Scribe

Taller 02 Ecuaciones Diferenciales

1. $U_1 = V_0 \rightarrow U_n = \frac{1}{3} U_{n-1}, \; n \geq 1$

$U_2 = \frac{1}{3} U_1$

$U_n = K^{n-1} U_1, \; K = \frac{1}{3}$

$U_3 = \frac{1}{3} U_2$

para $U_n = \left(\frac{1}{1 \times 10^6}\right) V_0 \Rightarrow \frac{\cancel{V_0}}{10^6} = \frac{1}{3}^{n-1} \cancel{V_0}$

$\log\left(\frac{1}{10^6}\right) = n-1 \log(1/3)$

$n = \dfrac{\log(1) - \log(10^6)}{\log(1) - \log(3)} + 1 \Rightarrow n = 13{,}57 \approx 14$

↳ R/ Después de 14 acciones.

2.

U: Población	$1000(r) = 25 \rightarrow U_n = U_{n-1}(1+r)$
n: año	$r = \frac{1}{40}$
r: Tasa	

$U_n = (1.025)^{n-1} U_1,$

$K = 1 + r$

Para $n = 15, \; U_1 = 200 \cdot 10^6$

$U_{15} = 1.025^{14} \cdot 200 \times 10^6 = 282.59 \cdot 10^6$ Personas para $U_1 = 200 \cdot 10^6$, $U_n = 150 \cdot 10^6$

$$150 \times \cancel{10^6} = (1.025)^{n-1} \cdot 200 \times \cancel{10^6}$$

$$\log(150/200) = (n-1)\log(1.025)$$

$$n = \frac{\log(150) - \log(200)}{\log(1.025)} + 1 = 54.53 \approx 55$$

R/ después de 55 años.

3. $U_n = 4U_{n-1} - 1$, $n \geq 2$, $k = 4$ $c = -1$

De manera que $U_n = kU_{n-1} + c$ entonces

$$U_n = k(kU_{n-2} + c) + c$$

$$\Rightarrow k^{n-1} U_1 + c\,[k^{n-2} + k^{n-3} + \dots 1]$$

entonces: P_n: $\sum_{i=2}^{n} k^{n-i} = \frac{(k^{n-1} - 1)}{k-1}$, $n \geq 2$, $k \neq 1$

Caso base: $n = 2$

$$k^{2-2} = k^0 = 1 \quad y \quad \frac{k^{2-1} - 1}{k-1} = \frac{\cancel{k-1}}{\cancel{k-1}} = 1$$

Por lo que se cumple para el caso base

Paso inductivo.

$$P_{n+1} = k^{n-1} + \overbrace{k^{n-2} + k^{n-3} + \dots + 1}$$

$$= k^{n-1} + P_n \quad , \qquad \text{donde } P_n = \frac{k^{n-1} - 1}{k - 1}$$

$$= k^{n-1} + (k^{n-1} - 1)/(k-1)$$

$$\Rightarrow \frac{k^{(n+1)-1} - k^{n-1} + k^{n-1} - 1}{k-1}$$

$$\Rightarrow \frac{k^{(n+1)-1} - 1}{k-1} \Big\} \text{ se cumple para } P_{n+1}$$

Entonces:

$$U_n = 4^{n-1} U_n - \frac{1}{3}(4^{-1} - 1)$$

De mismo modo:

$$V_n = 3^{n-1} U_n + 2(3^{n-1} - 1)$$

4. $U_n = -4U_{n-1} - 3, \quad n \geq 1$

Siguiendo la misma demostración realizada en el punto 3

$\cdot U_n = -4^{n-1} U_1 + \frac{3}{5} (-4^{n-1} - 1)$

• De mismo modo para

$U_n = -2U_{n-1} + 13, \quad n \geq 1$

$U_n = -2^{n-1} U_1 - \frac{13}{3} (-2^{n-1} - 1)$

5. $U_n = 3U_{n-1} + 5, \quad n \geq 1, \quad U_0 = 1$

$U_n = 3^{n-1+1} U_0 + \frac{5}{2} (3^{n-1+1} - 1)$

↳ existe un termino adicional para llegar al enesimo termino

$\Rightarrow 3^n U_0 + \frac{5}{2} (3^n - 1)$

$= \frac{1}{2} [7(3^n) - 5$ /

6. $U_1 = 7$

$U_2 = U_1 + 2^0 (10)$

$U_3 = U_2 + 2^1 (10)$

$\left.\begin{matrix} \\ \\ \end{matrix}\right\}$ $U_n = U_{n-1} + 5(2^{n-1})$

$n \geq 2$

↳ Derivado de los ejemplos

Hallar solución general:

$U_n = U_{n-1} + 5(2^{n-1})$

$= (U_{n-1} + 5[2^1 + 2^2 + 2^{n-1}])$

$\sum_{i=1}^{n-1} 2^i \Rightarrow 2^n - 2$

$U_n = U_1 + 5(2^n - 2)$

$\Rightarrow 7 + 5(2)(2^{n-1} - 1)$

$U_n = 7 - 10[1 - 2^{n-1}]$

7. $U_0 = 400$ M
$t = 3$ años
$i = 0.21$
X = Pago anual.

para los 3 años

Donde, $U_n = U_{n-1}(1+i) - x$

Resolviendo, para $K = 1.21$ y $C = -X$

$$U_n = (1.21)^n 400 - \frac{x(1.2^n - 1)}{0.21}, \text{ con } n = 3$$

$$U_3 = 0 = (1.21)^3 400 - x(1.21^3 - 1) / 0.21$$

$X = 192.87 \cdot 10^6$ pagando 192.87 millones anuales, se salda en 3 años

8. $rn = 200(0.1)n$.
$\Rightarrow 202\ n$

donde
$202n - 1000 = 0$
$n = 1000/202$
$n = 7.92 \simeq 8 \Rightarrow$ Desde el mes 8.

Entonces: para el café:

$U_n = (202\ n - 1600) + U_{n-1}$, $n \geq 8$

$U_7 = 0$ $(U_0 < n < 7)$

Para $n=12$:

$U_{12} = 202[6(13) - 23] - 5(1600)$

$U_{12} = 21007$

$U_n = U_7 - (n-7)1600 + 202[3+4+ \cdots + n]$

$\hookrightarrow \frac{1}{2}n(n+1) - 28$

$U_n = 101(n^2+n) - (n-7)1600 - 5656$

$U_{24} = 202[12(25) - 23] - 17(1600)$

$U_{24} = 28754$

9. $U_0 = 2000$, $U_n = U_{n-1}(1.05) + 100$

$U_n = (1.05)^n 2000 + \frac{100}{0.05}(1.05^n - 1)$

$= 2000[2(1.05)^n - 1]$

Para $n=10$

$U_{10} = 4515.58 \simeq 4.515$ arbolos.

de modo que, para la productividad.

$$P = \frac{(U_{16} - U_0)}{U_0} \cdot 100 \Rightarrow 125.45\%$$

↳ mejora

10.

$U_n = 3U_{n-1} = n$, $U_1 = 5 \longrightarrow U_2 = 17$

solución homogénea:

$U_n - 3U_n = 0$

$$n = \frac{3 \pm 3}{2} \quad \Bigg| \quad \frac{m_1 = 0}{m_2 = 3}$$

$U_n = 3^n$

Solución particular:

$a + bn - 3[a + b(n-1)] = n$

$(3b - 2a) + (-2bn) = n \longrightarrow b = -\frac{1}{2}$, $a = -\frac{3}{4}$

Así pues,

$U_n = A(3)^n - 3/4 - \frac{1}{2}$ para U_1

$A = \frac{25}{12}$

$$U_n = \frac{25}{12}(3^n) - \frac{3}{4} - \frac{1}{2}n$$

Scribe

11. $U_n = U_{n-1} + 2^n \to$ No homogenea de la forma k^n

1. $U_n - U_{n-1} = 0$

$m^2 - m = 0 \to m(m-1) = 0 \to m_1 = 0,\ m_2 = 1$

$U_n = A(1)^n = A$

2. $a2^n - a2^{n-1} = 2^n$

$a2^n[1 - 2^{-1}] = 2^n$

$a = 2 \longrightarrow U_n = 2^{n+1} + A$

$2(2^n) - 2(2^{n-1}) = 2^n$

$2^n[2 - 2^0] = {}^n$

$2^n 1 = 2^n$

$2^n = 2^n$ ✓

• $U_n = 2U_{n-1} + n$, no homogenea de la forma n donde $U_n = a + bn$

$U_n - 2U_{n-1} = n$

1. $U_n - 2U_{n-1} = 0$

$m(m-2) = 0 \to m_1 = 0,\ m_2 = 2$

$U_n = A(2^n)$

$2(a+bn) - 2(a + b(n-1)) = n$

$a - 2a + bn - 2bn + 2b = n$

$(2b - a) + (-bn) = n$ $\qquad U_n = -2 - n$

$b = -1$ $\qquad 2b - a = 0 \rightarrow a = -2$

$U_n = A(2^n) - 2 - n$

12. Si $U_n = k\,U_{n-1} + 5$ y $U_1 = 4$ y $U_2 = 17$

encuentre los valores de k y U_0.

