On the revival of Navitus Bay

An exploration into the mathematics of optimisation

Katie Murray, Ed Keall and James Arthur

Thursday 28th January 2020

Overview

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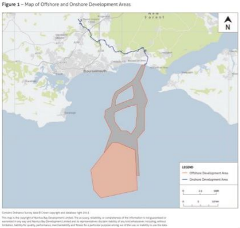
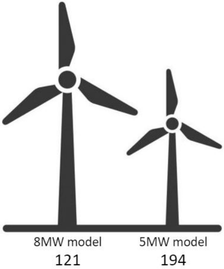
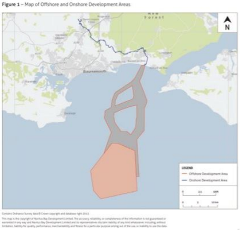
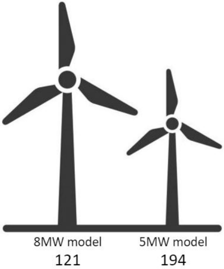




1. Proposed power output capacity between 900MW and 1200MW.

Introduction





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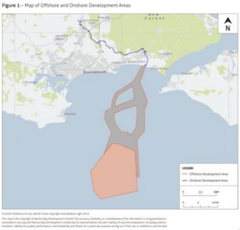

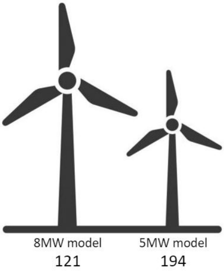

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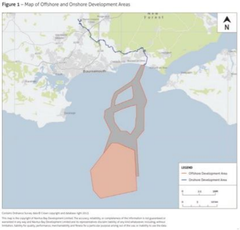

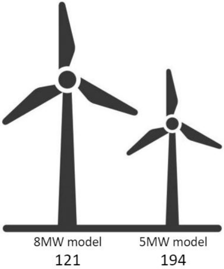

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4. This gives us a proposed area of 196km^2 and budget of 1.5 billion pounds.(*Navitus Bay Wind Park: Enviromental Statement*)

Our Optimisation Problem

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Now we could refine these questions to be as fine as a hair, but our constraints which we will show, make clear what we are optimising against.

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Constraints on number of turbines

	8MW Turbine	5MW Turbine
Rotor Diameter	167m	136m
Spacing	$1.074 km^2$	$0.7112 km^2$
Cost	£4 million	£2.5 million

Single Objective LP

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$$\max_x 8x_1 + 5x_2$$

$$\text{subject to} \quad \begin{aligned} 0.34175\pi x_1 + 0.2265\pi x_2 &\leq 196 \\ 4x_1 + 2.5x_2 &\leq 1500 \\ x_1 &\geq 0 \\ x_2 &\geq 0 \end{aligned}$$

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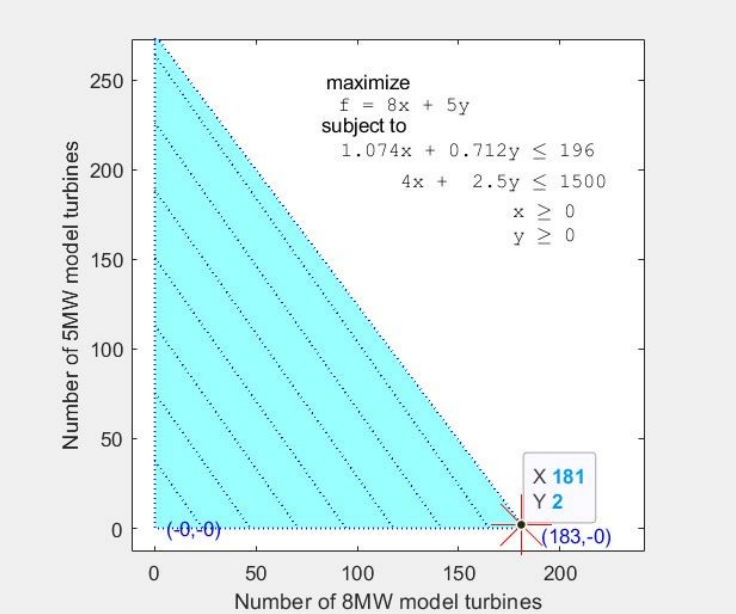
After implementing into MATLAB and using `intlinprog` we got the following solution. The optimal solution is 183 turbines with 181 being of the 8MW model and 2 of the 5MW model.

