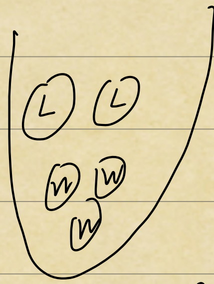
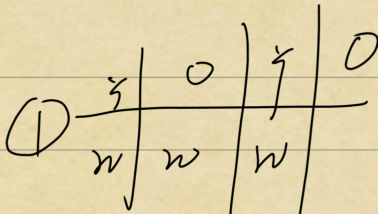


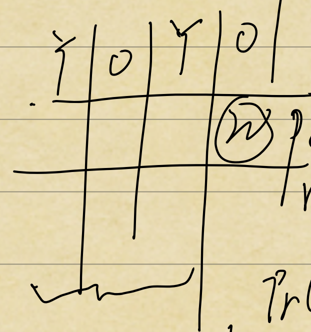
20 { 17 Lose, 3 win }

without replacement

① 

$\frac{3}{20}\times\frac{2}{19}\times\frac{1}{18}$

$= \frac{\binom{3}{3}\binom{17}{0}}{\binom{20}{3}} = \frac{1}{1140}$

② 

P {你先抽, 对手赢} =

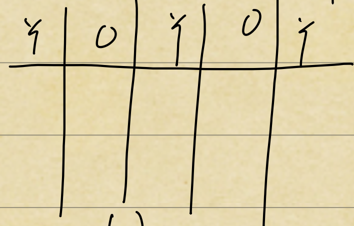
two win balls selected in 3 positions

Pr(前3个球有2个win且第四个是win)

$\frac{\binom{3}{2}\binom{17}{1}}{\binom{20}{3}} \times \frac{1}{17}$

$= \frac{1}{380}$

③ 第一个抓，赢的概率



Win 2nd → $\frac{(\ )}{\binom{20}{3}}$

3rd –

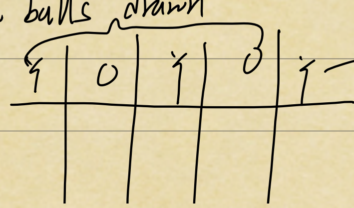
P(you win if draw first) (第1次不能赢)

$= \sum_{i=2}^{10} ($ $\underbrace{\text{if you draw first and win at ith draw}}_{B_i}$ $)$.

Note: On you ith draw, $2i-2$ balls have been previously drawn.

Your 3rd draw ⇒ 4个球被拿走了

4 balls drawn



→ 3rd draw

$\Rightarrow$ you want your ith draw to be winball
& in those $2i-2$ balls you want them have 2 winballs

$$P(B_i)=\frac{\binom{3}{2}\binom{17}{2i-4}}{\binom{20}{2i-2}}\times\frac{1}{20-(2i-2)}$$

$$\sum_{i=2}^{10}\frac{\binom{3}{2}\binom{17}{2i-4}}{\binom{20}{2i-2}}\times\frac{1}{22-2i}$$

$\downarrow$

1st draw can't win

let $\Rightarrow$ $k+1=i$

$\boxed{k=i-1}$ $i=k+1$

$$=\sum_{k=1}^{9}\frac{\binom{3}{2}\binom{17}{2k-2}}{\binom{20}{2k}}\cdot\frac{1}{20-2k}$$

④ 对手第一个抓你赢的概率

Work shop 3

x	-1	0	1
$f(x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$$\begin{cases} x=1 & f(x)=\frac{1}{4}\times 1=\frac{1}{4} \\ x=-1 & f(x)=\frac{1}{4} \\ x=0 & f(x)=\frac{1}{2} \end{cases}$$

mean $=(-\frac{1}{4})+(\frac{1}{4})=0$

X	1	2	3	4	5	6
$P_r(X)$	[illegible]	$\frac{5}{36}$	$\frac{1+3}{36}$	$\frac{1+2}{36}$	$\frac{1+1}{36}$	$\frac{1}{36}$

