

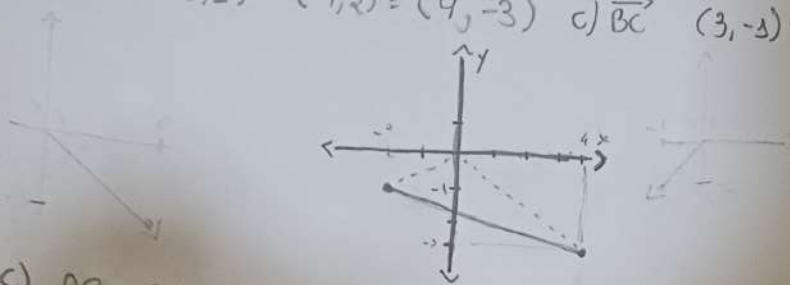
Lista - Parte 1 - Vetores

Eduardo Octavio de Paula Souza

$A(-1,2), B(3,-3), C(4,1)$

1) a) $AB = \frac{3}{2}, \frac{1}{2} = (1, 0.5)$ $AC = (\frac{3}{2}, \frac{3}{2})$ $BC = (\frac{7}{2}, 0)$

b) $\overrightarrow{BA} = (3, -1) - (-1, 2) = (4, -3)$ c) $\overrightarrow{BC} = (3, -1) - (4, 1) = (-1, -2)$



c) $AB = CD$
 $AB = D - C$

$-D = -C - AB$
 $D = C + AB$

$D = (4, 1) + (1, 0.5) = (5, 1.5)$

2) a) $(a, -2) = 13$

$\sqrt{a^2 + (-2)^2} = |13|$

$a^2 + 4 = 169$

$a^2 = 165$

$a = \pm\sqrt{165}$

b) $(2, -4, (a+1)) = 16$

$2^2 + (-4)^2 + (a+1)^2 = 16^2$

$4 + 16 + (a+1)^2 = 256$

$(a+1)^2 = 236$

$a+1 = \pm\sqrt{236}$

$a = -1 \pm \sqrt{236}$

c) $(a, \frac{1}{2}) = 1$

$\sqrt{a^2 + (\frac{1}{2})^2} = 1$

$a^2 + \frac{1}{4} = 1$

$a^2 = 1 - \frac{1}{4} = \frac{3}{4}$

$a = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$

$a = \pm\frac{\sqrt{3}}{2}$

$a = \pm\frac{\sqrt{3}}{2}$

$$d) AB = \sqrt{(3-a)^2 + (3-5)^2} \\ = \sqrt{(3-a)^2 + 4}$$

$$AC = \sqrt{(a-0)^2 + (2-5)^2} \\ = \sqrt{a^2 + 9}$$

$$BC = \sqrt{(0-3)^2 + (2-5)^2} \\ = \sqrt{18}$$

$$AB = AC$$

$$\sqrt{(3-a)^2 + 4} = \sqrt{a^2 + 9}$$

$$(3-a)^2 + 4 = a^2 + 9$$

$$9 - 6a + a^2 + 4 = a^2 + 9$$

$$-6a + 13 = 9$$

$$-6a = -4$$

$$a = \frac{-4}{-6} = \frac{2}{3}$$

$$3) \vec{u} = 2\vec{i} - 3\vec{j} - \vec{k} \text{ and } \vec{v} = \vec{i} - \vec{j} + 4\vec{k}$$

$$a) 2\vec{u} \cdot (-\vec{v})$$

$$2(2\vec{i} - 3\vec{j} - \vec{k}) \cdot (-\vec{i} + \vec{j} - 4\vec{k})$$

$$(4\vec{i} - 6\vec{j} - 2\vec{k}) \cdot (-\vec{i} + \vec{j} - 4\vec{k})$$

$$-4 - 6 + 8 = -2$$

$$b) (\vec{u} + \vec{v}) \cdot (\vec{u} - \vec{v})$$

$$(2\vec{i} - 3\vec{j} - \vec{k} + \vec{i} - \vec{j} + 4\vec{k}) \cdot (2\vec{i} - 3\vec{j} - \vec{k} - \vec{i} + \vec{j} - 4\vec{k})$$

