```
import numpy as np
import matplotlib.pyplot as plt


def euler(f, f0, n, hf, t_o = 0):
    t = t_o
    h = ((hf)-t_o)/n
    s0 = f0
    s, T = [],[]
    while t <= hf:
        ssq = s0 + h*f(t, s0)
        s.append(ssq)
        T.append(t)
        s0 = ssq
        t += h
    return s, T
def euler2(f, f0,g0, n, hf, t_o = 0):
  h = ((hf)-t_o)/n
  s, v, T = [f0], [g0], [t_o]
  for i in range(n):
    snext = s[i] + h*v[i]
    vnext = v[i] + h*f(T[i],s[i])
    s.append(snext)
    v.append(vnext)
    T.append(T[i]+h)
  return v,s, T

def rkuta2(f, f0, n, hf, t_o = 0):
    t = t_o
    h = ((hf)-t_o)/n
    s0 = f0
    s,T = [],[]
    while t<=hf:
        k1n = h*f(t, s0)
        k2n = h*f(t, s0 + 0.5*k1n)
        s.append(s0 + k2n)
        T.append(t)
```

```
        s0 += k2n
        t += h
    return s, T

def rk2_2(f, f0,g0, n, hf, t_o = 0):
  s = [f0]
  v = [g0]
  h = ((hf)-t_o)/n
  T = [t_o]
  for i in range(n):
    k1s = h*s[i]
    k1v = h*f(T[i],s[i])
    k2s = h * (v[i] + 0.5 * k1v)
    k2v = h * f(T[i] + 0.5 * h, s[i] + 0.5 * k1s)
    sp = s[i] + k2s
    vp = v[i] + k2v
    s.append(sp)
    v.append(vp)
    T.append(T[i] + h)
  return v,s, T

def rkuta4(f, f0, n, hf, t_o = 0):
    t = 0
    h = ((hf)-t_o)/n
    s0 = f0
    s,T = [],[]

    while t<=hf:
        k1n = h*f(t, s0)
        k2n = h*f(t, s0 + 0.5*k1n)
        k3n = h*f(t, s0 + 0.5*k2n)
        k4n = h*f(t, s0 + k3n)
        si = s0 + k1n/6 + k2n/3 + k3n/3 + k4n/6
        s.append(si)
        T.append(t)
        s0 = si

        t += h
```

```
    return s, T
def rk4_2(f, f0,g0, n, hf, t_o = 0):
  s = [f0]
  v = [g0]
  T = [t_o]
  h = ((hf)-t_o)/n
  for i in range(n):
    k1s = h*s[i]
    k1v = h*f(T[i],s[i])
    k2s = h * (v[i] + 0.5 * k1v)
    k2v = h * f(T[i] + 0.5 * h, s[i] + 0.5 * k1s)
    k3s = h * (v[i] + 0.5 * k2v)
    k3v = h * f(T[i] + 0.5 * h, s[i] + 0.5 * k2s)
    k4s = h * (v[i] + k3v)
    k4v = h * f(T[i] + 0.5 * h, s[i] + k3s)
    sp = s[i] + (k1s + 2 * k2s + 2 * k3s + k4s) / 6
    vp = v[i] + (k1v + 2 * k2v + 2 * k3v + k4v) / 6
    s.append(sp)
    v.append(vp)
    T.append(T[i] + h)
  return  v,s, T


def StormerV(f, f0,g0, n, h, t_o = 0):
  p, q = np.zeros(n), np.zeros(n)
  T = [t_o, t_o + h]
  p[0], q[0] = f0, g0
  p[1] = p[0] + q[0]*h + h**2*f(T[0],p[0])*0.5
  for i in range(1,n-1):
    p[i+1] = p[i]+q[i]*h +f(T[i],p[i])*h**2
    q[i+1] = q[i] + f(T[i],p[i])*h
    T.append(T[i] + h)

  return list(p), list(q), T

def SEuler(f,g, f0,g0, n, h, t_o = 0):
  p, q= np.zeros(n+1), np.zeros(n+1)
  T = [t_o]
```

```
    T = [t_0]
    p[0], q[0] = f0, g0
    for i in range(n):
      p[i+1] = p[i] + f(T[i],q[i]) * h
      q[i+1] = q[i] + g(T[i],p[i+1]) * h
      T.append(T[i] + h)

    return list(p), list(q), T

def dynamics_solve(f, f0 = [1],  n = 100,hf = 100, D = 1, t_o = 0, method = "Euler"):
    method_dict = {
        "Euler": euler,
        "RK2": rkuta2,
        "RK4": rkuta4
    }
    if method not in method_dict:
        return None

    selected_method = method_dict[method]
    vsol = [selected_method(f[0], f0[0], n, hf, t_o)[-1]]
    for i in range(D):
        vsol.append(selected_method(f[i], f0[i], n, hf, t_o)[0])

    return vsol
def hamiltonian_solve(f,g, f0 = [1],g0=[1],  n = 100, h = 0.1,hf = 100, D = 1, t_o = 0, method = "Euler"):
  if method == "Euler":
    if D==1:
      vsolp, vsolq, T = euler2(f[0], f0[0], g0[0], n, hf, t_o)
      return (vsolp), (vsolq), (T)
    else:
      vsolp, vsolq, T = euler2(f[0], f0[0], g0[0], n, hf, t_o)
      for i in range(D):
        eul = euler2(f[i], f0[i], g0[i], n, hf, t_o)
        vsolp.append(eul[0])
        vsolq.append(eul[1])

      return vsolp, vsolq, T
  elif method == "RK2":
    if D==1:
```

```
  if D==1:
    vsolp, vsolq, T = rk2_2(f[0], f0[0], g0[0], n, hf, t_o)
    return vsolp, vsolq, T
  else:
    vsolp, vsolq, T = rk2_2(f[0], f0[0], g0[0], n, hf, t_o)
    for i in range(D):
      eul = rk2_2(f[i], f0[i], g0[i], n, hf, t_o)
      vsolp.append(eul[0])
      vsolq.append(eul[1])

    return vsolp, vsolq, T
elif method == "RK4":
  if D==1:
    vsolp, vsolq, T = rk4_2(f[0], f0[0], g0[0], n, hf, t_o)
    return vsolp, vsolq, T
  else:
    vsolp, vsolq, T = rk4_2(f[0], f0[0], g0[0], n, hf, t_o)
    for i in range(D):
      eul = rk4_2(f[i], f0[i], g0[i], n, hf, t_o)
      vsolp.append(eul[0])
      vsolq.append(eul[1])

    return vsolp, vsolq, T
elif method == "SV":
  solp, solq, T = StormerV(f[0], f0[0], g0[0], n, h, t_o)
  if D == 1:
    return solp, solq, T
  else:
    for i in range(D):
      sverle = StormerV(f[i], f0[i], g0[i], n, h, t_o)
      solp.append(sverle[0])
      solq.append(sverle[1])
    return solp, solq, T
elif method == "SE":
  solp, solq, T = SEuler(f[0], g[0], f0[0], g0[0], n, h, t_o)
  if D ==1:
    return solp, solq, T
  else:
```

```
    for i in range(D):
      seu = SEuler(f[i], g[i], f0[i], g0[i], n, h, t_o)
      solp.append(seu[0])
      solq.append(seu[1])

  return solp, solq, T
else:
  return "None"
```

Project 1

1. Code tests for Euler, RK2, and RK4 using simple population model

$$\frac{dP}{dt}(t) = (B - D)P(t),\ P(0) \equiv P_o$$

For this population model, $B_1 = 0.57, D_1 = 0.88$ & $B_2 = 0.99, D_2 = 0.45$.

