Analysis of Voting Systems Using Monte Carlo Simulation

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Abstract

The goal of this project was to examine the effectiveness of various voting systems via Monte Carlo simulation. We used simulated data by randomly generating utility values for each voter-candidate pair to make many datasets of "voters," and then we ran every type of election on these datasets to see which system is most likely to elect the socially preferred candidate. We found that the score voting system is the most effective, although plurality is least vulnerable to strategic manipulation.

Introduction

I. Systems

The systems we test in this experiment are plurality, runoff, Borda, Condorcet, and score voting. Our primary question for this project is: which of these electoral systems is the best?

Plurality is the simplest system: each person casts one vote for their favorite candidate and the candidate with the most votes wins. However, this means that sometimes a candidate who is rather unpopular overall can win the election as in the case of the 1998 Minnesota gubernatorial election when Jesse Ventura won with only 37% of the popular vote.

Runoff is similar to plurality except that, after a first round of voting, the candidate with the least votes is eliminated and the election is repeated with the remaining candidates. In our experiment, we used exhaustive ballot runoff where this elimination is continued until one candidate receives a majority of the votes (eliminating only the bottom candidate in each round).

In a Borda count system, voters don't vote for a single candidate, rather they rank the candidates and each candidate receives points respective to their rank. In general, for an election with n candidates, if a certain voter ranks a candidate in i *th* place, that candidate receives $(n + 1 - i)$ points. The points are then tallied and the candidate with the most points wins the election.

A Condorcet system runs pairwise popular elections between every possible pair of candidates. If a single candidate beats all others in these pairwise elections, that candidate is the winner. The one issue with this system is that it can sometimes fail to return a winner (the Condorcet Paradox). To illustrate, suppose there are three candidates, A, B, and C. It is possible for A to beat B, B to beat C, and C to beat A, in which case the aforementioned issue arises.

Score voting when each voter gives each candidate a score on some scale (almost like a Yelp review) and the candidate with the highest average score wins. One noteworthy difference of this system is that it does not require any ranking of the candidates, since a voter can give multiple candidates the same score.

II. Strategic Voting

Sometimes, voters do not cast votes in line with their true preferences. One might ask how this could be beneficial, but it can be shown to increase the expected utility gain for a given voter; it is essentially “selfish voting.” For example, suppose in a plurality system, there are three candidates: A, B, and C. If A or B are preferred by 95% of the population, C doesn’t stand much of a chance of winning at all. Thus, for a voter in the given system, if C is their first choice, they are better off voting for their second choice (either A or B) because casting a vote for C would basically be a waste, while casting a vote for their second favorite candidate would improve the chances of their least favorite candidate not winning the election. In this way, voting strategically can help an individual, but it often means that the outcome of the election is less preferable for the society as a whole.

III. Criteria

To determine which voting system is best, we examined two criteria: how effective each system is at electing the socially preferred candidate assuming all voters are honest, and which system is least vulnerable to strategic manipulation.

Methods

I. Data

To simulate an “election,” we generated datasets with a specified number of candidates and voters. Each voter was assigned a private utility value for each candidate. These values were drawn randomly from a distribution that was specific to a candidate. Generally speaking, it is safe to assume that for a large enough population, the utility values for a given candidate will follow a normal distribution. In such a “population,” the candidate with the highest sum of private utility values would be the socially preferred candidate.

II. Strategic

When it came to strategic voting, we used the Myerson-Weber strategy which is an algorithm that suggests how a strategic voter would behave given the electoral systems and the voter’s private utility values for each candidate. One key component of this strategy is *perceived pivot probability,* which is essentially the probability that a voter will be the deciding vote between a given candidate pair. Clearly, pivot probabilities cannot be known exactly, and it is up to a voter to try to approximate that. However, we made the assumption that the strategic voters knew the distributions of private utility values for each candidate and thus could calculate the exact pivot probabilities. To test for the influence of strategic voting, we compared results from elections where there were 0%, 25%, or 50% strategic voters.

