Lab Test

Name : _____ Student Number : _____

This is an open-book test. You are permitted to use any notes, computer or calculator **except no access is permitted to any lab/workshop-related material, including the sample lab test and answers, either online or any kind of copy. You may access general material on R or Mathematica. You are not permitted to communicate with other students during the test.** Please write your answers in the boxes provided. **The total marks are 100. Your raw mark of this exam will be multiplied with 0.1 before being added to your final subject mark.**

Use Mathematica and/or R to complete the questions, as appropriate. Simplify results when possible. Give fractions or 4 decimal places for numerical answers.

Question 1.

Let X be a random variable with the pmf

$$Pr(X = k) = \frac{k^2}{73810}, \quad k = 1, 2, \ldots, 60.$$

(a). Find the probability $\Pr(30 \leq X \leq 50)$. [6]

Answer:

(b). Find the mean $E(X)$. [5]

Answer:

(c). Find the mean $E(1/X^2)$. [5]

Answer:

(d). Let $M_X(t)$ be the moment generating function of X. Find $M_X(-0.05)$. [6]

Answer:

Question 2.

Let a continuous random variable X have the following pdf

$$f(x) = \begin{cases} e^{-2x} + e^{-3x} + e^{-6x}, & x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a). Find the cdf $F(x)$ of X. **[6]**

Answer:

(b). Find the probability $\Pr(0.5 < X < 0.9)$. **[5]**

Answer:

(c). Find the mgf of X, $M_X(t)$. **[6]**

Answer:

(d). Find the mean $E(X)$. **[5]**

Answer:

(e). Find the third moment $E(X^3)$. **[6]**

Answer:

(f). Let $Y = e^X$.

    (i) Find the range (i.e., possible values) of Y. **[5]**

    Answer:

    (ii) Find the pdf $g(y)$ of Y. **[6]**

    Answer:

Question 3.

The R dataset named "cars" gives the stopping distance (feet) in the variable "dist" of 50 cars from the 1920's at varying speeds (miles per hour), recorded in the variable "speed". *Hint:* To access the dataset the R command "attach(cars)" will enable you refer to dist and speed without prefixing them with the cars dataframe and the $ sign.

(a). Do boxplots of the two variables and use these to briefly describe the two distributions. **[6]**

Answer:

(b). Find the minimum, quartiles, median and maximum for the "dist" variable. **[5]**

Answer:

(c). Find the mean and standard deviation of the "speed" variable. **[5]**

Answer:

(d). Plot dist versus speed (i.e., dist on y-axis, speed on x-axis). Comment on the plot. **[6]**

Answer:

(e). Find the intercept and slope of the line of best fit for dist versus speed. **[5]**

Answer:

(f). Plot the residuals of the fitted model versus speed. Comment on the plot. **[6]**

Answer:

(g). Find the predicted value of the stopping distance when the speed is 30. **[6]**

Answer:

Total marks = 100

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