

1 CONSUMER

1.1 Optimisation problem

$$\max_{K_t^s, C_t, L_t^s, I_t} U_t = \beta E_t [U_{t+1}] + (1 - \eta)^{-1} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{1-\eta} \quad (1.1)$$

s.t. :

$$C_t + I_t = \pi_t + K_{t-1}^s r_t + L_t^s W_t - \psi K_{t-1}^s \left(-\delta + K_{t-1}^{s-1} I_t \right)^2 \quad (\lambda_t^c) \quad (1.2)$$

$$K_t^s = I_t + K_{t-1}^s (1 - \delta) \quad \left(\lambda_t^{\text{CONSUMER}^2} \right) \quad (1.3)$$

1.2 First order conditions

$$-\lambda_t^{\text{CONSUMER}^2} + \beta \left((1 - \delta) E_t \left[\lambda_{t+1}^{\text{CONSUMER}^2} \right] + E_t \left[\lambda_{t+1}^c \left(r_{t+1} - \psi \left(-\delta + K_t^{s-1} I_{t+1} \right)^2 + 2\psi K_t^{s-1} I_{t+1} \left(-\delta + K_t^{s-1} I_{t+1} \right) \right) \right] \right) \quad (1.4)$$

$$-\lambda_t^c + \mu C_t^{-1+\mu} (1 - L_t^s)^{1-\mu} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} = 0 \quad (C_t) \quad (1.5)$$

$$\lambda_t^c W_t + (-1 + \mu) C_t^\mu (1 - L_t^s)^{-\mu} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} = 0 \quad (L_t^s) \quad (1.6)$$

$$\lambda_t^{\text{CONSUMER}^2} + \lambda_t^c \left(-1 - 2\psi \left(-\delta + K_{t-1}^{s-1} I_t \right) \right) = 0 \quad (I_t) \quad (1.7)$$

2 FIRM

2.1 Optimisation problem

$$\max_{K_t^d, L_t^d, Y_t, \pi_t} \Pi_t = \pi_t \quad (2.1)$$

s.t. :

$$Y_t = Z_t K_t^{d\alpha} L_t^{d^{1-\alpha}} \quad \left(\lambda_t^{\text{FIRM}^1} \right) \quad (2.2)$$

$$\pi_t = Y_t - L_t^d W_t - r_t K_t^d \quad \left(\lambda_t^{\text{FIRM}^2} \right) \quad (2.3)$$

2.2 First order conditions

$$-\lambda_t^{\text{FIRM}^2} r_t + \alpha \lambda_t^{\text{FIRM}^1} Z_t K_t^{d-1+\alpha} L_t^{d^{1-\alpha}} = 0 \quad (K_t^d) \quad (2.4)$$

$$-\lambda_t^{\text{FIRM}^2} W_t + \lambda_t^{\text{FIRM}^1} Z_t (1 - \alpha) K_t^{d\alpha} L_t^{d-\alpha} = 0 \quad (L_t^d) \quad (2.5)$$

$$-\lambda_t^{\text{FIRM}^1} + \lambda_t^{\text{FIRM}^2} = 0 \quad (Y_t) \quad (2.6)$$

$$1 - \lambda_t^{\text{FIRM}^2} = 0 \quad (\pi_t) \quad (2.7)$$

2.3 First order conditions after reduction

$$-r_t + \alpha Z_t K_t^{d-1+\alpha} L_t^{d^{1-\alpha}} = 0 \quad (K_t^d) \quad (2.8)$$

$$-W_t + Z_t (1 - \alpha) K_t^{d\alpha} L_t^{d-\alpha} = 0 \quad (L_t^d) \quad (2.9)$$

3 EQUILIBRIUM

3.1 Identities

$$K_t^d = K_{t-1}^s \quad (3.1)$$

$$L_t^d = L_t^s \quad (3.2)$$

4 EXOG

4.1 Identities

$$Z_t = e^{\epsilon_t^Z + \phi \log Z_{t-1}} \quad (4.1)$$

5 Equilibrium relationships (after reduction)

$$K_{t-1}^s - K_t^d = 0 \quad (5.1)$$

$$-r_t + \alpha Z_t K_t^{d-1+\alpha} L_t^{s1-\alpha} = 0 \quad (5.2)$$

$$-W_t + Z_t (1 - \alpha) K_t^{d\alpha} L_t^{s-\alpha} = 0 \quad (5.3)$$

$$-Y_t + Z_t K_t^{d\alpha} L_t^{s1-\alpha} = 0 \quad (5.4)$$

$$-Z_t + e^{\epsilon_t^Z + \phi \log Z_{t-1}} = 0 \quad (5.5)$$

$$\beta \left(\mu E_t \left[\left(r_{t+1} - \psi \left(-\delta + K_t^{s-1} I_{t+1} \right)^2 + 2\psi K_t^{s-1} I_{t+1} \left(-\delta + K_t^{s-1} I_{t+1} \right) \right) C_{t+1}^{-1+\mu} (1 - L_{t+1}^s)^{1-\mu} \left(C_{t+1}^\mu (1 - L_{t+1}^s)^{1-\mu} \right)^{-\eta} \right] \right) \quad (5.6)$$