Feasible Region



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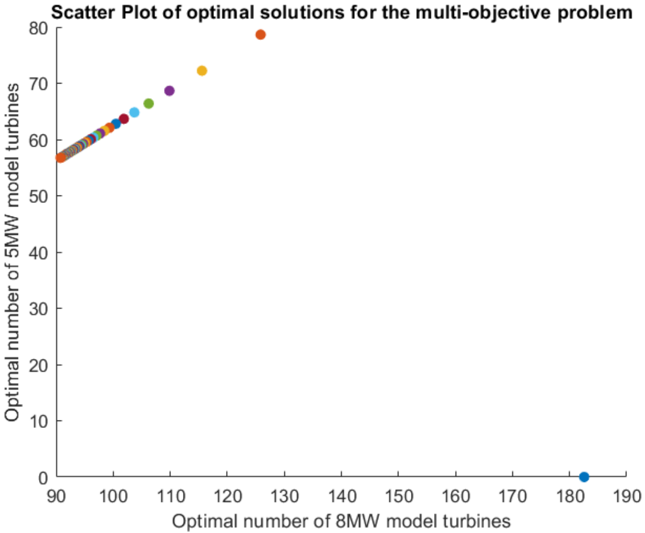
Multiobjective Front



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Data

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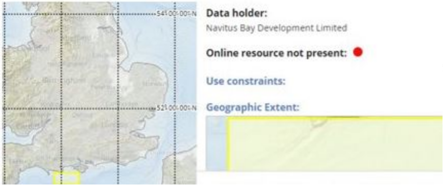

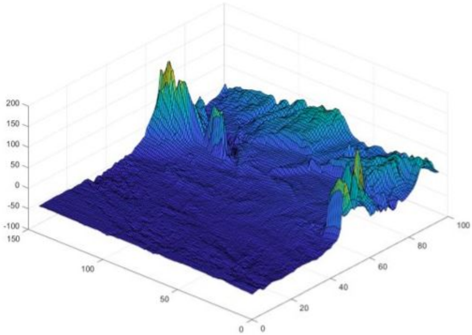

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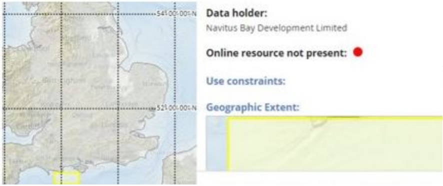

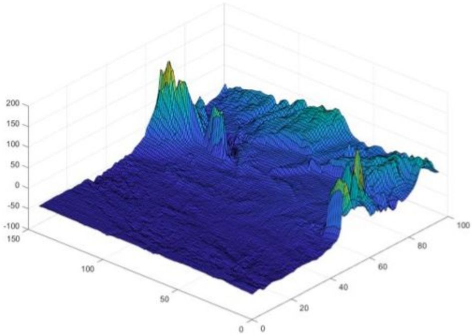

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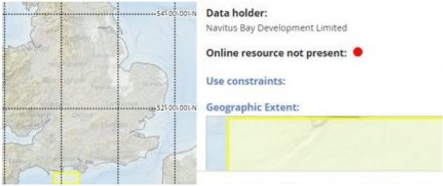
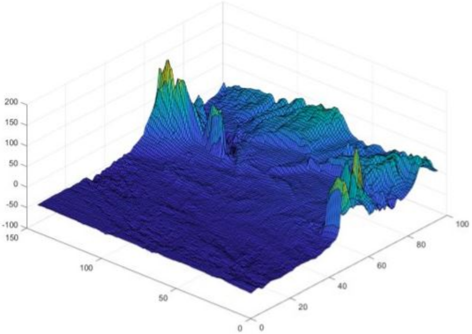
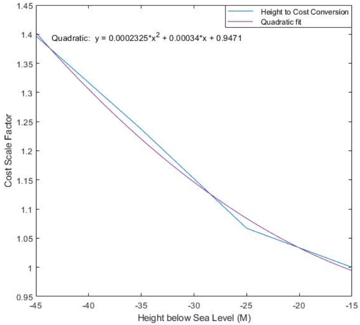
- Bathemetry data was sourced from GEBCO (*Data for Navitas Bay Wind Farm*)