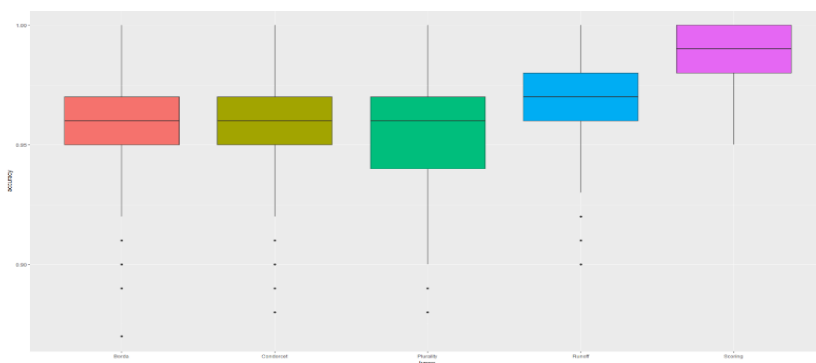
III. Simulation

The Monte Carlo method is a common method used to estimate unknown probabilities by repeated sampling. We chose this method since our aim was to estimate the probabilities that each of these systems would elect the socially preferred candidate. A single sample was 100 datasets generated as described above. We ran each election on each dataset and found the percentage of times that each system succeeded in electing the socially preferred candidate. Then we repeated this experiment 1000 times to get a good idea of the percentage range for each system. This design was also used to test our datasets with strategic voters.

Results

The graphs below compare results for each system under the assumption that all voters are honest. We show the results for elections with three, four, and five candidates.

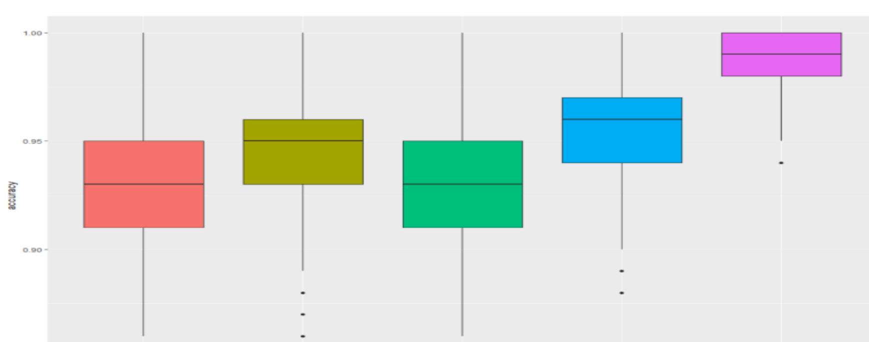
Three candidates



	Plurality <dbl>	Runoff <dbl>	Borda <dbl>	Condorcet <dbl>	Scoring <dbl>
min	0.8800	0.9000	0.8800	0.8800	0.9400
1st quartile	0.9400	0.9600	0.9500	0.9500	0.9800
median	0.9600	0.9700	0.9600	0.9600	0.9900
mean	0.9567	0.9674	0.9591	0.9602	0.9892
3rd quartile	0.9700	0.9800	0.9700	0.9700	1.0000
max	1.0000	1.0000	1.0000	1.0000	1.0000

6 rows

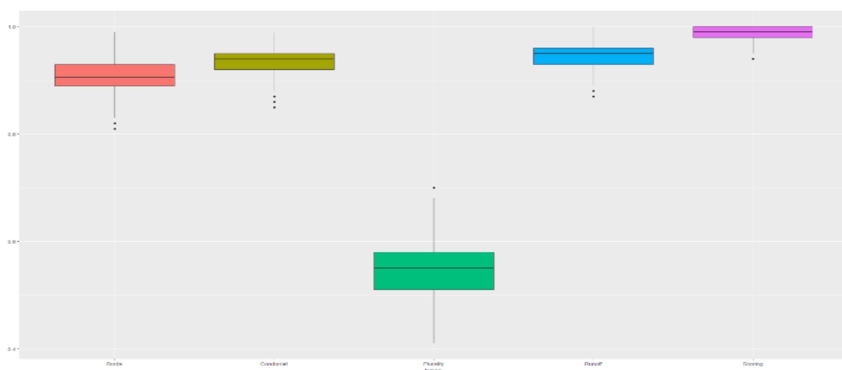
Four candidates:



	Plurality <dbl>	Runoff <dbl>	Borda <dbl>	Condorcet <dbl>	Scoring <dbl>
min	0.8500	0.880	0.8500	0.8600	0.940
1st quartile	0.9100	0.940	0.9100	0.9300	0.980
median	0.9300	0.960	0.9300	0.9500	0.990
mean	0.9314	0.956	0.9316	0.9454	0.988
3rd quartile	0.9500	0.970	0.9500	0.9600	1.000
max	1.0000	1.000	1.0000	1.0000	1.000

6 rows

Five candidates:



	Plurality <dbl>	Runoff <dbl>	Borda <dbl>	Condorcet <dbl>	Scoring <dbl>
min	0.4100	0.8700	0.810	0.8500	0.9400
1st quartile	0.5100	0.9300	0.890	0.9200	0.9800
median	0.5500	0.9500	0.905	0.9400	0.9900
mean	0.5472	0.9476	0.904	0.9358	0.9882
3rd quartile	0.5800	0.9600	0.930	0.9500	1.0000
max	0.7000	1.0000	0.990	0.9900	1.0000

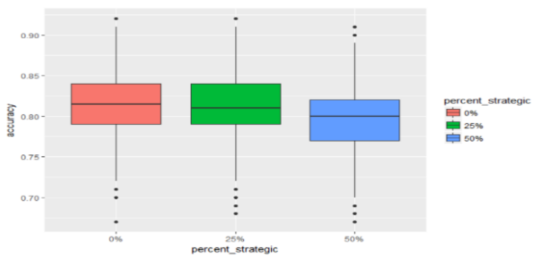
6 rows

Here we can see the Scoring system always gets a best performance when the voters are honest. The plurality system seems to be the worst and it falls further behind as the number of candidates increases.

The following graphs show results for each system comparing the percentage of success for populations with 0%, 25%, and 50% strategic voters. We also ran these simulations with 3, 4, and 5 candidates.

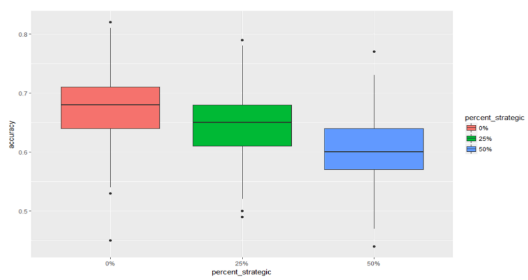
Plurality

Three candidates:



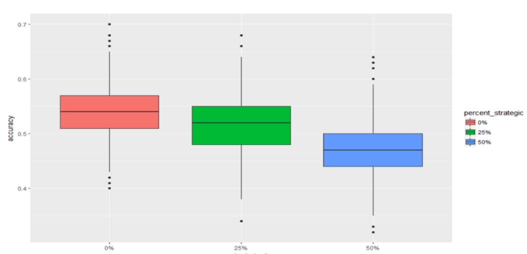
	X0.	X25.	X50.
	<dbl>	<dbl>	<dbl>
min	0.6700	0.6800	0.6700
1st quartile	0.7900	0.7900	0.7700
median	0.8150	0.8100	0.8000
mean	0.8146	0.8129	0.7955
3rd quartile	0.8400	0.8400	0.8200
max	0.9200	0.9200	0.9100

Four candidates:



	X0.	X25.	X50.
	<dbl>	<dbl>	<dbl>
min	0.4500	0.490	0.4400
1st quartile	0.6400	0.610	0.5700
median	0.6800	0.650	0.6000
mean	0.6759	0.646	0.6019
3rd quartile	0.7100	0.680	0.6400
max	0.8200	0.790	0.7700