- The following results were obtained for various h (step sizes), and it's shown how given h sufficiently small, one can obtain an arbitrarily good agreement with the function $P(t) = A\exp((B - D)t)$, the exact solutions of the previous function, by graphing a series of plots obtained by the different methods (Euler, RK2, RK4), in contrast with the exact solutions, first using B_1, D_1

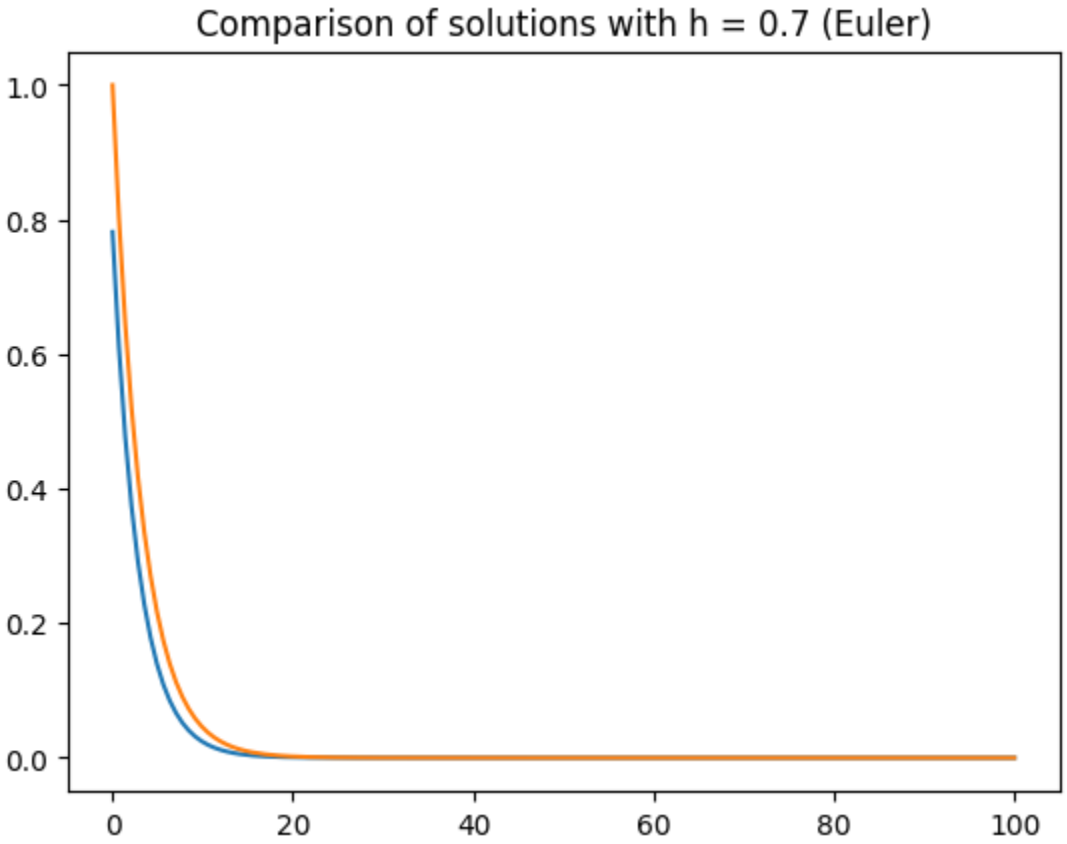
The blue line is the approximation and the orange line is the analytical solution.

```
b1 = 0.57
b2 = 0.99
d1 = 0.88
d2 = 0.45

sol = dynamics_solve([lambda t,x: ((b1-d1)*x)], n  = 142)
```

```
plt.title("Comparison of solutions with h = 0.7 (Euler)")
y =[np.exp((b1-d1)*t) for t in sol[0]]
plt.plot(sol[0],sol[1])
plt.plot(sol[0],y)
```

```
[<matplotlib.lines.Line2D at 0x7cf4710d1360>]
```

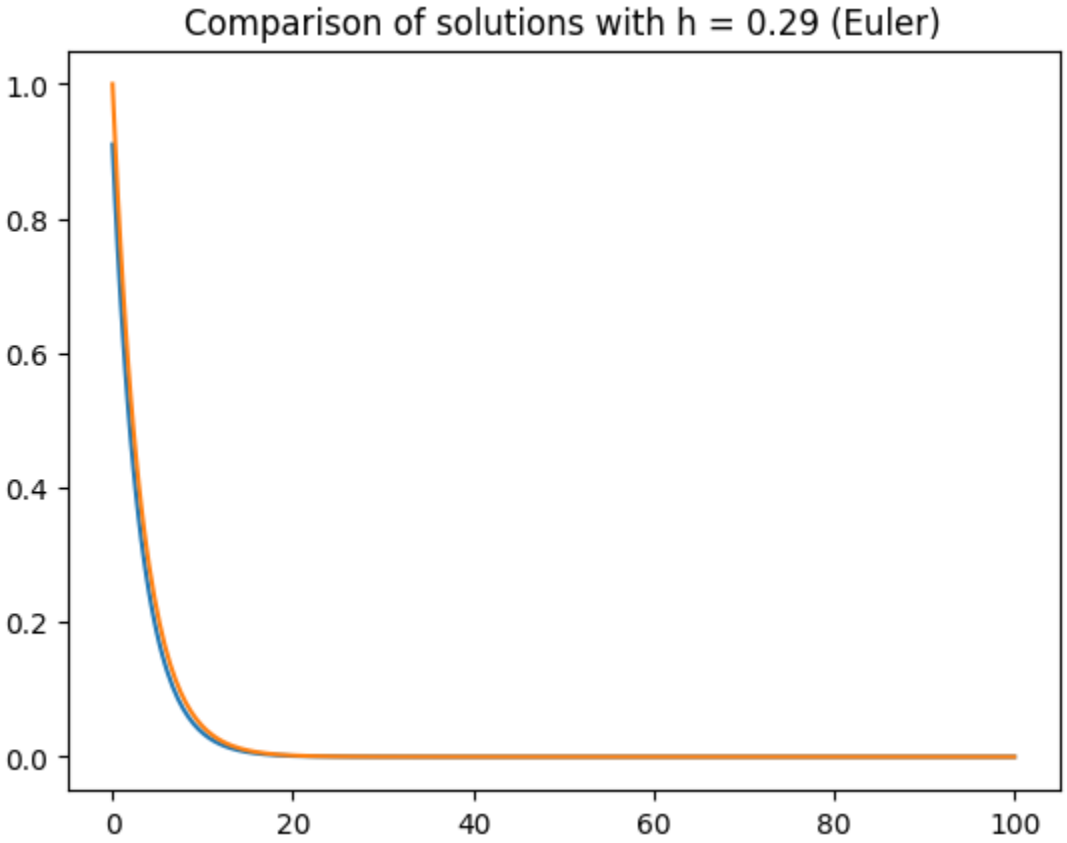



```
sol = dynamics_solve([lambda t,x: ((b1-d1)*x)], n=345)

plt.title("Comparison of solutions with h = 0.29 (Euler)")
y =[np.exp((b1-d1)*t) for t in sol[0]]
plt.plot(sol[0],sol[1])
```

```
plt.plot(sol[0],y)
```

```
[<matplotlib.lines.Line2D at 0x7cf4840b0580>]
```