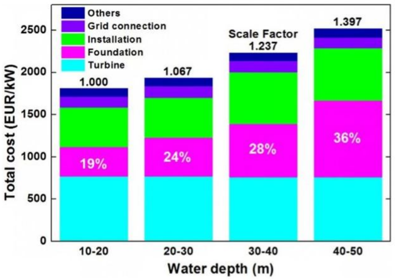




From Depth to Cost

- Using data from a paper on offshore wind turbine costs, we created a quadratic model from depth to cost.(Oh et al. 2018)





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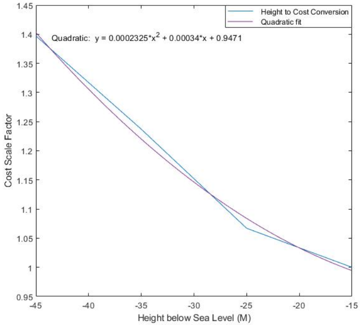
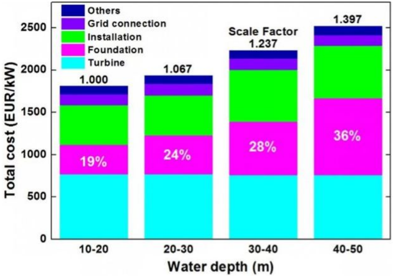
- Using this model the bathymetry dataset was changed to show cost of location.





Conversion of Map to Metric

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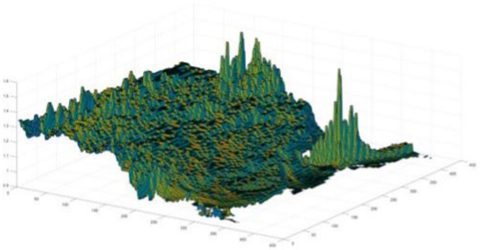

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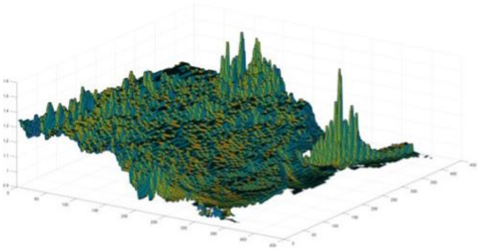

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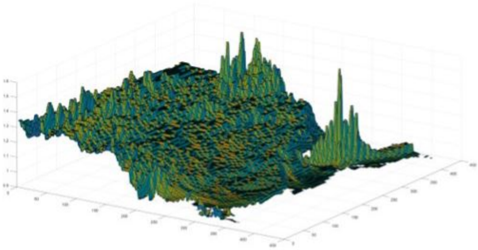

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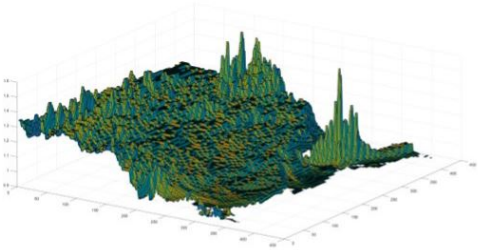
- To reduce processing, this matrix was averaged to 100m $\times$ 100m cells



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Random Placement

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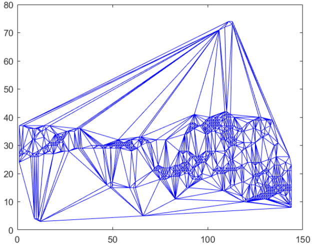

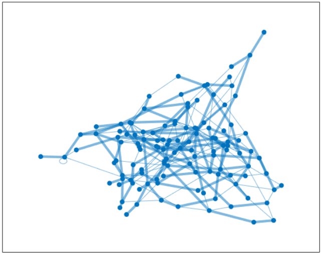

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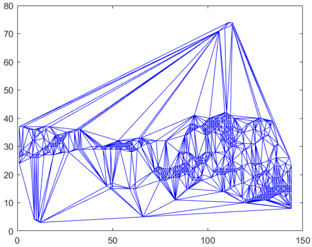

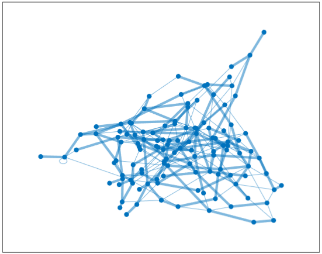

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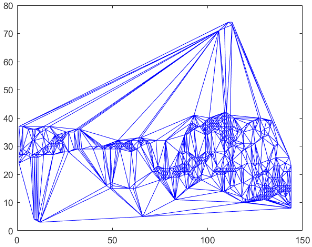
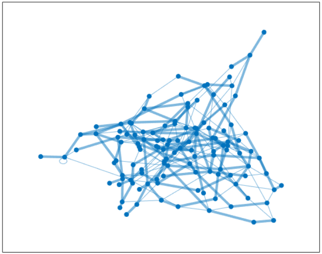
- We then started programming an algorithm.