$\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{5}{6}\times 2$ (两个骰子排序，(5,1)(1,5)不同)

$\frac{11}{36}$ $\quad \frac{1}{36}+\frac{8}{36}=\frac{9}{36}$ $\quad \frac{1+6}{36}$ $\quad \frac{1+4}{36}$ $\quad \frac{1+2}{36}$

Y	0	1	2	3	4	5
Pr	$\frac{6}{36}$	$\boxed{\frac{10}{36}}$	$\frac{8}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{2}{36}$

6种

3.1 hypergeometric $= \dfrac{\binom{5}{x}\binom{95}{10-x}}{\binom{100}{10}}$ $\quad$ Hyper(5, 95, 10)

3.2 $\quad 1-\dfrac{\binom{95}{10}}{\binom{100}{10}}$

$E(X) = np = 10\times\frac{5}{100} = 0.5$

$= n\frac{N_1}{N_1+N_2}$

$\sigma^2 = Var(X) =$

$Var(X+4) = 16$

$Var(X) = Var(X+4) = 16$

5. 2b 2w X: # draws until last balk ball is obtined

$X = 2, 3, 4$

$\frac{4321}{2\times 2}$

$\frac{4!}{2!2!}$

2	3	4
$\frac{1}{\binom{4}{2}}$	$\frac{\binom{3}{1}\binom{1}{1}}{\binom{4}{2}} \times \frac{1}{2}$	$\frac{\binom{3}{2}\binom{1}{1}}{\binom{4}{3}} \times 1$
$\frac{1}{6}$	$\frac{4}{6} \times \frac{1}{2}$	$\frac{2}{4} \times 1$
$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$

$\frac{}{2\times 3}$ $\frac{}{4\times 5}$

$\frac{}{3\times 4}$

$P_r(X \geq 1) = 1 - f(x=0) = 1 - \frac{1}{2} = \frac{1}{2}$

$P_r(X \geq 4) = 1 - f(x \leq 3) = 1 - f(0) - f(1) - f(2) - f(3) = 1 - \frac{1}{2} - \frac{1}{6} - \frac{1}{12} - \frac{1}{20}$

$= \frac{1}{2} - \frac{2}{12} - \frac{1}{12} - \frac{1}{20}$

$= \frac{1}{2} - \frac{3}{12} - \frac{1}{20}$

$= \frac{3}{12} - \frac{1}{20}$

$= \frac{15}{60} - \frac{3}{60} = \frac{12}{60} = \frac{1}{5}.$

$P(X \geq 4 \mid X \geq 1) \frac{\frac{1}{5}}{\frac{1}{2}} = \frac{2}{5}$

$E(X) = \sum_{x=0}^{\infty} \frac{x}{(x+1)(x+2)} =$

$$F_{X|A}(x) = \begin{cases} 1 & x \geq b \\ \frac{F_X(x) - F_X(a)}{F_X(b) - F_X(a)} \text{(歪的老的)} & a \leq x < b \\ 0 & \text{therwise} \end{cases}$$

$-\left[e^{-x}\Big|_0^x\right]$

$$f_{X|A}(x) = \begin{cases} \frac{f_X(x)}{P(A)} & a \le x < b \\ 0 & \text{otherwise} \end{cases} \qquad \int_0^{x} e^{-x} = 1 - e^{-x}$$

$$f_{X|X>1} = \frac{f_X(x)}{\Pr(X>1)} = \frac{e^{-x}}{\int_1^{+\infty} e^{-x}} = \frac{e^{-x}}{-\int_1^{+\infty} -e^{-x}} = \frac{e^{-x}}{-[e^{-x}]_1^{+\infty}} = \frac{e^{-x}}{e^{-1}}$$

$$= e^{-x} \times e^{1} = e^{-x+1} \quad (x>1)$$

$$F_X = \frac{F_X(x)-F_X(1)}{F_X(+\infty)-F_X(1)} = \frac{F_X(x)}{F(+\infty)} = \frac{1-e^{-x}}{e^{-1}}$$

$$\int_1^x \frac{e^{-x}}{e^{-1}}\,dx = e\int_1^x e^{-x}\,dx = -e\left[e^{-x}\Big|_1^x\right] = -e\left[e^{-x} - e^{-1}\right]$$

$$= 1 - e^{-x+1}$$

Let X ~ Exponential(1).