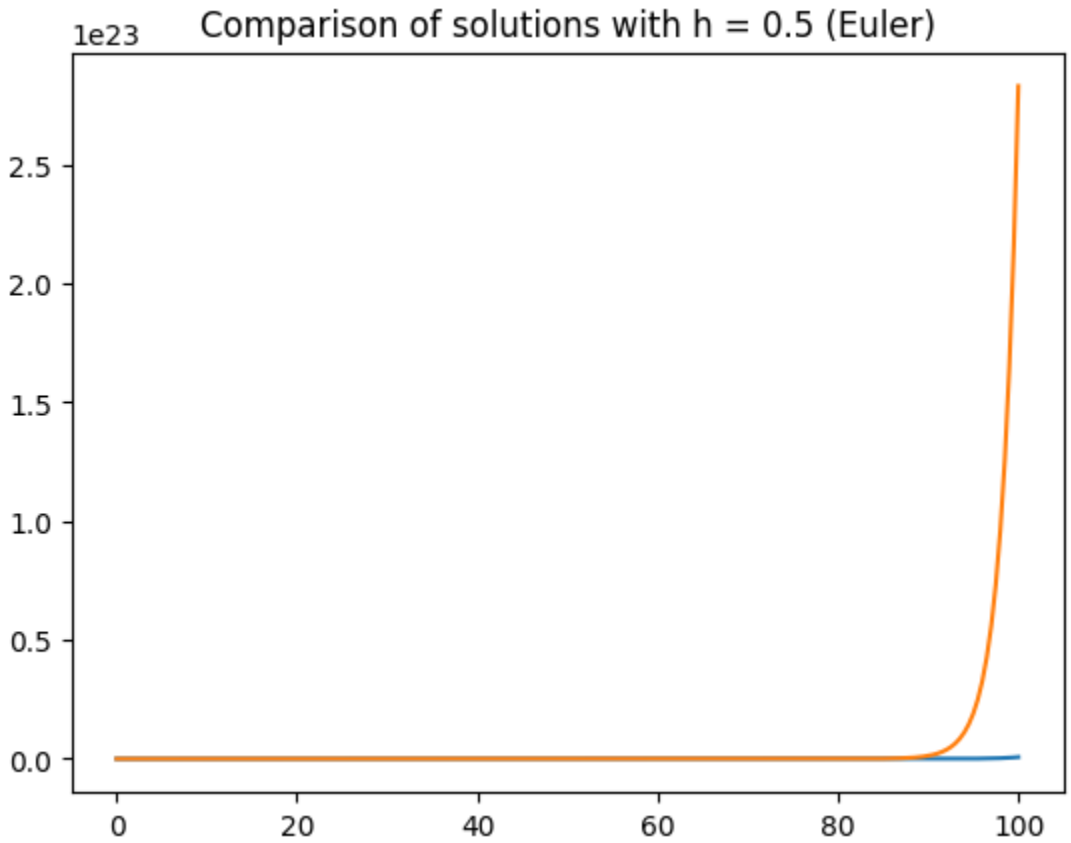


Then using B_2, D_2.

```
sol = dynamics_solve([lambda t,x: ((b2-d2)*x)], n = 200)

plt.title("Comparison of solutions with h = 0.5 (Euler)")
y =[np.exp((b2-d2)*t) for t in sol[0]]
plt.plot(sol[0],sol[1])
plt.plot(sol[0],y)
```

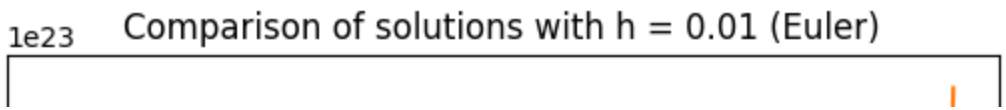
```
[<matplotlib.lines.Line2D at 0x7cf470f71c00>]
```

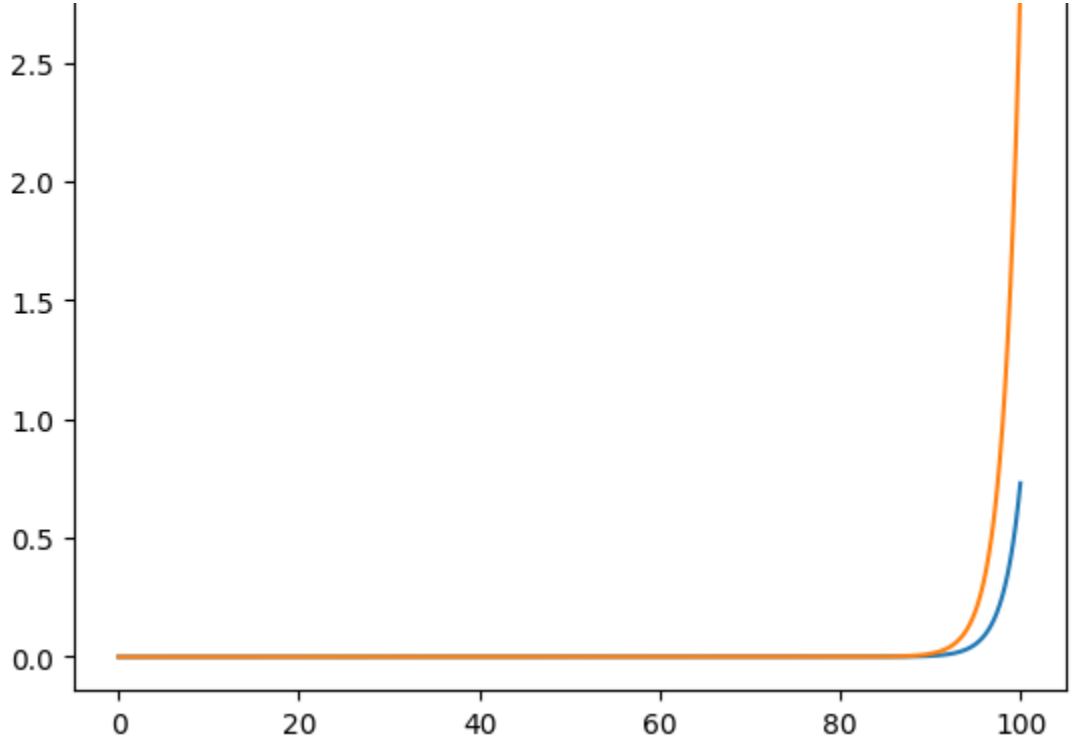



```
sol = dynamics_solve([lambda t,x: ((b2-d2)*x)], n = 1000)

plt.title("Comparison of solutions with h = 0.01 (Euler)")
y =[np.exp((b2-d2)*t) for t in sol[0]]
plt.plot(sol[0],sol[1])
plt.plot(sol[0],y)
```

```
[<matplotlib.lines.Line2D at 0x7cf470feb7c0>]
```

