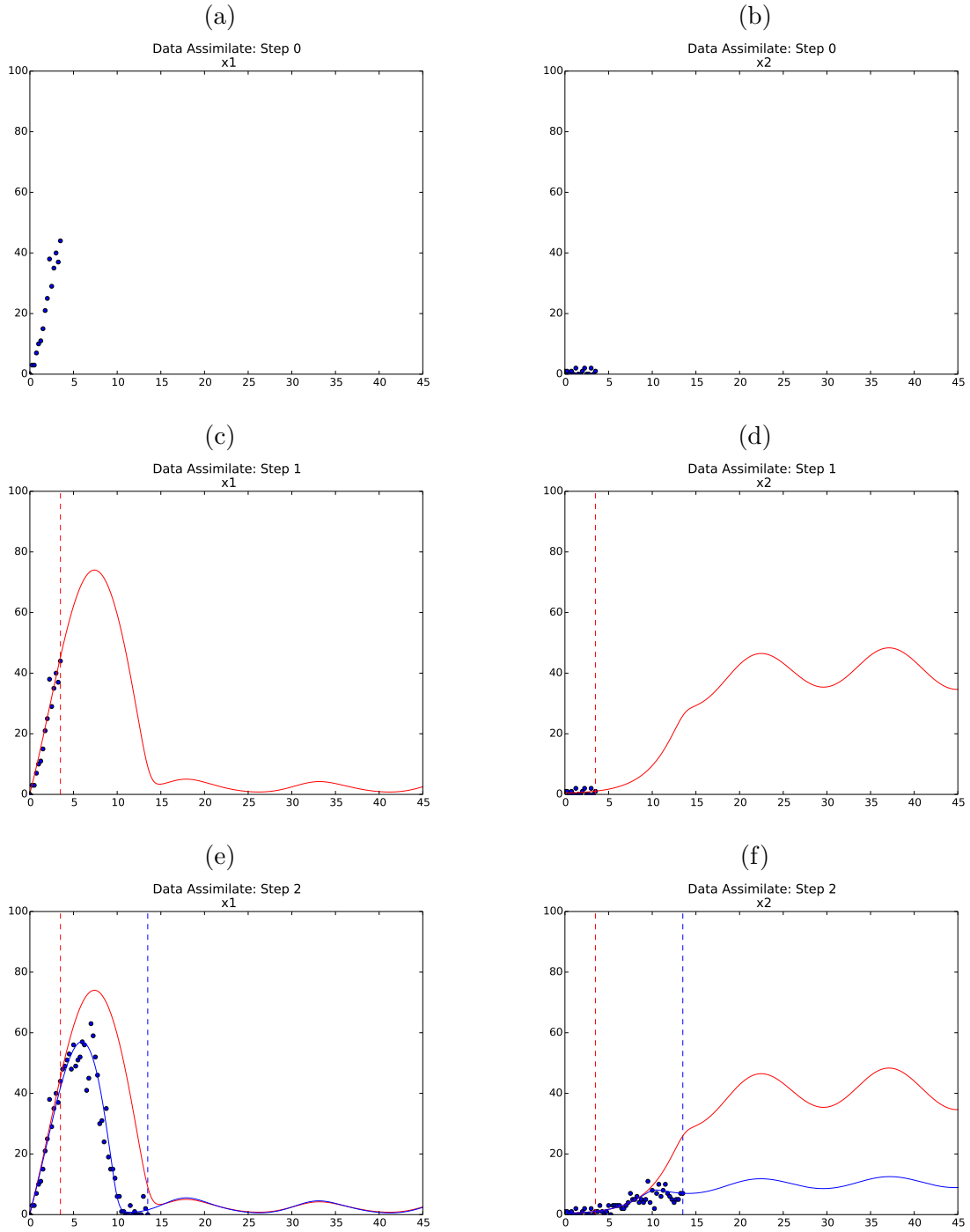


Figure 1: Data for (a)  $x_1$ , (b)  $x_2$ , (c) data with fitted model for  $x_1$ , (d) data with fitted model for  $x_2$ , (e) additional data and updated fitted model for  $x_1$  and (f) additional data and updated fitted model for  $x_2$ .