$$(-1 + \mu) C_t^\mu (1 - L_t^s)^{-\mu} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} + \mu W_t C_t^{-1+\mu} (1 - L_t^s)^{1-\mu} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} = 0 \quad (5.7)$$

$$I_t - K_t^s + K_{t-1}^s (1 - \delta) = 0 \quad (5.8)$$

$$U_t - \beta E_t [U_{t+1}] - (1 - \eta)^{-1} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{1-\eta} = 0 \quad (5.9)$$

$$-C_t - I_t + Y_t + K_{t-1}^s r_t - r_t K_t^d - \psi K_{t-1}^s \left(-\delta + K_{t-1}^{s-1} I_t \right)^2 = 0 \quad (5.10)$$

6 Steady state relationships (after reduction)

$$-r_{ss} + \alpha Z_{ss} K_{ss}^{d-1+\alpha} L_{ss}^{s1-\alpha} = 0 \quad (6.1)$$

$$-K_{ss}^d + K_{ss}^s = 0 \quad (6.2)$$

$$-W_{ss} + Z_{ss} (1 - \alpha) K_{ss}^{d\alpha} L_{ss}^{s-\alpha} = 0 \quad (6.3)$$

$$-Y_{ss} + Z_{ss} K_{ss}^{d\alpha} L_{ss}^{s1-\alpha} = 0 \quad (6.4)$$

$$-Z_{ss} + e^{\phi \log Z_{ss}} = 0 \quad (6.5)$$

$$\beta \left(\mu \left(r_{ss} - \psi \left(-\delta + I_{ss} K_{ss}^s \right)^{-1} \right)^2 + 2\psi I_{ss} K_{ss}^s \left(-\delta + I_{ss} K_{ss}^s \right)^{-1} \right) C_{ss}^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} \left(C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} - \mu \left(-1 - 2\psi \right) \left(C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} = 0 \quad (6.6)$$

$$(-1 + \mu) C_{ss}^\mu (1 - L_{ss}^s)^{-\mu} \left(C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} + \mu W_{ss} C_{ss}^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} \left(C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} = 0 \quad (6.7)$$

$$I_{ss} - K_{ss}^s + K_{ss}^s (1 - \delta) = 0 \quad (6.8)$$

$$U_{ss} - \beta U_{ss} - (1 - \eta)^{-1} \left(C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{1-\eta} = 0 \quad (6.9)$$

$$-C_{ss} - I_{ss} + Y_{ss} - r_{ss} K_{ss}^d + r_{ss} K_{ss}^s - \psi K_{ss}^s \left(-\delta + I_{ss} K_{ss}^s \right)^{-1} = 0 \quad (6.10)$$

7 Calibrating equations

$$-0.36 Y_{ss} + r_{ss} K_{ss}^d = 0 \quad (7.1)$$

8 Parameter settings

$$\beta = 0.99 \quad (8.1)$$

$$\delta = 0.025 \quad (8.2)$$

$$\eta = 2 \quad (8.3)$$

$$\mu = 0.3 \quad (8.4)$$

$$\phi = 0.95 \quad (8.5)$$

$$\psi = 0.8 \quad (8.6)$$

9 Steady-state values

	Steady-state value
r	0.0351
C	0.7422
I	0.2559
K^d	10.2368
K^s	10.2368
L^s	0.2695
U	-136.2372
W	2.3706
Y	0.9981
Z	1

10 Model parameters

	Value
α	0.36
β	0.99
δ	0.025
η	2
μ	0.3
ϕ	0.95
ψ	0.8

11 The solution of the 1st order perturbation

Matrix P

$$\begin{matrix} K_t^s & Z_t \end{matrix} \begin{pmatrix} K_{t-1}^s & Z_{t-1} \\ 0.9658 & 0.0863 \\ 0 & 0.95 \end{pmatrix}$$

Matrix Q

$$\begin{matrix} K^s \\ Z \end{matrix} \begin{pmatrix} \epsilon^Z \\ 0.0908 \\ 1 \end{pmatrix}$$