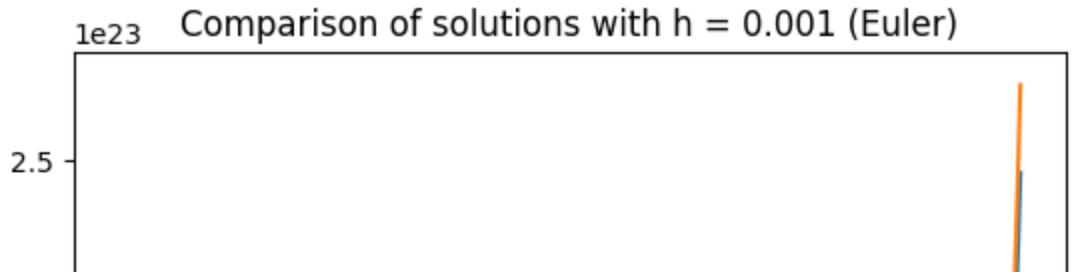


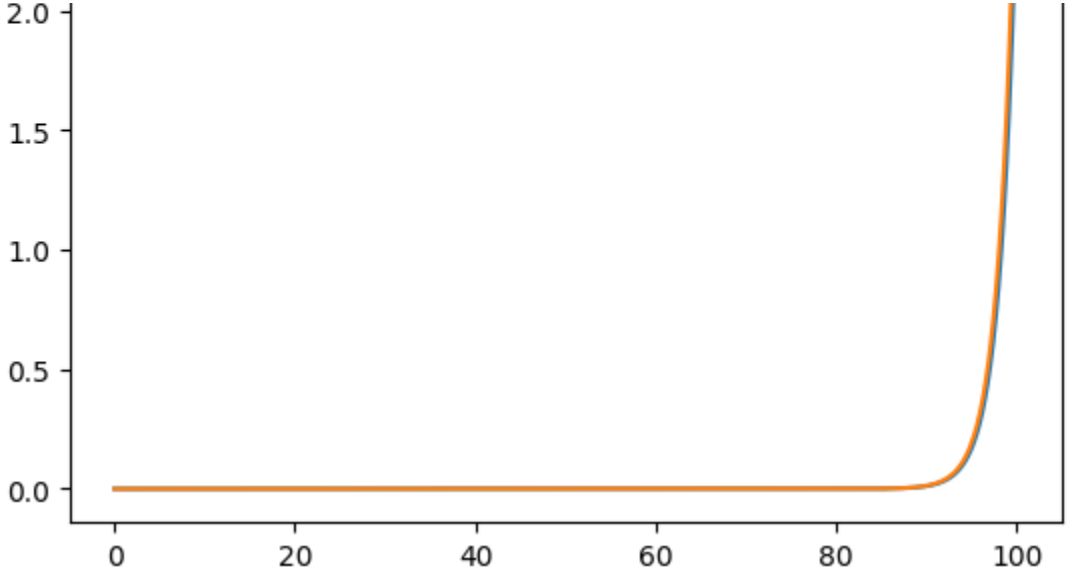



```
sol = dynamics_solve([lambda t,x: ((b2-d2)*x)], n = 10000)

plt.title("Comparison of solutions with h = 0.001 (Euler)")
y =[np.exp((b2-d2)*t) for t in sol[0]]
plt.plot(sol[0],sol[1])
plt.plot(sol[0],y)
```

```
[<matplotlib.lines.Line2D at 0x7cf470d807c0>]
```





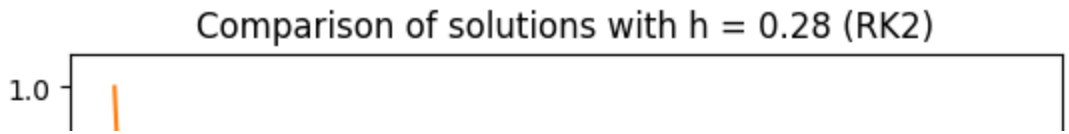
It can be shown that for values of h sufficiently close to 0, there will be an increasingly better aproximation using Euler. Particularly, $h << |B - D|$ for convergence to start appearing.

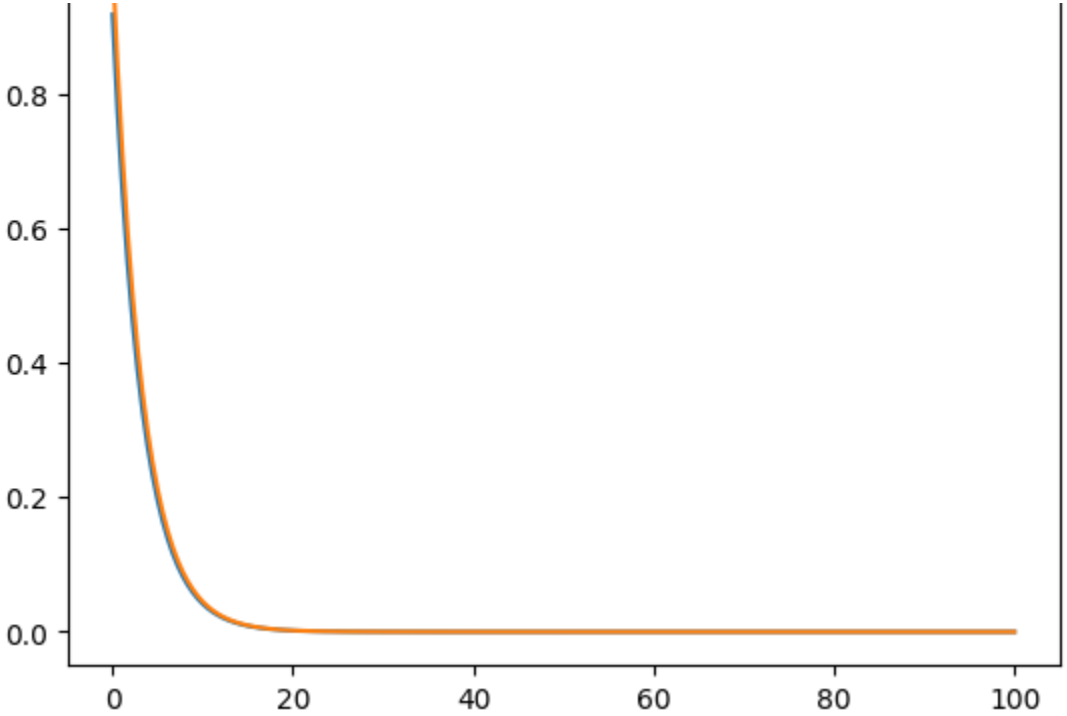
This process holds true for RK2.

```
sol = dynamics_solve([lambda t,x: ((b1-d1)*x)], n = 360, method = "RK2")
```

```
plt.title("Comparison of solutions with h = 0.28 (RK2)")
y =[np.exp((b1-d1)*t) for t in sol[0]]
plt.plot(sol[0],sol[1])
plt.plot(sol[0],y)
```

```
[<matplotlib.lines.Line2D at 0x7cf470dde500>]
```