a. Find the conditional PDF and CDF of X given X > 1.
b. Find E[X|X > 1].
c. Find Var(X|X > 1).

b. $$E[X|X>1] = \int_1^{\infty} x f_{X|A(x)}(x)\,dx = e\int_1^{\infty} x e^{-x}\,dx =$$

$$e\int^{\infty} x(-e^{-x})'\,dx = e\left[-e^{-\infty}\infty - (-e^{-1}) - \int_1^{\infty}(-e^{-x})\,dx\right]$$

$$= e\left[e^{-1} + \int_1^\infty e^{-x}dx\right]$$

$$= \left[1 + e\left[-e^{-x}\Big|_1^\infty\right]\right]$$

$$= \left[1 + e\left[0 + e^{-1}\right]\right] = 2.$$

c. Find Var(X | X>1)

$$= E(X^2|X>1) - E^2(X|X>1)$$

$$= \int_1^\infty x^2 f_{X|A}(x)\,dx - 4$$

$$= \int_1^\infty x^2 \frac{e^{-x}}{e^{-1}}\,dx - 4$$

$$= e\int_1^\infty x^2 e^{-x}\,dx - 4$$

$$e\int_1^\infty x^2 (-e^{-x})'\,dx - 4$$

$$= e\left[\lim_{x\to\infty} x^2(-e^{-x}) - (-e^{-1}) - 2\int_1^\infty x(-e^{-x})\,dx\right] - 4$$

$$= e\left[e^{-1} - 2\int_1^\infty x\,(e^{-x})'\,dx\right] - 4$$

$$= e\left[e^{-1} - 2\left[\lim_{x\to\infty} x(e^{-x}) - e^{-1} - \int_1^\infty e^{-x}dx\right]\right] - 4$$

$$= \left[1 - 2e\left[0 - e^{-1} - \left(-e^{-x}\Big|_1^\infty\right)\right]\right] - 4$$

$$= \left[1 + 2 + 2e(0 + e^{-1})\right] - 4$$

= [illegible]

Example 5.21 Let X and Y be two jointly continuous random variables with joint PDF

$$f_{XY}(x,y) = \begin{cases} \frac{x^2}{4} + \frac{y^2}{4} + \frac{xy}{6} & 0 \leq x \leq 1, 0 \leq y \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

For $0 \leq y \leq 2$, find

a. the conditional PDF of X given $Y = y$;
b. $P(X < \frac{1}{2} | Y = y)$.

a. $f_{X|Y}(x|Y=y) \quad \dfrac{f_{XY}(x,y)}{f_Y(x,y)} = \dfrac{\frac{x^2}{4}+\frac{y^2}{4}+\frac{xy}{6}}{\int_0^1 \frac{x^2}{4}+\frac{y^2}{4}+\frac{xy}{6}\,dx}$

$$\frac{\frac{x^2}{4}+\frac{y^2}{4}+\frac{xy}{6}}{\frac{1}{12}x^3\Big|_0^1 + \frac{y^2}{4}x\Big|_0^1 + \frac{x^2}{12}y\Big|_0^1} = \frac{\frac{x^2}{4}+\frac{y^2}{4}+\frac{xy}{6}}{\frac{1}{12}+\frac{y^2}{4}+\frac{1}{12}y}, = \frac{3x^2+3y^2+2xy}{3y^2+y+1}$$

