MAST90105 Methods of Mathematical Statistics, Semester 1 2019

Lab Test

Name :____________________ Student Number : ____________________

This is an open-book test. You are permitted to use any notes, computer or calculator **except no access is permitted to any lab/workshop-related material, including the sample lab test and answers, either online or any kind of copy. You may access general material on R or Mathematica. You are not permitted to communicate with other students during the test.** Please write your answers in the boxes provided. **The total marks are 100. Your raw mark of this exam will be multiplied with 0.1 before being added to your final subject mark.**

Use Mathematica and/or R to complete the questions, as appropriate. Simplify results when possible. Give fractions or 4 decimal places for numerical answers.

Question 1.

Let X be a random variable with the pmf

$$Pr(X = k) = \frac{k^2}{73810}, \quad k = 1, 2, \ldots, 60.$$

(a). Find the probability $\Pr(30 \leq X \leq 50)$. [6]

Answer: 3437/7381 or 0.4657. In R: `x = 1:60;  pmf = x^2/73810;  p=sum(pmf[30:50])`

(b). Find the mean $E(X)$. [5]

Answer: 5490/121 or 45.3719. In R: `m1=sum(x*pmf)`

(c). Find the mean $\mathrm{E}(1/X^2)$. [5]

Answer: 6/7381 or 0.0008129. In R: `m2 = sum(pmf/x^2)`

(d). Let $M_X(t)$ be the moment generating function of X. Find $M_X(-0.05)$. [6]

Answer: 0.1262. In R: `mgf = sum(pmf*exp(-0.05*y))`

Question 2.

Let a continuous random variable X have the following pdf

$$f(x) = \begin{cases} e^{-2x} + e^{-3x} + e^{-6x}, & x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a). Find the cdf $F(x)$ of X. [6]

Answer:

$$F(x) = \begin{cases} 1 - \frac{1}{2}e^{-2x} - \frac{1}{3}e^{-3x} - \frac{1}{6}e^{-6x}, & x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

(b). Find the probability $\Pr(0.5 < X < 0.9)$. [5]

Answer: $P(0.5 < X < 0.9) = 0.1608$.

(c). Find the mgf of X, $M_X(t)$. [6]

Answer:

$$M_X(t) = \frac{1}{(2-t)} + \frac{1}{(3-t)} + \frac{1}{(6-t)}\,.$$

(d). Find the mean $E(X)$. [5]

Answer: $7/18$ or 0.3889.

(e). Find the third moment $E(X^3)$. [6]

Answer: $49/108$ or 0.4537.

(f). Let $Y = e^X$.

(i) Find the range (i.e., possible values) of Y. [5]

Answer: $1 < x < \infty$.

(ii) Find the pdf $g(y)$ of Y. [6]

Answer:

$$g(y) = \begin{cases} y^{-3} + y^{-4} + y^{-7}, & y > 1, \\ 0, & \text{otherwise.} \end{cases}$$

Question 3.

The R dataset named "cars" gives the stopping distance (feet) in the variable "dist" of 50 cars from the 1920's at varying speeds (miles per hour), recorded in the variable "speed". *Hint:* To access the dataset the R command "attach(cars)" will enable you refer to dist and speed without prefixing them with the cars dataframe and the $ sign.

(a). Do boxplots of the two variables and use these to briefly describe the two distributions. [6]

> Answer: any two sensible comments such as: speed is roughly symmetrically distributed while distance has an outlier. In R: `boxplot(c(dist, speed))`

(b). Find the minimum, quartiles, median and maximum for the "dist" variable. [5]

> Answer: Minimum is 2.000, first quartile is 26.00, median is 36.00, third quartile is 56.00, maximum is 120.0. In R: `summary(dist)`

(c). Find the mean and standard deviation of the "speed" variable. [5]

> Answer: the mean is 15.40 and the standard deviation is 5.288. In R: `mean(speed); sd(speed)`

(d). Plot dist versus speed (i.e., dist on y-axis, speed on x-axis). Comment on the plot. [6]

> Answer: Stopping distance increases with speed but there is a lot of variation in stopping distances for similar speed. In R: `plot(speed, dist)`

(e). Find the intercept and slope of the line of best fit for dist versus speed. [5]

> Answer: intercept is -17.5791, slope is 3.9324. In R: `out=lm(dist~speed); out$coef`

(f). Plot the residuals of the fitted model versus speed. Comment on the plot. [6]

> Answer: The residuals are more scattered for higher values of speed. In R: `plot(out$residuals, speed)`

(g). Find the predicted value of the stopping distance when the speed is 30. [6]

> Answer: 100.3932. In R: `sum(out$coef*c(1,30))`

`Total marks = 100`

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