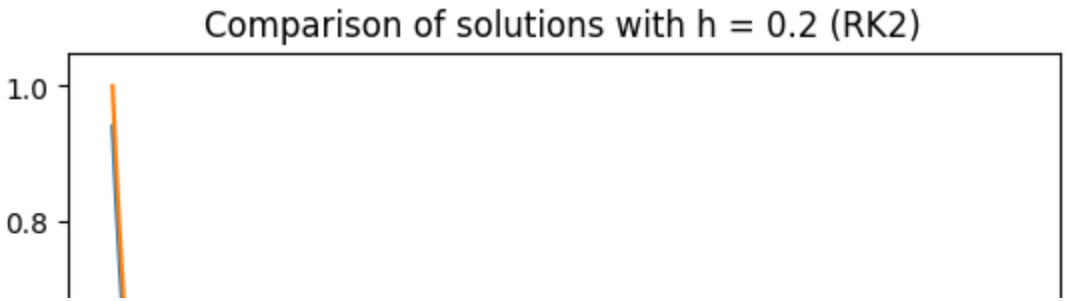


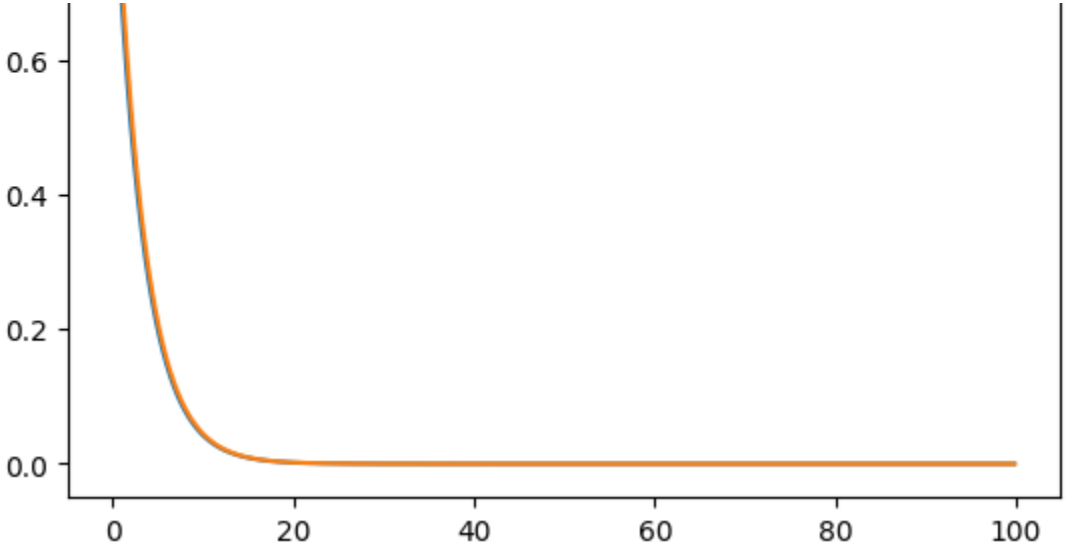



```
sol = dynamics_solve([lambda t,x: ((b1-d1)*x)], n = 500, method = "RK2")

plt.title("Comparison of solutions with h = 0.2 (RK2)")
y =[np.exp((b1-d1)*t) for t in sol[0]]
plt.plot(sol[0],sol[1])
plt.plot(sol[0],y)
```

```
[<matplotlib.lines.Line2D at 0x7cf470d17820>]
```