Five candidates:

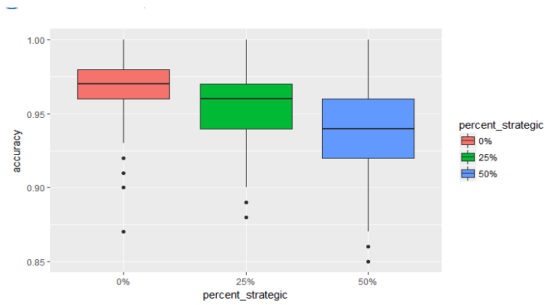


	X0.	X25.	X50.
	<dbl>	<dbl>	<dbl>
min	0.4000	0.3400	0.3200
1st quartile	0.5100	0.4800	0.4400
median	0.5400	0.5200	0.4700
mean	0.5429	0.5174	0.4702
3rd quartile	0.5700	0.5500	0.5000
max	0.7000	0.6800	0.6400

Here we see that strategic voting does not affect the results under plurality too much. However, the overall success rate of the system is still relatively low.

Runoff:

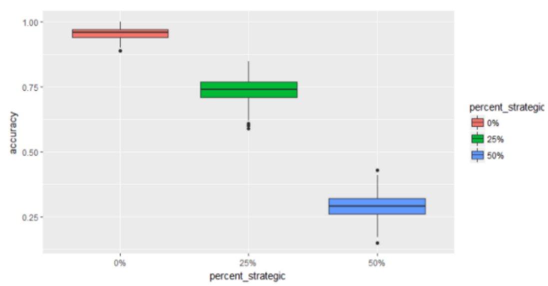
Three candidates:



	X0. <dbl>	X25. <dbl>	X50. <dbl>
min	0.870	0.8800	0.8500
1st quartile	0.960	0.9400	0.9200
median	0.970	0.9600	0.9400
mean	0.968	0.9575	0.9386
3rd quartile	0.980	0.9700	0.9600
max	1.000	1.0000	1.0000

1-6 of 6 rows

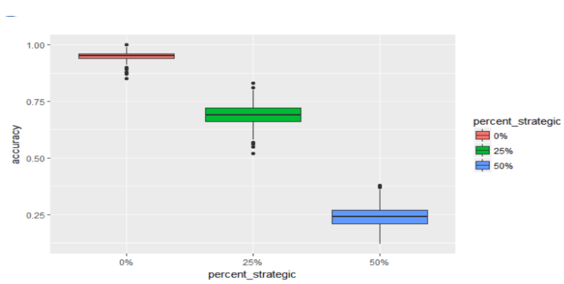
Four candidates:



	X0. <dbl>	X25. <dbl>	X50. <dbl>
min	0.8900	0.5900	0.1500
1st quartile	0.9400	0.7100	0.2600
median	0.9600	0.7400	0.2900
mean	0.9557	0.7362	0.2871
3rd quartile	0.9700	0.7700	0.3200
max	1.0000	0.8500	0.4300

1-6 of 6 rows

Five candidates:



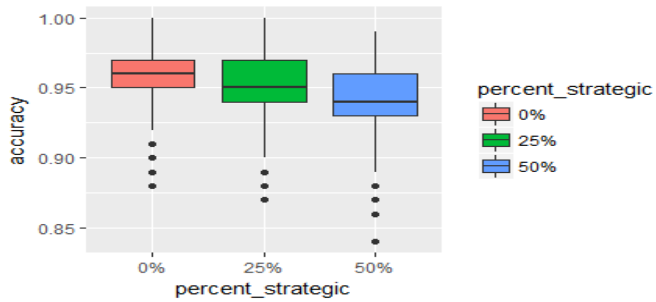
	X0. <dbl>	X25. <dbl>	X50. <dbl>
min	0.850	0.5200	0.1200
1st quartile	0.940	0.6600	0.2100
median	0.950	0.6900	0.2400
mean	0.948	0.6919	0.2382
3rd quartile	0.960	0.7200	0.2700
max	1.000	0.8300	0.3800

1-6 of 6 rows

Runoff initially has better success when voters are honest, but as the percent of strategic voters increases, the success rate of runoff decreases greatly, especially with more candidates.

