Theorem 0.0.1. Let S be a cyclic quotient singularity. Let $\widetilde{S}$ be minimal resolution . Let E be an exceptional divisor. Let $\widetilde{X}$ be the variety obtained by blowing up a general point on E. Then X is obtained as the blowdown of the strict transform of the E_i. Then we can extend this contraction to the deformation family and its associated toric degenerations.

Given the assumptions above we can assume that S is a $\frac{1}{r}(1, s)$ singularity and has a fan Σ with rays (a, b) and $(c - d, -d)$ with $a, b, c, d > 0$. We can also assume that the ray $(1, 0)$ corresponds to E in the minimal resolution. We also note that r is equal to the determinant of the rays, $ad - b(c - d)$.

We note that X has a torus acting on it. This means that it can be written as a polyhedral divisor $\mathcal{S}$.

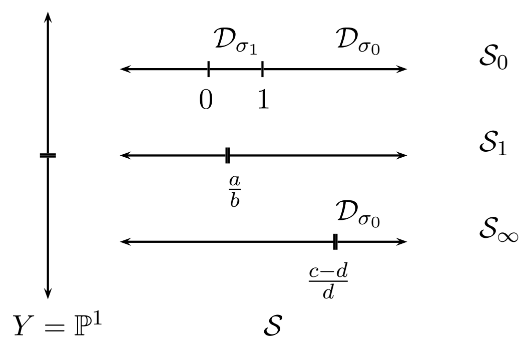

Figure 1: Divisorial fan associated to X.

Here the tail fan is $\mathbb{Z}_{\geq 0}$. When constructing the equivariant toric degeneration, we take the Minkowski sum of $\frac{c-d}{d}$ and the cone $[0, 1]$. After summation this is the toric variety with rays (a, b), $(c, -d)$, $(c - d, -d)$. Call this X_Σ. The cone $(c, -d)$, $(c - d, -d)$ is a T-singularity. The cone (a, b), $(c, -d)$ is a $\frac{1}{t}(1, u)$ singularity with once again $t = bc + ad$. Labeling these three rays v_1, v_2, v_3 we get the relation $d^2 v_1 - r v_2 + t v_3 = 0$. This implies that the Cox Ring $\mathcal{R}(X_\Sigma)$ is $\mathbb{A}^3_{<x,y,z>}$ with a quotient of $\mathbb{C}^*$ with weights $(d^2, -r, t)$ and a finite group action μ. It is easy to show that $|\mu| = \text{hcf(b,d)} = e$ as (x, ey) is clearly a sub lattice which contains all the vertices and $(1, 0)$ is in the lattice as b and d are coprime. This is a well defined quotient except along $x = z = 0$. To construct the deformation, we take the d-fold veronese embedding getting

$$\frac{\mathbb{C}[y_1, y_2, y_3, y_4]}{y_2^d - y_3 y_4} \text{ with weights } (d, b, -r, t)$$

Here the b occurs as $t - r = db$. We can now construct the deformation family by the following equation

$$\frac{\mathbb{C}[y_1, y_2, y_3, y_4]}{\lambda y_1^b + \mu y_2^d - y_3 y_4} \text{ with weights } (d, b, -r, t)$$

At $\lambda = 0$ this is our original variety. At $\mu = 0$ we get the variety corresponding to the Minkowski sum of the cones $\frac{a}{b}$ and the cone $[0, 1]$. This is the toric variety with rays $(c - d, -d)$, $(a + b, b)$, (a, b). Note that the index of the sublattice generated by these rays is still equal to $\mathrm{hcf}(b, d)$. All that remains is to show that this is the desired complexity one variety. From the above polyhedral divisor we get that $\mathcal{R}(X) = \frac{\mathbb{C}[y_1, y_2, y_3, y_4]}{\lambda y_1^b + \mu y_2^d - y_3 y_4}$. We now have to calculate the weights of the torus action on it. The Weil divisors on X satisfy the relations given by the rows of the following matrix.

$$\begin{pmatrix} b & 0 & -1 & -1 \\ 0 & c & -1 & -1 \\ a & d & 0 & 1 \end{pmatrix}$$

To calculate the grading is equivalent to finding a relation between the columns of the matrix. It is easy to verify that the weights $d, b, -r, t$ give the corresponding column sum of 0. To verify that the same finite group acts on the fiber we see that if we do Gaussian elimination. We get a row $(d, -b, 0, 0)$ if these two numbers are not coprime this means that we have torsion in the class of size $\mathrm{hcf}(b, d)$. So μ action extends across the family.

A brief remark on other log terminal cyclic extractions. The only cyclic extractions $f : X \to S$ from a cyclic quotient singularity result from blowing up infinitely near points of E.