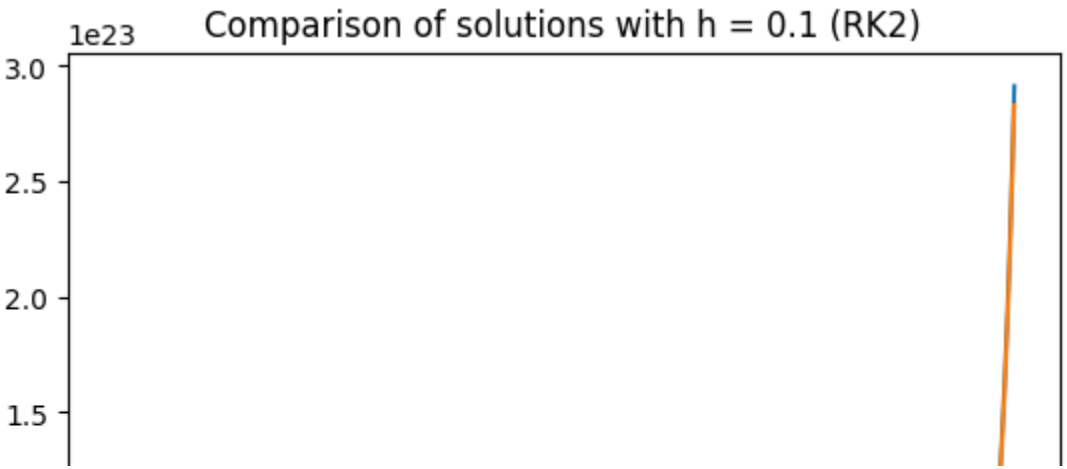


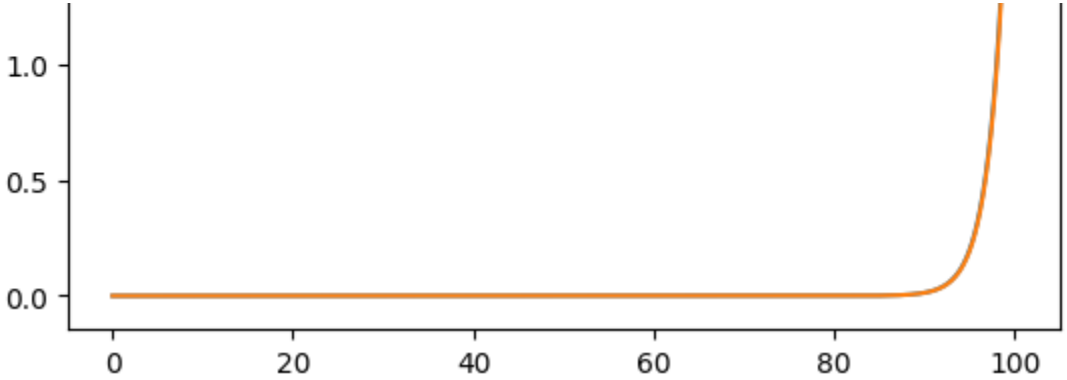



```
sol = dynamics_solve([lambda t,x: ((b2-d2)*x)], n = 1000, method = "RK2")

plt.title("Comparison of solutions with h = 0.1 (RK2)")
y =[np.exp((b2-d2)*t) for t in sol[0]]
plt.plot(sol[0],sol[1])
plt.plot(sol[0],y)
```

```
[<matplotlib.lines.Line2D at 0x7cf470ecc820>]
```

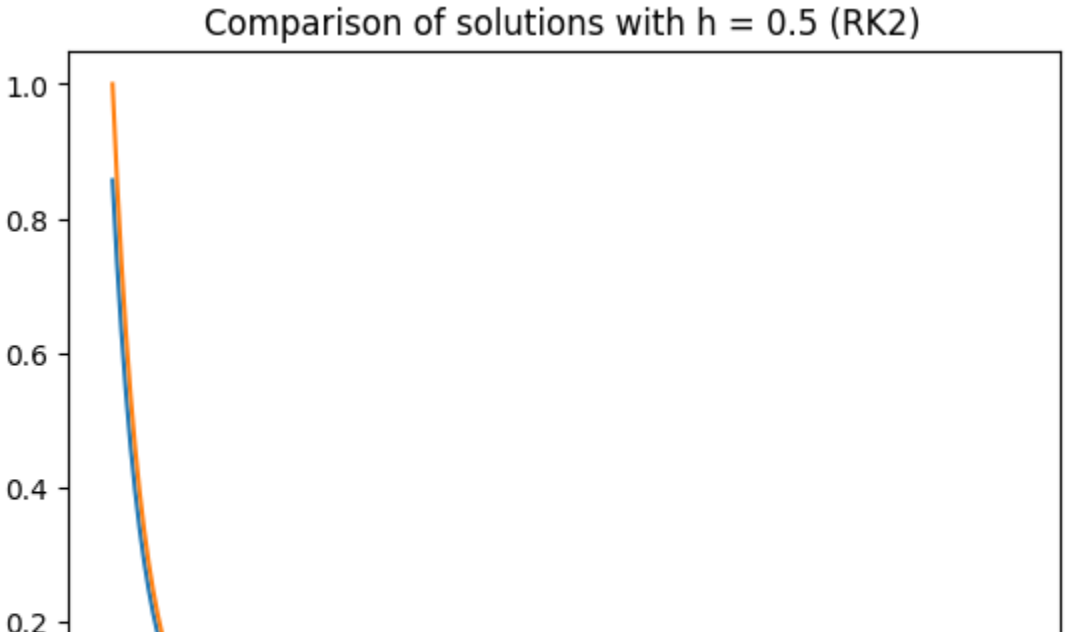





```
sol = dynamics_solve([lambda t,x: ((b1-d1)*x)], n = 200, method = "RK2")

plt.title("Comparison of solutions with h = 0.5 (RK2)")
y =[np.exp((b1-d1)*t) for t in sol[0]]
plt.plot(sol[0],sol[1])
plt.plot(sol[0],y)
```

```
[<matplotlib.lines.Line2D at 0x7cf470c80190>]
```



```
sol = dynamics_solve([lambda t,x: ((b1-d1)*x)], n = 5000, method = "RK2")

plt.title("Comparison of solutions with h = 0.007 (RK2)")

y =[np.exp((b1-d1)*t) for t in sol[0]]
plt.plot(sol[0],sol[1])
plt.plot(sol[0],y)
```

And RK4.

```
sol = dynamics_solve([lambda t,x: ((b1-d1)*x)], n = 357, method = "RK4")


plt.title("Comparison of solutions with h = 0.28 (RK4)")
y =[np.exp((b1-d1)*t) for t in sol[0]]
plt.plot(sol[0],sol[1])
plt.plot(sol[0],y)
```

```
sol = dynamics_solve([lambda t,x: ((b1-d1)*x)], n = 2000, method = "RK4")

plt.title("Comparison of solutions with h = 0.05 (RK4)")
y =[np.exp((b1-d1)*t) for t in sol[0]]
plt.plot(sol[0],sol[1])
plt.plot(sol[0],y)
```

```
sol = dynamics_solve([lambda t,x: ((b1-d1)*x)], n = 3000, method = "RK4")

plt.title("Comparison of solutions with h = 0.03 (RK4)")

y =[np.exp((b1-d1)*t) for t in sol[0]]
plt.plot(sol[0],sol[1])
plt.plot(sol[0],y)
```

For RK4, there were cases (where $h < 0.28$) in which the values of the analytic solution and the approximation were indistinguishable.

2. Code tests for Euler, RK2, Symplectic Euler, Stormer Verlet and RK4 for the system:

$$\frac{dp}{dt}(t) + kx = 0$$
$$\frac{dx}{dt}(t) - \frac{p}{m} = 0$$

Comienza a programar o generar con IA.

Tests for fixed N of oscillation periods

We shall compare the graphs of the solution using Symplectic Euler and the analytical version

The blue line is the approximation and the orange line is the analytical solution

Symplectic Euler

```
sola = hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0=[1], n= 100, h= 0.5, D = 1, t_o = 0, method =
cosx = [np.cos(i) for i in sola[2]]
sinx = [3*np.sin(i) for i in sola[2]]
plt.title("solution for h=0.5")
plt.plot(sola[1],sola[0])
plt.plot(cosx,sinx)
```

```
sola = hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0=[1], n = 100, h = 0.1,D = 1, t_o = 0, method
cosx = [np.cos(i) for i in sola[2]]
sinx = [3*np.sin(i) for i in sola[2]]
plt.title("solution for h=0.1")
plt.plot(sola[1],sola[0])
plt.plot(cosx,sinx)
```

```
sola = hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0=[1], h = 0.01, n = 1000, D = 1, t_o = 0, meth
cosx = [np.cos(i) for i in sola[2]]
sinx = [3*np.sin(i) for i in sola[2]]
plt.title("solution for h=0.01")
plt.plot(sola[1],sola[0])
plt.plot(cosx,sinx)
```

We can observe convergence in h=0.1 already.

