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Week 1- Functions

Tags	

Odd and Even Functions

Even	f(x)=f(-x)
Odd	f(-x)=-f(x)

Equations

Point-Slope Equation

$y - y_1 = m(x - x_1)$

Slope-Intercept Form

$y = mx + b$

Standard Form

$ax + by = c$

Function Types

Algebraic

Addition, subtraction, multiplication, division, rational powers, roots

Rational

$f(x) = \frac{p(x)}{q(x)}$

Root

$f(x) = x^{\frac{1}{n}}$ where $n > 1$

Transcendental

$f(x) = \sin(x)$

$f(x) = \cos(x)$

$f(x) = \tan(x)$

$f(x) = \cot(x)$

$f(x) = \sec(x)$

$f(x) = b^x$

$f(x) = \log_a(x)$

Piecewise

$$f(x) = \begin{cases} -x, \ x < 0 \\ x, \ x \geq 0 \end{cases}.$$

Periodic

$f(x) = A\sin(B(x{-}\alpha) + C$`

α - Horizontal/phase shift

A - Vertical shift of $|A|$

B - Changes the period to $\frac{2\pi}{|B|}$

C - Vertical Shift

One-to-One

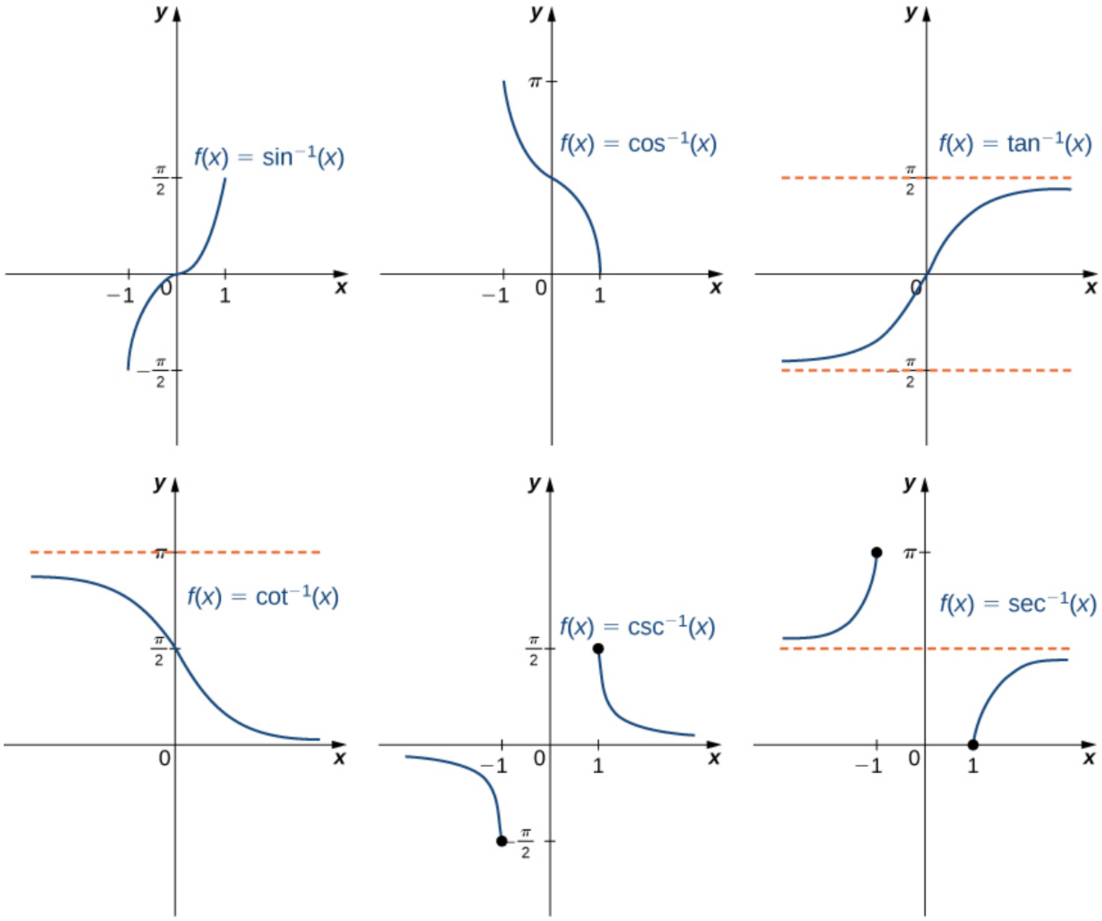
$f(x_1) \neq f(x_2)$ when $x_1 \neq x_2$

Only one-to-one if it passes horizontal line test

Restricting Domains

A function that is not one-to-one doesn't have an inverse across the whole domain.

However, by restricting domains we can make the function one-to-one



Function	Domain	Range
$\sin^{-1}(x)$	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
$\cos^{-1}(x)$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$
$\cos^{-1}(x)$	$-\infty \leq x \leq \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
$\cot^{-1}(x)$	$-\infty \leq x \leq \infty$	$0 < y < \pi$
$\csc^{-1}(x)$	$x \geq 1$	$-\pi \leq y \leq \pi$ and $y \neq 0$
$\sec^{-1}(x)$	$x \geq 1$	$0 \leq y \leq \pi$ and $y \neq \pi/2$

$\sin(\sin^{-1}(y)) = y$ if $-1 \leq y \leq 1$

$\sin^{-1}(\sin(x)) = x$ if $-\pi \leq x \leq \pi$.

