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Week 2 - Limits

Tags

Secant

$m_{sec} = \frac{f(x)-f(a)}{x-a}$

Average Velocity

$v_{average} = \frac{s(t_1)-s(t_2)}{t_1-t_2}$

- The instantaneous velocity is the average velocity when t_1 approaches t_2

One-Sided Limits

If limits from left ≠ limits from right, there is no limit (DNE)

Theorem 2.3 Infinite Limits from Positive Integers

$f(x) = \frac{1}{(x-a)^n}$ has infinite limits at a

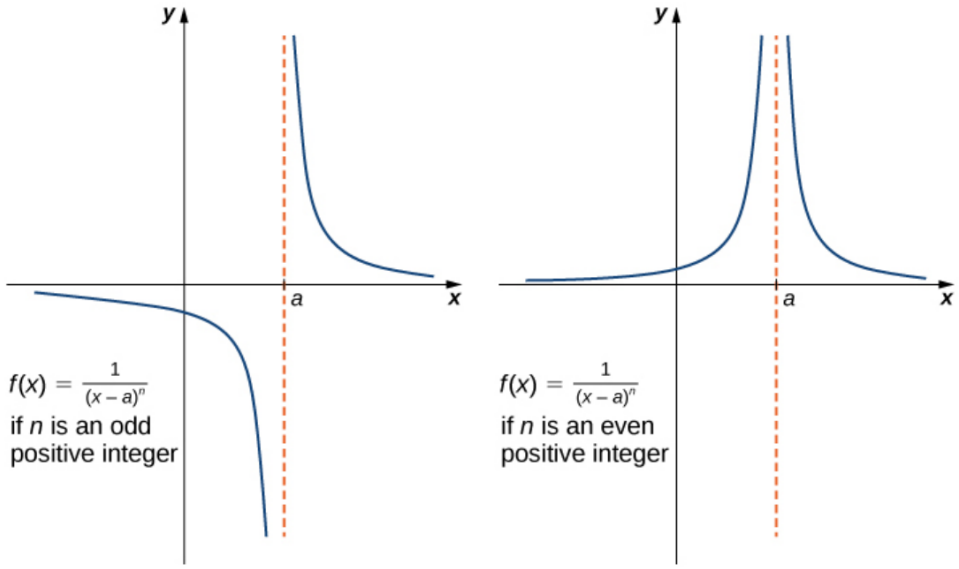


Figure 2.20 The function $f(x) = 1/(x-a)^n$ has infinite limits at a.

The one-sided limit values depends on whether n is odd or even

Theorem 2.4 Basic Limit Results

$\lim_{x \to a} x = a$

$\lim_{x \to a} c = c$

Theorem 2.6

$\lim_{x \to a} p(x) = p(a)$

$\lim_{x \to a} \frac{p(x)}{q(x)} = \frac{p(a)}{q(a)}$ when $q(a) \neq 0$

Theorem 2.7 - Squeeze

If $f(x) \leq g(x) \leq h(x)$

and $\lim_{x \to a} f(x) = L = \lim_{x \to a} h(x)$

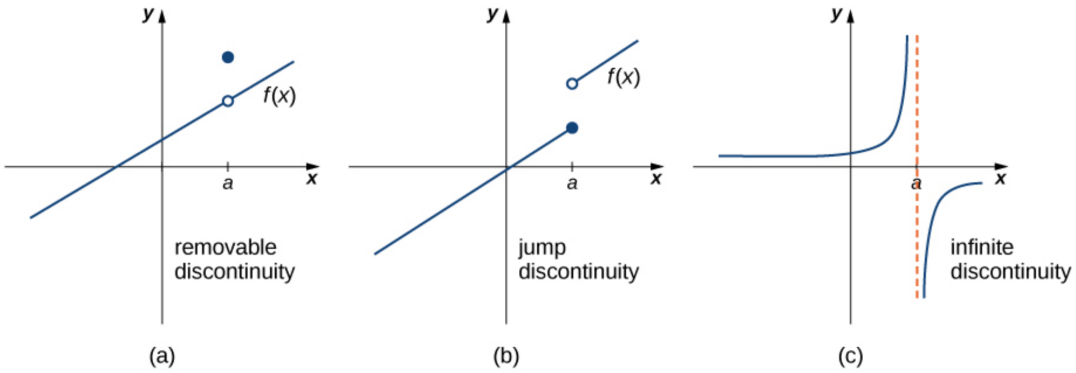
then $\lim_{x \to a} g(x) = L$

Requirements for Continuity

1. $f(a)$ is defined

2. $\lim_{x \to a} f(x)$ exists
3. $\lim_{x \to a} f(x) = f(a)$

Types of Discontinuities



Removable Discontinuity	If $\lim_{x \to a} f(x)$ exists
Jump Discontinuity	The left limit and right limit exist, but are not equivalent
Infinite Discontinuity	If either the left or right limit go to $\pm\infty$

- A function is continuous from the right if $\lim_{x \to a^+} = f(a)$
- Continuous from the left if $\lim_{x \to a^-} = f(a)$

Theorem 2.9 Composite Function

If $\lim_{x \to a} g(x) = L$ then

$\lim_{x \to a} f(g(x)) = f(\lim_{x \to a} g(x)) = f(L)$

Theorem 2.10 Continuity of Trigonometric Functions

Prove that $\lim_{x \to a} \cos(x) = \cos(a)$

$\lim_{x \to a} \cos(x)$

$= \lim_{x \to a} \cos((x - a) + a)$

$= \lim_{x \to a} (\cos(x - a)\cos(a) - \sin(x - a)\sin(a))$

$= \cos(\lim_{x \to a}(x - a))\cos(a) - sin(\lim_{x \to a}(x - a))\sin(a)$

$= \cos(0)\cos(a) - \sin(a)\sin(a)$

$= 1\cos(a) - 0\sin(a) = \cos(a)$

Therefore trigonometric functions are always continuous

Intermediate Value Theorem

IF $f(x)$ IS CONTINUOUS

If there exists some z such that $f(a) \leq z \leq f(b)$

Then there is some c such that $f(c) = z$ and $a \leq c \leq b$