Stormer Verlet

```
sola =hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0=[1], h= 0.1, n = 100, D = 1, t_o = 0, method =
plt.title("Solution for h=0.1")
cosx = [1.75*np.cos(i) for i in sola[2]]
sinx = [np.sin(i) for i in sola[2]]

plt.plot(sola[1],sola[0])
plt.plot(cosx,sinx)
```

```
sola =hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0=[1], h= 0.05, n = 1000, D = 1, t_o = 0, method
plt.title("Solution for h=0.05")
cosx = [1.75*np.cos(i) for i in sola[2]]
sinx = [np.sin(i) for i in sola[2]]

plt.plot(sola[1],sola[0])
plt.plot(cosx,sinx)
```

```
sola =hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0=[1], h= 0.001, n = 10000, D = 1, t_o = 0, meth
plt.title("Solution for h=0.001")
cosx = [1.75*np.cos(i) for i in sola[2]]
sinx = [np.sin(i) for i in sola[2]]
plt.plot(sola[1],sola[0])
plt.plot(cosx,sinx)
```

```
sola =hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0 = [1], h= 0.0005, n = 10000, D = 1, t_o = 0, m

plt.title("Solution for h=0.0005")
cosx = [1.75*np.cos(i) for i in sola[2]]
sinx = [np.sin(i) for i in sola[2]]
plt.plot(sola[1],sola[0])
plt.plot(cosx,sinx)
```

Euler

```
solap =hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0 = [1], n = 100,hf = 10, D = 1, t_o = 0, metho

cosx = [1*np.cos(i) for i in solap[2]]
sinx = [1*np.sin(i) for i in solap[2]]
plt.title("solution for h = 0.1")
plt.plot(solap[0],solap[1])
plt.plot(cosx,sinx)
```

```
solap =hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1], g0 = [1], n = 1000,hf = 10, D = 1, t_o = 0, met

cosx = [1*np.cos(i) for i in solap[2]]
sinx = [1*np.sin(i) for i in solap[2]]
plt.title("solution for h = 0.01")
plt.plot(solap[0],solap[1])
plt.plot(cosx,sinx)
```

```
solap =hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0=[1], n = 10000,hf = 10, D = 1, t_o = 0, metho

cosx = [1*np.cos(i) for i in solap[2]]
sinx = [1*np.sin(i) for i in solap[2]]
plt.title("solution for h = 0.001")
plt.plot(solap[0],solap[1])
plt.plot(cosx,sinx)
```

Runge-Kutta 2

```
solap =hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0 = [1], n = 100,hf = 10, D = 1, t_o = 0, metho

cosx = [1*np.cos(i) for i in solap[2]]
sinx = [1*np.sin(i) for i in solap[2]]
plt.title("solution for h = 0.1")
plt.plot(solap[0],solap[1])
plt.plot(cosx,sinx)
```

```
solap =hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0 = [1], n = 1000,hf = 10, D = 1, t_o = 0, meth

cosx = [1*np.cos(i) for i in solap[2]]
sinx = [1*np.sin(i) for i in solap[2]]
plt.title("solution for h = 0.01")
plt.plot(solap[0],solap[1])
plt.plot(cosx,sinx)
```

```
solap =hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0 = [1], n = 10000,hf = 10, D = 1, t_o = 0, met
```

```
cosx = [1*np.cos(i) for i in solap[2]]
sinx = [1*np.sin(i) for i in solap[2]]
plt.title("solution for h = 0.01")
plt.plot(solap[0],solap[1])
plt.plot(cosx,sinx)
```

Runge-Kutta 4

```
solap =hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0 = [1], n = 100,hf = 10, D = 1, t_o = 0, metho
```

```
cosx = [1*np.cos(i) for i in solap[2]]
sinx = [1*np.sin(i) for i in solap[2]]
plt.title("solution for h = 0.1")
plt.plot(solap[0],solap[1])
plt.plot(cosx,sinx)
```

```
solap =hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0 = [1], n = 1000,hf = 10, D = 1, t_o = 0, meth

cosx = [1*np.cos(i) for i in solap[2]]
sinx = [1*np.sin(i) for i in solap[2]]
plt.title("solution for h = 0.01")
```

```
plt.title("solution for h = 0.01")
plt.plot(solap[0],solap[1])
plt.plot(cosx,sinx)
```

```
solap =hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0 = [1], n = 10000,hf = 10, D = 1, t_o = 0, met

cosx = [1*np.cos(i) for i in solap[2]]
sinx = [1*np.sin(i) for i in solap[2]]
plt.title("solution for h = 0.001")
plt.plot(solap[0],solap[1])
plt.plot(cosx,sinx)
```

Tests for increasing N, given h=0.1

Symplectic Euler

```
solap =hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0 = [1], n = 100,hf = 100, D = 1, t_o = 0, meth

plt.title("Solution for n=100")
```

```
plt.plot(sola[1],sola[0])
cosx = [2*np.cos(i) for i in sola[2]]
sinx = [np.sin(i) for i in sola[2]]
plt.plot(cosx, sinx)
```

```
solap =hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0 = [1], n = 500,hf = 100, D = 1, t_o = 0, meth

plt.title("Solution for n=500")
plt.plot(sola[1],sola[0])
cosx = [2*np.cos(i) for i in sola[2]]
sinx = [np.sin(i) for i in sola[2]]
```

```
plt.plot(cosx, sinx)
```

```
solap =hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0 = [1], n = 2000,hf = 100, D = 1, t_o = 0, met

plt.title("Solution for n=2000")
plt.plot(sola[1],sola[0])
cosx = [2*np.cos(i) for i in sola[2]]
sinx = [np.sin(i) for i in sola[2]]
plt.plot(cosx, sinx)
```

Stormer Verlet

```
solverl =hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0 = [1], n = 100,hf = 100, D = 1, t_o = 0, me

cosx = [1.75*np.cos(i) for i in solverl[2]]
sinx = [np.sin(i) for i in solverl[2]]
plt.title("solution for n=100")
plt.plot(solverl[1],solverl[0])
plt.plot(cosx,sinx)
```

```
solverl =hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0 = [1], n = 500,hf = 100, D = 1, t_o = 0, me

cosx = [1.75*np.cos(i) for i in solverl[2]]
sinx = [np.sin(i) for i in solverl[2]]
plt.title("solution for n=500")
plt.plot(solverl[1],solverl[0])
plt.plot(cosx,sinx)
```

```
solverl=hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0 = [1], n = 1000,hf = 100, D = 1, t_o = 0, me
cosx = [1.75*np.cos(i) for i in solverl[2]]
sinx = [np.sin(i) for i in solverl[2]]
plt.title("solution for n=1000")
plt.plot(solverl[1],solverl[0])
plt.plot(cosx,sinx)
```

```
solverl=hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0 = [1], n = 5000,hf = 100, D = 1, t_o = 0, me
cosx = [1.75*np.cos(i) for i in solverl[2]]
sinx = [np.sin(i) for i in solverl[2]]
plt.title("solution for n=5000")
plt.plot(solverl[1],solverl[0])
plt.plot(cosx,sinx)
```