Dado que $U_1 = 4$ y $U_2 = 17$, se puede calcular k para U_2:

sustituyendo: U

$U_2 = k \cdot U_1 + 5$

$U_2 = k \cdot 4 + 5$

$17 = k \cdot 4 + 5$

$k = \frac{17 - 5}{4} = 3 \rightarrow k = 3$

→ Calcular U_6 por medio de la siguiente propiedad:

$$U_n = kU_{n-1} + C$$

$$U_n = k^{n-1}U_1 + \frac{C(k^{n-1}-1)}{k-1}$$

$$U_6 = kU_5 + 5$$

$$U_6 = k^5 U_1 + \frac{5(k^5-1)}{k-1}$$

$$U_6 = 243.4 + \frac{5(242)}{2} = \boxed{1547}$$

$$U_6 = k.U_5 + 5$$

$$U_6 = 3.524 + 5$$

$$\boxed{U_6 = 1547}$$

13. $U_n = \frac{U_{n-1}}{U_{n-2}}$, $U_2 = \frac{U_1}{U_0}$

Dado que $U_1 = \frac{1}{6}$, se debe hallar U_2

es $n \geq 2$ es posible retroceder en la relación de ~~..~~ recurrencia

$$U_1 = \frac{U_0}{U_{-1}} \rightsquigarrow U_0:$$

$$U_0 = U_1(U_{-1}) = U_0 = \left(\frac{1}{6}\right) U_{-1}$$

Con esos valores es posible hallar U_2:

$$U_2 = \frac{U_1}{U_0} \quad U_2 = \frac{\cancel{\frac{1}{6}}}{\cancel{\frac{1}{6}}(U_{-1})} \quad U_2 = \frac{1}{U_{-1}}$$

14. $U_n = U_{n-1} + 2a_{n-2}$

$k = n-2$

$U_n - U_{n-1} - 2U_{n-2} = 0$

$k+1 = n-1$

$k+2 = n$

Solución homogenea:

$m^2 - m - 2 = 0$

$(m-2)(m+1)=0 \rightarrow m_1=-1,\ m_2=2$

$U_n = A(-1)^n + B(2)^n$

$$\frac{U_n}{U_{n+1}} = \frac{A(-1)^n + B(2)^n}{A(-1)^{n-1} + B(2)^{n-1}}$$

$$\hookrightarrow \frac{B(-2)^n + A}{2B(-2)^n - A}$$

$$\lim_{n \to \infty} \frac{U_n}{U_{n+1}} = \lim_{n \to \infty} \frac{B(-2)^n + A}{2B(-2)^n - A}$$

Casos: n par o impar

Caso n par. $n = 2k$

$$\lim_{k \to \infty} \frac{\cancel{\ln(4)\,B(4)^k}}{2\ln(4)\,\cancel{B(4)^k}} = \lim_{k \to \infty} = \frac{1}{2}$$

Caso impar) $2k+1 = n$

$$\lim_{k\to\infty} = \frac{-2B(4)^k + A}{-4B(4)^k - A}$$

L.H

$$\Rightarrow \lim_{k\to\infty} \frac{-2\ln(4)B(4)^k}{-4\ln(4)B(4)^k} = \lim_{k\to\infty} \frac{1}{2} = \frac{1}{2}$$

