1.

$$tr(XY) = \sum_{i=1}^{n} (XY)_{ii} = \sum_{i=1}^{n}\sum_{j=1}^{m} x_{ij} y_{ji} = \sum_{j=1}^{m}\sum_{i=1}^{n} y_{ji} x_{ij} = \sum_{j=1}^{m} (YX)_{jj} = tr(YX)$$

2.

Let $M = E(y)$ so $E(Ay) = AM$.

$$\begin{aligned} Var\, Ay &= E\left[(Ay-AM)(Ay-AM)^T\right] \\ &= E\left[A(y-M)[A(y-M)]^T\right] \\ &= E\left[A(y-M)(y-M)^T A^T\right] \\ &= AE\left[(y-M)(y-M)^T\right]A^T \\ &= A(Var\, y)A^T \end{aligned}$$

3.

(a) $y \sim MVN(M, V)$ $\quad Ay \sim MVN(AM, AVA^T)$

$$AM = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 2 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -8 \\ 15 \\ -7 \end{bmatrix}$$

$$AVA^T = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 2 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 4 \end{bmatrix}\begin{bmatrix} 1 & -3 & 2 \\ -3 & 2 & -1 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 19 & -11 & 18 \\ -11 & 23 & -20 \\ 18 & -20 & 40 \end{bmatrix}$$

(b)

$$E[y^TAy] = tr(AV) + M^TAM = 11 + 55 = 66$$

$$AV = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 2 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 0 & -3 & 5 \\ -7 & 0 & -2 \\ 5 & 3 & 11 \end{bmatrix} \qquad tr(AV) = 11$$

$$M^TAM = \begin{bmatrix} -4 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & -3 & 2 \\ -3 & 2 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 1 \end{bmatrix} = 55$$

(c) y^TAy does not have a noncentral chi-square distribution because

$$AV \cdot AV = \begin{bmatrix} 0 & -3 & 5 \\ -7 & 0 & -2 \\ 5 & 3 & 11 \end{bmatrix}\begin{bmatrix} 0 & -3 & 5 \\ -7 & 0 & -2 \\ 5 & 3 & 11 \end{bmatrix} = \begin{bmatrix} 46 & 15 & 61 \\ -10 & 15 & -57 \\ 34 & 18 & 140 \end{bmatrix} \neq AV,$$

then AV is not idempotent

, which does not satisfy the requirement of noncentral chi-square distribution.

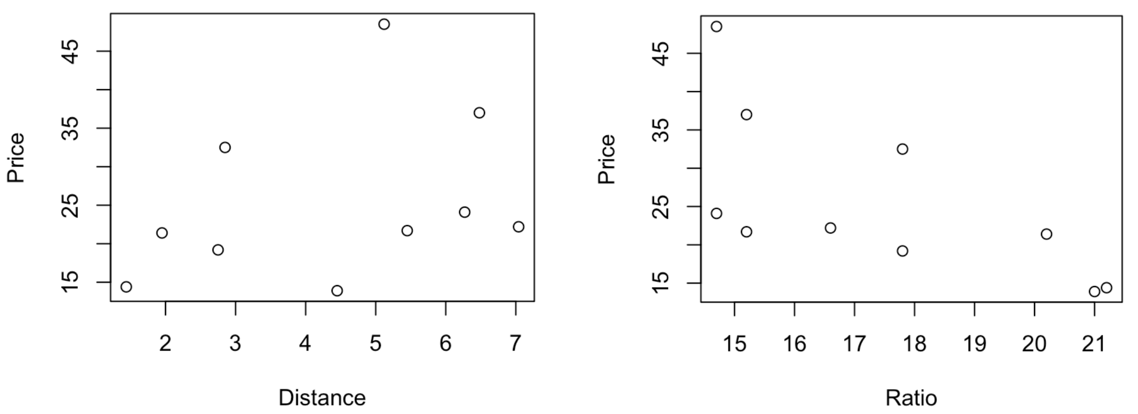
4.

(a)

```
Price=c(37.0,32.5,48.5,13.9,14.4,24.1,22.2,21.7,19.2,21.4)
Distance=c(6.48,2.85,5.12,4.45,1.44,6.27,7.04,5.45,2.75,1.95)
Ratio=c(15.2,17.8,14.7,21.0,21.2,14.7,16.6,15.2,17.8,20.2)


#(a)
plot(Distance,Price)

plot(Ratio,Price)
```



(b)

$$y=\begin{bmatrix}37.0\\32.5\\48.5\\13.9\\14.4\\24.1\\22.2\\21.7\\19.2\\21.4\end{bmatrix}_{1\times10} \quad X=\begin{bmatrix}1 & 6.48 & 15.2\\1 & 2.85 & 17.8\\1 & 5.12 & 14.7\\1 & 4.45 & 21.0\\1 & 1.44 & 21.2\\1 & 6.27 & 14.7\\1 & 7.04 & 16.6\\1 & 5.45 & 15.2\\1 & 2.75 & 17.8\\1 & 1.95 & 20.2\end{bmatrix}_{10\times3} \quad \beta=\begin{bmatrix}\beta_0\\\beta_1\\\beta_2\end{bmatrix}_{3\times1} \quad \varepsilon=\begin{bmatrix}\varepsilon_1\\\varepsilon_2\\\varepsilon_3\\\varepsilon_4\\\varepsilon_5\\\varepsilon_6\\\varepsilon_7\\\varepsilon_8\\\varepsilon_9\\\varepsilon_{10}\end{bmatrix}_{1\times10}$$

$$y=X\beta+\varepsilon$$

(c) Yes, The model is a full rank model. we can see X as a matrix made of three column vectors (x_1, x_2, x_3), and these three vectors can not be expressed as $a_1x_1+a_2x_2+a_3x_3=0$ when $a_1, a_2, a_3 \neq 0$. So, it is a full rank model. The rank of X is equal to the number of its columns.

$[x_1 \; x_2 \; x_3]\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}=0$ the equation only can be satisfied when $a_1=a_2=a_3=0$

```
> rankMatrix(X)
[1] 3
```

(d)

```
> #(d)
> y=matrix(Price,10,1)
> constant=c(rep(1,10))
> X=cbind(matrix(constant),matrix(Distance),matrix(Ratio))
> b=solve(t(X)%*%X,t(X)%*%y)
> b
           [,1]
[1,] 102.058797
[2,]  -1.937970
[3,]  -3.903698
```

(e)

```
> y_hat=c(1,3,17.5)%*%b
> y_hat
         [,1]
[1,] 27.93018
```

So the predicted price of house would be $2793018s.