Exponential

$f(x) = b^x$

$f(x) = e^x$ is the only exponential to have the gradient at $x = 0$ equal to 1

Logarithmic

$f(x) = \log_a(x)$ iff $b^y = x$

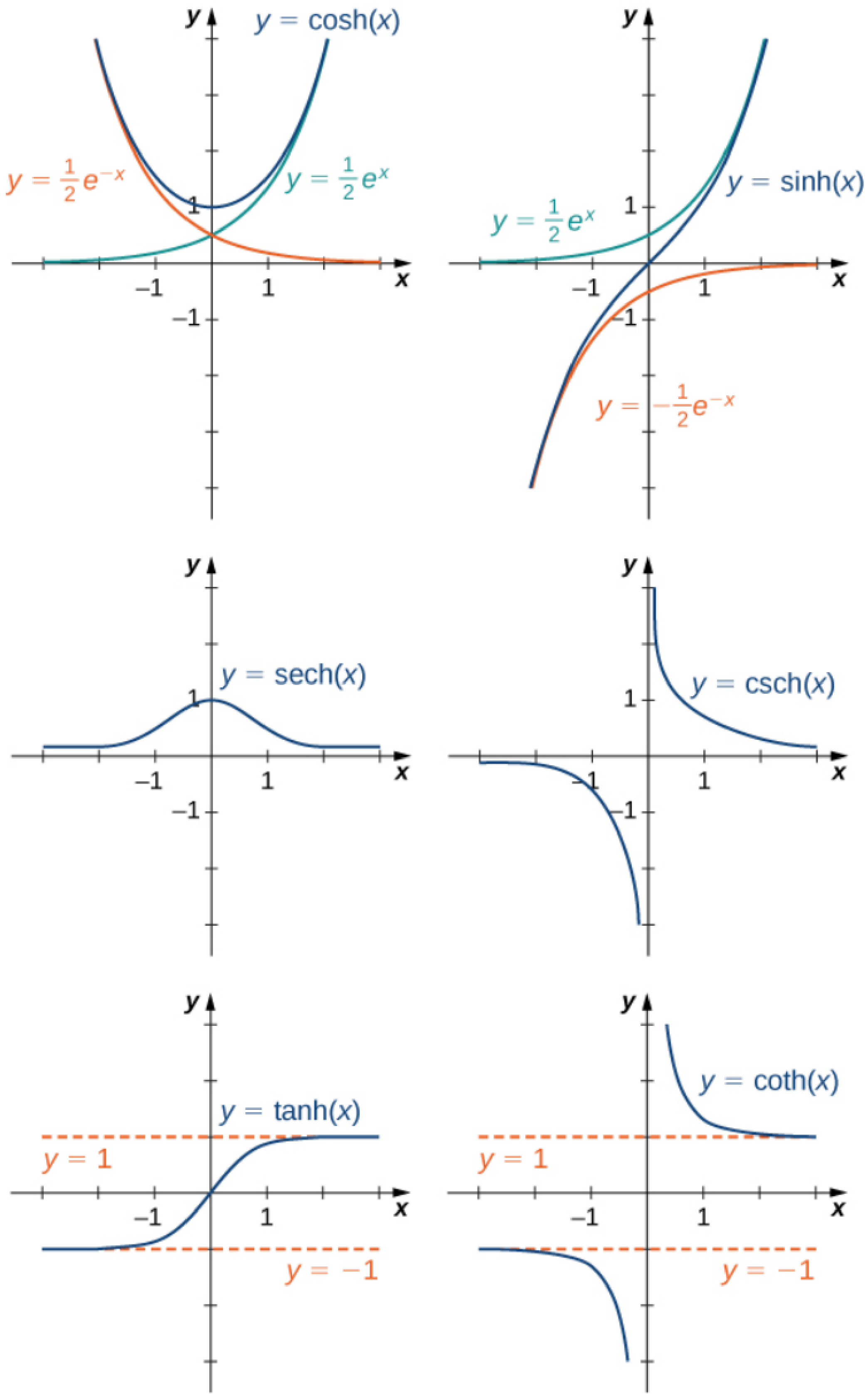
Change of base formulae

$a^x = b^{x \log_b(a)}$

$log_a(x) = \frac{\log_b(x)}{\log_b(a)}$

Hyperbolic

Function	Equation
$\cosh(x)$	$\frac{e^x + e^{-x}}{2}$
$\sinh(x)$	$\frac{e^x - e^{-x}}{2}$
$\tanh(x)$	$\frac{\sinh(x)}{\cosh(x)}$


y = cosh(x)
y = ½e^{−x}
y = ½e^x
y = sinh(x)
y = ½e^x
y = −½e^{−x}
y = sech(x)
y = csch(x)
y = tanh(x)
y = 1
y = −1
y = coth(x)
y = 1
y = −1

Identities

1. $\cosh^2(x) - \sinh^2(x) = 1$
2. $1 - \tanh^2(x) = \frac{1}{\cosh^2(x)}$
3. $\sinh(x \pm y) = \sinh(x)\cosh(y) \pm \cosh(x)\sinh(y)$
4. $\cosh(x \pm y) = \cosh(x)\cosh(y) \pm \sinh(x)\sinh(y)$

Inverse Hyperbolics

$\sinh^{-1}(x)$	$\ln(x + \sqrt{x^2 + 1})$
$\cosh^{-1}(x)$	$\ln(x + \sqrt{x^2 - 1})$
$\tanh^{-1}(x)$	$\frac{1}{2}\ln(\frac{1+x}{1-x})$

$\sinh^{-1}(x) = \ln(x + \sqrt{x^2 + 1})$