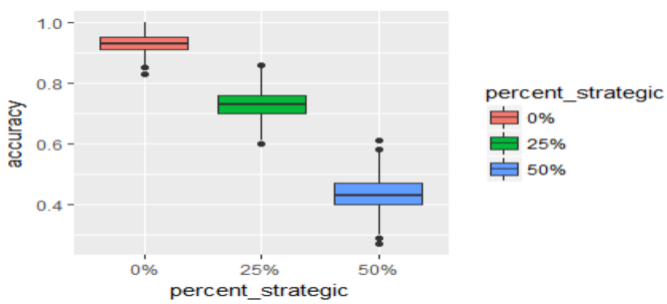
Borda:

Three candidates:



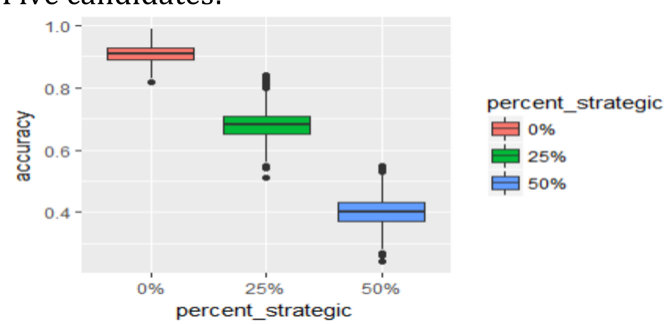
	X0. <dbl>	X25. <dbl>	X50. <dbl>
min	0.8800	0.8700	0.8400
1st quartile	0.9500	0.9400	0.9300
median	0.9600	0.9500	0.9400
mean	0.9581	0.9523	0.9405
3rd quartile	0.9700	0.9700	0.9600
max	1.0000	1.0000	0.9900

Four candidates:



	X0. <dbl>	X25. <dbl>	X50. <dbl>
min	0.8300	0.6000	0.2700
1st quartile	0.9100	0.7000	0.4000
median	0.9300	0.7300	0.4300
mean	0.9303	0.7326	0.4336
3rd quartile	0.9500	0.7600	0.4700
max	1.0000	0.8600	0.6100

Five candidates:

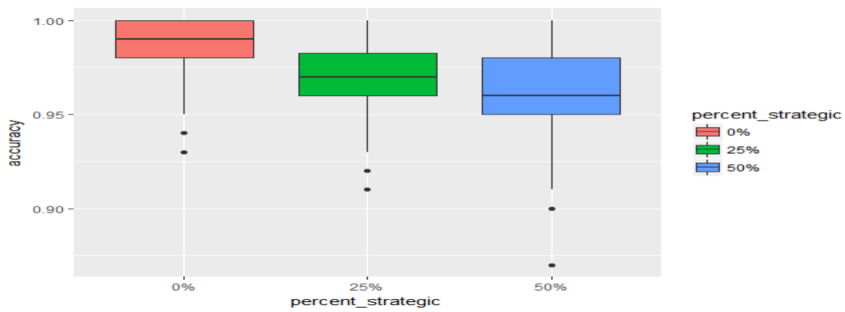


	X0. <dbl>	X25. <dbl>	X50. <dbl>
min	0.8200	0.5100	0.2400
1st quartile	0.8900	0.6500	0.3700
median	0.9100	0.6800	0.4000
mean	0.9057	0.6803	0.3995
3rd quartile	0.9300	0.7100	0.4300
max	0.9900	0.8400	0.5500

Borda exhibits the same trends as runoff, but it is less affected by strategic voting.

