Exercise 8 Solution

1. What is the Mean time to failure values of different RAID systems?
   - a. RAID 0 with 2 disks
   - b. RAID 2 with 2 disks
   - c. RAID 1 with 2 disks
   - d. RAID 1 with 3 disks
   - e. RAID 3 with 3 disks
   - f. RAID 4 with 3 disks
   - g. RAID 5 with 3 disks
   - h. RAID 6 with 5 disks

   Solution: Let's label the probability of one disk failure as p, and the mean time to failure of one individual disk is MTTF. P is between 0 and 1.

   RAID 0 with 2 disks and RAID 2 with 2 disks – The system fails if one of the disks fails. The probability that one of the two disks fails (disk A or disk B) is $p + p = 2p$. So, as failure probability doubles mean time to failure is halved = $\frac{1}{2} \times MTTF$.

   RAID 1 with 2 disks - The system fails if both of the disks fail at the same time. The probability that both disks fail (disk A and disk B) is $p * p = p^2$ as p is between 0 and 1, mean time to failure of the system will increase to $MTTF^2$.

   RAID 1 with 3 disks - The system fails if three of the disks fail at the same time. The probability that all disks fail (disk A and disk B and disk C) is $p * p * p = p^3$. So, the mean time to failure of the system is accordingly $MTTF^3$.

   RAID 3 with 3 disks- The system fails if 2 of the 3 disks fail at the same time. The probability that 2 disks fail is $p * p = p^2$. There are 3 different possible combinations of 2 disk failures (A, B; or A, C; or B, C), so the probability that any of the 2 disks out of these 3 disks fail is $3p^2$. The mean time to failure of the system is $\frac{1}{3} \times MTTF^2$.

   RAID 6 with 5 disks - The system fails if 3 out of the 5 disks fail at the same time. The probability that 3 disks fail is $p * p * p = p^3$. There are 10 different possible combinations of 3 disks failures out of 5 disks (A,B,C; or A,B,D; or A,B,E; A,C,D; or A,C,E; or A,D,E; or B,C,D; or B,C,E; or B,D,E; or C,D,E), so the probability that any of the 3 disks out of these 5 disks fail is $10p^3$. Mean time to failure of the system is then $\frac{1}{10} \times MTTF^3$.

2. Which of the following RAID configurations that we saw in class has the lowest disk space utilization? Your answer needs to have explanations with calculations for each case.
   - (1) RAID 0 with 2 disks
   - (2) RAID 1 with 2 disks
   - (3) RAID 3 with 3 disks

   Where does this lack of utilization of space go, i.e., where we can use such a configuration as it has some benefits gained due to the loss of space utilization?

   Solution:
   - In case 1, the space utilization is 100% because the two disks store contiguous blocks of a file in RAID 0.
   - In case 2, the space utilization is 50% because RAID 1 uses mirroring. MTTF increases so for cases where a disk can fail easily this is good. The system operates even when a disk fails.
   - In case 3, the space utilization is (3-1)/3=66.7 because RAID 3 uses one disk for storing parity data.

   Case 2 has the lowest disk space utilization with an explanation as given underlined above as a part of the answer.

3. Aries Example: After a crash, we find the following log. What will be the analysis, redo, and undo phases?

   10 T1: UPDATE P1 (OLD: YYY NEW: ZZZ)
   15 T2: UPDATE P3 (OLD: UUU NEW: VVV)
   20 BEGIN CHECKPOINT
   25 END CHECKPOINT (XACT TABLE=[[T1,10],[T2,15]]; DPT=[[P1,10],[P3,15]])
   30 T1: UPDATE P2 (OLD: WWW NEW: XXX)
   35 T1: COMMIT
   40 T2: UPDATE P1 (OLD: ZZZ NEW: TTT)
   45 T2: ABORT
   50 T2: CLR P1(ZZZ), undonextLSN=15

   Solution:

   Analysis phase:

   Scan forward through the log starting at LSN 20.
   LSN 25: Initialize XACT table with T1 (LastLSN 10) and T2 (LastLSN 15). Initialize DPT to P1 (RecLSN 10) and P3 (RecLSN 15).
   LSN 30: Add (T1, LSN 30) to XACT table. Add (P2, LSN 30) to DPT.
   LSN 35: Change T1 status to ”Commit” in XACT table. Set LastLSN=35 for T1 in XACT table.
   LSN 40: Set LastLSN=40 for T2 in XACT table.
   LSN 45: Change T2 status to ”Abort” in XACT table. Set LastLSN=45 for T2 in XACT table.
   LSN 50: Set LastLSN=50 for T2 in XACT table.

   Redo phase:

   Scan forward through the log starting at LSN 10.
   LSN 10: Read page P1, check PageLSN stored in the page. If PageLSN<10, redo LSN 10 (set value to ZZZ) and set the page’s PageLSN=10.
   LSN 15: Read page P3, check PageLSN stored in the page. If PageLSN<15, redo LSN 15 (set value to VVV) and set the page’s PageLSN=15.
   LSN 30: Read page P2, check PageLSN stored in the page. If PageLSN<30, redo LSN 30 (set value to XXX) and set the page’s PageLSN=30.
   LSN 40: Read page P1 if it has been flushed, check PageLSN stored in the page. It will be 10. Redo LSN 40 (set value to TTT) and set the page’s PageLSN=40.
   LSN 50: Read page P1 if it has been flushed, check PageLSN stored in the page. It will be 40. Redo LSN 45 (set value to ZZZ) and set the page’s PageLSN=50.

   Undo phase:

   T2 must be undone. Put LSN 50 in ToUndo.
   LSN 50: Put LSN 15 in ToUndo
   LSN 15: Undo LSN 15 - write a CLR for P3 with ”set P3=UUU” and undonextLSN=NULL. Write UUU into P3.