Matrix R

$$\begin{matrix} r_t \\ C_t \\ I_t \\ K_t^d \\ L_t^s \\ U_t \\ W_t \\ Y_t \end{matrix} \begin{pmatrix} K_{t-1}^s & Z_{t-1} \\ -0.7408 & 1.2972 \\ 0.4748 & 0.5545 \\ -0.3661 & 3.4511 \\ 1 & 0 \\ -0.1575 & 0.5426 \\ 0.0418 & 0.0644 \\ 0.4167 & 0.7547 \\ 0.2592 & 1.2972 \end{pmatrix}$$

Matrix S

$$\begin{matrix} r \\ C \\ I \\ K^d \\ L^s \\ U \\ W \\ Y \end{matrix} \begin{pmatrix} \epsilon^Z \\ 1.3655 \\ 0.5837 \\ 3.6328 \\ 0 \\ 0.5711 \\ 0.0678 \\ 0.7944 \\ 1.3655 \end{pmatrix}$$

12 Model statistics

12.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
r	0.0351	0.1814	0.0329	Y
C	0.7422	0.0783	0.0061	Y
I	0.2559	0.4741	0.2248	Y
K^d	10.2368	0.0422	0.0018	Y
K^s	10.2368	0.0422	0.0018	Y
L^s	0.2695	0.0749	0.0056	Y
U	-136.2372	0.009	0.0001	Y
W	2.3706	0.1047	0.011	Y
Y	0.9981	0.1781	0.0317	Y
Z	1	0.1303	0.017	Y

12.2 Correlation matrix

	r	C	I	K^d	K^s	L^s	U	W	Y	Z
r	1	0.908	0.99	-0.193	0.09	0.996	0.932	0.942	0.973	0.985
C		1	0.958	0.235	0.498	0.94	0.998	0.996	0.981	0.967
I			1	-0.054	0.228	0.998	0.974	0.98	0.996	0.999
K^d				1	0.96	-0.11	0.175	0.147	0.04	-0.021
K^s					1	0.173	0.445	0.418	0.319	0.26
L^s						1	0.959	0.967	0.989	0.996
U							1	1	0.991	0.981
W								1	0.994	0.986
Y									1	0.998
Z										1

12.3 Cross correlations with the reference variable (Y)

	$\sigma[\cdot]$ rel. to $\sigma[Y]$	Y_{t-6}	Y_{t-5}	Y_{t-4}	Y_{t-3}	Y_{t-2}	Y_{t-1}	Y_t	Y_{t+1}	Y_{t+2}	Y_{t+3}	Y_{t+4}	Y_{t+5}
r_t	1.018	0.013	0.109	0.228	0.373	0.545	0.745	0.973	0.631	0.353	0.132	-0.037	-0.161
C_t	0.439	-0.2	-0.107	0.021	0.189	0.402	0.665	0.981	0.761	0.564	0.392	0.245	0.121
I_t	2.662	-0.058	0.039	0.164	0.319	0.508	0.733	0.996	0.688	0.431	0.222	0.057	-0.07
K_t^d	0.237	-0.499	-0.503	-0.48	-0.422	-0.321	-0.17	0.04	0.319	0.504	0.612	0.66	0.659
K_t^s	0.237	-0.503	-0.48	-0.422	-0.321	-0.17	0.04	0.319	0.504	0.612	0.66	0.659	0.623
L_t^s	0.42	-0.029	0.067	0.19	0.341	0.524	0.74	0.989	0.666	0.401	0.187	0.019	-0.107
U_t	0.05	-0.171	-0.077	0.052	0.218	0.428	0.684	0.991	0.751	0.54	0.359	0.207	0.081
W_t	0.588	-0.157	-0.062	0.066	0.232	0.439	0.692	0.994	0.745	0.528	0.343	0.188	0.062
Y_t	1	-0.105	-0.008	0.119	0.28	0.479	0.718	1	0.718	0.479	0.28	0.119	-0.008
Z_t	0.732	-0.074	0.023	0.148	0.306	0.499	0.729	0.998	0.699	0.448	0.242	0.078	-0.049

12.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6
r	0.71	0.466	0.266	0.104	-0.022	-0.116
C	0.745	0.521	0.329	0.169	0.038	-0.066
I	0.712	0.468	0.268	0.107	-0.019	-0.114
K^d	0.96	0.863	0.728	0.572	0.408	0.246
K^s	0.96	0.863	0.728	0.572	0.408	0.246
L^s	0.71	0.466	0.265	0.103	-0.022	-0.116
U	0.735	0.505	0.311	0.15	0.02	-0.08
W	0.73	0.498	0.303	0.142	0.013	-0.087
Y	0.718	0.479	0.28	0.119	-0.008	-0.105
Z	0.713	0.471	0.271	0.11	-0.016	-0.111

12.5 Variance decomposition

	ϵ^Z
r	1
C	1
I	1
K^d	1
K^s	1
L^s	1
U	1
W	1
Y	1
Z	1