$0 \leq x \leq 1$

$$\text{PDF} = f_{X|Y}(x|y) \begin{cases} \dfrac{3x^2+3y^2+2xy}{3y^2+y+1} & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

b. $P(x < \frac{1}{2} | Y = y)$

$$\int_0^{\frac{1}{2}} \frac{3x^2+3y^2+2xy}{3y^2+y+1}\,dx = \frac{1}{3y^2+y+1}\int_0^{\frac{1}{2}} (3x^2+3y^2+2xy)\,dx$$

$$= \frac{1}{3y^2+y+1}\left[x^3\Big|_0^{\frac{1}{2}} + 3y^2x\Big|_0^{\frac{1}{2}} + x^2y\Big|_0^{\frac{1}{2}}\right]$$

$$= \frac{1}{3y^2+y+1}\left[\frac{1}{8} + \frac{3}{2}y^2 + \frac{1}{4}y\right]$$

Example 5.22
Let X and Y be as in Example 5.21. Find $E[X|Y=1]$ and $Var(X|Y=1)$.

$f_{xy} = \frac{x^2}{4} + \frac{y^2}{4} + \frac{xy}{6} \qquad 0 \le x \le 1 \qquad 0 \le y \le 2$

$E(X|Y=1) = \int_0^1 x f_{X|Y}(x|y=1)\,dx$

条件PDF

$f_{X|Y}(x|y=1) = \frac{f(x=x, y=1)}{f_Y(y=1)} = \frac{\frac{x^2}{4}+\frac{1}{4}+\frac{x}{6}}{\frac{2}{12}+\frac{1}{4}}$

写法 → 边缘分布PDF

$f_Y(y) = \int_0^1 \frac{x^2}{4}+\frac{y^2}{4}+\frac{xy}{6}\,dx = \frac{1}{12}x^3\Big|_0^1 + \frac{y^2}{4}x\Big|_0^1 + \frac{x^2}{12}y\Big|_0^1 = \frac{1}{12}+\frac{y^2}{4}+\frac{1}{12}y$

$= \int_0^1 x \cdot \frac{\frac{x^2}{4}+\frac{1}{4}+\frac{x}{6}}{\frac{5}{12}}\,dx = \frac{1}{5}\int_0^1 x(3x^2+3+2x)\,dx$

$= \frac{1}{5}\int_0^1 3x^3\,dx + \frac{1}{5}\int_0^1 3x\,dx + \frac{1}{5}\int_0^1 2x^2\,dx$

$= \frac{3}{5}\int_0^1 x^3\,dx + \frac{3}{5}\int_0^1 x\,dx + \frac{2}{5}\int_0^1 x^2\,dx$

$\frac{3}{20}x^4\Big|_0^1 + \frac{3}{10}x^2\Big|_0^1 + \frac{2}{15}x^3\Big|_0^1$

$= \frac{3}{20} + \frac{3}{10} + \frac{2}{15} =$

$= 0.15 + 0.3 + \frac{2}{15} = 0.45 + \frac{2}{15} = \frac{45}{100} + \frac{2}{15}$

$\frac{1}{12}$

$\frac{9}{20} + \frac{1}{15}$

$= \frac{27}{60} + \frac{8}{60} = \frac{35}{60}$

$$E[X^2|Y=1] = \int_0^1 x^2 f_{X|Y=1}(x|y=1) = \frac{1}{5}\int_0^1 x^2(3x^2+3+2x)\,dx$$

$$= \frac{1}{5}\int_0^1 3x^4 + 3x^2 + 2x^3\,dx$$

$$= \frac{3}{5}\left[\frac{1}{5}x^5\right] + \frac{1}{5}\left(x^3\Big|_0^1\right) + \left[\frac{2}{20}x^4\Big|_0^1\right]$$

$$= \frac{3}{25} + \frac{1}{5} + \frac{2}{20} = \frac{3}{25} + \frac{5}{25} + \frac{1}{10} = \frac{8}{25} + \frac{5}{50}$$