Euler

```
solap = hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0 = [1], n = 100,hf = 10, D = 1, t_o = 0, meth

cosx = [1.5*np.cos(i) for i in solap[2]]
sinx = [np.sin(i) for i in solap[2]]
plt.title("solution for n = 100")
plt.plot(solap[0],solap[1])
plt.plot(cosx,sinx)
```

```
solap =hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0 = [1], n = 100,hf = 20, D = 1, t_o = 0, metho
cosx = [1.5*np.cos(i) for i in solap[2]]
sinx = [np.sin(i) for i in solap[2]]
plt.title("solution for n = 200")
plt.plot(solap[0],solap[1])
plt.plot(cosx,sinx)
```

Runge-kutta 2

```
solap =hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0 = [1], n = 100,hf = 10, D = 1, t_o = 0, metho
cosx = [1.5*np.cos(i) for i in solap[2]]
sinx = [1*np.sin(i) for i in solap[2]]
plt.title("solution for n = 100")
plt.plot(solap[0],solap[1])
plt.plot(cosx,sinx)
```

```
solap =hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0 = [1], n = 100,hf = 20, D = 1, t_o = 0, metho
cosx = [1.5*np.cos(i) for i in solap[2]]
sinx = [1*np.sin(i) for i in solap[2]]
plt.title("solution for n = 200")
plt.plot(solap[0],solap[1])
plt.plot(cosx,sinx)
```

Runge-Kutta 4

```
solap =hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0 = [1], n = 100,hf = 100, D = 1, t_o = 0, meth

cosx = [1*np.cos(i) for i in solap[2]]
sinx = [1*np.sin(i) for i in solap[2]]
plt.title("solution for n = 100")
plt.plot(solap[0],solap[1])
plt.plot(cosx,sinx)
```

```
solap =hamiltonian_solve([lambda t,q: -q*3],[lambda t,p: p/3], f0 = [1],g0 = [1], n = 100,hf = 20, D = 1, t_o = 0, metho

cosx = [1*np.cos(i) for i in solap[2]]
sinx = [1*np.sin(i) for i in solap[2]]
plt.title("solution for n = 200")
plt.plot(solap[0],solap[1])
plt.plot(cosx,sinx)
```

We start to see convergence for $n > 100$, fixed h for Stormer-Verlet and Symplectic Euler, but for Euler it seems to gain energy for fairly low instances of n. So far the worst (most deviating) one for this case is Euler. It starts to diverge with fairly low values

Howver, for increasingly lower values for h, there seems to be closed orbits for either Stormer/Symplectic Euler or Euler/RK2/RK4. This makes sense since the value of q_{i+1}, p_{i+1} depends linearly wrt h, n for Euler, so increasing n "gives" energy, but reducing h keeps the results fairly stable.

Out of the non symplectic methods the least bad one was Runge-Kutta 4, mainly because it takes more time correcting losses of energy.

Energy manipulation of different methods

I'll be checking the total energy of the system

$$E = \frac{p^2}{2m} + \frac{kx^2}{2}$$

Symplectic Euler

```
sola = SEuler(lambda t,p: p/3, lambda t,q: -3*q,f0=0,g0 = 1, h=0.1, n=1000)
eK = [(sola[0][i]**2)/6 + (sola[1][i]**2)*3/2 for i in range(len(sola[0]))]
plt.plot(sola[2],eK)
plt.ylim(ymin =0)
```

Stormer Verlet

```
sola =StormerV(lambda t,p: -3*p,f0=0,g0=1, h=0.1, n=2000)
eK = [(sola[0][i]**2)/6 + (sola[1][i]**2)*3/2 for i in range(len(sola[0]))]

plt.plot(sola[2],eK)
plt.ylim(ymin =0)
```

Euler

```
sola =euler2(lambda t,p: -3*p,f0=0,g0=1, hf=1000, n=100)
eK = [(sola[0][i]**2)/6 + (sola[1][i]**2)*3/2 for i in range(len(sola[0]))]
plt.plot(sola[2],eK)
```

Runge Kutta 2

```
sola =rk2_2(lambda t,p: -3*p,f0=0,g0=1, hf=90, n=100)
eK = [(sola[0][i]**2)/6 + (sola[1][i]**2)*3/2 for i in range(len(sola[0]))]
plt.plot(sola[2],eK)
```

Runge Kutta 4

```
sola =rk4_2(lambda t,p: -3*p,f0=0,g0=1, hf=100, n=100)
eK = [(sola[0][i]**2)/6 + (sola[1][i]**2)*3/2 for i in range(len(sola[0]))]
plt.plot(sola[2],eK)
```

Comparing Euler, RK2, RK4

```
sola =euler2(lambda t,p: -3*p,f0=0,g0=1, hf=100, n=100)
sola2 =rk2_2(lambda t,p: -3*p,f0=0,g0=1, hf=100, n=100)
sola3 =rk4_2(lambda t,p: -3*p,f0=0,g0=1, hf=100, n=100)
eK = [(sola[0][i]**2)/6 + (sola[1][i]**2)*3/2 for i in range(len(sola[0]))]
plt.plot(sola[2],eK)
eK2 = [(sola2[0][i]**2)/6 + (sola2[1][i]**2)*3/2 for i in range(len(sola[0]))]
plt.plot(sola[2],eK2)
eK3 = [(sola3[0][i]**2)/6 + (sola3[1][i]**2)*3/2 for i in range(len(sola[0]))]
plt.plot(sola[2],eK3)
```

Removing Euler

```
plt.plot(sola[2],eK2)
plt.plot(sola[2],eK3)
```

Removing RK2

```
plt.plot(sola[2],eK)
plt.plot(sola[2],eK3)
```

Removing RK4

```
plt.plot(sola[2],eK)
plt.plot(sola[2],eK2)
```

We see that for Stormer Verlet we get the expected result of total energy being conserved for high time signatures. There seems to be net energy at the beginning, which is attributed to boundary conditions. For Euler there are fluctuations in energy but these fluctuations seem to be bounded. The other methods blow up wrt time in different levels. The worst one is Euler, followed by RK2 and RK4 seems to emulate well the presence of changing energies, but still, the graph makes it seem as if the system is getting

energy out of nowhere.

However, for RK4 the condition of energy conservation is met after a while. That means the total energy gets progressively lower with respect to time (unlike Euler/RK2 where energy diverges).