A Dynamic Vaccine Uptake Model with Known Disinformation Shocks

Edward de Vries

December 2025

1 Overview

We model a discrete-time vaccination campaign in which a policymaker allocates a finite communication budget to promote vaccine uptake over a short horizon. Campaign spending influences the probability that an unvaccinated individual chooses to vaccinate, with diminishing returns to advertising. Vaccinations accumulate into a stock of protected individuals, which generates a flow of Vaccinated Person-Months that can be mapped into QALYs for cost-effectiveness analysis.

This version extends the baseline model by allowing a *known disinformation shock*: at a user-chosen time period, disinformation temporarily reduces the responsiveness of vaccine uptake to advertising. The shock timing and duration are assumed to be known when optimising the spending plan.

2 Time, Population, and Per-Capita Budget

Time is discrete and indexed by

$$t = 1, 2, \ldots, T,$$

where each period can be interpreted as a month.

The population is fixed and of size $N > 0$. Let:

- ω_t denote the cumulative number of vaccinated individuals at the *start* of period t,
- $U_t = N - \omega_t$ denote the number of unvaccinated individuals at the start of period t.

The initial condition is

$$\omega_1 = \omega_{\text{start}}, \qquad U_1 = N - \omega_{\text{start}}.$$

The policymaker has a fixed total *per-capita* budget $B > 0$ to allocate over the T periods. Thus total currency spending is NB.

3 Budget Shares and Per-Capita Spending

Let

$$f_t \geq 0, \quad t = 1, \ldots, T$$

denote the share of the total per-capita budget allocated to period t. Budget feasibility requires:

$$\sum_{t=1}^{T} f_t = 1.$$

Per-capita campaign spending in period t is

$$S_t = Bf_t,$$

and total currency spending in period t is $NS_t = NBf_t$.

4 Campaign Effectiveness and Flexible Returns to Advertising

Spending is translated into an effectiveness index via

$$C_t = S_t^{\rho},$$

where $\rho \in (0, 1]$ controls concavity:

- $\rho = 1$: linear returns to spending in the effectiveness index;
- $0 < \rho < 1$: diminishing returns to spending (concave in S_t).

5 Baseline Vaccination Probability

In the baseline model, the probability that an unvaccinated individual vaccinates in period t is an exponential saturation function:

$$p_t = 1 - \exp(-\beta C_t) = 1 - \exp\left(-\beta S_t^{\rho}\right),$$

where $\beta > 0$ is an advertising responsiveness parameter.

6 Known Disinformation Shock

We introduce a disinformation shock that temporarily reduces the responsiveness of uptake to advertising.

6.1 Shock specification

Let:

- $\tau \in \{1, \ldots, T\}$ be the user-chosen *start period* of the shock,
- $L \in \{1, 2, \ldots\}$ be the *duration* of the shock (in periods),
- $\delta \in [0, 1]$ be the *percentage reduction* in responsiveness, interpreted as a proportional reduction in β.

Define the indicator for being in the shock window:

$$\mathbb{1}_t^{\text{shock}} = \begin{cases} 1 & \text{if } \tau \le t \le \min\{\tau + L - 1, T\}, \\ 0 & \text{otherwise.} \end{cases}$$

We then define a time-varying effective responsiveness parameter:

$$\beta_t = \beta\big(1 - \delta \mathbb{1}_t^{\text{shock}}\big).$$

Thus, during the shock window, $\beta_t = \beta(1 - \delta)$; outside the window, $\beta_t = \beta$.

6.2 Vaccination probability under disinformation

With the shock, the vaccination probability becomes:

$$p_t = 1 - \exp(-\beta_t C_t) = 1 - \exp(-\beta_t S_t^{\rho}) \, .$$

The key modelling interpretation is that disinformation reduces the marginal impact of advertising spend on uptake *for a limited number of periods*. The shock is assumed to be *known in advance* when choosing $\{f_t\}$.

7 Vaccination Dynamics

In each period t, unvaccinated individuals decide whether to vaccinate independently with probability p_t. The expected number of new vaccinations (the flow) in period t is

$$V_t = p_t U_t = p_t (N - \omega_t).$$

The vaccinated stock evolves according to

$$\omega_{t+1} = \omega_t + V_t = \omega_t + p_t (N - \omega_t), \qquad t = 1, \ldots, T,$$

with $\omega_1 = \omega_{\text{start}}$ and p_t defined using β_t as above.

8 Vaccinated Person-Months (VPMs)

Because ω_t is the number vaccinated at the *start* of period t, the Vaccinated Person-Months produced in period t are:

$$\text{VPM}_t = \omega_t.$$

Total VPMs over the horizon are:

$$\text{VPM}_{\text{total}} = \sum_{t=1}^{T} \omega_t.$$

This objective rewards earlier vaccination because earlier vaccination generates more protected months.