Por lo tanto: el límite es igual a $\frac{1}{2}$

15. Encuentre el término de la siguiente secuencia: $-3, 21, 3, 129, 147$

Se pueden expresar

$U_1 = -3 \quad U_2 = 21 \quad U_3 = 3 \quad U_4 = 129 \quad U_5 = 147$

$$\left.\begin{aligned} \hookrightarrow U_2 &= 21 = (-1)^2(-3)^2 + 12 = 21 \\ U_3 &= (-1)^3(-3)^3 + (-24) = 3 \\ U_4 &= (-1)^4(-3)^4 + 48 = 129 \end{aligned}\right\}$$

$$\checkmark \; (-1)^n U_1^n + (-1)^{n-1} 2^n U_1$$

Scribe

16. $U_n = 4^n + 3 \cdot 2^n$ solución particular

$U_{n} = -6U_{n-1} + 8U_{n-2} = 0$

Polinomio
$r^2 + 6r + 8 \rightsquigarrow M_1 = 4 \quad M_2 = 2$

$A(4)^n + B(2)^n \to$ solución general

$10 = A4 + B2$
$28 = A16 + B4$
$\Big\}\quad A8 = 8;\quad A = 1 \qquad \dfrac{10-4}{2} = B$

$3 = B$

Solución particular

$4^n + 3 \cdot 2^n = U_n$

$U_6 = 4^6 + 3 \cdot 2^6 = 4288$

17. $U_{n+2} + 2U_{n+1} + U_n = 0$ $\quad U_1 = -1$

$U_2 = -2$

Polinomio

$m^2 + 2m + 1 \Rightarrow (x+1)^2 = \quad m_1 = -1$

Sol general:

$A(-1)^2 + Bn(-1)^2$

$$\left.\begin{aligned} -1 &= -A - B \\ -2 &= A + 2B \end{aligned}\right\} \quad \begin{aligned} -3 &= B \\ 4 &= A \end{aligned}$$

solución particular:

$4(-1)^n + (-3)n(-1)^n$

Scribe

18

$U_n - 5U_{n-1} + 6U_{n-2} = f(n)$, $f(n) = 2$

→ hallamos el polinomio

$$\left.\begin{array}{l} m^2 - 5m + 6 = 0 \\ \Rightarrow (m-2)(m-3) = 0 \end{array}\right\} \begin{array}{l} m_1 = 2 \\ m_2 = 3 \end{array}$$

$U_n = A(2)^n + B(3)^n$ → Parte homogénea

Solución: $a + bn$

Reescribir

$U_n = a + bn$

$U_{n-1} = a + b(n-1)$

$U_{n-2} = a + b(n-2$

$a + bn - 5(a + b(n-1)) + 6(a + b(n-2$

$\Rightarrow 2a + 2bn - 7b$

$\Rightarrow \underbrace{2(a+bn)}_{1} - \underbrace{7b}_{0} = 2$

$7b = 0$

$b = 0$

$a + \underbrace{bn}_{0} = 1$

$a = 1$ ✓

$U_n = A(2)^n + B(3)^n + 1$

$f(n) = n.$

sabemos que la solución de la parte homogenea es:

$$U_n = A(2)^n + B(3)^n.$$

$$\left.\begin{aligned} U_n &= an + b \\ U_{n-1} &= a(n-1) + b \\ U_{n-2} &= a(n-2) + b \end{aligned}\right\} \text{ siendo } a \text{ y } b \text{ constantes}$$

Remplazar

$$an + b - 5(a(n-1) + b) + 6(a(n-2) + b) = n.$$

$$\Rightarrow (\underbrace{2a}_{1})n + (\underbrace{12b - 7a}_{0}) = n$$

$$\boxed{a = \frac{1}{2}} \qquad 12b - 7\left(\frac{1}{2}\right) = 0 \qquad b = \frac{7}{24}$$

$$U_n = A(2)^n + B(3)^n + \frac{19}{24}.$$

Scribe

$F(n) = 5^n$

solución homogenea:

$U_n = A(2)^n + B(3)^n$

Solución alternativa

Remplazar

$$\left.\begin{array}{l} U_n = a5^n \\ U_{n-1} = a5^{n-1} \\ U_{n-2} = a5^{n-2} \end{array}\right\} \quad a5^n - 5(a5^{n-1}) + 6(a5^{n-2})$$

$$\Rightarrow 6\,5^{n-2} \times a \Rightarrow 5^n \times \underbrace{\left(\frac{6}{25} \times a\right)}_{1}$$

$$\frac{6}{25}a = 1 \qquad \boxed{a = \frac{25}{6}}$$

$$U_n = A(2)^n + B(3)^n + \frac{25}{6}$$

$f(n) = 1 + n^2$

solución homogénea

$U_n = A(2)^n + B(3)^n$

Alternativa

$U_n = a + bn + cn^2$

$U_{n-1} = a + b(n-1) + c(n-1)^2$

$U_{n-2} = a + b(n-2) + c(n-2)^2$

Reescribir:

$a + bn + cn^2 - 5(a + b(n-1) + c(n-1)^2)$
$+ 6(a + b(n-2) + c(n-2)^2)$

$2a + 2bn + 2cn^2 - 7b - 14cn + 19c = 1 + n^2$

$2bn + 2cn^2 - 14cn - 7b + 19c$ $+2a$

$\to 2c + 17c$

$2c(1+n^2) + \underbrace{(2bn - 14cn - 7b + 17c + 2a)}_{0}$

$c = \frac{1}{2}$

$b(2n-7) + c(-14n+17) + 2a = 0$

$a = 8$; $b = \frac{7}{2}$

$c = \frac{1}{2}$

$A(2)^n + B(3)^n + 12$

Solución

19.

Resuelve la siguiente ecuación, usando la función generatriz:

$U_n - 3U_{n-1} + 4U_{n-2} = 0$, dado $U_0 = 0$

$U_1 = 20$, $n \geq 2$ ↳ $U_n = 3U_{n-1} - 4U_{n-2}$

Sea:

$G(x) = U_0 + U_1 X + U_2 x^2 + \ldots$

↳ $0 + 20x + U_2 x^2 + \ldots$

$U_2 = 3U_1 - 4U_0$

$U_3 = 3U_2 - 4U_1$

$U_4 = 3U_3 - 4U_2$, etc

Sustituyendo:

$G(x) = 0 + 20x + (3U_1 - 4U_0)x^2 + (3U_2 - 4U_1)x^3$

$\Rightarrow 0 + 20x + (3U_1 x^2 + 3U_2 x^3 + \ldots)$
$- (4U_0 x^2 + 4U_1 x^3 + \ldots)$

Scribe

$\Rightarrow 0 + 20x + 3x\,(U_1 x + U_2 x^2 + \dots)$
$- 4x^2\,(U_0 + U_1 x + U_2 x^2 + \dots)$

$\Rightarrow 0 + 20x + 3x\,(G(x) - U_0) - 4x^2 G(x)$

$\Rightarrow 0 + 20x + 3x\,(G(x)) - 4x^2 G(x) \rightarrow U_0 = 0$

$G(x) = 20x + 3x\,(G(x)) - 4x^2 G(x)$

$G(x) - 3x(G(x)) = 20x - 4x^2 G(x)$

$G(x) - 3x(G(x)) + 4x^2 G(x) = 20x$

$G(x)(1 - 3x + 4x^2) = 20x$

$$G(x) = \frac{20x}{1 - 3x + 4x^2}$$

$$G(x) = \frac{20x}{\left(2x - \left(\frac{3 + i\sqrt{7}}{4}\right)\right)\left(2x - \left(\frac{3 - i\sqrt{7}}{4}\right)\right)}$$

Scribe

Fracciones parciales

$$\frac{A}{\left(2x-\left(\frac{3-\sqrt{7}i}{4}\right)\right)}+\frac{B}{\left(2x-\left(\frac{3+\sqrt{7}\,i}{4}\right)\right)}$$

$$\frac{20x}{\left(2x-\left(\frac{3+\sqrt{7}i}{4}\right)\right)\left(2x-\left(\frac{3-\sqrt{7}\,i}{4}\right)\right)}=$$

$$\frac{\left(2x-\left(\frac{3+\sqrt{7}}{4}\,i\right)\right)A+\left(2x-\left(\frac{3-\sqrt{7}\,i}{4}\right)\right)B}{\left(2x-\left(\frac{3+\sqrt{7}i}{4}\right)\right)\left(2x-\left(\frac{3-\sqrt{7}\,i}{4}\right)\right)}$$

Resultado

$A = 5 + \frac{15i}{\sqrt{7}}$ $\quad B = 5 - \frac{15\,i}{\sqrt{7}}$

Por fracciones parciales

$$\frac{5+\frac{15i}{\sqrt{7}}}{\left(2x-\left(\frac{3+\sqrt{7}\,i}{4}\right)\right)}+\frac{5-\frac{15i}{\sqrt{7}}}{\left(2x-\left(\frac{3-\sqrt{7}\,i}{4}\right)\right)}=G(x)$$

$$\left(\frac{5+5,67i}{2x-0,75+0,66i}\right)+\left(\frac{5-5,67i}{2x-0,75-0,66i}\right)=G(x)$$