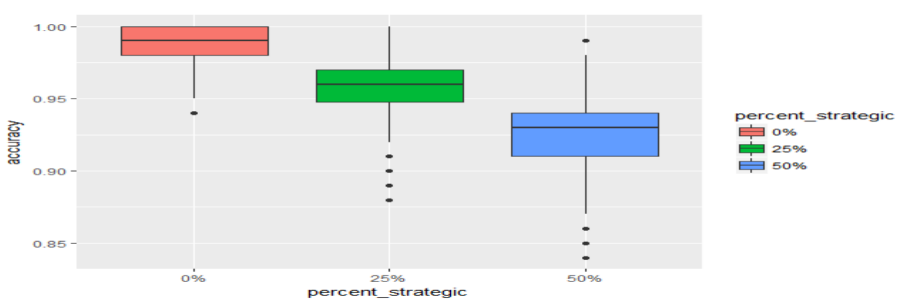
Scoring:

Three candidates:



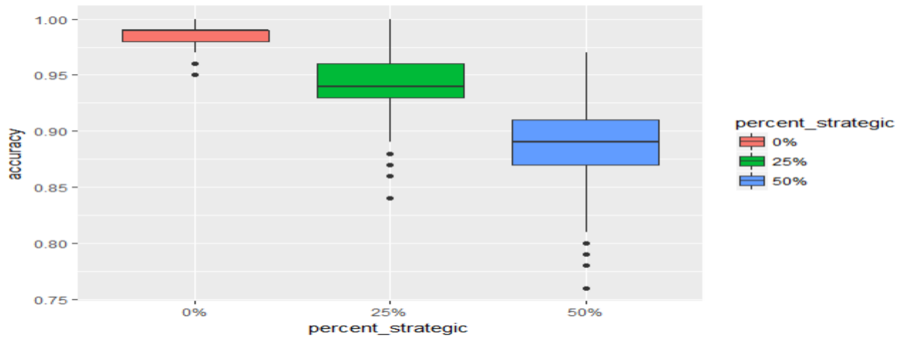
	X0. <dbl>	X25. <dbl>	X50. <dbl>
min	0.9300	0.9100	0.870
1st quartile	0.9800	0.9600	0.950
median	0.9900	0.9700	0.960
mean	0.9895	0.9733	0.962
3rd quartile	1.0000	0.9825	0.980
max	1.0000	1.0000	1.000

Four candidates:



	X0. <dbl>	X25. <dbl>	X50. <dbl>
min	0.9400	0.8800	0.840
1st quartile	0.9800	0.9475	0.910
median	0.9900	0.9600	0.930
mean	0.9883	0.9571	0.926
3rd quartile	1.0000	0.9700	0.940
max	1.0000	1.0000	0.990

Five candidates



	X0. <dbl>	X25. <dbl>	X50. <dbl>
min	0.9500	0.840	0.7600
1st quartile	0.9800	0.930	0.8700
median	0.9900	0.940	0.8900
mean	0.9861	0.942	0.8919
3rd quartile	0.9900	0.960	0.9100
max	1.0000	1.000	0.9700

Score also is vulnerable to strategic manipulation, but significantly less so than runoff or Borda.

***There are no graphs displayed for Condorcet since the best voting strategy in Condorcet is to vote honestly. This is due to the fact that Condorcet runs pairwise elections between each candidate.*

Conclusion

Taking into account the criteria of success that we are looking for, it seems that score voting is the best. It is the most successful system in a population of honest voters and although it is more vulnerable to strategic manipulation than plurality is, score voting still had much higher success rates even with up to 50% strategic voters. The main reason to rule out Condorcet despite its relatively high success rate and invulnerability to strategic manipulation is the fact that it does not always produce a winner.

It must be understood however, that the results we obtained are not perfectly like the real world. In real world elections, people will not always vote honestly and we cannot assume that strategic voters will know the exact pivot probabilities. Rather, strategic voters will base their actions on *perceived* pivot probabilities, which means that media influence and popular opinion could be more important for determining the behavior of actual voters in a real world election. Thus, this project provides us with some initial insight as to how elections work but more research would be necessary to better understand real world results.