$$= \frac{21}{50}$$

$$E(X^2|Y=1) - E^2(X|Y=1) = \frac{21}{50} - \left(\frac{7}{12}\right)^2 =$$

$$f_Y(y) = \int_0^{+\infty} 2e^{-x-2y}\,dx = \int_0^{+\infty} 2e^{-x}\cdot e^{-2y}\,dx = 2e^{-2y}\int_0^{+\infty} e^{-x}\,dx$$

$$= 2e^{-2y}\left(-e^{-x}\Big|_0^{+\infty}\right) = 2e^{-2y}[0+1) = 2e^{-2y}$$

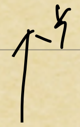
$$f_X(x) = \int_0^{+\infty} 2e^{-x-2y}\,dy = \int_0^{+\infty} 2e^{-x}\cdot e^{-2y}\,dy = 2e^{-x}\int_0^{+\infty} e^{-2y}\,dy = e^{-x}\left(-e^{-2y}\right)\Big|_0^{+\infty}$$

$$= e^{-x}$$

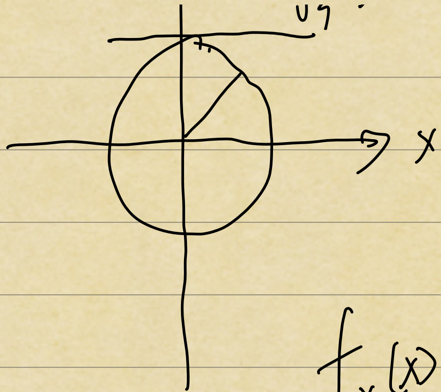
$$f_{XY}(x,y) = \begin{cases} 8xy & 0<x<y<1 \\ 0 & \text{otherwise} \end{cases} \Rightarrow f_Y(y) = \int_0^1 8xy\,dx = 8y\int_0^1 x\,dx = 4y\,x^2\Big|_0^1 = 4y$$

$$f_X(x) = \int_x^1 8xy\,dy = 8x\int_x^1 y\,dy = 4x\,y^2\Big|_x^1 = 4x(1-x^2) = 4x - 4x^3$$

$f(x)\cdot f(y) \neq 8xy$ Not independent



$$f_Y(y) = \int^{\sqrt{1-y^2}} \frac{1}{\pi}\,dx = \frac{1}{\pi}x\Big|^{\sqrt{1-y^2}} = \frac{2}{\pi}\sqrt{1-y^2}$$



$\int_{-\sqrt{1-y^2}}^{\sqrt{}}$ ∵ $1-\sqrt{1-y^2}$

$x^2 \le 1-y^2$

$x^2+y^2 \le 1$ ⇒ $y^2 \le 1-x^2$.

$$f_X(x)=\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{1}{\pi}\,dy = \frac{2}{\pi}\sqrt{1-x^2}.$$

Not:

$$f_{X|Y}(x|y)=\frac{f(x,y)}{f_Y(y)}=\frac{\frac{1}{\pi}}{\frac{2}{\pi}\sqrt{1-y^2}}=\frac{1}{2\sqrt{1-y^2}}\quad \{x^2+y^2\le 1.\}$$

$$-\frac{1}{\sqrt{1-y^2}} \le x \le \frac{1}{\sqrt{1-y^2}}$$

~~错~~求 $f_{X|Y}$ 给 x的 range
求 $f_{Y|X}$ 给 y的 range
错在上面 给值的 range

☆

Example 5.25

Let X and Y be two independent Uniform(0, 1) random variables. Find $P(X^3+Y>1)$.

Law of Total Probability:

$$P(X^3+Y>1)=\int_{-\infty}^{+\infty} P(X^3+Y>1 \mid X=x)\, f_X(x)\,dx=\int_0^1 P(x^3+Y>1)\,dx$$

$$=\int_0^1 P(Y>1-x^3)\,dx$$

$$\int_0^1 \left(1-P(Y\le 1-x^3)\right)=\int_0^1\left(1-\int_0^{1-x^3} 1\,dy\right)dx=\int_0^1\left[1-(1-x^3)\right]dx$$

$$=\int_0^1 x^3\,dx=\frac{1}{4}x^4\Big|_0^1=\frac{1}{4}$$

$= \int_0 \ldots$

☆

Example 5.26

Suppose $X \sim Uniform(1,2)$ and given $X = x$, Y is an exponential random variable with parameter $\lambda = x$, so we can write

$$Y|X = x \quad \sim \quad Exponential(x).$$

We sometimes write this as

$$Y|X \quad \sim \quad Exponential(X).$$

a. Find EY.
b. Find $Var(Y)$.