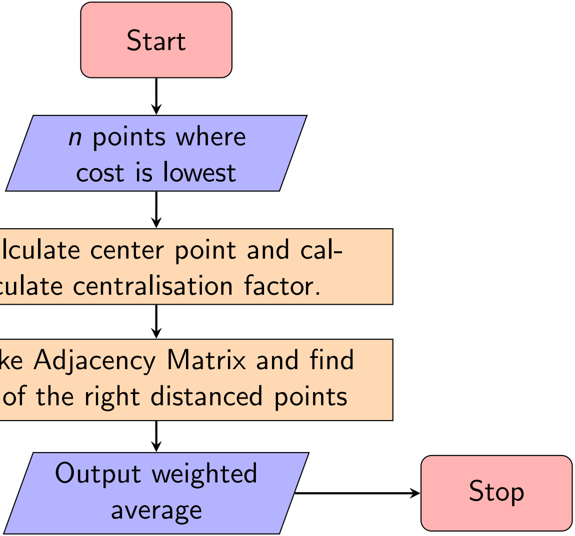


Figure: The Algorithm for finding optimal placement

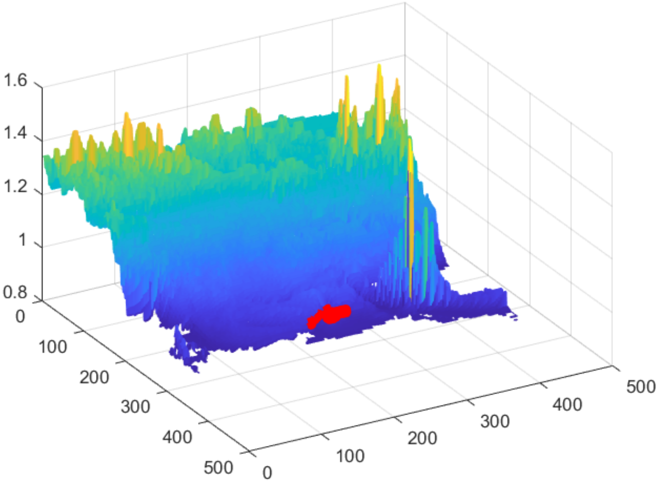
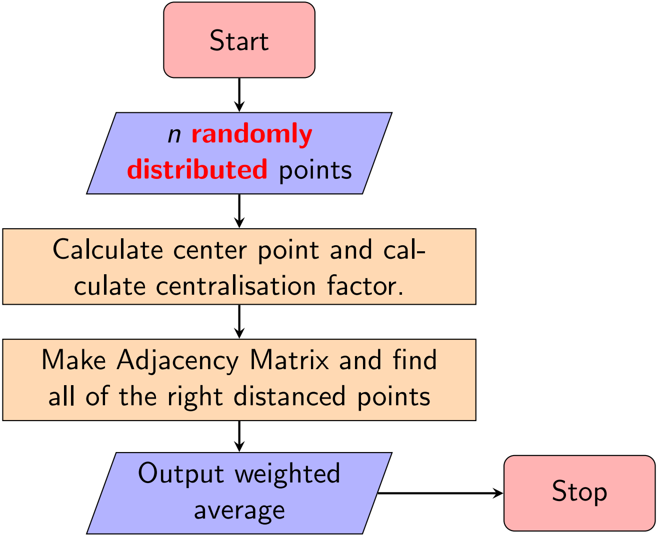


Figure: A not quite perfect plot of the surface



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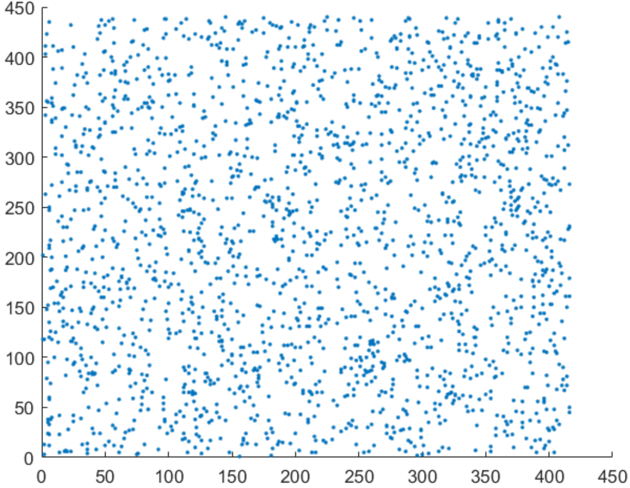

Figure: 2000 Points of uniform placement over the seabed.

