We wish to show that given $(S, C) \to X$ a log terminal cyclic extraction on a normal affine toric surface then this admits a toric degeneration such that the contraction extends over the total space. This map can be constructed via the minimal resolution. If you take the minimal resolution $\widetilde{X}$ then there exists an exceptional curve E such that the blowup of a general point on E is the minimal resolution of S.

In particular we can assume that X is a $\frac{1}{r}(1, s)$ singularity and has a fan Σ with rays (a, b) and $(c-d, -d)$ with $a, b, c, d > 0$. We can also assume that the ray $(1, 0)$ corresponds to E in the minimal resolution. We also note that r is equal to the determinant of the rays, $ad - b(c-d)$.

We note that S has a torus acting on it. Via the deformation theory of complexity one varieties we see that one of the equivariant toric degeneration is the toric variety with rays (a, b), $(c, -d)$, $(c-d, -d)$. Call this Y. The cone $(c, -d)$, $(c-d, -d)$ is a T-singularity. The cone (a, b), $(c, -d)$ is a $\frac{1}{t}(1, u)$ singularity with once again $t = bc + ad$. Labelling these three rays v_1, v_2, v_3 we get the relation $d^2 v_1 - r v_2 + t v_3 = 0$. So writing out the Cox ring we have

$$\mathcal{R}(Y) = \mathbb{C}[x_1, x_2, x_3]$$

With a $\mathbb{C}^*$ action with weights $(d^2, -r, t)$. Taking the d fold veronese embedding of this gets us

$$\frac{\mathbb{C}[y_1, y_2, y_3, y_4]}{y_2^d - y_3 y_4} \text{ with weights } (d, b, -r, t)$$

Here the b occurs as $t - r = db$. This lets us construct the deformation family by considering

$$\frac{\mathbb{C}[y_1, y_2, y_3, y_4]}{\lambda y_1^b + \mu y_2^d - y_3 y_4} \text{ with weights } (d, b, -r, t)$$

All that remains is to show that this is the desired complexity one variety. This is equivalent to finding the Smith Normal Form of a a matrix

$$\begin{pmatrix} b & -1 & -1 & 0 \\ 0 & -1 & -1 & d \\ a & 0 & 1 & c \end{pmatrix}$$