Law of total E:

$E[Y] = \int_{-\infty}^{+\infty} E[Y|X=x] f_X(x)\,dx = E[E[Y|X]]$

Law of total prob: $P(A) = \int_{-\infty}^{+\infty} P(A|X=x) f_X(x)\,dx$

law total variance: $Var(Y) = E(Var(Y|X)) + Var[E(Y|X)]$

$$EY = \int_{-\infty}^{+\infty} E(Y|X=x) f_X(x)\,dx = \int_1^2 \frac{1}{x}\,dx = \ln x \Big|_1^2 = \ln 2 - \ln 1 = \ln 2$$

$$Var(Y) = E(Y^2) - E^2(Y) = E(Y^2) - (\ln 2)^2$$

total Eexpectation

换成X的期望了

$f_X(x)$

f(x)为条件函数

1.不+f(x)为条件函数

$$= E[E[Y^2|X]] - (\ln 2)^2$$

$$= E\left[\frac{2}{X^2}\right] - (\ln 2)^2$$

$$= \int_1^2 \frac{2}{x^2}\,dx - (\ln 2)^2 = 2(-x^{-1})\Big|_1^2 - (\ln 2)^2$$

$$= 2(-2^{-1}+1) - (\ln 2)^2$$

$$= 2\left(-\frac{1}{2}+1\right) - (\ln 2)^2$$

$$= 1 - (\ln 2)^2$$

$$E[Y^2|X] - E^2(Y|E) = Var(Y|E)$$

$$E[Y^2|X] = \frac{1}{X^2} + \frac{1}{X^2} = \frac{2}{X^2}$$

只要一换成条件式，后面的全要转为条件函数！（外层全部平过）

$E(Y) = E[E(Y|X)]$ 外面那层是dx $= \int_{-\infty}^{+\infty} E(Y|X=x) f_X(x)\,dx$.

$Var(Y) = E[Var(Y|X)] + Var(E(Y|X))$

λ, X

$$= E\left(\frac{1}{X^2}\right) + E\left(\frac{1}{X^2}\right) - E^2\left(\frac{1}{X}\right)$$

$$= E\left(\frac{2}{X^2}\right) - E^2\left(\frac{1}{X}\right) \qquad E\left(\frac{1}{X}\right) = \int_1^2 \frac{1}{x}\,dx$$

$$= \int_1^2 \frac{2}{x^2}\,dx - (\ln 2)^2$$

Example 5.28

Let X and Y be two independent $Uniform(0,1)$ random variables, and $Z = XY$. Find the CDF and PDF of Z.

$$P(Z \le z) = P(XY \le z) = P\left(X \le \frac{z}{y}\right)$$

$$f(x,y) = \int_0^x \int_0^y 1\,dx\,dy$$

$$f(x,y) = \int_0^1 dy \int_0^{\frac{z}{y}} 1\,dx = \begin{cases} \frac{z}{y} \ge 1 \Rightarrow z \ge y \Rightarrow \int_0^z dy \int_0^1 1\,dx = \int_0^z dy = z \\ \frac{z}{y} < 1 \Rightarrow z < y \Rightarrow \int_z^1 dy \int_0^{\frac{z}{y}} 1 = \int_z^1 \frac{z}{y}\,dy \end{cases}$$