9 Policymaker's Optimisation Problem (with known shock)

The policymaker chooses $\{f_t\}_{t=1}^{T}$ to maximise total VPMs:

$$\max_{\{f_t\}} \quad \sum_{t=1}^{T} \omega_t,$$

subject to:

$$
\begin{aligned}
&\sum_{t=1}^{T} f_t = 1, \quad f_t \geq 0 \ \forall t, \\
&S_t = B f_t, \\
&C_t = S_t^{\rho}, \quad \rho \in (0, 1], \\
&\beta_t = \beta\big(1 - \delta \mathbb{1}_t^{\text{shock}}\big), \\
&p_t = 1 - \exp(-\beta_t C_t) = 1 - \exp(-\beta_t S_t^{\rho})\,, \\
&\omega_{t+1} = \omega_t + p_t(N - \omega_t), \quad t = 1, \ldots, T, \quad \omega_1 = \omega_{\text{start}}.
\end{aligned}
$$

Because $\{\beta_t\}$ is known, this remains a deterministic constrained optimisation problem: the shock changes the payoff to spending in affected periods, which can shift optimal spending across time.

10 Conversion to QALYs and Cost-Effectiveness

Let $\kappa > 0$ denote QALYs per Vaccinated Person-Month (VPM). Total QALYs are:

$$\text{QALYs}_{\text{total}} = \kappa \sum_{t=1}^{T} \omega_t.$$

Because total currency cost is NB, the implied cost per QALY is:

$$\text{Cost per QALY} = \frac{NB}{\kappa \sum_{t=1}^{T} \omega_t}.$$

11 Connection to Numerical Implementation

For parameters $(N, T, B, \beta, \rho, \omega_{\text{start}})$ and shock parameters (τ, δ, L), the computational steps are:

1. Choose a candidate budget-share vector $f = (f_1, \ldots, f_T)$ on the simplex.
2. Compute $S_t = B f_t$ and $C_t = S_t^{\rho}$.
3. Construct the known β_t path using $\beta_t = \beta(1 - \delta \mathbb{1}_t^{\text{shock}})$.
4. Compute $p_t = 1 - \exp(-\beta_t C_t)$.
5. Simulate the state path via $\omega_{t+1} = \omega_t + p_t(N - \omega_t)$.
6. Evaluate the objective $\sum_{t=1}^{T} \omega_t$ (or $\kappa \sum_{t=1}^{T} \omega_t$ for QALYs).

A standard constrained optimiser (e.g. sequential quadratic programming) can then solve for the spending plan $\{f_t\}$.

12 Model Limitations

This framework is intentionally stylised. Key limitations include:

12.1 Advertising effectiveness does not improve as the unvaccinated pool shrinks

In the current specification,

$$p_t = 1 - \exp(-\beta_t S_t^{\rho}),$$

the mapping from spending to vaccination probability does *not* depend on the size or composition of the remaining unvaccinated population U_t.

In reality, when the remaining unvaccinated share is low, outreach can become more targeted (e.g. focused on specific hesitant groups or geographic pockets), potentially *increasing* the marginal effectiveness of spending. A richer specification could allow β_t (or the functional form of p_t) to depend on $u_t = U_t/N$, for example by letting $\beta_t = \beta(u_t)$ or by incorporating targeting technologies that change returns to advertising as u_t falls.

12.2 Homogeneous population and identical vaccination response

The population is assumed homogeneous, and every unvaccinated individual in period t shares the same vaccination probability p_t. This omits heterogeneity in:

- baseline willingness to vaccinate,
- responsiveness to advertising and to disinformation,
- access constraints and logistical barriers,
- risk profiles (which affect the mapping from vaccination to health outcomes).

In practice, uptake dynamics are often driven by distinct subgroups (e.g. highly responsive early adopters vs. persistent refusers). Extending the model to multiple types (indexed by i) with type-specific stocks $\omega_{i,t}$ and probabilities $p_{i,t}$ would capture this more realistically.

12.3 Interpreting the model's "spend less during a shock" implication

Because a disinformation shock is modelled as an exogenous reduction in β_t, the optimisation problem can mechanically favour shifting spend away from shock periods: when advertising is temporarily less productive, the model reallocates budget toward periods with higher expected marginal conversions.

However, this result should not be interpreted as a general policy recommendation. In real settings, communication has strategic and reputational roles beyond immediate conversions. Reducing visible outreach during a misinformation episode could:

- be misread by the public as uncertainty, retreat, or implicit validation of the disinformation narrative,
- allow false claims to diffuse unchallenged, potentially lowering future responsiveness in a way not captured by a temporary β_t shock,
- ignore the possibility that effective policy during a shock involves *counter-messaging*, trust-building, or targeted interventions whose returns differ from baseline advertising.

In short, the model treats disinformation as a temporary shift in the *technology* of persuasion, whereas real-world optimal response may involve a different communication mix, strategic signalling considerations, and endogenous belief dynamics. Capturing these channels would require modelling beliefs, trust, and media feedback explicitly (e.g. allowing disinformation to affect future parameters and letting communication influence the evolution of beliefs).

13 Conclusion

This model provides a compact framework for analysing optimal vaccine promotion over time under a per-capita budget, diminishing returns to advertising, and a *known disinformation shock* that temporarily reduces advertising responsiveness. The extension is implemented by allowing a time-varying β_t that is scaled down for a user-chosen window, leaving the underlying stock–flow vaccination dynamics unchanged.