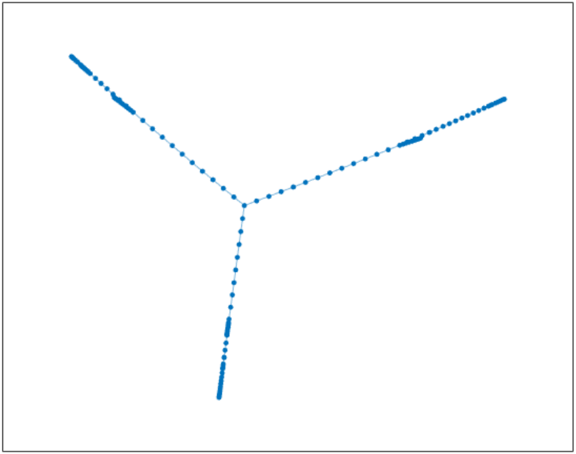

Figure: The MST of all the optimal points.

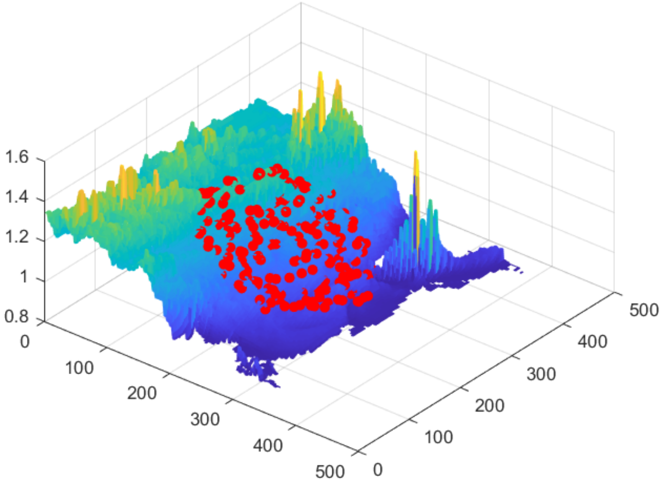

Figure: The locations of the optimal points on the surface

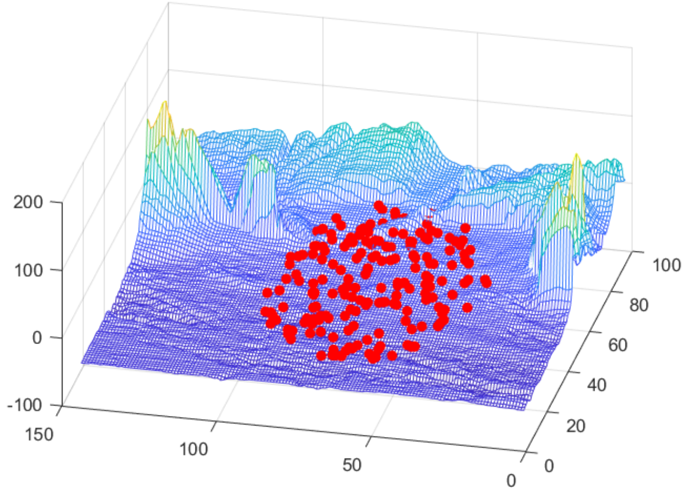

Figure: The points of optimal placement for the 183 turbines

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- We also found the optimal spacing of them in the bay, with respect to relevant physical factors.
- If given more time we could implement an 'annoyance' factor, which have the mathematics of, but omitted as it wasn't included in the code.
- Even though we may not see Navitus Bay come to fruition, it is still a lovely mathematical problem to explore optimisation with. □

References

Commitee, Navitus Bay. *Navitus Bay Wind Park: Enviromental Statement*. URL: https://infrastructure.planninginspectorate.gov.uk/wp-content/ipc/uploads/projects/EN010024/EN010024-001044-6.3%20NonTechnical%20Summary.pdf. (accessed: 13.01.2021).

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