↳ No se puede aplicar expansión por teorema fracionado

$$u_n - 3u_{n-1} + 4u_{n-2} = 0$$

P.C $\quad \lambda^2 - 3\lambda + 4 = 0$

$$\lambda = \frac{3 \pm \sqrt{9 - 4(4)(1)}}{2}$$

$$\lambda = \frac{3 \pm \sqrt{9-16}}{2}$$

$$\lambda = \frac{3 \pm \sqrt{7}\,i}{2}$$

$$\lambda = \frac{3 + \sqrt{7}\,i}{2} \qquad \lambda = \frac{3 - \sqrt{7}\,i}{2}$$

$$\lambda = \frac{3}{2} + \frac{\sqrt{7}}{2} i \longrightarrow \lambda = a + bi$$

$$a = \frac{3}{2} \qquad b = \frac{\sqrt{7}}{2}$$

$$\sqrt{a^2 + b^2} = \sqrt{\frac{9}{4} + \frac{7}{4}} = 2$$

$$\cos\theta = \frac{3}{4} \qquad \sin\theta = \frac{\sqrt{7}}{4}$$

$$\theta \approx 0{,}723 = \cos^{-1}\left(\frac{3}{4}\right)$$

$$u(n) = 2^n \left[C_1 \cos(0{,}723n) + C_2 \sin(0{,}723n) \right]$$

$u(0) = 0 \qquad u(1) = 20$

$$u(0) = 1(C_1(1) + C_2(0)) = 0$$

$$C_1 = 0$$

$$u(1) = 2\left[C_1 \cos(0{,}723) + C_2 \sin(0{,}723) \right] = 20$$

$$2\left[\cancel{C_1 \left(\tfrac{3}{4}\right)}^{\,C_1 = 0} + C_2\left(\frac{\sqrt{7}}{4}\right) \right] = 20$$

$$C_2\left(\frac{\sqrt{7}}{4}\right) = 10$$

$$C_2 = \frac{40}{\sqrt{7}}$$

$$u(n) = 2^n \left[\frac{40}{\sqrt{7}} \cdot \sin\left(\sin^{-1}\left(\frac{\sqrt{7}}{4}\right) n \right) \right]$$

20. $U_m = U_{n-1} + U_{m-2}$

polinomio $\rightarrow$ $m^2 - m - 1 = 0$

$m_1 = \frac{1 + \sqrt{5}}{2}$ $\quad m_2 \frac{1 \sqrt{5}}{2}$

$U_m = m_1 \left(\frac{1 + \sqrt{5}}{2}\right)^n + m_2 \left(\frac{1 - \sqrt{5}}{2}\right)$

$A + B = 0 \rightarrow A = -B$

$A\left(\frac{1 + \sqrt{5}}{2}\right) - A\left(\frac{1 - \sqrt{5}}{2}\right) = 1$

$\sqrt{5} A = 1 \longrightarrow A_1 = \frac{\sqrt{5}}{5}$

$B = -\frac{\sqrt{5}}{5}$

$U_m = \frac{\sqrt{5}}{5} \left[\left(\frac{1 + \sqrt{5}}{2}\right)^n - \left(\frac{1 - \sqrt{5}}{2}\right)^n \right]$

21.

solução homogénea

$$U_n - 2U_{n-1} = 0$$

$$m^2 - 2m = 0$$

$$m(m-2) = 0 \rightarrow$$

$$m_1 = 0$$

$$m_2 = 2$$

$$U_n = \boxed{A(2)^n}$$

sol particular

$$F_p \sim 3^n$$

$$F_p(n) = \alpha 3^n$$

$$\alpha 3^n - 2\alpha 3^{n-1} = 3^n$$

$$\alpha 3^n - \frac{2}{3}\alpha 3^n = 3^n$$

$$\frac{\alpha}{3} 3^n = 3^n$$

$$\alpha = 3.$$

$$\Rightarrow F_p(n) = 3 \cdot 3^n = \boxed{3^{n+1}}$$

solução geral

$$A(2)^n + 3^{n+1}$$

$$U_0 = A + 3 = 1 \rightarrow A = -2$$

$$U_n = -2(2)^n + 3^{n+1}$$

$$U_n = -2^{n+1} + 3^{n+1}$$

$$U_n = 3^{n+1} - 2^{n+1}.$$