$$= z - z\ln z$$

1 $\quad Z \geq Y$

$$Z\int_Z^1 dy$$

$$Z\,(\ln y)\Big|_Z^1 = Z\,(0-\ln z)$$

$$= -Z\ln z$$

$$f_{X,Y}(x,y) = \begin{cases} 6e^{-(2x+3y)} & x,y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$f_X(x) = \int_0^{+\infty} 6e^{-(2x+3y)}dy = 2e^{-2x}\int_0^{+\infty} 3e^{-3y}dy = 2e^{-2x}\left[-e^{-3y}\Big|_0^{+\infty}\right] = 2e^{-2x}$$

$$f_Y(x,y) = 3e^{-3y}\int_0^{+\infty} 2e^{-2x}dx = 3e^{-3y}\left[-e^{-2x}\Big|_0^{+\infty}\right] = 3e^{-3y}$$

$$E[Y|X>2] = E[Y] = \int_0^{+\infty} y\left[e^{-3y}\right]' = y(-3e^{-3y}) - 0 - \int_0^{+\infty} -e^{-3y}$$

$$= -(\cancel{\;\;}) = \left[\frac{1}{3}e^{-3y}\right]_0^{+\infty}$$

$$= \frac{1}{3}$$

$P(X>Y)$

$$= P(Y<X) = \int_0^{+\infty}dx\int_0^x 6e^{-(2x+3y)}dy = \int_0^{+\infty}2e^{-2x}dx\int_0^x 3e^{-3y}dy$$

$$= \int^{+\infty} 2e^{-2x}\left[-e^{-3y}\Big|_0^x\right]dx$$

$$= \int_0^{+\infty} 2e^{-2x}(1-e^{-3x})\,dx$$

$$= \int_0^{+\infty} 2e^{-2x}\,dx - \int_0^{+\infty} 2e^{-5x}\,dx$$

$$= (-e^{-2x}) - (-\tfrac{2}{5}e^{-5x})$$

$$= 1 - \frac{2}{5}$$

$$= \frac{3}{5}$$

$$\int_0^{x} e^{-2x} + e^{-3x} + e^{-6x} = -\frac{1}{2}e^{-2x}\Big|_0^x - \frac{1}{3}e^{-3x}\Big|_0^x - \frac{1}{6}e^{-6x}\Big|_0^x$$

$$= -\frac{1}{2}e^{-2x} - (-\tfrac{1}{2}) - \frac{1}{3}e^{-x} - (-\tfrac{1}{3}) - \frac{1}{6}e^{-6x} - (-\tfrac{1}{6})$$

$$=$$

$$E(e^{tx}) = \int_0^{+\infty} e^{tx}\cdot(e^{-2x} + e^{-3x} + e^{-6x})\,dx$$

$$= \int_0^{+\infty} e^{(t-2)x}\,dx + \int_0^{+\infty}$$

$P(Y \le y) = P(e^{X} \le y) = P(X \le \ln y)$ $(y > 1)$

$\{ 0 \quad \text{other.}$

$X \sim F_X(x)$ CDF $\Rightarrow$ $Y = g(X) \Rightarrow Pr(Y \le y) = Pr(g(X) \le y)$

(1) $= Pr(X \le g^{-1}(y))$

CDF: $x^3 \quad 0<x<1$

$y = x^5 \quad , \; 0<y<1$

$Pr(X^5 \le y) = Pr(X \le y^{\frac{1}{5}}) = (y^{\frac{1}{5}})^3 = y^{\frac{3}{5}}$

PdF: $\frac{3}{5} y^{-\frac{2}{5}}$

(2) $X = F(x) = (1+x)^2/4 \quad -1<x<1 \qquad y = x^2 \Rightarrow (0<y<1)$

$Pr(X^2 \le y) = Pr(-\sqrt{y} \le X \le \sqrt{y}) = Pr(X \le \sqrt{y}) - Pr(X \le -\sqrt{y}) = F(\sqrt{y}) - F(-\sqrt{y}) = \sqrt{y}$

~~sum of MGF = product of MGF~~

$\frac{X - E(X)}{\sqrt{Var(X)}} \sim N(0,1)$