$$(3\vec{i} - 4\vec{j} + 3\vec{k}) \cdot (\vec{i} - 2\vec{j} - 5\vec{k}) =$$

$$(3 - 8 - 15) = -20$$

$$c) (\vec{u} + 3\vec{v}) \cdot (\vec{v} - 2\vec{u})$$

$$(2\vec{i} - 3\vec{j} - \vec{k} + 3\vec{i} - 3\vec{j} + 12\vec{k}) \cdot (\vec{i} + \vec{j} + 4\vec{k} - 4\vec{i} + 6\vec{j} + 2\vec{k})$$

$$(5\vec{i} - 6\vec{j} + 11\vec{k}) \cdot (-3\vec{i} + 5\vec{j} + 6\vec{k})$$

$$(-15 - 30 + 66) = 21$$

$$4) |\vec{u}| = 4, |\vec{v}| = 3 \text{ e } 60^\circ$$

$$a) |\vec{u} - \vec{v}| = \sqrt{|\vec{u}|^2 + |\vec{v}|^2 - 2|\vec{u}||\vec{v}|\cos 60^\circ}$$

$$\sqrt{4^2 + 3^2 - 2(4)(3) \cdot \cos 60^\circ}$$

$$\sqrt{16 + 9 - 24 \cdot \frac{1}{2}}$$

$$\sqrt{25 - 12} = \sqrt{13}$$

$$b) A = |\vec{u}| |\vec{v}| \cdot \sin 60^\circ$$

$$A = 4 \cdot 3 \cdot \frac{\sqrt{3}}{2}$$

$$A = \frac{12 \cdot \sqrt{3}}{2} = \frac{6\sqrt{3}}{2}$$

$$c) |4\vec{u} \cdot \vec{v}|$$

$$|4\vec{u}| \cdot |\vec{v}| \cdot \sin 60^\circ$$

$$16 \cdot 3 \cdot \frac{\sqrt{3}}{2}$$

$$\frac{48\sqrt{3}}{2} = 24\sqrt{3}$$

$$5) a) \cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|} = \frac{(1, -2, 1) \cdot (-4, 2, -2)}{\sqrt{1^2 + (-2)^2 + 1^2} \cdot \sqrt{(-4)^2 + 2^2 + (-2)^2}} = \frac{-4 - 4 - 2}{\sqrt{6} \cdot 2\sqrt{6}} = \frac{-10}{12} = -\frac{5}{6}$$

$$\cos \theta = -\frac{5}{6}$$

$$b) \vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ -4 & 2 & -2 \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ -4 & 2 & -2 \end{vmatrix} = (+4\hat{i} - 4\hat{j} + 2\hat{k}) - (+8\hat{k} + 2\hat{i} - 2\hat{j}) = +4\hat{i} - 4\hat{j} + 2\hat{k} - 8\hat{k} - 2\hat{i} + 2\hat{j} = 2\hat{i} - 2\hat{j} - 6\hat{k}$$

$$c) A = |\vec{u} \times \vec{v}| = \sqrt{2^2 + (-2)^2 + (-6)^2} = \sqrt{4 + 4 + 36} = \sqrt{44} = 2\sqrt{11}$$

$$\begin{array}{r} 44 \div 2 \\ 22 \div 2 \\ 11 \div 11 \end{array}$$

Eduardo Octavio de Paula Sargen

Pov

,2)

B=

$$d)(2\vec{u} + \vec{v})$$

$$(2, -4, 2) + (-4, 2, -2)$$

$$(-2, -2, 0)$$

$$(\vec{u} - \vec{v})$$

$$(1, -2, 1) - (-4, 2, -2)$$

$$(+5, -4, 3)$$

$$\begin{vmatrix} i & j & k & i & j \\ -2 & -2 & 0 & -2 & -2 \\ +5 & -4 & -3 & -5 & -4 \end{vmatrix}$$

$$+6i + 0 + 8k - (-10k + 0 + 6j)$$

$$6i + 8k + 10k - 6j$$

$$18k - 6j + 6i$$